Generation and annihilation of scalar particles due to a curved expanding and contracting space-time.

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Abstract

While a theory calculating a cosmological generation of particles in a case of the expanding space-time is quite developed, we study here a theory of a cosmological generation of particles in a case of a space-time which is expanding and then contracting back. The simplest case of fields studied in this connection is a scalar field. We will show in our paper that the quantum scalar field has delocalized in the conformal time η particle-like modes u_k^{in} and two localized in the conformal time modes u_0^{in} and u_1^{in} for our choosen scale factor $C(\eta)$. The vacuum for these states $|0, out\rangle$ defined through massive modes u_k^{out} and through modes u_0^{out} and u_1^{out} is the same as the vacuum $|0, in\rangle$. A detector shows that there are no mass particles and no localized states for $\eta \rightarrow +\infty$ for non-accelerating case. For $\eta \rightarrow -\infty$ a Minkowski space-time is realized, as it is realized also in the *out* case. The quantum field has delocalized in the conformal time η particle-like modes u_k^{out} which in the -out region have k-dependent phase shifts with respect to the quantum field delocalized in the conformal time η particle-like modes u_k^{in} in the -in region. The phase shift of delocalized modes (k-particles) is due to scattering in the gravitational field leading to expansion and contraction of the space. Thus while in the expansion phase there is present generation of particles, due to nonpresence of particles in $\eta \rightarrow +\infty$ conformal time it is clear that in the phase of contraction of the scale factor there is present annihilation of particles from their peak state, where they are occurring from the generation process.

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1 Introduction.

Quantum mechanics in a non-relativistic case [1] was generalized to the relativistic case, see [2] and [3], in a Minkowski space-time. Then generalizing the relativistic case in the Minkowski space-time lead to a natural question how can be quantum theory generalized for curved space-times. In [4] B. S. De Witt's "Quantum Field Theory In Curved Space-time" there is reviewed the Quantum field theory, and how it predicts a number of unusual physical effects in non-Minkowskian manifolds (flat or curved) that have no immediate analogs in Minkowski space-time. He review the following examples: (1) The Casimir effect; (2) Radiation from accelerating conductors; (3) Particle production in manifolds with horizons, including both stationary black holes and black holes formed by collapse. In the latter examples curvature couples directly to matter through the stress tensor and induces the creation of real particles. However, it also induces serious divergences in the vacuum stress. These divergences are analyzed, and methods for handling them are reviewed. In 1967 A. D. Sakharov in [5] considered the hypothesis which identifies the action of space-time depending on R (R is the invariant of the Ricci tensor) in the Einstein theory of gravitation with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, he consider the metrical elasticity of space as a sort of level displacement effect (emergent space-time effect). Recent development of emergent space-time is described in [6] - [8]. This problem, similar to condensed matter theory of solids, is connected with calculation of expected values of quantum fields in vacuum and is still developing.

In [9] N. D. Birrell and P.C.W. Davies study the cosmological creation of particles. Calculation of expected values of quantum fields in vacuum is important and here in this paper we use the way of [9] to calculate cosmological generation of particles in some special case. The fact that quantum fluctuations of the inflaton field can back-react on space-time and form classical inhomogeneities, giving rise to CMB anisotropies and big structures like galaxies and clusters, is a property of Quantum Field Theory in curved space, and one of the great successes of inflation [10]. Authors of [11] have studied the evolution of a massive, non-interacting and non-minimally coupled scalar field in the exponentially expanding de Sitter space, which has relevance for the early and late time Universe. The Bunch-Davies vacuum state in the de Sitter space-time has the de Sitter invariance of this state which is independent of the details of renormalization and can be understood as a manifestation of covariant conservation. The behavior of the modes in the Bunch-Davies state can be interpreted to constantly go through a particle creation process as indicated by a nontrivial Bogolubov transformation.

