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Decision-Making Analysis Based on Hesitant Fuzzy N -Soft ELECTRE-I Approach

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Abstract

For the formal [expression](#) of uncertain data, hesitant fuzzy set theory [has established itself](#) as a distinguished [model](#) because it has a broad use in multi-attribute decision-making problems. With the [incorporation of features from \$N\$ -soft sets](#), a useful [framework](#) referred to as hesitant fuzzy N -soft sets [has acquired an even greater appeal](#). [This model](#) integrates and associates the hesitant environment [with](#) information regarding the existence of grades or star ratings. In this research article, we introduce a multi-attribute decision-making technique known as hesitant fuzzy N -soft ELECTRE-I, which computes the decision-maker assessments in an adjustable and formative manner. [The](#) proposed method also improves the [robustness](#) and accuracy of the decisions [relying](#) on grades or star ratings. [Thus it lays](#) a bedrock for [subsequent analyses](#) and applications. We [justify](#) the relevance and convenience of [the](#) proposed technique by [testing](#) it [in](#) actually existing scenarios. Finally, we give a comparison of [this novel](#) methodology with [the](#) HFNS-TOPSIS method.

Keywords: Hesitant fuzzy sets, N -soft sets, hesitant fuzzy N -soft sets, decision-making, HFNS-ELECTRE-I.

1 Introduction

Certain types of uncertainties arise in several areas of engineering and decision-making. To handle such uncertainties, probability theory, fuzzy set theory [47], soft sets [30] and their associated models have been suggested as suitable mathematical means [2]. The research about generalization of fuzzy sets, especially hesitant fuzzy sets (henceforth, HFSs), is [growing rapidly](#) [11, 39, 40]. [Meanwhile, the literature has been providing](#) novel applications to several branches of decision-making [25, 27, 29, 48]. For the complicated situations with hesitancy in [the submission of the memberships of the](#) objects of a reference set, HFSs [have an edge over](#) fuzzy sets [22]. HFSs are concerned with [multiple membership degrees](#) of the elements, which [are formally expressed by a certain collection of](#) feasible values between “0 and 1” [43]. [Since their inception,](#)

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in contexts of decision-making where the decision-makers are irresolute and hesitant, they have the option to choose HFSs as a precise modelization [45, 49]. Hesitancy has been linked to several other frames of reference and a number of hybrid models. For example, Ding et al. [20] proposed the concept of interval-valued hesitant fuzzy TODIM method for dynamic emergency responses. Recently, Akram et al. [4] have proposed a novel hybrid model which is the combination of HFSs and N -soft sets [21], referred to as hesitant fuzzy N -soft sets (henceforth, HFNSSs), that processes vague information with a concern for two expressions of data. Put intuitively, this model provides information in terms of “which specific grades are assigned to the objects, when the evaluations of the attributes are graded, and partial membership degrees are allowed, in the presence of hesitancy”. With the passage of time, new models based on N -soft sets were introduced by various researchers. In this regard, Akram et al. [6] developed a new approach of decision-making known as Intuitionistic fuzzy N -Soft rough sets. Kamaci and Petchimuthu [26] worked on bipolar N -Soft set theory with its applications. And Ali and Akram [12] proposed a decision-making method based on fuzzy N -soft expert sets. For further information on the current development of this field, the readers are referred to [3, 10, 18, 28, 31, 32, 33, 46] as a sample of recent references.

As a functional aspect to determine a finite number of decisions among the “objects of the reference set” (with a respect for a multiplicity of evaluation attributes or criteria), multi-criteria decision-making (henceforth, MCDM) or multi-attribute decision-making (henceforth, MADM) have a strong appeal. A decision-making approach, ELECTRE “Elimination and choice translating reality”, was proposed by Benayoun et al. [15]. It is a distinctive MCDM or MADM approach, in which the decision-maker prefers to have distinct criteria’s having a stable accumulation related to the structure of evaluation attributes confined with a total of the requirements. Roy [36] drawn-out the fluctuation of different objects of the reference set by introducing the improved version of ELECTRE known as ELECTRE-I. In the literature, several researchers generalized and defined other innovative methods and applied them on modified versions of fuzzy sets and its extensions. Sevkli [38] explored an application for supplier selection based on the fuzzy ELECTRE method. Rouyendegh and Erkan [34, 35] used the fuzzy ELECTRE technique to recruit academic staff and select the best project. As a novel approach of ELECTRE, Vahdani et al. [42] put forward an intuitionistic fuzzy ELECTRE method. Motivated by a similar concern, Vahdani and Hadipour [41] developed another approach of ELECTRE known as the interval-valued fuzzy ELECTRE method. Aytac et al. [14] put the fuzzy ELECTRE-I approach to practice in a case-study with catering firms. Asghari et al. [13] also applied the fuzzy ELECTRE-I method for the interpretation and analysis of mobile payment models. Adeel et al. [1] contributed to the detection and estimation of severity of tumor cells with the aid of a fuzzy N -soft model of graded decision-making. Akram et al. [7] introduced hybrid TOPSIS and ELECTRE-I solutions with Pythagorean fuzzy information for risk evaluation in failure modes and effects analysis. With the passage of time, various additional generalizations and extensions of the ELECTRE-I approach were investigated in order to solve difficulties associated with actual cases [17, 19, 23, 24, 44]. Recently, Sarwar et al. [37] introduced an integrated rough ELECTRE-II approach for risk evaluation and effects analysis in automatic manufacturing process. Akram et al. [5, 8] proposed the idea of hesitant Pythagorean fuzzy ELECTRE-II method for multi-criteria decision-making problems and hesitant fuzzy N -soft ELECTRE-II model as a new framework for decision-making. When dealing with a large number of “objects of a reference set” and selecting a set of acceptable items without creating preference outranking of different

reference set objects, ELECTRE-I is considered as one of the most proficient approaches.

The goal of this study is to suggest a new ELECTRE-I method that operates in a hesitant fuzzy N -soft environment, referred to as the hesitant fuzzy N -soft ELECTRE-I (henceforth, HFNS-ELECTRE-I) method. In the proposed method, the perceptions from some members about the concerned set are encapsulated by HFNSSs. The accumulation of the HFNS-ELECTRE-I method is expressed by “HFNS concordance” and “HFNS discordance” sets, whereas the ranking of the “objects of the reference set” derives from items known as “HFNS concordance” and “HFNS discordance” levels. The proposed method pinpoints the most dominant object from the reference set and eliminates the irrelevant choices.

The following summary gives the structure of this paper.

Section 2 provides the basic concept of HFNSS and some examples. Section 3 proposes the new approach of decision-making referred as HFNS-ELECTRE-I. Section 4 is based on the concept of proposed method, and illustrates it with relevant applications. Section 5 compares the proposed model with the HFNS-TOPSIS method and discusses its merits. Section 6 puts an end to this paper with some concluding remarks.

2 Hesitant Fuzzy N -Soft Sets

In this section, we review some basic concepts and describe them by real life example.

Definition 2.1. [21] Let O be the “required set of objects” under consideration and P be the “set of attributes”, $T \subseteq P$. Let $G = \{0, 1, 2, \dots, N - 1\}$ be the set of ordered grades, where $N \in \{2, 3, \dots\}$. A triple (F, T, N) is called an N -soft set on O , “if F is mapping from T to $2^{O \times G}$, with the property that for each $t \in T$ and $o \in O$, there exists a unique $(o, g_t) \in O \times G$ ”, such that $(o, g_t) \in F(t)$, $g_t \in G$.

When $(o, g_t) \in F(t)$, “we interpret that o belongs to the set of t -approximations of O with grade g_t ”.

Definition 2.2. [4] Let O be the “required set of objects” and P be the “set of attributes”, $T \subseteq P$. A triple (\tilde{h}_f, T, N) is called HFNSS, when \tilde{h}_f is a mapping defined as

$$\tilde{h}_f : O \times T \rightarrow G \times \mathcal{P}^*([0, 1]).$$

“When $\tilde{h}_f(o, t) = (g, \tilde{h}_{f_t}(o))$, it is interpreted that $\tilde{h}_{f_t}(o)$ is a non-empty set formed by values in $[0, 1]$, which denote the possible membership degrees of the element $o \in O$ to the subset of t -approximations of O (or options approximated by t) with grade g ”.

Example 2.3. In drama industry, the top rating drama serials totally depend upon the star ratings as well as grades by the huge amount of audience, casting agencies, popularity of major characters, climax of story and several other, that are considered as the evaluation attributes. For the evaluation of top rating drama serials, let $D_s = \{d_{s1}, d_{s2}, d_{s3}, d_{s4}, d_{s5}, d_{s6}\}$ be the “reference set” of drama serials, and P be the “set of attributes, evaluation of drama serials by major criteria”. The collection of evaluation attributes is defined by the subset $T \subseteq P$, where $T = \{t_1, t_2, t_3, t_4\}$. The following is a breakdown of how drama serials are rated in terms of stars:

- Four stars “★★★★” correspond to ‘highest rating’,

- Three stars “★★★” correspond to ‘higher rating’,
- Two stars “★★” correspond to ‘high rating’,
- One star “★” corresponds to ‘average rating’,
- Hole “○” corresponds to ‘low rating’.

This rating evaluation by stars can be identified by $G = \{0, 1, 2, 3, 4\}$, where

- “★★★★” are identified by 4,
- “★★★” are identified by 3,
- “★★” are identified by 2,
- “★” is identified by 1,
- “○” is identified by 0.

