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Multi-criteria optimization scheduling of surgical units

A case study at AOU-Policlinico Umberto I

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Abstract Nowadays, Operating Rooms (ORs) scheduling and planning represent one of the most challenging aspects in healthcare systems management. In particular, one of the main concern is doubtless the under-use of the surgery rooms, that one can tackle by modelling different management decision aspects. The number of ORs available in the operating theatre and their daily opening time, the ideal OR Utilization Rate (*UR*) that the hospital aims to fulfil, the possibility to transfer patients between different surgical units (SUs) are just some of them. Taking into account these management and economic factors, we develop a multi-criteria integer linear optimization model for the advance scheduling of patients for the *Azienda Ospedaliera - Universitaria Policlinico Umberto I* of Rome, among the largest public hospitals in Europe for by total area occupied. The problem is formulated according to an open scheduling strategy that, for each surgical case placed on a waiting list, determines the date, time, and operating room resources needed. Extensive tests were carried out on real data collected during twelve weeks in two SUs, each of them consisting of two ORs. The aim of the study is to evaluate the impact of different policies of opening times on the hospital Key Performance Indicators (KPIs) of interest. The results show a relevant improvement of the ORs usage of the case study.

Keywords Operating Room Planning · Mastery Surgery Schedule · Multi-criteria Optimization · Health Care Management

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1 Introduction

Especially in the large public hospitals, the management of clinical processes is the weakness of the production chain that is often based on the “maximum production” concept applied to the single unit instead of focusing on the optimization of the entire flow along the chain. To obtain an overall improvement on the management of the hospital core activities, it is important to merge the Operations Management approaches with Operation Research models Agnetis et al. (2019).
Surgical units (SUs) are the beating heart of a healthcare structure. Providing around the 60% of all hospital admissions (Fügener et al. (2017), Gupta (2007)), Operating Room (OR) scheduling and planning gained in the last decades a crucial role in the management of the whole structure activities. As stated in the literature Cardoen et al. (2009), Guerriero and Guido (2011), Gür and Eren (2018), Samudra et al. (2016), Zhu et al. (2019), and in the recent and comprehensive review of Rahimi and Gandomi (2020), three levels of action can be defined in the OR scheduling process: *strategic*, *tactical* and *operational*. The strategic level covers all those decisions demanding a long-term planning horizon (from several months up to one year), such as capacity planning, allocation or case-mix problem (CMP). By forecasting the hospital demand, the aim is to efficiently distribute the available resources and the budget among the surgical specialties (see Fügener et al. (2017), Marques and Captivo (2015), Koppka et al. (2018)). The tactical level, or *Master Surgical Scheduling* problem (MSSP), is a simple cyclic and repetitive schedule used to assign OR times to groups of surgeons according to their specific needs. It is a medium-term decision level and it usually controls an interval of three or four months. Doubtless, this stage plays a key role in the decision-making process by ensuring the operational guidelines for short-term decisions, and a wide range of approaches have been tested in the literature (Aringhieri et al. (2015), Guido and Conforti (2016), Mannino et al. (2012), Penn et al. (2017)). According to the MSSP output, the operational level solves the Surgical Process Scheduling Problem, that can be further divided in the sub-problems of *advance* (*OR-day*) and *allocation* (*OR-day-time*) scheduling. In this final phase, the resources and the patients in the waiting list are assigned to specific surgical slots and thus associated with specific ORs, days and starting times (Agnetis et al. (2013), Day et al. (2012)).
Furthermore, the OR scheduling problems generally adopt one of the following scheduling strategies: block strategy, open strategy or modified block strategy. In the first, OR capacity is divided into blocks and then assigned exclusively to a specific surgical group. This means that surgeries can only be allocated to blocks associated with surgeons of the same specialty. Instead, an open strategy guarantees a more flexible solution in which no block assignment exists. It allows more different surgical specialties to be allocated in the same OR session without any priority, and following a scheduling principle (e.g. the ”first come first served”). Intuitively, both strategies have their advantages and disadvantages. In the case of block scheduling, if an OR block is assigned to one surgeon group, others cannot perform any surgery in it even if the slot is free. On the contrary, due to its flexible arrangements, the open scheduling leads most of the times to long waiting periods. To overcome these drawbacks, in the last decades various modified block scheduling strategies that combine the strengths of the previous two strategies have been provided.
As concerning the patients, the literature classifies patients in two major classes: elective and non-elective, and inpatients or outpatients. The first classification represents respectively those patients whose surgeries can be planned in advance and arrivals are unexpected, hence need to be performed urgently (Marques and Captivo (2015), Riet and Demeulemeester (2014)). Instead, inpatients have to stay overnight in the clinic, whereas the resignations of outpatients are signed on the same arrival day.
From exact approaches to heuristics and meta-heuristics, a variety of methods have been applied to ORs scheduling and planning, according to the size and complexity of the problem under analysis (Cardoen et al. (2010), Freeman et al. (2015), Rahimi and Gandomi (2020), Samudra et al. (2016), Zhu et al. (2019)).

However, the operating theatre has become highly dynamic with complex decision-making processes which involve many participants, such as OR managers, head doctors, surgeons and patients. Due to conflicting priorities and preferences, it is hence very hard to satisfy all stakeholders’ interests and propose a unique simple method capable to improve all performance measures. To find a compromise and take care of all these concerns simultaneously, a considerable number of multi-criteria approaches have been proposed in the last years (see Cardoen et al. (2010), Gul et al. (2011), Meskens et al. (2012), Rachuba (2017)).
It is also important to note that when sharing the same ORs and surgery facilities among more than one SU, the need for planning overnights staying of inpatients strongly interacts with the OR schedule. In this paper, we address an open schedule for elective surgeries, including both inpatients or outpatients, of a set of ORs located in different SUs. The model proposed takes into

account multiple performance criteria related to both an OR management improvement and the preservation of the service quality offered to the patients.