However this is not visible in the semi-classical back-reaction: it bears no sign of a density from classical particles and implies strictly $w = -1$ for the equation of state. Then there are in [9] interesting results to calculate cosmological generation of particles in case of expanding space-time using Quantum Field Theory in Curved Space-times. On dimensional grounds it is generally believed that quantum effects of gravity should be important at least when the space-time curvature becomes comparable to the Planck length $(\frac{\hbar G}{c^3})^{\frac{1}{2}} \approx 10^{-33}cm$. For less extreme space-time curvature, one hopes that the semi-classical approximation will be valid at least in many situations. This later case of semi-classical approximation assumption to be valid will be used in our paper. From [9] study of the cosmological creation of particles in case of expanding space-time from the vacuum state in conformal time $\eta \rightarrow -\infty$ one may expect that there exists the cosmological annihilation of particles in case of contracting space-time to the same vacuum state as the vacuum state in conformal time $\eta \rightarrow -\infty$.

We study here the case of space-time which is expanding and then contracting back to the same state. Our aim is not only to calculate cosmological generation of particles in case of expanding period of such a space-time, but also to study what happens in the contracting period. The simplest case of fields studied is a scalar field. This field will be considered in this paper.

2 Cosmological generation and annihilation of particles: in- and out- Minkowski space-time.

As in [9] we consider a 2-dimensional case of space-time, in which the coordinates (time, space) are (t, x) . In the Robertson-Walker space-time the metric tensor is given by the element ds :

$$ds^2 = dt^2 - a(t)^2 dx^2. \quad (1)$$

In standard stationary cosmological model of the spatial expansion there is a time dependent factor $a(t)$ which is described by the form $a_{scm}(t) = a_{scm}(t_0) \exp(H(t-t_0))$ [13]. The scale factor $C(\eta) \equiv a^2(\eta)$ has an exponential form too. Here H is the Hubble constant, and η is a conformal time defined below, it is dependent on time t .

In our case the space sections of the space homogeneously expanding (or contracting) are in correspondence with the time dependent factor $a(t)$ in the equation (1). Let us define the conformal time η by $d\eta \equiv \frac{dt}{a(t)}$. Then the

equation (1) has the form:

$$ds^2 = a^2(t)(d\eta^2 - dx^2) = C(\eta)(d\eta^2 - dx^2). \quad (2)$$

where $C(\eta)$ is a conformal scale factor.

Our aim is to study an influence of the conformal scale factor in the form describing a curved expanding and contracting space-time on generation and annihilation of scalar particles due to this space-time:

$$C(\eta) = A - B \tanh^2(\rho\eta). \quad (3)$$

Here A and B are constants, we will consider possible values of these constants such that the conformal scale factor will be positive for all conformal times η , $C(\eta) > 0$. We assume that the constant ρ is positive. It describes the width of the peak in the scale factor corresponding to expanding and then shrinking space, $C(\eta\rho)$, which is located at the conformal time $\eta = 0$. From (3) we can see that:

$$C(\eta\rho) \rightarrow A - B \quad (4)$$

for $\eta \rightarrow \pm\infty$. We will consider the case $A > B$ and $B > 0$, e.i. the conformal scale factor is positive in (3), and describes space-time in which the scale factor $C(\eta)$ is smoothly expanding and smoothly contracting from the value $A - B$ to the value $A - B$.

As in [9] we will consider a scalar field $\phi(\bar{x})$, here $\bar{x} \equiv (t, \mathbf{x})$ with $\mathbf{x}$ a vector of (3-) space coordinates, with nonzero mass which is minimally coupled ($\xi = 0$) to the gravitational field, and study cosmological generation and annihilation of modes of this field. Note that in the dimension of space-time $n = 2$ the conformal coupling, which in the n -dimensional space-time has the form ($\xi = \frac{1}{4}[\frac{(n-2)}{(n-1)}]$), and a minimal coupling to the Ricci scalar R invariant are the same, they have the value 0. In general the coupling term of the scalar field to the gravitational field has the form:

$$(-\xi R)\phi^2(\bar{x}). \quad (5)$$

From the variational principle we find the general equation for the scalar field $\phi(\bar{x})$:

$$[\square + m^2 + \xi R]\phi(\bar{x}) = 0. \quad (6)$$

Here the operator $\square$ is given by:

$$\square \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu. \quad (7)$$

Let us take the full set $u_i(\bar{x})$ of solutions of the equation (6) which is orthogonal in the sense of the scalar product defined as:

$$(\phi_1, \phi_2) = -i \int_{\Sigma} \phi_1(x) \overleftrightarrow{\partial}_\mu \phi_2^*(x) [-g_\sigma(\bar{x})]^{\frac{1}{2}} d\Sigma^\mu. \quad (8)$$