The star ratings obtained from relevant information is presented in Table 1, and affiliated 5-soft set is analyzed in Table 2.

Table 1: Tabular representation of star ratings occupied from concerned data

D_s/T	t_1	t_2	t_3	t_4
d_{s1}	★★★	★★	★★	○
d_{s2}	★★★★	★★★	★★★	★
d_{s3}	★★★★	★	★★	★★
d_{s4}	★★	★★★	★★★★	★★★
d_{s5}	★	★★★★	★★★	★★
d_{s6}	○	○	★★★	★★★★

Table 2: Tabular representation of the associated 5-soft set

$(F, T, 5)$	t_1	t_2	t_3	t_4
d_{s1}	3	2	2	0
d_{s2}	4	3	3	1
d_{s3}	4	1	2	2
d_{s4}	2	3	4	3
d_{s5}	1	4	3	2
d_{s6}	0	0	3	4

These star ratings and their grades are sufficient, when it is cited from actual and authentic information without any hesitancy or ambiguity . It refers to the model given in Definition 2.1, but the cases when the estimations and judgments are uncertain and hesitant, we may need to use HFNSS which is more compatible and flexible about the knowledge, that how the grades are given to drama serials under hesitant situations. The following HF5SS is defined as

$$\begin{aligned}
\bar{h}_f(t_1) &= \left\{ \left\langle \frac{(d_{s1}, 3)}{\{0.47, 0.50, 0.59\}} \right\rangle, \left\langle \frac{(d_{s2}, 4)}{\{0.73, 0.80\}} \right\rangle, \right. \\
&\quad \left\langle \frac{(d_{s3}, 4)}{\{0.75, 0.78, 0.81, 0.90\}} \right\rangle, \left\langle \frac{(d_{s4}, 2)}{\{0.29, 0.38, 0.40\}} \right\rangle, \\
&\quad \left. \left\langle \frac{(d_{s5}, 3)}{\{0.10, 0.15\}} \right\rangle, \left\langle \frac{(d_{s6}, 0)}{\{0.02, 0.03, 0.04, 0.07\}} \right\rangle \right\}, \\
\bar{h}_f(t_2) &= \left\{ \left\langle \frac{(d_{s1}, 2)}{\{0.30, 0.34, 0.40, 0.42\}} \right\rangle, \left\langle \frac{(d_{s2}, 3)}{\{0.46, 0.50, 0.70\}} \right\rangle, \right. \\
&\quad \left\langle \frac{(d_{s3}, 1)}{\{0.15, 0.25\}} \right\rangle, \left\langle \frac{(d_{s4}, 3)}{\{0.31, 0.38, 0.42\}} \right\rangle, \\
&\quad \left. \left\langle \frac{(d_{s5}, 4)}{\{0.80, 0.89, 0.90, 0.95\}} \right\rangle, \left\langle \frac{(d_{s6}, 0)}{\{0.03, 0.05\}} \right\rangle \right\}, \\
\bar{h}_f(t_3) &= \left\{ \left\langle \frac{(d_{s1}, 2)}{\{0.34, 0.38, 0.40\}} \right\rangle, \left\langle \frac{(d_{s2}, 3)}{\{0.45, 0.67, 0.70, 0.71\}} \right\rangle, \right. \\
&\quad \left\langle \frac{(d_{s3}, 2)}{\{0.29, 0.31, 0.39, 0.40\}} \right\rangle, \left\langle \frac{(d_{s4}, 4)}{\{0.77, 0.89\}} \right\rangle, \\
&\quad \left. \left\langle \frac{(d_{s5}, 3)}{\{0.46, 0.49, 0.69\}} \right\rangle, \left\langle \frac{(d_{s6}, 3)}{\{0.50, 0.51, 0.62, 0.66\}} \right\rangle \right\}, \\
\bar{h}_f(t_4) &= \left\{ \left\langle \frac{(d_{s1}, 0)}{\{0.01, 0.03, 0.04\}} \right\rangle, \left\langle \frac{(d_{s2}, 1)}{\{0.13, 0.21, 0.27, 0.28\}} \right\rangle, \right. \\
&\quad \left\langle \frac{(d_{s3}, 2)}{\{0.30, 0.38, 0.41\}} \right\rangle, \left\langle \frac{(d_{s4}, 3)}{\{0.48, 0.52, 0.68, 0.70\}} \right\rangle, \\
&\quad \left. \left\langle \frac{(d_{s5}, 2)}{\{0.29, 0.37, 0.41\}} \right\rangle, \left\langle \frac{(d_{s6}, 4)}{\{0.77, 0.89, 0.90, 0.92\}} \right\rangle \right\}.
\end{aligned}$$

3 Hesitant Fuzzy N -Soft ELECTRE-I Approach

This section introduces the HFNS-ELECTRE-I decision-making approach. This method is basically used, “for the selection of best object from reference set in the interest”, that it concludes the predominate realizations in “decision-making”. We devised this strategy and introduced it into the network of HFNSSs.

Let $O = \{o_1, o_2, \dots, o_p\}$ be the reference set of objects and $T = \{t_1, t_2, \dots, t_q\}$ be the set of “evaluation attributes, evaluation of objects by standard attributes”. The knowledge perturbed with related data is equipped in form of star ratings and its affiliated numbers.

(i). The information for the “reference set of objects” $o_j \in O$ given by the “evaluation attributes” t_k is literally based on a HFNSS, such that $\bar{h}_f(o_j, t_k) = \langle g_{jk}, \bar{h}_{f_{jk}} \rangle$. Table 3 presents a general tabular representation of a HFNS “decision matrix”, which distinguishes star ratings with associated grades and hesitant fuzzy information.

Table 3: Tabular representation of a general HFNSS

(\bar{h}_f, T, N)	t_1	t_2	\dots	t_q
o_1	$\langle g_{11}, \bar{h}_{f_{11}} \rangle$	$\langle g_{12}, \bar{h}_{f_{12}} \rangle$	\dots	$\langle g_{1q}, \bar{h}_{f_{1q}} \rangle$
o_2	$\langle g_{21}, \bar{h}_{f_{21}} \rangle$	$\langle g_{22}, \bar{h}_{f_{22}} \rangle$	\dots	$\langle g_{2q}, \bar{h}_{f_{2q}} \rangle$
\vdots	\vdots	\vdots	\dots	\vdots
o_p	$\langle g_{p1}, \bar{h}_{f_{p1}} \rangle$	$\langle g_{p2}, \bar{h}_{f_{p2}} \rangle$	\dots	$\langle g_{pq}, \bar{h}_{f_{pq}} \rangle$

Given this, the count of HFEs in the fundamental HFNS decision-matrix may be distinct. To achieve equal cardinality of all HFEs, we reflect the smallest or maximum value in order to achieve standardization or confidence. The decision-maker wields such power, allowing to express pessimism or optimism. The grades and hesitant values of each object in reference set ($o_j \in O$, $j = 1, 2, \dots, p$) across all characteristics ($t_k \in T$, $k = 1, 2, \dots, q$) are defined as follows in the course:

$$\langle g_{jk}, \bar{h}_{f_{jk}} \rangle = \langle g_{jk}, \{\lambda_{jk}^1, \lambda_{jk}^2, \dots, \lambda_{jk}^r\} \rangle. \quad (1)$$

By using the relation (1) in Table 3, we can get the “optimistic or pessimistic HFNS decision matrix”.

(ii). The weights $w_k \in (0, 1]$ are nominated conceding to the “choice of decision-makers and importance of evaluation attributes”. We suppose that the nominated weights $w = (w_1, w_2, \dots, w_q) \in (0, 1]$ are normalized, and meet the criteria such as $\sum_{k=1}^q w_k = 1$.

(iii). The weighted “pessimistic or optimistic HFNS decision matrix” is computed as follows:

$$\begin{aligned} W &= [\langle g'_{jk}, \bar{h}'_{f_{jk}} \rangle]_{p \times q} \\ &= [\langle g'_{jk}, \{\lambda_{jk}^1, \lambda_{jk}^2, \dots, \lambda_{jk}^r\} \rangle]_{p \times q} \\ &= [w_k g_{jk}, w_k \{\lambda_{jk}^1, \lambda_{jk}^2, \dots, \lambda_{jk}^r\}]_{p \times q} \end{aligned}$$

(iv). The HFNS “concordance set” is represented as

$$Y_{uv} = \{1 \leq k \leq q; 1 \leq l \leq r | \langle g'_{uk} \geq g'_{vk}, \lambda'_{uk} \geq \lambda'_{vk} \rangle, “u \neq v; u, v = 1, 2, \dots, p”\}.$$

(v). The HFNS “concordance indices” are computed as

$$\langle g_{y_{uv}}, y_{uv} \rangle = \langle \sum_{k \in g'_{uk} \geq g'_{vk}} w_k, \sum_{k \in \lambda'_{uk} \geq \lambda'_{vk}} w_k \rangle,$$

ans the HFNS “concordance matrix” is calculated and represented by Y .

$$Y = \begin{pmatrix} -- & \langle g_{y_{12}}, y_{12} \rangle & \langle g_{y_{13}}, y_{13} \rangle & \dots & \langle g_{y_{1p}}, y_{1p} \rangle \\ \langle g_{y_{21}}, y_{21} \rangle & -- & \langle g_{y_{23}}, y_{23} \rangle & \dots & \langle g_{y_{2p}}, y_{2p} \rangle \\ \langle g_{y_{31}}, y_{31} \rangle & \langle g_{y_{32}}, y_{32} \rangle & -- & \dots & \langle g_{y_{3p}}, y_{3p} \rangle \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \langle g_{y_{p1}}, y_{p1} \rangle & \langle g_{y_{p2}}, y_{p2} \rangle & \langle g_{y_{p3}}, y_{p3} \rangle & \dots & -- \end{pmatrix}.$$