In particular, we develop a multi-criteria integer linear optimization model for elective patients scheduling at *Azienda Ospedaliera - Universitaria Policlinico Umberto I of Rome* (later simply denoted as *AOU-Policlinico*). It is worth noticing that the aforementioned is the largest European hospital considering the total area and, with a total of 1235 beds (at 31/12/2018), the third in Italy by capacity (AOU-Policlinico (2017, 2018)) with more than 30 operating rooms. Therefore, it represents a great opportunity and is more than qualified for effective application of Operations Management tools. Acting at the operational levels, the proposed model provides a feasible weekly allocation scheduling. Following an open scheduling strategy, for each surgical case placed on a waiting list, we determine a scheduled date, a time and the OR resources needed. Our aim is to integrate this approach into a wider scheme of functional reorganization of the whole surgical area of the hospital.
Starting from the data provided by the hospital, we performed a preliminary analysis to identify the most crucial items to intervene on. We examined the clinical processes in-depth to redefine the relationships between different medical units, optimize the use of clinical resources and increase consequently the whole surgical area efficiency. The study led us to consider two SUs geographically close to each other, consisting of two ORs each, and to formulate a model encompassing the core activities and objectives. We want to obtain a surgeries schedule that allows to use the OR at an ideal utilization rate (UR) defined by the policy of the hospital and balance the distribution of the ORs daily opening time. Furthermore, the movements of patients among different wards is considered as a drawback by the hospital and therefore also falls within the optimization criteria too.
Far from being an innovative theory-oriented article, our main goal is to develop a scalable linear model able to solve real-world problems that have to be faced when dealing with a large facility. We show that the use of optimization provides a great improvement in the management of the ORs and can lead to several changes in the strategic policy of the AOU-Policlinico.

The paper is organized as follows. In section 2 we describe the problem developed thanks to a larger collaboration between the *Department of Computer, Control and Management Engineering "Antonio Ruberti"* and the AOU-Policlinico. In Section 3 we formulate

Fig. 1 Map of the main enclosure of AOU-Policlinico (from AOU-Policlinico (2018)).

the problem as a Multi-criteria Integer Linear Programming problem (MILP). The real case problem of AOU-Policlinico is then presented in Section 4.1, where data and basics stats on the current use of the AOU-Policlinico SUs and ORs are analyzed. Finally, in Section 5 we show the results obtained using the optimization model on different scenarios. Section 6 summarises the research outcomes and gives some ideas for future works.

2 The Case Study of AOU-Policlinico of Rome

The AOU-Policlinico, built at the end of the 20th century, is the hospital that covers the largest geographic area in Europe. It consists of "pavilions" spread over 300 000 square meters and distributed in 54 buildings; 46 of them are located in a large enclosure (see Picture 1) and 8 outside of it (AOU-Policlinico (2018)). It has numerous (more than 30) operating rooms (ORs) located in Surgery Units (SUs) that can accommodate high specialized or generic clinical surgeries related to various diseases.

2.1 Surgical Units and Operating Rooms

The SUs are located into building, which cover different clinical specialties for both inpatients and outpatients and have their own wards. We assume that the OR dedicated to general surgeries are interchangeable and there is no preference between them. For both the mathematical model and case study, we consider schedules of only clinical SUs performing general surgeries since they are the only ones that allows for improvement of the patient waiting list management. Indeed, specialized SUs usually are dedicated to a small group of surgeons and highly skilled teams and are associated with wards having a limited number of beds. Since crew tournaments are not affecting and influencing the outlined hospital goals in determining an optimal schedule, we do not include this aspect and any surgeon preferences in the

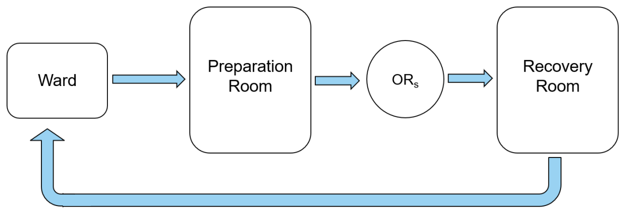

Fig. 2 Patient's workflow.

model.
Each OR is instead characterized by the weekly opening days (usually 5 days per weeks from Monday to Friday) and by the opening time per day (usually starting at 8:00 am). We assume that the opening time does not vary depending on the day of the week and it is established at the strategic level for each room. Opening times value considered at AOU-Policlinico are reported into details in Table 3 of Section 4, where different possible scenario are analysed.

2.2 Patients workflow management

Patients come both from clinical wards (elective), with different availability of beds for inpatients, and from Emergency Department. A recently implemented planning policy at AOU-Policlino provides a fixed prior allocation of SU beds dedicated to elective patients (AOU-Policlinico (2018)). Thus, we can assume that the schedule is needed only for this class of patients.
We take into account the possibility of transferring patients from one SU to the other to perform the surgery in a specific OR. However, whatever the OR where the intervention is performed, the pre and post-operative hospitalization of the patients remain in the ward (hence in the SU) where they have been admitted. A schematic patient's workflow is reported in Figure 2. The patient is transferred from the clinic department to the SU. Here, he first enters in the preoperative *preparation room*, where the preliminary operations for the surgery are made. In the meantime, the OR is set up for the specific surgery. Then, the patient enters the OR where a team of surgeons performs the scheduled surgery. Once the surgery is concluded, the patient is transferred to the *recovery room* for observation until his full awakening, while the OR needs to be properly cleaned and sanitized before a new surgery can start. Different surgeries may require longer or shorter overnights staying. We introduce the week availability of beds in each SU as a constraint into the model. However, this information was not available on the historical data that we could access and therefore it was not implemented in the case study, which is the main reason why we used an heuristic approach to account for this aspect, as described in Section 4. The formulated model also takes into account weekly and daily precedence constraints for surgeries that present special needs or a mandatory day of scheduling.