Here $d\Sigma^\mu = n^\mu d\Sigma$ where n^μ is a time-like vector orthogonal to a space-like hyper-surface Σ , which has unit amplitude. Hyper-surface is chosen to be a surface of the Cauchy type in the space-time. Note that this scalar product does not depend on the hyper-surface Σ . The orthogonality of the full set of solutions $u_i(\bar{x})$ (note that it will be denoted as u_i in the future) may be expressed now as:

$$(u_i, u_j) \approx \delta_{ij}, \quad (9)$$

$$(u_i^*, u_j^*) \approx -\delta_{ij},$$

$$(u_i, u_j^*) = 0.$$

The index i goes through all states (modes) of the orthogonal full set of solutions $u_i(\bar{x})$. The field $\phi(\bar{x})$ may be expressed through modes u_i and u_i^* as:

$$\phi(\bar{x}) = \sum_i [a_i u_i(\bar{x}) + a_i^\dagger u_i^*(\bar{x})]. \quad (10)$$

Here $\dagger$ denotes conjugation of the operator ($a_i^\dagger$) to the operator a_i .

Now the covariant quantisation in this theory is based on the relations:

$$[a_i, a_j^\dagger] = \delta_{ij}, \quad (11)$$

for all i, j . Here i and j indices are denoting all modes and δ_{ij} is the Kronecker delta and brackets $[..., ...]$ are usual known brackets.

The vacuum state $|0\rangle$ in the Fock space is defined as:

$$a_i |0\rangle = 0 \quad (12)$$

for every i state. In a curved space-time, where the Poincare group is not the group of symmetry, Killing vectors do not exist, and thus we are not able to define time-like solutions. We will not consider some special cases where Killing vectors it is possible to define. In the general case there are not privileged coordinate systems which would play that role in the quantum field theory which they play in the Minkowski space-time. Thus in the general case there is impossible to find for the wave function of the field $\phi(\bar{x})$ an expression through modes u_i and u_i^* as in (10). This corresponds to a general idea of the general theory of relativity that the choose of the coordinate system is not important from the physical point of view.

Let us consider another orthogonal set of solutions $\overline{u_j(x)}$ of the equation (6). The field $\phi(\bar{x})$ may be again expressed through the orthogonal set of solutions:

$$\phi(\bar{x}) = \sum_i [\overline{a_i u_i(\bar{x})} + \overline{a_i^\dagger u_i^*(\bar{x})}]. \quad (13)$$

The vacuum state $|\bar{0}\rangle$ in the Fock space-time for this expansion of the field $\phi(\bar{x})$ is defined as:

$$\bar{a}_i |\bar{0}\rangle = 0 \quad (14)$$

for every i state. There is defined a new Fock space for the expansion (13) of the field $\phi(\bar{x})$. Due to the fact that both solutions are the full set of solutions of the equation (6), it is possible to express modes $\bar{u}_i$ through the first set of solutions:

$$\bar{u}_i = \sum_j \alpha_{ji} u_j + \beta_{ji} u_j^*. \quad (15)$$

and

$$u_i^* = \sum_j \alpha_{ji}^* \bar{u}_i - \beta_{ji}^* \bar{u}_i^*. \quad (16)$$