(vi). The HFNS “discordance set” is represented as

$$Z_{uv} = \{1 \leq k \leq q; 1 \leq l \leq r | \langle g'_{uk} \leq g'_{vk}, \lambda'_{uk} \leq \lambda'_{vk} \rangle, “u \neq v; u, v = 1, 2, \dots, p”\}.$$

(vii). The HFNS “discordance indices” are computed as

$$\langle g_{z_{uv}}, z_{uv} \rangle = \left\langle \frac{\max_{k \in g'_{uk} \leq g'_{vk}} \sqrt{(g'_{uk} - g'_{vk})^2}}{\max_k \sqrt{(g'_{uk} - g'_{vk})^2}}, \frac{\max_{k \in \lambda'_{uk} \leq \lambda'_{vk}} \sqrt{\frac{1}{r} \sum_{l=1}^r (\lambda'_{uk} - \lambda'_{vk})^2}}{\max_k \sqrt{\frac{1}{r} \sum_{l=1}^r (\lambda'_{uk} - \lambda'_{vk})^2}} \right\rangle,$$

and the HFNS “discordance matrix” is computed and represented by Z .

$$Z = \begin{pmatrix} -- & \langle g_{z_{12}}, z_{12} \rangle & \langle g_{z_{13}}, z_{13} \rangle & \cdots & \langle g_{z_{1p}}, z_{1p} \rangle \\ \langle g_{z_{21}}, z_{21} \rangle & -- & \langle g_{z_{23}}, z_{23} \rangle & \cdots & \langle g_{z_{2p}}, z_{2p} \rangle \\ \langle g_{z_{31}}, z_{31} \rangle & \langle g_{z_{32}}, z_{32} \rangle & -- & \cdots & \langle g_{z_{3p}}, z_{3p} \rangle \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \langle g_{z_{p1}}, z_{p1} \rangle & \langle g_{z_{p2}}, z_{p2} \rangle & \langle g_{z_{p3}}, z_{p3} \rangle & \cdots & -- \end{pmatrix}.$$

(viii). To obtain the required positions of several “objects of reference set”, we calculate the dawn characters known as “HFNS concordance” and “HFNS discordance” levels given as

$$\langle \bar{g}_y, \bar{y} \rangle = \left\langle \frac{1}{p(p-1)} \sum_{\substack{u=1 \\ u \neq v}}^p \sum_{\substack{v=1 \\ u \neq v}}^p g_{y_{uv}}, \frac{1}{p(p-1)} \sum_{\substack{u=1 \\ u \neq v}}^p \sum_{\substack{v=1 \\ u \neq v}}^p y_{uv} \right\rangle,$$

$$\langle \bar{g}_z, \bar{z} \rangle = \left\langle \frac{1}{p(p-1)} \sum_{\substack{u=1 \\ u \neq v}}^p \sum_{\substack{v=1 \\ u \neq v}}^p g_{z_{uv}}, \frac{1}{p(p-1)} \sum_{\substack{u=1 \\ u \neq v}}^p \sum_{\substack{v=1 \\ u \neq v}}^p z_{uv} \right\rangle.$$

(ix). Based on the HFNS “concordance level”, the HFNS concordance “dominance matrix” is computed and represented by R .

$$R = \begin{pmatrix} -- & \langle g_{r_{12}}, r_{12} \rangle & \langle g_{r_{13}}, r_{13} \rangle & \cdots & \langle g_{r_{1p}}, r_{1p} \rangle \\ \langle g_{r_{21}}, r_{21} \rangle & -- & \langle g_{r_{23}}, r_{23} \rangle & \cdots & \langle g_{r_{2p}}, r_{2p} \rangle \\ \langle g_{r_{31}}, r_{31} \rangle & \langle g_{r_{32}}, r_{32} \rangle & -- & \cdots & \langle g_{r_{3p}}, r_{3p} \rangle \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \langle g_{r_{p1}}, r_{p1} \rangle & \langle g_{r_{p2}}, r_{p2} \rangle & \langle g_{r_{p3}}, r_{p3} \rangle & \cdots & -- \end{pmatrix}.$$

where,

$$\langle g_{r_{uv}}, r_{uv} \rangle = \begin{cases} \langle 1, 1 \rangle & \langle g_{r_{uv}} \geq \bar{g}_y, y_{uv} \geq \bar{y} \rangle; \\ \langle 0, 0 \rangle & \langle g_{r_{uv}} < \bar{g}_y, y_{uv} < \bar{y} \rangle. \end{cases}$$

(x). Based on the HFNS “discordance level”, the HFNS discordance “dominance matrix” is computed and represented by S .

$$S = \begin{pmatrix} -- & \langle g_{s_{12}}, s_{12} \rangle & \langle g_{s_{13}}, s_{13} \rangle & \cdots & \langle g_{s_{1p}}, s_{1p} \rangle \\ \langle g_{s_{21}}, s_{21} \rangle & -- & \langle g_{s_{23}}, s_{23} \rangle & \cdots & \langle g_{s_{2p}}, s_{2p} \rangle \\ \langle g_{s_{31}}, s_{31} \rangle & \langle g_{s_{32}}, s_{32} \rangle & -- & \cdots & \langle g_{s_{3p}}, s_{3p} \rangle \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \langle g_{s_{p1}}, s_{p1} \rangle & \langle g_{s_{p2}}, s_{p2} \rangle & \langle g_{s_{p3}}, s_{p3} \rangle & \cdots & -- \end{pmatrix}.$$

where,

$$\langle g_{s_{uv}}, s_{uv} \rangle = \begin{cases} \langle 1, 1 \rangle & \langle g_{s_{uv}} < \bar{g}_z, z_{uv} < \bar{z} \rangle; \\ \langle 0, 0 \rangle & \langle g_{s_{uv}} \geq \bar{g}_z, z_{uv} \geq \bar{z} \rangle. \end{cases}$$

(xi). The aggregated HFNS “dominance matrix” is computed and represented by T .

$$T = \begin{pmatrix} -- & \langle g_{t_{12}}, t_{12} \rangle & \langle g_{t_{13}}, t_{13} \rangle & \cdots & \langle g_{t_{1p}}, t_{1p} \rangle \\ \langle g_{t_{21}}, t_{21} \rangle & -- & \langle g_{t_{23}}, t_{23} \rangle & \cdots & \langle g_{t_{2p}}, t_{2p} \rangle \\ \langle g_{t_{31}}, t_{31} \rangle & \langle g_{t_{32}}, t_{32} \rangle & -- & \cdots & \langle g_{t_{3p}}, t_{3p} \rangle \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \langle g_{t_{p1}}, t_{p1} \rangle & \langle g_{t_{p2}}, t_{p2} \rangle & \langle g_{t_{p3}}, t_{p3} \rangle & \cdots & -- \end{pmatrix}.$$

where, $\langle g_{t_{uv}}, t_{uv} \rangle$ is represented as

$$\langle g_{t_{uv}}, t_{uv} \rangle = \langle g_{r_{uv}} g_{s_{uv}}, r_{uv} s_{uv} \rangle.$$

(xii). Lastly, “to rank the objects of reference set on account to the outranking values of aggregated HFNS dominance matrix T ”. For every pair of objects, “there exists multiple directed edges from o_u to o_v if and only if $\langle g_{t_{uv}}, t_{uv} \rangle = \langle 1, 1 \rangle$ ”. The consecutive cases appear as follows:

1. There happens the different occurrence of directed bold and dotted edges from o_u to o_v , to indicate the choice of o_u over o_v .
2. There happens the different occurrence of directed bold and dotted edges from o_u to o_v and o_v to o_u , to determine the indifference of o_u and o_v .
3. There does not happens any dotted and bold edge between o_u and o_v , to determine the incomparability of o_u and o_v .

4 Applications of HFNS-ELECTRE-I Approach

This section justifies the proposed method by fixing it on actually existing scenarios to identify the top star ratings of drama serials and dominated seven star airlines.

4.1 Top Star Ratings of Drama Serials

The evaluation of top star ratings of drama serials in industry is often treated as interlaced and puzzling process, specially having the hesitancy to estimate the drama serials by “evaluation attributes”. For interpretation, we continue exploring Example 2.3.

For the ease of readers, and to promote and apply the proposed algorithm in better way, the original experimental data of “Top Star Ratings of Drama Serials” is obtained from the drama serial ratings website (<https://www.oyeyeah.com/dramaratings/>), in which latest news, drama reviews and ratings of drama serials are discussed in a formal way, whereas the hesitation fuzzy N -soft set or its associated grades/numbers are obtained from the original experimental data of drama serial ratings which are prescribed in form of stars. These star ratings and its corresponding grades are explained in Tables 1 and 2. Moreover, the HF5SS defined in Example 2.3 have the format as shown in Table 4.

(i). The tabular representation of HF5S “decision matrix” is represented by Table 4.