2.3 Surgery times

Assume that the following times, obtained either from historical data or from a physician prediction, are available for each type of surgery.

- *Setup time*: the time needed to set up the OR for the surgery;
- *Surgical time*: the duration time of the surgery; this time component, considered as the real operative time (Davila (2013)), suffers from a great variability deriving from several causes, such as the surgery complexity and specialty, the patient conditions and response and possible complications that may arise during the surgery;
- *Cleaning time*: the time for cleaning and sanitizing the OR.
- *Dismiss time*: the time spent by the patient in the *recovery room*;

Note that, for each surgery i, the overall time of use of an OR can be obtained by summing up the Setup, Surgical and Cleaning times. We will refer to this sum as *Occupation time*. Table 1 summarizes all time component presented above and specifies how their values have been set in the case study, in agreement with the medical staff. Both historical operations data and doctor's surgery times estimation have been used to infer the values. More details can bee found in Section 4.1. The GANTT diagram shown in Figure 3 describes the surgical phases of a generic operation with the four times above set to 20, 60, 30, 20 respectively. Note that the time overlapping between the room cleaning and the patient awaking phase do not generate a system incompatibility since the two procedures are carried out in different rooms.

2.4 Hospital goals

When dealing with OR scheduling and planning, most of the performance metrics concern under and over-utilization of the resources. Moreover, among the multi-criteria models, almost two-thirds of the mathematical programs show at least a utilization related metric (see the review Cardoen et al. (2010)). AOU-Policlinico

Table 1 Surgery times: definitions and type of values.

Time Name	Description	Assumptions
Setup time	the room is prepared for a specific surgery	constant
Surgical time	a team of surgeons performs the surgery on the patient	obtained from historical data and/or doctors' experience
Cleaning time	the OR is properly cleaned and sanitized post-surgery	constant
Dismiss time	the time patient remains in the recovery room	constant
Occupation time	Overall OR occupation time	sum of the *Setup*, *Surgical* and *Cleaning time*

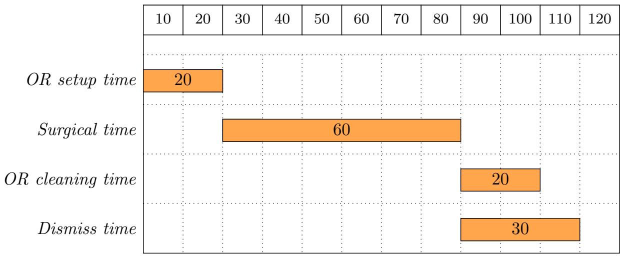

Fig. 3 GANTT representation of the time components of a generic surgery on a time period of two hours (120 minutes).

shares this optimization goal.
The policy of the AOU-Policlinico is to set a target utilization rate (*UR*) for each OR. A portion of the OR time capacity is hence set aside to manage either possible emergency surgeries on non-elective cases or to face uncertainties that may occur during the working day and that may cause scheduled surgeries to be extended beyond their scheduled duration. This approach is justified since emergency operations seem to be performed more efficiently on elective ORs instead of predefined rooms (Wullink et al. (2008), Hans and Vanberkel (2012)).
After a lengthy discussion with the governance of AOU-Policlinico, we identified as one of the aim of the project the definition of ORs optimized workloads that are as close as possible to the ideal *UR*, while avoiding occasional ORs daily closures. Indeed, the difference in the cost for opening an OR for just a few days or for the full week is negligible, so that when an OR works for a day the policy of AOU-Policlinico is to make it working all the days of the week. On the contrary, the hospital is interested in checking if the closure of an OR is possible for all the week.
In order to gain more flexibility in the ORs use, AOU-Policlinico agreed to consider the transfers of patients in charge of one SU to ORs located in a different SU. However, these transfers between SUs located in separate buildings require an effort both in terms of personnel needed and patients comfort. Thus, we identify as further criterion the minimization of the number of transfers between SUs. Hence, we formulate a multi objective optimization problem, as discussed into details in Section 3.4.

3 Model Definition

In this section, we give a formal definition of the multi-criteria optimization model for the weekly ORs scheduling problem at AOU-Policlinico. The model plans the daily ORs surgeries inserted in the patients weekly waiting list and it has been designed as a mathematical support tool for the planning decisions of the top healthcare administrator and the medical staff of the SUs. In this section, we present in details the formulation of the multi-criteria integer linear program (MILP): parameters and decision variable are introduced respectively in Section 3.1 and 3.2, then Section 3.3 reports the restrictions identified by the doctors due to the resources capacity. The optimization criteria used are reported in Section 3.4. Ultimately, we report the final MILP formulation which uses a weighted scalarization technique to find non-dominated point of the multi objective problem.

Notation. We use lower cases to denotes vectors x and x_i to denote the $i-$th component of vector x. Calligraphic characters, such as $\mathcal{I}$, denote instead the sets.