Bogoljubov coefficients α_{ij} and β_{ij} are expressed as:

$$\alpha_{ij} \approx (\bar{u}_i, u_j) \quad (17)$$

and

$$\beta_{ij} \approx (\bar{u}_i, u_j^*).$$

Two Fock spaces based on two different sets u_i and $\bar{u}_i^*$ are different if $\beta_{ij} \neq 0$. In this case:

$$a_i |0\rangle = \sum_j \beta_{ji}^* |\bar{1}_j\rangle \neq 0. \quad (18)$$

Here $|\bar{1}_j\rangle \equiv a_j^\dagger |\bar{0}\rangle$. Number $N_i = a_i^\dagger a_i$ of particles of the i -th type in its mean value has the form:

$$\langle \bar{0} | N_i | \bar{0} \rangle = \langle \bar{0} | a_i^\dagger a_i | \bar{0} \rangle = \sum_j |\beta_{ji}|^2. \quad (19)$$

This is the number of particles of modes u_i in the vacuum of modes $\bar{u}_j$.

3 Quantisation of the scalar field.

The density L of the Lagrangian of the scalar field in our gravitational field is:

$$L = \frac{1}{2} [-g(\bar{x})]^{\frac{1}{2}} g^{\mu\nu}(\bar{x}) \phi(\bar{x})_{,\mu} \phi(\bar{x})_{,\nu} - [m^2 - \xi R(\bar{x})] \phi^2(\bar{x}), \quad (20)$$

here $\phi(\bar{x})$ is a scalar field, m is a mass of this field. There exist term $(-\xi R \phi^2(\bar{x}))$ which describes an interaction of the scalar field with the gravitation field through Ricci scalar $R(\bar{x})$. The ξ is a constant. The action S has the form:

$$S = \int L d^n \bar{x}. \quad (21)$$

Here a point $(t, \mathbf{x})$ of the n -dimensional space-time with t time and with $\mathbf{x}$ space coordinates is denoted by $\bar{x}$. In the case of two dimensional space-time we will use coordinates (t, x) . Here the x denotation is the space 1-d vector $\mathbf{x}$ in this two dimensional space-time. Equation for the scalar field $\phi(\bar{x})$ is obtained from the condition of zero for the first variation of the action S :

$$\delta S = 0. \quad (22)$$

The equation for the scalar field $\phi(\bar{x})$ is:

$$[\square + m^2 + \xi R(\bar{x})]\phi(\bar{x}) = 0. \quad (23)$$

Here the operator $\square$ is defined as above:

$$\square\phi(\bar{x}) = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi(\bar{x}). \quad (24)$$

The parameter ξ has the value $\xi = 0$ corresponding to the minimal coupling, and the value $\xi = \frac{1}{4}[\frac{(n-2)}{(n-1)}]$ for a conformal coupling, in our case $n = 2$.

4 Modes of the scalar field which exist in conformal times in between far past and far future.

As in [9], we see that our scale factor does not depend on the space coordinate. Thus space transformations are symmetry transformations in this space-time. Note that it is the $n = 2$ space-time in which conformal and minimal coupling are equivalent, see also [9] for discussion of conformal and minimal couplings. In modes of the scalar field $\phi(\bar{x})$ in (10) there is dependence on the conformal time and on the space coordinate x segregated:

$$u_k(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(ikx) \chi_k(\eta). \quad (25)$$

From (23) and (25) we obtain the equation for $\chi_k(\eta)$:

$$[\frac{d^2}{d\eta^2} + (k^2 + C(\eta)m^2)]\chi_k(\eta) = 0 \quad (26)$$

taking into account that $\xi = 0$. This equation (26) we are able to solve exactly. Besides delocalized modes we will find localized modes.

5 Solution of the equation (26).

The equation (26) in its explicit form taking into account the scale factor form (4) is:

$$\left[\frac{d^2}{d\eta^2} + (k^2 + Am^2 - Bm^2 \tanh^2(\eta\rho))\right]\chi_k(\eta) = 0. \quad (27)$$

Let us introduce a dimensionless variable $\delta \equiv \eta \cdot \rho$. Note that $\frac{d}{d\eta} = \rho \cdot \frac{d}{d\delta}$ and that $\frac{d^2}{d\eta^2} = \rho^2 \cdot \frac{d^2}{d\delta^2}$. Now the equation (27) takes the form (we have multiplied the equation (27) by $-\frac{1}{2\rho^2}$):

$$\left[-\frac{1}{2} \frac{d^2}{d\delta^2} - \frac{1}{2} \left(\frac{k^2}{\rho^2} + \frac{Am^2}{\rho^2} - \frac{Bm^2}{\rho^2} \tanh^2(\delta) \right)\right]\chi_k(\delta) = 0. \quad (28)$$