Table 4: Tabular representation of HF5S decision matrix

Drama serials	Evaluation attributes	
	t_1	t_2
d_{s1}	$\langle 3, \{0.47, 0.50, 0.59\} \rangle$	$\langle 2, \{0.30, 0.34, 0.40, 0.42\} \rangle$
d_{s2}	$\langle 4, \{0.73, 0.80\} \rangle$	$\langle 3, \{0.46, 0.50, 0.70\} \rangle$
d_{s3}	$\langle 4, \{0.75, 0.78, 0.81, 0.90\} \rangle$	$\langle 1, \{0.15, 0.25\} \rangle$
d_{s4}	$\langle 2, \{0.29, 0.38, 0.40\} \rangle$	$\langle 3, \{0.31, 0.38, 0.42\} \rangle$
d_{s5}	$\langle 1, \{0.10, 0.15\} \rangle$	$\langle 4, \{0.80, 0.89, 0.90, 0.95\} \rangle$
d_{s6}	$\langle 0, \{0.02, 0.03, 0.04, 0.07\} \rangle$	$\langle 0, \{0.03, 0.05\} \rangle$
Drama serials	Evaluation attributes	
	t_3	t_4
d_{s1}	$\langle 2, \{0.34, 0.38, 0.40\} \rangle$	$\langle 0, \{0.01, 0.03, 0.04\} \rangle$
d_{s2}	$\langle 3, \{0.45, 0.67, 0.70, 0.71\} \rangle$	$\langle 1, \{0.13, 0.21, 0.27, 0.28\} \rangle$
d_{s3}	$\langle 2, \{0.29, 0.31, 0.39, 0.40\} \rangle$	$\langle 2, \{0.30, 0.38, 0.41\} \rangle$
d_{s4}	$\langle 4, \{0.77, 0.89\} \rangle$	$\langle 3, \{0.48, 0.52, 0.68, 0.70\} \rangle$
d_{s5}	$\langle 3, \{0.46, 0.49, 0.69\} \rangle$	$\langle 2, \{0.29, 0.37, 0.41\} \rangle$
d_{s6}	$\langle 3, \{0.50, 0.51, 0.62, 0.66\} \rangle$	$\langle 4, \{0.77, 0.89, 0.90, 0.92\} \rangle$

Table 5: Tabular representation of optimistic HF5S decision matrix by extending the maximal values

Drama serials	Evaluation attributes	
	t_1	t_2
d_{s1}	$\langle 3, \{0.47, 0.50, 0.59, 0.59\} \rangle$	$\langle 2, \{0.30, 0.34, 0.40, 0.42\} \rangle$
d_{s2}	$\langle 4, \{0.73, 0.80, 0.80, 0.80\} \rangle$	$\langle 3, \{0.46, 0.50, 0.70, 0.70\} \rangle$
d_{s3}	$\langle 4, \{0.75, 0.78, 0.81, 0.90\} \rangle$	$\langle 1, \{0.15, 0.25, 0.25, 0.25\} \rangle$
d_{s4}	$\langle 2, \{0.29, 0.38, 0.40, 0.40\} \rangle$	$\langle 3, \{0.31, 0.38, 0.42, 0.42\} \rangle$
d_{s5}	$\langle 1, \{0.10, 0.15, 0.15, 0.15\} \rangle$	$\langle 4, \{0.80, 0.89, 0.90, 0.95\} \rangle$
d_{s6}	$\langle 0, \{0.02, 0.03, 0.04, 0.07\} \rangle$	$\langle 0, \{0.03, 0.05, 0.05, 0.05\} \rangle$
Drama serials	Evaluation attributes	
	t_3	t_4
d_{s1}	$\langle 2, \{0.34, 0.38, 0.40, 0.40\} \rangle$	$\langle 0, \{0.01, 0.03, 0.04, 0.04\} \rangle$
d_{s2}	$\langle 3, \{0.45, 0.67, 0.70, 0.71\} \rangle$	$\langle 1, \{0.13, 0.21, 0.27, 0.28\} \rangle$
d_{s3}	$\langle 2, \{0.29, 0.31, 0.39, 0.40\} \rangle$	$\langle 2, \{0.30, 0.38, 0.41, 0.41\} \rangle$
d_{s4}	$\langle 4, \{0.77, 0.89, 0.89, 0.89\} \rangle$	$\langle 3, \{0.48, 0.52, 0.68, 0.70\} \rangle$
d_{s5}	$\langle 3, \{0.46, 0.49, 0.69, 0.69\} \rangle$	$\langle 2, \{0.29, 0.37, 0.41, 0.41\} \rangle$
d_{s6}	$\langle 3, \{0.50, 0.51, 0.62, 0.66\} \rangle$	$\langle 4, \{0.77, 0.89, 0.90, 0.92\} \rangle$

(ii). w_k represent the normalized weights nominated to evaluation attributes, given as follows:

$$w_k = (0.2701, 0.2526, 0.2432, 0.2341).$$

(iii). The “weighted optimistic HF5S decision matrix” is computed in Table 6.

Table 6: Tabular representation of weighted optimistic HF5S decision matrix

Drama serials	Evaluation attributes	
	t_1	t_2
d_{s1}	$\langle 0.8103, \{0.1269, 0.1351, 0.1594, 0.1594\} \rangle$	$\langle 0.5052, \{0.0758, 0.0859, 0.1010, 0.1061\} \rangle$
d_{s2}	$\langle 1.0804, \{0.1972, 0.2161, 0.2161, 0.2161\} \rangle$	$\langle 0.7578, \{0.1162, 0.1263, 0.1768, 0.1768\} \rangle$
d_{s3}	$\langle 1.0804, \{0.2026, 0.2107, 0.2188, 0.2431\} \rangle$	$\langle 0.2526, \{0.0379, 0.0631, 0.0631, 0.0631\} \rangle$
d_{s4}	$\langle 0.5402, \{0.0783, 0.1026, 0.1080, 0.1080\} \rangle$	$\langle 0.7578, \{0.0783, 0.0960, 0.1061, 0.1061\} \rangle$
d_{s5}	$\langle 0.2701, \{0.0270, 0.0405, 0.0405, 0.0405\} \rangle$	$\langle 1.0104, \{0.2021, 0.2248, 0.2273, 0.2400\} \rangle$
d_{s6}	$\langle 0, \{0.0054, 0.0081, 0.0108, 0.0189\} \rangle$	$\langle 0, \{0.0076, 0.0126, 0.0126, 0.0126\} \rangle$
Drama serials	Evaluation attributes	
	t_3	t_4
d_{s1}	$\langle 0.4864, \{0.0827, 0.0924, 0.0973, 0.0973\} \rangle$	$\langle 0, \{0.0023, 0.0070, 0.0094, 0.0094\} \rangle$
d_{s2}	$\langle 0.7296, \{0.1094, 0.1629, 0.1702, 0.1727\} \rangle$	$\langle 0.2341, \{0.0304, 0.0492, 0.0632, 0.0655\} \rangle$
d_{s3}	$\langle 0.4864, \{0.0705, 0.0754, 0.0948, 0.0973\} \rangle$	$\langle 0.4682, \{0.0702, 0.0890, 0.0960, 0.0960\} \rangle$
d_{s4}	$\langle 0.9728, \{0.1873, 0.2164, 0.2164, 0.2164\} \rangle$	$\langle 0.7023, \{0.1124, 0.1217, 0.1592, 0.1639\} \rangle$
d_{s5}	$\langle 0.7296, \{0.1119, 0.1192, 0.1678, 0.1678\} \rangle$	$\langle 0.4682, \{0.0679, 0.0866, 0.0960, 0.0960\} \rangle$
d_{s6}	$\langle 0.7296, \{0.1216, 0.1240, 0.1508, 0.1605\} \rangle$	$\langle 0.9364, \{0.1803, 0.2083, 0.2107, 0.2154\} \rangle$

(iv). The HF5S “concordance set” is computed in Table 7.

Table 7: Tabular representation of HF5S concordance set

v	1	2	3
Y_{1v}	—	$\langle \{ \}, \{ \} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$
Y_{2v}	$\langle \{1, 2, 3, 4\}, \{1, 2, 3, 4\} \rangle$	—	$\langle \{1, 2, 3\}, \{2, 3\} \rangle$
Y_{3v}	$\langle \{1, 3, 4\}, \{1, 4\} \rangle$	$\langle \{1, 4\}, \{1, 4\} \rangle$	—
Y_{4v}	$\langle \{2, 3, 4\}, \{2, 3, 4\} \rangle$	$\langle \{2, 3, 4\}, \{3, 4\} \rangle$	$\langle \{2, 3, 4\}, \{2, 3, 4\} \rangle$
Y_{5v}	$\langle \{2, 3, 4\}, \{2, 3, 4\} \rangle$	$\langle \{2, 3, 4\}, \{2, 4\} \rangle$	$\langle \{2, 3, 4\}, \{2, 3\} \rangle$
Y_{6v}	$\langle \{3, 4\}, \{3, 4\} \rangle$	$\langle \{3, 4\}, \{4\} \rangle$	$\langle \{3, 4\}, \{3, 4\} \rangle$
v	4	5	6
Y_{1v}	$\langle \{1\}, \{1\} \rangle$	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$
Y_{2v}	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1, 3\}, \{1, 3\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$
Y_{3v}	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 4\}, \{1, 4\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$
Y_{4v}	—	$\langle \{1, 3, 4\}, \{1, 3, 4\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$
Y_{5v}	$\langle \{2\}, \{2\} \rangle$	—	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$
Y_{6v}	$\langle \{4\}, \{4\} \rangle$	$\langle \{3, 4\}, \{4\} \rangle$	—

(v). The HF5S “concordance matrix” is computed and represented by Y .