3.1 Parameters

A list $\mathcal{I}$ of all elective surgery procedures planned for the week is given (*Week Surgery Program*). Each element i of the list $\mathcal{I}$ comes with the number g_i of hospitalization days and a fixed time parameter t_i, that is the *surgery total occupation time*. The latter is defined as the sum of preparation, surgical and cleaning time and it is inferred from the historical registers (for more detail see Table 1).
Among all the surgeries $i \in \mathcal{I}$, some of them can have

different priorities. These priorities can depend on patients care cases (urgency or routine), medical staff needs or hospital wards space requirements. Here we distinguish between:

- *day priority*: surgeries that have to be scheduled as the first surgery of the day because of patients requirements (e.g. child, elder) or other doctors commitments. The day priority surgeries are inserted in a list $\mathcal{I_D} \subset \mathcal{I}$;
- *mandatory day*: operations that, due to some patients or doctors needs, are bind to a specific day of the week. The list of these surgeries is $\mathcal{I_M} \subset \mathcal{I}$ and for each $i \in \mathcal{I_M}$ its mandatory surgery day is denoted by the parameter $\widehat{d_i}$.

Let $\mathcal{C}$ be the set of the available SUs and $\mathcal{S}$ the set of all accessible ORs. $\mathcal{D} = \{1,2,3,4,5\}$ is the set of the days of the week (Monday to Friday) while $\mathcal{P} = \{1, ..., p^{\max}\}$ represents the possible schedule positions of the surgery within a day. One can observe that the effective number of positions available in a day actually depends on the opening time O_s of the OR s and on the duration of the surgeries scheduled in day d, so that we should have $p^{\max}_{sd}$. For sake of simplicity, we assume that the number $p^{\max}$ is constant on all the days of the week and for all the ORs. We overestimated it considering the highest possible number of daily positions as follows:

$$p^{\max} = \left\lfloor \frac{\max\limits_{s \in S} O_s}{\min\limits_{i \in I} t_i} \right\rfloor \in \mathbb{Z} \tag{1}$$

For each day $d \in \mathcal{D}$ of the week the number of beds available for SU $c \in \mathcal{C}$ is given by B^{dc}. This parameter allows us to take into account the occupation of the beds caused by surgeries performed in the previous weeks, requiring a number of overnight stay greater than $|\mathcal{D}|= 5$.
To guarantee the feasibility of the problem we assume that $|\mathcal{I_D}| \leq |\mathcal{D}| \times |\mathcal{S}|$ and, for each day $d \in \mathcal{D}$, that $\sum_{i \in \mathcal{I_M} | \hat{d}_i = d} t_i \leq \sum_{s \in \mathcal{S}} O_s$.
As previously stated, we do not consider in the model the stochastic nature of the surgery times. Following the policy of the AOU-Policlinico, unforeseen events that may cause delays in the schedules are taken into account by considering a limitation on the maximum daily percentage of OR workload. In particular, the total daily occupation time for an OR, given by the sum of all the occupation times of the surgeries performed in that room on that day, must not exceed the percentage workload U of the daily opening time O_s.
Besides, each surgery must be associated with the SU where the patient has been hospitalized. Hence, we introduce the parameter $a_{ic} \in \{0,1\}$, with $i \in \mathcal{I}$ and $c \in \mathcal{C}$, which means that the surgery i "originates" from unit c. We have that:

$$a_{ic} = \begin{cases} 1 & \text{if patient undergoing surgery } i \\ & \text{is hospitalized in SU } c \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Similarly, we introduce the parameter $f_{sc} \in \{0,1\}$, with $s \in \mathcal{S}$ and $c \in \mathcal{C}$ to indicate that the OR s is located in the building of the SU c, namely:

$$f_{sc} = \begin{cases} 1 \text{ if OR } s \text{ is located in SU } c \\ 0 \text{ otherwise.} \end{cases} \tag{3}$$

For sake of clarity, the complete list of the sets and parameters used in the model is reported in Table 2.

Table 2 List of sets and parameters used in the model.

Set	Description
$\mathcal{I}$	Set of all elective surgeries planned for the week
$\mathcal{I_D}$	Subset of surgeries with a day priority
$\mathcal{I_M}$	Subset of surgeries with a mandatory day
$\mathcal{C}$	Set of the SUs
$\mathcal{S}$	Set of the ORs
$\mathcal{D}$	Set of the days of the week
$\mathcal{P}$	Set of the time slots of a surgery day

Parameters	Description
t_i	Surgery total occupation time
g_i	Number of hospitalization days required by the surgery
B^{dc}	Number of beds available for SU c on day d
U	Maximum daily percentage of ORs workload
O_s	Total daily opening time of OR s
a_{ic}	Boolean value representing if the surgery i origins in the unit c
f_{sc}	Boolean value representing if the SU s belong to the unit c
$\widehat{d_i}$	Mandatory day to perform the surgery $i \in I_M$
$p^{\max}$	Maximum number of surgeries per day obtained as in (1)

3.2 Variables

We use positional assignment binary activation variables for the problem formulation. For each surgery i we need to decide where ($s \in \mathcal{S}$) and when ($d \in \mathcal{D}$) it is scheduled, together with its position ($p \in \mathcal{P}$) in the daily schedule. Therefore, we introduce a binary variable called *surgery assignment variable* x_i^{sdp} indicating if the surgery i occurs in the OR $s \in \mathcal{S}$ on day d at position p.

$$x_i^{sdp} = \begin{cases} 1 & \text{if } i \text{ is performed in OR } s \text{ on day } d \\ & \text{as the } p^{th} \text{surgery} \\ 0 & \text{otherwise.} \end{cases}$$

We will refer to the triple (sdp) with the term *slot*, since it identifies exactly when and where the surgery i is performed. We can say that $x_i^{sdp} = 1$ denotes that surgery i is performed in slot (sdp). We further denote by x the vector of dimension $|\mathcal{I}| \times |\mathcal{S}| \times |\mathcal{D}| \times |\mathcal{P}|$ made up of elements x_i^{sdp}.
We also need to consider a further binary *OR weekly*

opening variable z_s stating if the OR s is active during the week, i.e. at least one surgery is performed in s on any day $d \in \mathcal{D}$.

$$z_s = \begin{cases} 1 & \text{if at least one surgery is performed in } s \text{ during the week} \\ 0 & \text{otherwise.} \end{cases}$$

We denote with z the vector of dimension $|\mathcal{S}|$ made up of elements z_s.