This type of equation was solved in [14] where the corresponding equation has the form:

$$-\frac{1}{2} \frac{d^2}{d\delta^2} \chi_k(\delta) + (3 \tanh^2(\delta) - 1) \chi_k(\delta) = \frac{\omega_k^2}{M^2} \chi_k(\delta). \quad (29)$$

To identify both equations (28) and (29) we have to put:

$$\frac{1}{2} \frac{Bm^2}{\rho^2} \equiv 3, \quad (30)$$

and

$$+1 + \frac{\omega_k^2}{M^2} \equiv \frac{1}{2} \left(\frac{k^2}{\rho^2} + \frac{Am^2}{\rho^2} \right),$$

e.i.:

$$\begin{aligned} m^2 &= \frac{6\rho^2}{B}, \\ \frac{\omega_k^2}{M^2} &= -1 + \frac{1}{2} \left(\frac{k^2}{\rho^2} + \frac{6A}{B} \right) = \\ &= \frac{1}{2} \left(\frac{k^2}{\rho^2} + \frac{6A - 2B}{B} \right). \end{aligned} \quad (31)$$

In the next sections we describe delocalized and localized modes for the equation (28) using (31). Here k denotes a mode.

6 Localized modes.

Using results from [14] and equations from the previous section we find that there are two localized modes. The first one, we will denote it as 0 mode, has its zero energy:

$$\omega_0^2 = 0, \quad (32)$$

and its wave function is:

$$\chi_0(\delta) = \frac{1}{\cosh^2(\delta)}. \quad (33)$$

The corresponding value of the k_0 is found from (31):

$$k_0^2 = -\frac{6A - 2B}{B}\rho^2. \quad (34)$$

As we can see (Note that $\frac{6A-2B}{B} = \frac{2A-2B}{B} + \frac{4A}{B}$) from (34) the momentum k_0 is imaginary, it is given by:

$$k_0 = \pm i \sqrt{\frac{6A - 2B}{B}}\rho \equiv \pm i K_0. \quad (35)$$

Here we define K_0 (which is the amplitude of the momentum) and assume that $K_0 > 0$. Let us write the wave function for the 0 mode from (25) and (35). It has either a form:

$$u_{k_0,+}(\eta, x) = \frac{A_+}{\sqrt{2\pi}} \exp(ik_0x) \chi_{k_0}(\eta) = \frac{1}{\sqrt{2\pi}} \chi_{k_0}(\eta) (A_+ \exp(+K_0x) \quad (36)$$

or a form:

$$u_{k_0,-}(\eta, x) = \frac{A_-}{\sqrt{2\pi}} \exp(ik_0x) \chi_{k_0}(\eta) = \frac{1}{\sqrt{2\pi}} \chi_{k_0}(\eta) (A_- \exp(-K_0x). \quad (37)$$

Here A_+ and A_- are constants. In x space the mode 0 cannot be infinitely increasing on its boundaries $x \rightarrow \pm\infty$. Thus the wave functions (36) and (37) has to be chosen in such a way as to equal probability of an observation of the 0 mode at some point x_0 going from the left and going from the right to this point, defined by $u_{k_0,-}(\eta, x_0) = u_{k_0,+}(\eta, x_0)$, e.i. where:

$$A_- \exp(-K_0x_0) = A_+ \exp(K_0x_0). \quad (38)$$

Thus the point x_0 is given by $\exp(2K_0x_0) = \frac{A_-}{A_+}$. Note that the resulting probability density is in the point x_0 continuous, also probability of momentum is continuous due to the fact that $K_0 \neq 0$. From this and from the