$$Y = \begin{pmatrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} -- & \langle 0.0000, 0.0000 \rangle & \langle 0.4958, 0.4958 \rangle \\ \langle 1.0000, 1.0000 \rangle & -- & \langle 0.7659, 0.4958 \rangle \\ \langle 0.7474, 0.5042 \rangle & \langle 0.5042, 0.5042 \rangle & -- \\ \langle 0.7299, 0.7299 \rangle & \langle 0.7299, 0.4773 \rangle & \langle 0.7299, 0.7299 \rangle \\ \langle 0.7299, 0.7299 \rangle & \langle 0.7299, 0.4867 \rangle & \langle 0.7299, 0.4958 \rangle \\ \langle 0.4773, 0.4773 \rangle & \langle 0.4773, 0.2341 \rangle & \langle 0.4773, 0.4773 \rangle \end{matrix} \\ \hline & \begin{matrix} 4 & 5 & 6 \end{matrix} \\ \begin{matrix} \langle 0.2701, 0.2701 \rangle & \langle 0.2701, 0.2701 \rangle & \langle 0.5227, 0.5227 \rangle \\ \langle 0.5227, 0.5227 \rangle & \langle 0.5133, 0.5133 \rangle & \langle 0.7659, 0.7659 \rangle \\ \langle 0.2701, 0.2701 \rangle & \langle 0.5042, 0.5042 \rangle & \langle 0.5227, 0.5227 \rangle \\ -- & \langle 0.7474, 0.7474 \rangle & \langle 0.7659, 0.7659 \rangle \\ \langle 0.2526, 0.2526 \rangle & -- & \langle 0.7659, 0.7659 \rangle \\ \langle 0.2341, 0.2341 \rangle & \langle 0.4773, 0.2341 \rangle & -- \end{matrix} \end{pmatrix}.$$

(vi). The HF5S “discordance set” is computed in Table 8.

Table 8: Tabular representation of HF5S discordance set

v	1	2	3
Z_{1v}	--	$\langle \{1, 2, 3, 4\}, \{1, 2, 3, 4\} \rangle$	$\langle \{1, 3, 4\}, \{1, 4\} \rangle$
Z_{2v}	$\langle \{ \}, \{ \} \rangle$	--	$\langle \{1, 4\}, \{1, 4\} \rangle$
Z_{3v}	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{1, 2, 3\}, \{2, 3\} \rangle$	--
Z_{4v}	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1, \}, \{1\} \rangle$
Z_{5v}	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 3\}, \{1, 3\} \rangle$	$\langle \{1, 4\}, \{1, 4\} \rangle$
Z_{6v}	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$
v	4	5	6
Z_{1v}	$\langle \{2, 3, 4\}, \{2, 3, 4\} \rangle$	$\langle \{2, 3, 4\}, \{2, 3, 4\} \rangle$	$\langle \{3, 4\}, \{3, 4\} \rangle$
Z_{2v}	$\langle \{2, 3, 4\}, \{3, 4\} \rangle$	$\langle \{2, 3, 4\}, \{2, 4\} \rangle$	$\langle \{3, 4\}, \{4\} \rangle$
Z_{3v}	$\langle \{2, 3, 4\}, \{2, 3, 4\} \rangle$	$\langle \{2, 3, 4\}, \{2, 3\} \rangle$	$\langle \{3, 4\}, \{3, 4\} \rangle$
Z_{4v}	--	$\langle \{2\}, \{2\} \rangle$	$\langle \{4\}, \{4\} \rangle$
Z_{5v}	$\langle \{1, 3, 4\}, \{1, 3, 4\} \rangle$	--	$\langle \{3, 4\}, \{4\} \rangle$
Z_{6v}	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$	--

(vii). The HF5S “discordance matrix” is computed and represented by Z .

$$Z = \begin{pmatrix} \begin{array}{ccc} 1 & 2 & 3 \\ \hline -- & \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle \\ \langle 0.0000, 0.0000 \rangle & -- & \langle 0.4634, 0.3791 \rangle \\ \langle 0.5395, 0.4458 \rangle & \langle 1.0000, 1.0000 \rangle & -- \\ \langle 0.3846, 0.3483 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 0.9600 \rangle \\ \langle 1.0000, 0.8261 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle \\ \langle 0.8653, 0.6844 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle \\ \hline 4 & 5 & 6 \\ \langle 1.0000, 1.0000 \rangle & \langle 0.9352, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle \\ \langle 0.8667, 0.7830 \rangle & \langle 0.3117, 0.4411 \rangle & \langle 0.6500, 0.7558 \rangle \\ \langle 0.9352, 1.0000 \rangle & \langle 0.9352, 0.9163 \rangle & \langle 0.4334, 0.5568 \rangle \\ -- & \langle 0.9352, 1.0000 \rangle & \langle 0.3089, 0.7418 \rangle \\ \langle 1.0000, 0.5550 \rangle & -- & \langle 0.4634, 0.5511 \rangle \\ \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle & -- \end{array} \end{pmatrix}.$$

(viii). The HF5S “concordance level” $\langle \bar{g}_y, \bar{y} \rangle = \langle 0.5577, 0.5000 \rangle$, and HF5S “discordance level” $\langle \bar{g}_z, \bar{z} \rangle = \langle 0.8009, 0.7981 \rangle$ are computed.

(ix). The HF5S concordance “dominance matrix” is computed and represented by R .

$$R = \begin{pmatrix} \begin{array}{cccccc} -- & \langle 0, 0 \rangle & \langle 0, 1 \rangle \\ \langle 1, 1 \rangle & -- & \langle 1, 0 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 1, 1 \rangle \\ \langle 1, 1 \rangle & \langle 0, 1 \rangle & -- & \langle 0, 0 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 1, 1 \rangle & \langle 1, 0 \rangle & \langle 1, 1 \rangle & -- & \langle 1, 1 \rangle & \langle 1, 1 \rangle \\ \langle 1, 1 \rangle & \langle 1, 0 \rangle & \langle 1, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & -- \end{array} \end{pmatrix}.$$

(x). A HF5S discordance “dominance matrix” is computed and represented by S .

$$S = \begin{pmatrix} \begin{array}{cccccc} -- & \langle 0, 0 \rangle \\ \langle 1, 1 \rangle & -- & \langle 1, 1 \rangle & \langle 0, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle \\ \langle 1, 1 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 1 \rangle & -- & \langle 1, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 0 \rangle & -- \end{array} \end{pmatrix}.$$

(xi). An aggregated HF5S “dominance matrix” is computed and represented by T .

$$T = \begin{pmatrix} \begin{array}{cccccc} -- & \langle 0, 0 \rangle \\ \langle 1, 1 \rangle & -- & \langle 1, 0 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 1, 1 \rangle \\ \langle 1, 1 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 1 \rangle \\ \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & -- & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & -- \end{array} \end{pmatrix}.$$

(xii). Lastly, to determine the dominant position of drama serials, owing to star rated characters of “aggregated HF5S dominance matrix”, a multiple edges directed graph for each pair of drama serials is presented in Figure 1.

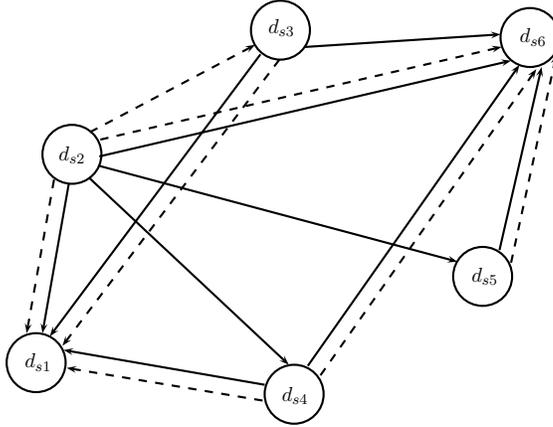


Figure 1: Outranking relation of drama serials

For ranking the positions, different drama serials are represented by circles, the grading evaluations are represented by dotted directed edges, and the evaluations under hesitant environment are represented by bold directed edges.

1. d_{s1} is incomparable to other $d_{sj}'s$, because there is no directed edge from d_{s1} to $d_{sj}'s$, where $j = 1, 2, \dots, 6$.
2. d_{s2} is preferred over all other $d_{sj}'s$, because there are several bold and dotted directed edges from d_{s2} to other $d_{sj}'s$.
3. Similarly, d_{s3} is preferred over d_{s1} and d_{s6} .
4. Similarly, d_{s4} is preferred over d_{s1} and d_{s6} .
5. Similarly, d_{s5} is preferred over d_{s6} .
6. Finally, d_{s6} is incomparable to others.

Hence, d_{s2} is considered as the most dominated drama serial due to the highest and top star ranking.

4.2 Dominated Seven Star Airline

In 1999 the SKYTRAX proposed the global airline quality rating programme. The audit office awards star ratings after a thorough expert study of an airline's quality requirements. A typical standard rating is based on an examination of 500 to 800 product and service delivery evaluation elements. This includes onboard requirements for all appropriate cabin or aircraft types, as well as airport services at the airline's hub. Each airline is evaluated on the basis of its front-line product, as well as service quality onboard and in the airport, which are regarded evaluation qualities. The actuality of delivered product and service provided to clients is a key component of star rating. The current standards of an airline's home base operating airport are used to assign an airport rating.