3.3 Constraints

In this section we present the constraints considered in the model.

- Surgery assignment: each surgery must be assigned exactly to one slot (sdp).

$$\sum_{s\in\mathcal{S}}\sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}} x_i^{sdp} = 1 \qquad \forall\, i \in \mathcal{I} \tag{4}$$

- Slot assignment: a slot (sdp) can be assigned to at most one surgery.

$$\sum_{i\in\mathcal{I}} x_i^{sdp} \leq 1 \qquad \forall\, s \in \mathcal{S}, p \in \mathcal{P}, d \in \mathcal{D} \tag{5}$$

- Time utilization: For each OR $s \in \mathcal{S}$ on each day $d \in \mathcal{D}$, the daily occupation time must not exceed the maximum percentage of utilization of the OR s when it is working, namely $z_s = 1$.

$$\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} t_i \cdot x_i^{sdp} \leq U \cdot O_s \cdot z_s \qquad \forall\, s \in \mathcal{S}, d \in \mathcal{D} \tag{6}$$

- Position priority: the surgeries $i \in \mathcal{I}_{\mathcal{D}}$ must be scheduled as the first operation of the day in the OR s, namely

$$\sum_{s\in\mathcal{S}}\sum_{d\in\mathcal{D}} x_i^{sd1} = 1 \qquad \forall\, i \in \mathcal{I}_{\mathcal{D}} \ . \tag{7}$$

- Day priority: the surgeries $i \in \mathcal{I}_{\mathcal{M}}$, having a mandatory day priority $\widehat{d}_i$, must be allocated properly.

$$\sum_{s\in\mathcal{S}}\sum_{p\in\mathcal{P}} x_i^{s\widehat{d}_i p} = 1 \qquad \forall\, i \in \mathcal{I}_{\mathcal{M}} \tag{8}$$

- Beds availability: the number of beds used on day d in each SU c must not exceed the total number of beds available on that day. Note that the number of occupied beds is obtained by taking into account also patients who underwent a surgery in the previous days and having a number of post-hospitalization number of days greater than $d - t$. Besides, the patients transferred for the surgery must return to the SU they belong to, which is taken into account in the constraint by means of the parameter a_{ic} defined in (2).

$$\sum_{s\in\mathcal{S}}\sum_{p\in\mathcal{P}}\sum_{t=1}^{d}\sum_{i\in\mathcal{I}:\, g_i\geq d-t+1} a_{ic} x_i^{stp} \leq B^{dc} \qquad \forall\, d \in \mathcal{D}, c \in \mathcal{C} \tag{9}$$

We define the feasible region

$$\mathcal{F} = \Big\{ (x,z) \in \{0,1\}^{|I|\cdot|\mathcal{S}|\cdot|\mathcal{D}|\cdot|\mathcal{P}|} \times \{0,1\}^{|\mathcal{S}|} : (x,z) \text{ satisfy } (4)-(9) \text{ constraints} \Big\} .$$

3.4 Optimization criteria

According to the discussion on Section 2, three main optimization criteria are identified as follows.

$F1$: minimize the number of weekly open ORs. Thus, we aim to minimize the function

$$F_1(x,z) = \sum_{s\in\mathcal{S}} z_s \ .$$

$F2$: minimize the number of patients transferred between SUs. We consider the "cost" of transferring patients among SUs independent from the origin/destination. Aiming to minimize the overall number of transfers among SUs, we need to count the number of patients i hospitalized in c and undergoing the surgery in a OR s that is not located in the SU c. Thus, we minimize the following function

$$F_2(x,z) = \sum_{i\in\mathcal{I}}\sum_{c\in\mathcal{C}}\sum_{s\in\mathcal{S}} (1 - f_{sc}) \cdot a_{ic} \left(\sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}} x_i^{sdp} \right)$$

where a_{ic}, f_{sc} are defined in (2) and (3).

$F3$: minimize the maximum deviation between the actual workload and the AOU-Policlinico ideal one. The latter is obtained as the product of the maximum daily percentage workload U and the ORs opening time O_s. Therefore, we define the third objective as

$$F_3(x,z) = \max_{s\in\mathcal{S}} \left(|\mathcal{D}| \cdot U \cdot O_s \cdot z_s - \sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} t_i \cdot x_i^{sdp} \right) \ .$$

This criteria can be linearized using standard arguments by means of the the introduction of an auxiliary variable.

We observe that the three criteria might be at odds with each other. Thus, the problem is a truly multi objective integer optimization problem. We look for a Pareto optimal solution, namely a feasible point $(\widetilde{x}, \widetilde{z})$

such that there exists no other feasible point (x, z) satisfying $F_i(x, z) \le F_i(\tilde{x}, \tilde{z})$ for $i = 1, 2, 3$ and $F(x, z) \ne F(\tilde{x}, \tilde{z})$, being F the vector made up of the three components F_1, F_2, F_3. We define the *ideal* objective vector $F^{id} \in \mathbf{R}^3$ component-wise as

$$F_i^{id} = \min_{(x,z)\in\mathcal{F}} F_i(x, z) \qquad i = 1, 2, 3.$$

As usual in multi objective optimization, we consider the vector F^{id} as a reference vector and the feasible solutions $(\tilde{x}, \tilde{z})_i^\star$ $i = 1, 2, 3$ are Pareto optimal.
In order to tackle a multi-Criteria problem we adopt a weighted-sum scalarization technique (Yang (2014)). We hence consider a single objective integer optimization problems with the following objective function:

$$\min_{(x,z)\in\mathcal{F}} w_1 F_1(x, z) + w_2 F_2(x, z) + w_3 F_3(x, z), \tag{10}$$

where $w_i \ge 0$, for $i = 1, 2, 3$ are finite weights. It is well known (see Proposition 3.9 in Ehrgott (2005)) that if $w_i > 0$ for all i, then each optimal solution of Problem (10) is a Pareto solution for the MILP. In principle, one is interested in finding all the Pareto optimal solutions but this is known to be an hard optimization problem even in the case of two objectives (see e.g. De Santis et al. (2020) and references therein). Thus, we are interested in selecting one Pareto solution by fixing the weights w_i to suitable positive values. The choice of the weights w_i has been decided according to the policy of the AOU-Policlinico as reported in Section 5.

3.5 MILP formulation

We report in this section the weight formulation of (10) as a MILP problem. At first step, we introduce an auxiliary variable $u \in \mathbb{Z}_+$ to linearize the criterion $F3$. Note that, without loss of generality, we consider the set of all positive integers as domain for the new variable. Indeed, it is sufficient to consider integer value for the occupational time parameter t_i. Following standard arguments, we add the group of constraints presented below:

$$|\mathcal{D}| \cdot U \cdot O_s \cdot z_s - \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}} \sum_{p\in\mathcal{P}} t_i \cdot x_i^{sdp} \le u \qquad \forall\, s \in \mathcal{S} \tag{11}$$

and substitute $F3$ as u in (10).
Thus, the integer problem (10) can be written as the following MILP problem

$$\begin{aligned} \min_{(x,z,u)} \ & w_1 F_1(x, z) + w_2 F_2(x, z) + w_3 u, \\ s.t. \quad & |\mathcal{D}| \cdot U \cdot O_s \cdot z_s \quad + \\ & - \textstyle\sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}} \sum_{p\in\mathcal{P}} t_i \cdot x_i^{sdp} \le u \quad \forall\, s \in \mathcal{S} \\ & (x, z) \in \mathcal{F} \qquad u \in \mathbb{Z}_+ \quad . \end{aligned}$$

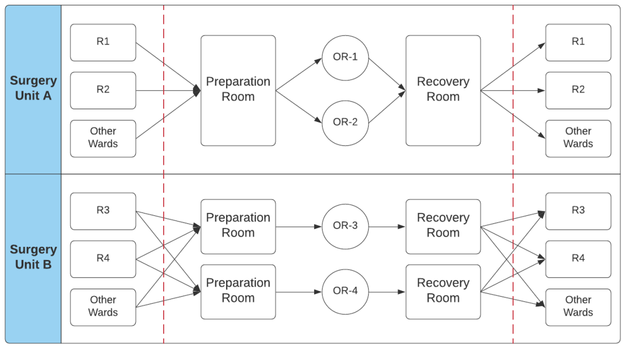

Fig. 4 Organization of the SU-a and SU-B in the case study and the patient flow at present.

4 Data analysis and model implementation

In this section, we describe the real case problem that we have considered within the project involving the AOU-Policlinico.
We consider two Surgical Units used for general surgeries managing two ORs each. This choice is motivated by the fact that the two SUs account for more than 30% of the overall number of clinical surgeries. Furthermore, the high heterogeneity in the complexity and surgery durations opens the way for different schedules. Indeed, the managing board of AOU-Policlinico was particularly interested in checking their performance KPI and exploring different management policy. As we mentioned before, clinical SUs can allocate surgeries without any preference, therefore, due to the possibility of transferring patient, they could provide a great improvement when using an optimization procedure. The chosen SUs are located close to each other, so they allow for easy transfer of patients between them. We denote the Surgical Units with the letters A and B (SUA-A and SU-B), the operating rooms as OR-1 and OR-2 (for SU-A), and OR-3 and OR-4 (for SU-B) and the respective wards as R1, R2 and R3, R4. The SUs have different facilities (different number of pre and post surgery rooms) that however do not affect the formulation of the mathematical model. In Figure 4 we report the structure of the two SUs and their patient flow throughout the wards. We recall that our model allows the transfer of patients between SUs, that however remain in charge for the hospitalization. Specifically, the surgery of a patient coming from ward *R1* (SU-A) can be performed in *OR-3* (SU-B) but the patient is then transferred back to his hospitalization ward in SU-A. The ORs are opened 5 days and their operating times are reported in Table 3.

Table 3 SUs actual opening time.

SU	OR	Opening days	Opening hours
SU-A	OR-1	Monday - Friday	8:00 - 20:00
	OR-2	Monday - Friday	8:00 - 16:00
SU-B	OR-3	Monday - Friday	8:00 - 16:00
	OR-4	Monday - Friday	8:00 - 16:00

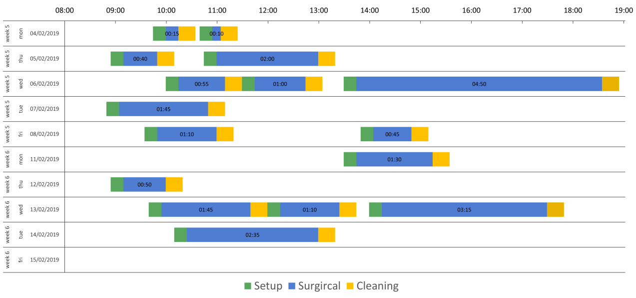

Fig. 5 GANTT chart of historical schedule planned for OR-3 (opening 8.00 - 16:00) during weeks 5 and 6.