condition that x_0 is real we have that the constants A_+ and A_- are either positive either negative, both simultaneously. Then the value of the point x_0 lies in the interval $-\infty < x_0 < +\infty$. The case in which the signs of constants A_+ and A_- are different does not correspond to a solution for which in x space the mode 0 cannot be infinitely increasing on its boundaries $x \rightarrow \pm\infty$. We see that the localized in the conformal time η mode is localized also in the x coordinate with a peak at the point x_0 . As we can see there is no oscillation in the conformal time η for this mode. Conformal time development is described by the function $\frac{1}{\cosh^2(\delta)}$. Note that the mode 0 for $\delta \rightarrow \pm\infty$ has its wave function $\frac{1}{\cosh^2(\delta)}$ with vanishing value. For $\delta \rightarrow 0$ has its wave function $\frac{1}{\cosh^2(\delta)}$ with maximum value 1. Thus it represents a mode which is of a localized nature in η and x coordinates. While the peak in the first variable is at the peak of the scale factor, e.i. of the expansion of space when it turns to shrinking phase from the expansion phase, the peak in the second variable x is at the point x_0 which is in our case an arbitrary quantity.

The second one localized mode, we will denote it as 1 mode, has its energy:

$$\frac{\omega_1^2}{M^2} = \frac{3}{2}, \quad (39)$$

and its $\chi_1(\delta)$ wave function is:

$$\chi_1(\delta) = \frac{\sinh(\delta)}{\cosh^2(\delta)}. \quad (40)$$

As we can see the function $\chi_1(\delta)$ is zero for $\delta = 0$. For $\delta \rightarrow \pm\infty$ it is vanishing.

The corresponding value of the k_1 is:

$$k_1^2 = \frac{-6A + 5B}{B} \rho^2 = \frac{-A - 5(A - B)}{B} \rho^2 < 0. \quad (41)$$

As we can see from (41) the momentum k_1 is imaginary, it is given by:

$$k_1 = \pm i \sqrt{\frac{6A - 5B}{B}} \rho \equiv \pm i K_1. \quad (42)$$

Here we assume that $K_1 > 0$. Let us write the wave function for the 1 mode from (25) and (42). It has either a form:

$$u_{k_1,+}(\eta, x) = \frac{B_+}{\sqrt{2\pi}} \exp(ik_1 x) \chi_{k_1}(\eta) = \frac{1}{\sqrt{2\pi}} \chi_{k_1}(\eta) (B_+ \exp(+K_1 x)) \quad (43)$$

or a form:

$$u_{k_1,-}(\eta, x) = \frac{B_-}{\sqrt{2\pi}} \exp(iK_1 x) \chi_{k_1}(\eta) = \frac{1}{\sqrt{2\pi}} \chi_{k_1}(\eta) (B_- \exp(-K_1 x)) \quad (44)$$

Here B_+ and B_- are constants. In x space the mode 1 cannot be infinitely increasing on its boundaries $x \rightarrow \pm\infty$ (again) for this case of imaginary k_1 . Thus the wave functions (43) and (44) has to be chosen in such a way as (for yhe mode 0) at some point x_1 defined by $u_{k_1,-}(\eta, x_1) = u_{k_1,+}(\eta, x_1)$, e.i. where:

$$B_- \exp(-K_1 x_1) = B_+ \exp(K_1 x_1). \quad (45)$$

The point x_1 is given by $\exp(2K_1 x_1) = \frac{B_-}{B_+}$. From this and from the condition that x_1 is real we have that the constants B_+ and B_- are either positive either negative, both simultaneously. Then the value of the point x_1 is in the interval $-\infty < x_1 < +\infty$. The case in which the signs of the constants B_+ and B_- is different does not correspond to a solution of the mode 1, for which in x space cannot be infinitely increasing on its boundaries $x \rightarrow \pm\infty$. We see that the localized in the conformal time η mode 1 is localized also in the x coordinate with a peak at the point x_1 in this case. As we can see there is no oscillation in the conformal time η for this mode. However the amplitude of the mode is given by the function $\chi_1(\delta) = \frac{\sinh(\delta)}{\cosh^2(\delta)}$. This amplitude has its maximum for η_{max} and has its minimum for η_{min} (both have the same absolute value, there is an inflection point in $\eta = 0$). These points can be found from the condition $\tanh^2(\eta_{min}, \eta_{max}) = \frac{1}{2}$. Note that the point x_0 from above and the point x_1 are in general different points. Conformal time development as described by the function $\frac{\sinh(\delta)}{\cosh^2(\delta)}$ for the mode 1 and for $\delta \rightarrow \pm\infty$ has its wave function with vanishing value. For $\delta \rightarrow 0$ has its wave function value 0, it is antisymmetric around this point in δ . Thus it represents a mode which is of a localized nature in η and x coordinates. There are two peaks (maximum and minimum) in the first variable and one peak in the second variable x , it is at the point x_1 which in our case is arbitrary.