For the ease of readers, and to promote and apply the proposed algorithm in better way, the original experimental data of "Dominated Seven Star Airline" is obtained from the SKY-TRAX (Certified Ratings) website (<https://skytraxratings.com/about-airline-rating>), which is a global airline quality rating programme, whereas the hesitation fuzzy N -soft set or its associated grades/numbers are obtained from the original experimental data of air-line ratings which are prescribed in form of stars. These star ratings and its associated grades are explained Table 9. For the evaluation of dominated seven star airlines, let $A_l = \{A_{l1}, A_{l2}, A_{l3}, A_{l4}, A_{l5}, A_{l6}, A_{l7}\}$ be the required set of airlines, and P be the set of attributes "evaluation of airlines by major criteria". The subset $T \subseteq P$ is considered as the set of evaluation attributes, such that $T = \{t_1, t_2, t_3\}$. The star rating evaluations of airlines are performed by the numbers as $G = \{0, 1, 2, 3, 4, 5, 6\}$, and described in the following way:

- Six stars "*****" correspond to 'very high Quality', and identified by 6.
- Five stars "*****" correspond to 'Good Quality', and identified by 5.
- Four stars "****" correspond to 'Fair rating', and identified by 4.
- Three stars "***" correspond to 'Average rating', and identified by 3.
- Two stars "**" correspond to 'Below Average Quality', and identified by 2.
- One star "*" corresponds to 'Lower Quality', and identified by 1.
- Hole "o" corresponds to 'Poor Quality', and identified by 0.

Table 9 defines the seven star ratings and accompanying 7-soft set obtained from relevant knowledge.

Table 9: The star ratings and its associated 7-soft set

A_l/T	t_1	t_2	t_3
A_{l1}	*****/5	*****/4	****/3
A_{l2}	**/2	*/1	*****/4
A_{l3}	*****/5	o/0	****/3
A_{l4}	**/2	*****/6	*****/5
A_{l5}	*****/4	*****/5	****/3
A_{l6}	****/3	*****/6	**/2
A_{l7}	o/0	**/2	*****/5

The hesitancy to estimate the airlines by evaluation attributes in format of HF7SS is shown in Table 10.

- (i). The tabular representation of "HF7S decision matrix" is represented by Table 10.

Table 10: Tabular representation of HF7S decision matrix

Airlines	Evaluation attributes		
	t_1	t_2	t_3
A_{I1}	$\langle 5, \{0.683, 0.872, 0.791\} \rangle$	$\langle 4, \{0.470, 0.391, 450\} \rangle$	$\langle 3, \{0.351, 0.320\} \rangle$
A_{I2}	$\langle 2, \{0.217, 0.301\} \rangle$	$\langle 1, \{0.145, 0.250, 0.269\} \rangle$	$\langle 4, \{0.465, 0.370\} \rangle$
A_{I3}	$\langle 5, \{0.656, 0.750\} \rangle$	$\langle 0, \{0.005, 0.019, 0.029\} \rangle$	$\langle 3, \{0.390, 0.435\} \rangle$
A_{I4}	$\langle 2, \{0.310, 0.331, 0.399\} \rangle$	$\langle 6, \{0.889, 0.979\} \rangle$	$\langle 5, \{0.691, 0.730, 0.735\} \rangle$
A_{I5}	$\langle 4, \{0.517, 0.620\} \rangle$	$\langle 5, \{0.745, 0.776, 0.877\} \rangle$	$\langle 3, \{0.441, 0.538\} \rangle$
A_{I6}	$\langle 3, \{0.447, 0.520\} \rangle$	$\langle 6, \{0.845, 0.956, 0.957\} \rangle$	$\langle 2, \{0.341, 0.340, 0.358\} \rangle$
A_{I7}	$\langle 0, \{0.017, 0.020\} \rangle$	$\langle 2, \{0.345, 0.306, 0.357\} \rangle$	$\langle 5, \{0.641, 0.758\} \rangle$

Table 11: Tabular representation of pessimistic HF7S decision matrix

Airlines	Evaluation attributes		
	t_1	t_2	t_3
A_{I1}	$\langle 5, \{0.683, 0.872, 0.791\} \rangle$	$\langle 4, \{0.470, 0.391, 0.450\} \rangle$	$\langle 3, \{0.351, 0.351, 0.320\} \rangle$
A_{I2}	$\langle 2, \{0.217, 0.217, 0.301\} \rangle$	$\langle 1, \{0.145, 0.250, 0.269\} \rangle$	$\langle 4, \{0.465, 0.465, 0.370\} \rangle$
A_{I3}	$\langle 5, \{0.656, 0.656, 0.750\} \rangle$	$\langle 0, \{0.005, 0.019, 0.029\} \rangle$	$\langle 3, \{0.390, 0.390, 0.435\} \rangle$
A_{I4}	$\langle 2, \{0.310, 0.331, 0.399\} \rangle$	$\langle 6, \{0.889, 0.889, 0.979\} \rangle$	$\langle 5, \{0.691, 0.730, 0.735\} \rangle$
A_{I5}	$\langle 4, \{0.517, 0.517, 0.620\} \rangle$	$\langle 5, \{0.745, 0.776, 0.877\} \rangle$	$\langle 3, \{0.441, 0.441, 0.538\} \rangle$
A_{I6}	$\langle 3, \{0.447, 0.447, 0.520\} \rangle$	$\langle 6, \{0.845, 0.956, 0.957\} \rangle$	$\langle 2, \{0.341, 0.340, 0.358\} \rangle$
A_{I7}	$\langle 0, \{0.017, 0.017, 0.020\} \rangle$	$\langle 2, \{0.345, 0.306, 0.357\} \rangle$	$\langle 5, \{0.641, 0.641, 0.758\} \rangle$

(ii). w_k represent the normalized weights nominated to evaluation attributes, given as follows:

$$w_k = (0.321, 0.333, 0.346).$$

(iii). The “weighted pessimistic HF7S decision matrix” is computed in Table 12.

Table 12: Tabular representation of weighted pessimistic HF7S decision matrix

Airlines	Evaluation attributes		
	t_1	t_2	t_3
A_{t1}	$\langle 1.6050, \{0.2192, 0.2799, 0.2539\} \rangle$	$\langle 1.3320, \{0.1565, 0.1302, 0.1499\} \rangle$	$\langle 1.0380, \{0.1214, 0.1214, 0.1107\} \rangle$
A_{t2}	$\langle 0.6420, \{0.0697, 0.0697, 0.0966\} \rangle$	$\langle 0.3330, \{0.0483, 0.0833, 0.0896\} \rangle$	$\langle 1.3840, \{0.1609, 0.1609, 0.1280\} \rangle$
A_{t3}	$\langle 1.6050, \{0.2106, 0.2106, 0.2408\} \rangle$	$\langle 0, \{0.0017, 0.0063, 0.0097\} \rangle$	$\langle 1.0380, \{0.1349, 0.1349, 0.1505\} \rangle$
A_{t4}	$\langle 0.6420, \{0.0995, 0.1063, 0.1281\} \rangle$	$\langle 1.9980, \{0.2960, 0.2960, 0.3260\} \rangle$	$\langle 1.7300, \{0.2391, 0.2526, 0.2543\} \rangle$
A_{t5}	$\langle 1.2840, \{0.1660, 0.1660, 0.1990\} \rangle$	$\langle 1.6650, \{0.2481, 0.2584, 0.2920\} \rangle$	$\langle 1.0380, \{0.1526, 0.1526, 0.1861\} \rangle$
A_{t6}	$\langle 0.9630, \{0.1435, 0.1435, 0.1669\} \rangle$	$\langle 1.9980, \{0.2814, 0.3183, 0.3187\} \rangle$	$\langle 0.6920, \{0.1180, 0.1176, 0.1239\} \rangle$
A_{t7}	$\langle 0, \{0.0055, 0.0055, 0.0064\} \rangle$	$\langle 0.6660, \{0.1149, 0.1019, 0.1189\} \rangle$	$\langle 1.7300, \{0.2218, 0.2218, 0.2623\} \rangle$

(iv). The HF7S “concordance set” is computed in Table 13.

Table 13: Tabular representation of HF7S concordance set

v	1	2	3	4
Y_{1v}	--	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2\} \rangle$	$\langle \{1\}, \{1\} \rangle$
Y_{2v}	$\langle \{3\}, \{3\} \rangle$	--	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{1\}, \{1\} \rangle$
Y_{3v}	$\langle \{1, 3\}, \{3\} \rangle$	$\langle \{1\}, \{1\} \rangle$	--	$\langle \{1\}, \{1\} \rangle$
Y_{4v}	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$	--
Y_{5v}	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{1, 2\}, \{1, 2, 3\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{1\}, \{1\} \rangle$
Y_{6v}	$\langle \{2\}, \{2, 3\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{2\}, \{2\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$
Y_{7v}	$\langle \{3\}, \{3\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{3\}, \{1\} \rangle$
v	5	6	7	--
Y_{1v}	$\langle \{1, 3\}, \{1\} \rangle$	$\langle \{1, 3\}, \{1\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	--
Y_{2v}	$\langle \{3\}, \{1\} \rangle$	$\langle \{3\}, \{3\} \rangle$	$\langle \{1\}, \{1\} \rangle$	--
Y_{3v}	$\langle \{1, 3\}, \{1\} \rangle$	$\langle \{1, 3\}, \{1, 3\} \rangle$	$\langle \{1\}, \{1\} \rangle$	--
Y_{4v}	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{2, 3\}, \{3\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$	--
Y_{5v}	--	$\langle \{1, 3\}, \{1, 3\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	--
Y_{6v}	$\langle \{2\}, \{2\} \rangle$	--	$\langle \{1, 2\}, \{1, 2\} \rangle$	--
Y_{7v}	$\langle \{3\}, \{3\} \rangle$	$\langle \{3\}, \{3\} \rangle$	--	--