4.1 Data analysis

Data have two main sources: the hospital discharge databases (HDD) and the hardcopy registers of the ORs, both accounting for privacy protection. The two sources have been combined to obtained the data needed for the implementation of the MILP model. We consider surgeries carried out in the first 12 weeks of the year 2019, from January 1st to March 30th, in the two clinical SUs of AOU-Policlinico described previously.
Data coming from the hospital discharge databases (HDD), and sent to the regional public authority (*Regione Lazio*), report:

- the type of the surgery to be performed;
- the type of the patient (elective or coming from the Emergency Department);
- the "hospitalization regime" (ordinary \ day surgery);
- the SU in which the patient is admitted to;
- ward admission and resignation dates.

The lists also report some information about the patient medical history. Additionally, from the ORs we obtained the hardcopies of the surgeries registers. These allowed us to deduce, for the 12 weeks under study and for each patient, the following information:

- *diagnosis-related group* (DRGs);
- date of surgery;
- time of arrival in the preparation room;
- starting time of surgery;
- ending time of the surgery;
- time of exiting from the OR.

By a careful analysis and cross-references between all the information, we have been able to obtain the detailed times of the patients' surgeries that we need: *preparation time*, *surgical time* and *OR cleaning time*. As regard the *preparation time* and the *OR cleaning time*, we consider them as fixed values respectively equal to 15 and 20 minutes. Indeed, according to the doctors and to the chief operating room nurse experience, these values appear to be independent from the type of surgery performed and lightly affected by uncertainty. The presence of outliers in available data appears to be due to other organizational and logistic problems that could be remove by a careful surgery planning. For the *surgical time* we instead simply rounded the effective duration time of the surgery.
For each of the 12 weeks under study (numbered from 1 to 12), we report in Table 4 the basic stats as follows:

- the number of surgeries performed in the SU-A and in the SU-B (#I SU-A and #I SU-B respectively);
- the *average surgical time*, namely the average duration of the surgeries scheduled for the week (Avg T) in minutes;
- the standard deviation of this times (StDev T) in minutes;
- the number of *underused* ORs (#ORs UU), namely their workload percentage follows below 50% of its operating time;
- the number of ORs in which the percentage of the daily workload exceeds 80% of the operating time, i.e. they are *overused* (#ORs OU).

For the numerical experiments, we assume that the number of surgeries performed in the week corresponds to the number of patient inserted in the SUs waiting lists.
We remark that the historical surgery planning is not feasible for our optimization model due to violation of the maximum target utilization rate $U = 80\%$ (Over Run). In some cases, the total occupation time of an OR even exceeds its opening time O_s. In addition, in the historical schedule the operating theatres are often under-utilized, registering a utilization rate of less than 50 % of the opening time. As a matter of fact, we report as example in Figure 5 the GANTT chart of the surgeries performed in the OR-3 during weeks 5 and 6. In this case, both the ORs under and over usage can be easily observed. Similar situations occur also for the other ORs and in other weeks. More details on the historical surgery schedule are reported in Section 5, where the planned schedule by the hospital are used as a baseline for the comparison with the model outputs schedules.

Table 4 Dataset stats.

Week	**#I SU-A**	**#I SU-B**	**Avg T**	**StDev T**	**#ORs UU**	**#ORs OU**
1	31	12	123	67	7	1
2	29	17	137	74	7	4
3	31	18	110	49	9	4
4	31	10	122	78	8	1
5	34	15	121	65	9	6
6	29	11	144	90	8	4
7	25	13	144	86	8	5
8	28	13	129	84	7	1
9	27	12	122	69	10	2
10	29	17	126	81	9	4
11	35	14	124	63	6	5
12	32	11	120	53	8	3

4.2 Implementation details

In the mathematical model we also considered constraints on bed availability (see (9) in Section 3.3). Unfortunately, the information about the hospitalization days needed by each surgery g_i was missing among the data. However, since AOU - Policlinico is a public hospital, its beds are mainly occupied by patients coming from the Emergency Department. This peculiarity leads to the need of defining a protection policy that guarantees a minimum availability of "elective beds". To this end, the AOU - Policlinico recently implemented a new strategy for the allocation of beds between emergency and elective patients: according to the regional funding plan, the hospital settled for elective surgeries a fixed percentage of beds in each ward. Looking to the data, we have observed that the number of patients in the waiting lists is always considerably smaller than the number of "elective beds" available in the two clinical wards. Thus, the lack of constraints (9) does not affect the solutions obtained by the model on the 12 weeks under analysis. Further, the data do not show any mandatory day for surgeries or any daily priorities, so that the corresponding constraints (7)-(8) are not active in the tests.

5 Scenario analysis and optimization results

In this section we present the results obtained by running the MILP model described in Section 3. The computational experiments are performed using an x64 MS Windows 10 machine with an Intel (R) Core (TM) i7-10510U CPU and 16 GB of RAM. The solution of the MILP is carried using the *IBM ILOG CPLEX v12.10* as optimizer. Different choice of the weights in (10) will obviously lead to different Pareto solutions. The

Table 5 List of sets and parameters used in the model.