We can say that localized modes 0 and 1 are localized in the $\delta = \eta\rho$ variable, e.i. in the η variable which is a conformal time, and in the x variable.

7 Delocalized modes - scalar particles.

Besides two localized modes described above there are delocalized modes (in the conformal time η) and x coordinates. These modes are indexed by the

momentum k , we will denote them as k modes. They have the energy ω_k from the solution of the equation (29):

$$\frac{\omega_k^2}{M^2} = \frac{q^2}{2} + 2 = \frac{1}{2} \left(\frac{k^2}{\rho^2} + \frac{6A - 2B}{B} \right). \quad (46)$$

Here we have to find a dependence of the momentum q (from the solution of the equation (29) according to [14]) on the momentum k . This dependence is found from the above equation of both expressions for the energy of k states:

$$\frac{q^2}{2} + 2 = \frac{k^2}{2\rho^2} + \frac{3A - B}{B}. \quad (47)$$

We find that:

$$q^2(k) = \frac{k^2}{\rho^2} + 6 \frac{A - B}{B}. \quad (48)$$

Then k modes have the energy ω_k :

$$\frac{\omega_k^2}{M^2} = \frac{q^2}{2} + 2 = \frac{k^2}{2\rho^2} + 3 \frac{A - B}{2B}. \quad (49)$$

Thus as we can see that there is the momentum k which can be find from $\frac{k^2 M^2}{2\rho^2}$ above. The second expression on the right hand side (49) is positive. We can define as a mass of these modes squared the positive quantity $3M^2 \frac{A-B}{2B}$. The mass of the k mode (we will speak about a particle) is real because the squared mass $3M^2 \frac{A-B}{B}$ is positive, this quantity is a positive quantity due to the fact that $A - B > 0$ and $B > 0$. As we see the mass of particles depends on parameters of the expanding space-time and on the mass m of the field (this mass is related to the parameter B of the scale conformal factor, see (30)). The wave function of the k mode is given from (25):

$$u_k(\eta, x) \approx \frac{1}{\sqrt{(2\pi)}} \exp(ikx) \chi_k(\delta = \rho\eta)$$

where

$$\chi_k(\delta) = \exp(iq(k)\rho\eta) [3 \tanh^2(\eta\rho) - 1 - \left(\frac{k^2}{\rho} + 3 \frac{A - B}{B} \right) - \quad (50)$$

$$-3i \sqrt{\frac{k^2}{\rho} + 3 \frac{A - B}{B}} \tanh(\eta\rho)].$$

For the conformal time $\eta \rightarrow \pm\infty$ we obtain from (50) that the wave function has a phase shift:

$$\chi_k(\delta) \rightarrow \exp(iq\rho\eta \pm \frac{i}{2} \Delta(q)), \quad (51)$$

where the quantity $q = q(k)$ is given from (48). The phase shift $\Delta(q)$ is given by:

$$\Delta(q) = -2 \arctan\left(\frac{3q}{2 - q^2}\right). \quad (52)$$

As we can see there is an oscillation type of a development in the conformal time η with the frequency $q(k)\rho$ and an oscillation type of a development in the space coordinate x with the momentum k for these (delocalized solutions) particles. Conformal time development is further described by a factor $(3 \tanh^2(\eta\rho) - 1 - (\frac{k^2}{\rho} + 3\frac{A-B}{B}) - 3i\sqrt{\frac{k^2}{\rho} + 3\frac{A-B}{B}} \tanh(\eta\rho))$.