(v). The HF7S “concordance matrix” is computed and represented by Y .

$$Y = \begin{pmatrix} \begin{array}{cccc} 1 & 2 & 3 & 4 \\ -- & \langle 0.6540, 0.6540 \rangle & \langle 1.0000, 0.6540 \rangle & \langle 0.3210, 0.3210 \rangle \\ \langle 0.3460, 0.3460 \rangle & -- & \langle 0.6790, 0.6790 \rangle & \langle 0.3210, 0.0000 \rangle \\ \langle 0.6670, 0.3460 \rangle & \langle 0.3210, 0.3210 \rangle & -- & \langle 0.3210, 0.3210 \rangle \\ \langle 0.6790, 0.6790 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 0.6790, 0.6790 \rangle & -- \\ \langle 0.6790, 0.6790 \rangle & \langle 0.6540, 1.0000 \rangle & \langle 0.6790, 0.6790 \rangle & \langle 0.3210, 0.3210 \rangle \\ \langle 0.3330, 0.6790 \rangle & \langle 0.6540, 0.6540 \rangle & \langle 0.3330, 0.3330 \rangle & \langle 0.6540, 0.6540 \rangle \\ \langle 0.3460, 0.3460 \rangle & \langle 0.6790, 0.6790 \rangle & \langle 0.6790, 0.6790 \rangle & \langle 0.3460, 0.0000 \rangle \end{array} \\ \hline \begin{array}{cccc} 5 & 6 & 7 & -- \\ \langle 0.6670, 0.3210 \rangle & \langle 0.6670, 0.3210 \rangle & \langle 0.6540, 0.6540 \rangle & -- \\ \langle 0.3460, 0.0000 \rangle & \langle 0.3460, 0.3460 \rangle & \langle 0.3210, 0.3210 \rangle & -- \\ \langle 0.6670, 0.3210 \rangle & \langle 0.6670, 0.6670 \rangle & \langle 0.3210, 0.3210 \rangle & -- \\ \langle 0.6790, 0.6790 \rangle & \langle 0.6790, 0.3460 \rangle & \langle 1.0000, 1.0000 \rangle & -- \\ -- & \langle 0.6670, 0.2526 \rangle & \langle 0.6540, 0.6540 \rangle & -- \\ \langle 0.3330, 0.3330 \rangle & -- & \langle 0.6540, 0.6540 \rangle & -- \\ \langle 0.3460, 0.3460 \rangle & \langle 0.3460, 0.3460 \rangle & -- & -- \end{array} \end{pmatrix}.$$

(vi). The HF7S “discordance set” is computed in Table 14.

Table 14: Tabular representation of HF7S discordance set

v	1	2	3	4
Z_{1v}	--	$\langle \{3\}, \{3\} \rangle$	$\langle \{1, 3\}, \{3\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$
Z_{2v}	$\langle \{1, 2\}, \{1, 2\} \rangle$	--	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$
Z_{3v}	$\langle \{1, 2, 3\}, \{1, 2\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$	--	$\langle \{1\}, \{2, 3\} \rangle$
Z_{4v}	$\langle \{1\}, \{1\} \rangle$	$\langle \{1\}, \{\} \rangle$	$\langle \{1\}, \{1\} \rangle$	--
Z_{5v}	$\langle \{1, 3\}, \{1\} \rangle$	$\langle \{3\}, \{\} \rangle$	$\langle \{1, 3\}, \{1\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$
Z_{6v}	$\langle \{1, 3\}, \{1\} \rangle$	$\langle \{3\}, \{3\} \rangle$	$\langle \{1, 3\}, \{1, 3\} \rangle$	$\langle \{2, 3\}, \{3\} \rangle$
Z_{7v}	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1\}, \{1\} \rangle$	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 2, 3\}, \{1, 2, 3\} \rangle$
v	5	6	7	--
Z_{1v}	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{2\}, \{2, 3\} \rangle$	$\langle \{3\}, \{3\} \rangle$	--
Z_{2v}	$\langle \{1, 2\}, \{1, 2, 3\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1\}, \{2, 3\} \rangle$	--
Z_{3v}	$\langle \{2, 3\}, \{2, 3\} \rangle$	$\langle \{2\}, \{2\} \rangle$	$\langle \{2, 3\}, \{2, 3\} \rangle$	--
Z_{4v}	$\langle \{1\}, \{1\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{3\}, \{\} \rangle$	--
Z_{5v}	--	$\langle \{2\}, \{2\} \rangle$	$\langle \{3\}, \{3\} \rangle$	--
Z_{6v}	$\langle \{1, 3\}, \{1, 3\} \rangle$	--	$\langle \{3\}, \{3\} \rangle$	--
Z_{7v}	$\langle \{1, 2\}, \{1, 2\} \rangle$	$\langle \{1, 2\}, \{1, 2\} \rangle$	--	--

(vii). The HF7S “discordance matrix” is computed and represented by Z .

$$Z = \begin{pmatrix} \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline -- & \langle 0.3463, 0.1933 \rangle & \langle 0.0000, 0.1817 \rangle & \langle 0.7186, 1.0000 \rangle \\ \langle 1.0000, 1.0000 \rangle & -- & \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle \\ \langle 1.0000, 1.0000 \rangle & \langle 0.3593, 0.4892 \rangle & -- & \langle 1.0000, 1.0000 \rangle \\ \langle 1.0000, 0.8795 \rangle & \langle 0.0000, 0.0000 \rangle & \langle 0.4820, 0.3643 \rangle & -- \\ \langle 0.9640, 0.6467 \rangle & \langle 0.2598, 0.0000 \rangle & \langle 0.1928, 0.1675 \rangle & \langle 1.0000, 1.0000 \rangle \\ \langle 0.9640, 0.6336 \rangle & \langle 0.4156, 0.1517 \rangle & \langle 0.3213, 0.2309 \rangle & \langle 1.0000, 1.0000 \rangle \\ \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 0.8024 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle \\ \hline 5 & 6 & 7 & -- \\ \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 0.4312, 0.4863 \rangle & -- \\ \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 0.5389, 1.0000 \rangle & -- \\ \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle & \langle 0.4312, 0.4936 \rangle & -- \\ \langle 0.92770.7669 \rangle & \langle 0.3092, 0.3111 \rangle & \langle 0.0000, 0.0000 \rangle & -- \\ -- & \langle 0.9624, 0.9270 \rangle & \langle 0.5389, 0.4165 \rangle & -- \\ \langle 1.0000, 1.0000 \rangle & -- & \langle 0.7793, 0.5968 \rangle & -- \\ \langle 1.0000, 1.0000 \rangle & \langle 1.0000, 1.0000 \rangle & -- & -- \end{array} \end{pmatrix}.$$

(viii). The HF7S “concordance level” $\langle \bar{g}_y, \bar{y} \rangle = \langle 0.5562, 0.5000 \rangle$, and HF7S “discordance level” $\langle \bar{g}_z, \bar{z} \rangle = \langle 0.7367, 0.7081 \rangle$ are computed.

(ix). The HF7S concordance “dominance matrix” is computed and represented by R .

$$R = \begin{pmatrix} \begin{array}{ccccccc} -- & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 1, 0 \rangle & \langle 1, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & -- & \langle 1, 1 \rangle & \langle 0, 0 \rangle \\ \langle 1, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 0 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle \\ \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & -- & \langle 1, 1 \rangle & \langle 1, 0 \rangle & \langle 1, 1 \rangle \\ \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & -- & \langle 1, 1 \rangle & \langle 1, 1 \rangle \\ \langle 0, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & -- & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & -- \end{array} \end{pmatrix}.$$

(x). A HF7S discordance “dominance matrix” is computed and represented by S .

$$S = \begin{pmatrix} \begin{array}{ccccccc} -- & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 0 \rangle \\ \langle 0, 0 \rangle & \langle 1, 1 \rangle & -- & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle \\ \langle 0, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 1 \rangle \\ \langle 0, 0 \rangle & -- \end{array} \end{pmatrix}.$$

(xi). An aggregated HF7S “dominance matrix” is computed and represented by T .

$$T = \begin{pmatrix} -- & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle \\ \langle 0, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle \\ \langle 0, 0 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 1 \rangle & \langle 1, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 0 \rangle & \langle 1, 1 \rangle \\ \langle 0, 1 \rangle & \langle 1, 1 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 0, 0 \rangle & -- & \langle 0, 1 \rangle \\ \langle 0, 0 \rangle & -- \end{pmatrix}.$$

(xii). Lastly, to determine the dominant position of airlines, owing to the star rated values of aggregated HF7S “dominance matrix”, a multiple edges directed graph for each pair of airlines is presented in Figure 2.

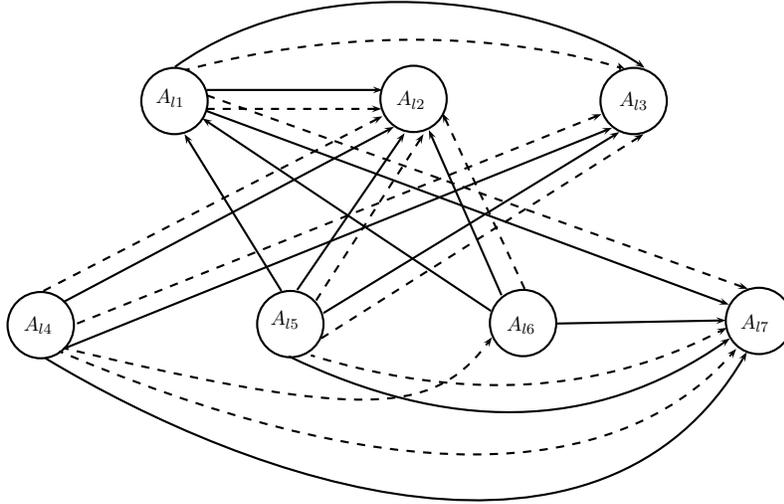


Figure 2: Outranking relation of airlines

For ranking the positions, different airlines are represented by circles, the grading evaluations are represented by dotted directed edges, and the evaluations under hesitant environment are represented by bold directed edges.

1. A_{l1} is preferred over A_{l2} , A_{l3} and d_{s7} .
2. A_{l2} is incomparable to others, because there is no directed edge from A_{l2} to $A_{lj}'s$, where $j = 1, 2, \dots, 7$.
3. Similarly, A_{l3} is incomparable to others.
4. A_{l4} is preferred over A_{l2} , A_{l3} , A_{l6} and d_{s7} .
5. Similarly, A_{l5} is preferred over A_{l1} , A_{l2} , A_{l3} and d_{s7} .

6. Similarly, A_{l6} is preferred over A_{l1} , A_{l2} and d_{s7} .
7. Finally, A_{l7} is incomparable to others.

Hence, A_{l4} and A_{l5} are considered as the most dominated airlines due to the highest ranking. Final, decision depends upon the decision-makers opinion either they considered the dotted directed edges or bold directed edges.

5 Comparative Analysis

This section presents a comparison of the HFNS-ELECTRE-I method proposed in this article with the HFNS-TOPSIS method [4]. We present both advantages and disadvantages.

1. HFNS-ELECTRE-I method (proposed):

- **Advantages**

- (a) The originality of HFNS-ELECTRE-I is based on the research of outranking relations, and it analyzes outranking relations among alternatives using concordance and discordance indexes. The satisfaction and dissatisfaction with which a decision maker prefers one choice over another can be measured using concordance and discordance indexes.
- (b) The HFNS-ELECTRE-I method contributes in the hesitant situation under graded evaluation, and points out the most dominant object from the reference set with the elimination of irrelevant choices (as calculated in subsection 4.1 in which d_{s2} is considered as the most dominated drama serial due to its highest and top star ranking). The proposed method also identifies a limited subset of desirable objects, which is beneficial in situations when one or more than one dominant objects are required with eliminated choices for others (as calculated in subsection 4.2, where both A_{l4} and A_{l5} are declared as the top airlines due to their highest ranking.)
- (c) The HFNS-ELECTRE-I method motivates to compute the decision-maker assessments in an adjustable and formative manner. Also, it improves the robustness and accuracy of decisions that rely on grades or star ratings. It has been demonstrated that this strategy may be used effectively in the selection process in real-world scenarios, allowing decision makers to rank the alternatives.

- **Disadvantages**

- (a) The application of the proposed approach is not intuitive, due to massive and dual calculations.
- (b) It does not necessarily identify a single dominant object that stands out among the whole set.
- (c) It does not provide an efficient ranking of objects.

2. HFNS-TOPSIS method [4]:

- **Advantages**

- (a) The HFNS-TOPSIS method selects the best object from a given set with final ranking.
 - (b) Its calculations are easy to tackle, even under massive collections of data.
 - (c) The HFNS-TOPSIS method may be rank reversal “that preference order changes, when a new object is added and an existing object is removed from the problem”.
- **Disadvantages**
 - (a) HFNS-TOPSIS method is not useful when one has to eliminate the choices with no scope.
 - (b) It is subject to the total reversal paradox, that its preference order relation for the alternatives has totally inverted behavior.
 - (c) Its calculations are insufficient to eliminate the unnecessary choices.

For the real comparison of HFNS-TOPSIS and HFNS-ELECTRE-I methods, we consider the Example 3.1 from [4]. We present and finalize the whole outline of HFNS-TOPSIS method in Table 15.

Table 15: Comparison of candidates by HFNS-TOPSIS method

Candidates	$\langle d_{gj}^+, d_{oj}^+ \rangle$	$\langle d_{gj}^-, d_{oj}^- \rangle$	C_j	Ranking
o_1	$\langle 0.3, 0.08 \rangle$	$\langle 1.5, 0.56 \rangle$	$\langle 0.9, 0.88 \rangle$	1
o_2	$\langle 1.6, 0.53 \rangle$	$\langle 0.4, 0.10 \rangle$	$\langle 0.2, 0.16 \rangle$	4
o_3	$\langle 1.3, 0.43 \rangle$	$\langle 0.7, 0.21 \rangle$	$\langle 0.3, 0.33 \rangle$	3
o_4	$\langle 0.7, 0.29 \rangle$	$\langle 1.2, 0.36 \rangle$	$\langle 0.6, 0.55 \rangle$	2

With the final rating of all the candidates, the selected candidate (lecturer) is determined from Table 15. Next, We present and finalize the whole outline of HFNS-ELECTRE-I method in Table 16.

Table 16: Comparison of candidates by HFNS-ELECTRE-I method

Comparison of candidates	$\langle g_{y_{uv}}, y_{uv} \rangle$	$\langle g_{z_{uv}}, z_{uv} \rangle$	$\langle g_{r_{uv}}, r_{uv} \rangle$	$\langle g_{s_{uv}}, s_{uv} \rangle$	$\langle g_{t_{uv}}, t_{uv} \rangle$	Ranking
(o_1, o_2)	$\langle 1.00, 1.0000 \rangle$	$\langle 0.00, 0.0000 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$o_1 \Rightarrow o_2$
(o_1, o_3)	$\langle 1.00, 1.0000 \rangle$	$\langle 0.00, 0.0000 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$o_1 \Rightarrow o_3$
(o_1, o_4)	$\langle 0.72, 0.7200 \rangle$	$\langle 0.76, 0.4829 \rangle$	$\langle 1, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$o_1 \rightarrow o_4$
(o_2, o_1)	$\langle 0.00, 0.0000 \rangle$	$\langle 1.00, 1.0000 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_2, o_3)	$\langle 0.35, 0.3500 \rangle$	$\langle 1.00, 1.0000 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_2, o_4)	$\langle 0.72, 0.3700 \rangle$	$\langle 1.00, 1.0000 \rangle$	$\langle 1, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_3, o_1)	$\langle 0.37, 0.0000 \rangle$	$\langle 1.00, 1.0000 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_3, o_2)	$\langle 0.65, 0.6500 \rangle$	$\langle 0.95, 0.8279 \rangle$	$\langle 1, 1 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_3, o_4)	$\langle 0.37, 0.3700 \rangle$	$\langle 1.00, 1.0000 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_4, o_1)	$\langle 0.28, 0.2800 \rangle$	$\langle 1.00, 1.0000 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	Incomparable
(o_4, o_2)	$\langle 1.00, 0.6300 \rangle$	$\langle 0.00, 0.0843 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$o_4 \Rightarrow o_2$
(o_4, o_3)	$\langle 0.63, 0.6300 \rangle$	$\langle 0.66, 0.7565 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 0 \rangle$	$\langle 1, 0 \rangle$	$o_4 \rightarrow o_2$

From Table 16, the most dominated and selected candidate (lecturer) is identified with elimination of others.

Table 17: Comparative Analysis

Methods	Ranking of candidate	Dominant candidate
HFNS-ELECTRE-I (proposed)	$o_1 > o_4 > o_3 = o_2$	o_1
HFNS-TOPSIS [4]	$o_1 > o_4 > o_3 > o_2$	o_1

As a result, we conclude that the results of our suggested approach and the HFNS-TOPSIS method [4] are identical, as shown in Table 17. In both techniques, the most dominated and best candidate (lecturer) are the same, but the HFNS-TOPSIS method offers the outranking relation of candidates, whilst the HFNS-ELECTRE-I method provides a subset of the most dominated candidates (lecturer) and excludes the irrelevant candidates.

6 Conclusion

ELECTRE has been addressed as one of the most well-known MADM technique, depending on the performance model, which has promoted a new choice of decision-making adopted to engage the context, in which the “objects of a reference set” are inadequate to analyze by decision-makers. In this research article, we have [contributed to enlarge its scope with a new MADM approach known as HFNS-ELECTRE-I. It has adapted the methodology so as to include data from sources](#) possessing star rating systems with a hesitant structure. Contrary to the classic approaches of ELECTRE, confined information is not employed in the [inspection](#) of the alternatives. The proposed method is [better suited for](#) structured knowledge, having grades or star ratings [in a](#) hesitant setting. When it comes to eliminating options and dealing with structures with several arguments, the HFNS-ELECTRE-I technique is [recommended in order](#) to produce more persuasive and consistent findings. The proposed approach is [reported](#) as a more systematic method than previously existing methods, in which alternatives that are [affected](#) by others are eliminated based on specified degrees and star ratings. Finally, we have [tested](#) our approach on actually existing scenarios and presented a comparative analysis with [the HFNS-TOPSIS method. In future research studies, different extended versions of ELECTRE method including ELECTRE-III, IV could be used for the selection problems of real life scenarios. Other MADM methods such as VIKOR, AHP, PROMETHEE could be extended to encompass the HFNS framework.](#)

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