8 Discussion.

Let us now calculate in the style as in [9] the ω_{in} and ω_{out} frequencies from above calculations. We find:

$$\omega_{in} = \omega_{out} = M \sqrt{\frac{k^2}{2\rho^2} + 3\frac{A-B}{2B}}. \quad (53)$$

The frequencies $\omega_{\pm}$ which are found from -in and -out frequencies are given by:

$$\omega_{+} = \omega_{in} = \omega_{out} \quad (54)$$

and

$$\omega_{-} = 0. \quad (55)$$

Note that the mass of particles generated in k-modes is $M\sqrt{3\frac{A-B}{2B}}$. The expression below the square root is positive: $A - B > 0$ and $B > 0$. There are two localized modes, 0 and 1.

In difference to the expanding case [9] of the space-time, the wave functions u_k^{in} and u_k^{out} are identical besides the phase factor in u_k^{out} . As is known from integrable systems the phase factor in scattered wave function enables us to calculate the potential of scattering, see [15] and [16], where it was discussed the Inverse Scattering Method, as a methodological paper with an example - soliton (kink) solution of the Sine-Gordon Equation and a solution of a breather type.

While in general we can express the in-function u_k^{in} through u_k^{out} as:

$$u_k^{in} = \alpha_k u_k^{out} + \beta_k u_{-k}^{out*}, \quad (56)$$

where the coefficients α_k and β_k are expressed as:

$$\alpha_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - \frac{i\omega_{in}}{\rho}) \Gamma(1 - \frac{i\omega_{out}}{\rho})}{\Gamma(1 - \frac{i\omega_{+}}{\rho}) \Gamma(1 - \frac{i\omega_{-}}{\rho})}, \quad (57)$$

and

$$\beta_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - \frac{i\omega_{in}}{\rho})\Gamma(\frac{i\omega_{out}}{\rho})}{\Gamma(\frac{i\omega_{-}}{\rho})\Gamma(1 + \frac{i\omega_{-}}{\rho})}, \quad (58)$$

from (53) - (55) we find:

$$|\alpha_k|^2 = \frac{\sinh^2(\frac{\pi\omega_+}{\rho})}{\sinh(\frac{\pi\omega_{in}}{\rho})\sinh(\frac{\pi\omega_{out}}{\rho})} = 1, \quad (59)$$

and

$$|\beta_k|^2 = \frac{\sinh^2(\frac{\pi\omega_-}{\rho})}{\sinh(\frac{\pi\omega_{in}}{\rho})\sinh(\frac{\pi\omega_{out}}{\rho})} = 0. \quad (60)$$

Thus the relation of the norm is satisfied:

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \quad (61)$$

We see that the quantum field in the vacuum state $|0, in\rangle$ defined through modes u_k^{in} and u_0^{in} and u_1^{in} , when detector showed that there are no particles, was the vacuum state for non-accelerating state. This was for $\eta \rightarrow -\infty$ and Minkowski space-time was realized. In the out region $\eta \rightarrow +\infty$ we have found that the Minkowski space-time is realized again, and that the vacuum state is the same: for the inertial observer there are no particles detected in this vacuum $|0, out\rangle$ which is identical with the vacuum $|0, in\rangle$. This is the reason why we speak about generation (in expanding part of the space-time described by our conformal factor) and annihilation (in contracting period of the space-time described by our conformal factor), in such a way that there are no particles in the limit $\eta \rightarrow +\infty$, there is only the -out vacuum identical to the -in vacuum. Particles generated during the expansion period up to $\eta = 0$ are annihilated in $\eta \rightarrow +\infty$, (due to gravity). In difference to the case in [9] we have found that besides the k-particles with nonzero mass there are present localized modes 0 and 1 evolving from $\eta \rightarrow -\infty$ as η is developing to $\eta \rightarrow +\infty$. So their development is present in the conformal time interval $\eta \rightarrow -\infty \leq \eta \leq \eta \rightarrow +\infty$.

Conflict of Interest.

The authors declare that they have no conflict of interest.

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