Parameters	**Value**
$\mathcal{I}$	Set of all elective surgeries in the week patient list
$\mathcal{I}_{\mathcal{D}}$	$\emptyset$
$\mathcal{I}_{\mathcal{M}}$	$\emptyset$
$\mathcal{C}$	$\{A, B\}$
$\mathcal{S}$	$\{1, 2, 3, 4\}$
$\mathcal{D}$	$\{1, 2, 3, 4, 5\}$
$\mathcal{P}$	$\{1, ..., p^{max}\}$
$p^{\max}$	see equation (1)
t_i	35 + surgery effective time (in minutes)
g_i	$= 0$
B^{dc}	$= \infty$
U	80%
O_s	fixed according to scenarios in Table 7
w_1, w_2, w_3	10,1,10

three objectives have different priorities for the AOU-Policlinico. Further the three objectives have different scale too. After several trials and following the the preferences of the AOU-Policlinico, for the numerical results the weights in (10) have been set as follows:

w_1	w_2	w_3
10	1	10

For sake of clarity, we report in Table 5 the parameters values settled in the model for the experiments.
We use three Key Performance Indicators (KPIs) to evaluate the models outputs:

- *UR*: the OR week average utilization rate in the open days, which is close to the ideal maximum utilization rate U when minimizing $F3$;
- *Cls*: the total number of inactive days during the week of the OR, which is minimized when using the objective $F1$;
- *Tr*: the number of patients transferred among SUs, which is minimized when using the objective $F2$.

As a first set of experiments we consider the actual setting of the ORs opening times O_s, hereafter denoted as *Scenario #0*, corresponding to $(12, 8, 8, 8)$ opening hours for the respective ORs (for a total of 36 hours per week). We use the historical planned schedules for the surgeries along the 12 weeks as benchmark for the optimized schedules. In particular, we compare the KPIs evaluated on the historical schedule with those obtained by optimizing each of the three single objectives $F1$ $F3$ $F2$, and the multi-criteria function obtained with the weight scalarization approach. This first set of results are reported and discussed in Section 5.1; by analysing them, we found that there were a large possibility of improvements on all the KPIs by changing the actual setting of the ORs opening times O_s. Indeed, one of the aims of AOU-Policlinico is to obtain some insight on the possibility of closing one OR or changing the operating times to obtain a better utilization rate. To this purpose, in the second set of experiments we analyse the MILP results on several different scenarios for the daily ORs opening times O_s. We report them in Table 7, where, for the sake of completeness, we also show Scenario #0 opening hours. The results of these experiments are discussed in Section 5.2.

5.1 Optimization results with current Operating times

In this section we analyse the results obtained on Scenario #0 by solving the single optimization problems related to each of the three objectives $F1$ $F2$ $F3$ and the multi-criteria problem. We use as baseline for comparison the historical schedule on the 12 weeks.

In particular, for each of the 12 weeks and for each of the four ORs, we report in Table 7 the week average utilization rate (*UR*) in the open days, the total number of inactive days throughout the week (*Cls*) as well as the number of transferred patients between the two SUs (*Tr*). We remark that when *Cls* is five, it means that the OR is closed for the entire week and its corresponding *UR* is equal to 0. Instead, for each week and SU, *Tr* describes the number of transferred patients moved to that SU.

We note that in all the experiments conducted the optimal solution was reached ($MIPGap = 0$ %) in a negligible computational time proving that the model can be solved efficiently. Indeed, solving to optimality one of the single objectives $F1$ or $F2$ requires on average half a second to be solved, while optimizing $F3$ needs about one second to produce the final schedule. On the other hand, the multi-objective model takes about five seconds to optimally solve each instance. Based on the results in Table 8, we observe that the historical schedules register in each week at least an OR where the *UR* falls below 50%. It is interesting to note that, although the SU-B (OR-3, OR-4) is opened 4 hours less than SU-A (OR-1, OR-2), the SU-B presents a higher number of daily closed ORs and an overall lower percentage of usage. This highlights an inefficient distribution of surgeries among the two SUs and justifies the considered possibility of transferring patients among them. Note that in the historical schedule there are no transfers of patients among SUs. Getting into details, the minimization of $F1$, namely the daily open ORs, presents a higher *UR* obtained by the closure of at least an OR on each week (the ninth week presents two closed ORs). Minimizing $F2$ obviously leads to a solution with the lowest number of transferred patients (only 5 over 524 total surgeries) and might be seen as a minimal change with respect to the historical schedules. It further presents the highest number of inactive days (33) distributed among the ORs. In this case, no week closure is planned, thus highlighting the conflict between the two objectives $F1$ and$F2$. Besides, the $F3$ optimization results are similar to those seen for $F1$, only differing in the choice of the weekly closed OR. Indeed, since $F3$ minimize the OR deviation from the ideal *UR*, it forces in some weeks the closure of the OR with the longest opening time (OR-1). Furthermore, for both $F1$ and $F3$ the number of patients transferred among SUs accounts for 50% of all the surgeries performed. The output schedules of the MILP leaves the evidence that it is possible to obtain a more uniform distribution in the ORs workload while deciding for the OR closure of the entire week. Without doubt, the multi-objective model balances the results satisfactorily while fulfilling all the hospital requirements. Indeed, an high average *UR* of the weekly opened ORs is obtained transferring only a small number of patients from one SU to the other (15% of the patients).

Figure 6 shows the aggregated performance obtained for the four ORs over the twelve weeks in terms of the different objective functions analyzed. Aside from the *UR*, we report the total number of closed ORs (*Weekly Cls*), the number of inactive days (*Daily Cls*) and the number of days showing an *UR* lower than 50% (*Days UR < 50%*).

On the basis of the results obtained by the MILP, the AOU-Policlinico could have decided to close one of the ORs with 8 hours of opening time while maintaining the same efficiency level for elective surgeries. However, there is still room for improvements. Note that, the closed OR in each week is not always the same and it can be either one belonging to SU-A or to SU-B. In particular, in 3 weeks we observe the closure of the OR-1, i.e. the one with an opening time equal to 12 hours. Given these observations, it is also interesting to check whether a different opening time schedule can give bet-

Table 6 KPIs computed for the historical schedule and the four optimized schedules.

Week	SU	OR	Historical use			OPT_*F1*			OPT_*F2*			OPT_*F3*			MILP		
			UR	*Cls*	*Tr*	*UR*	*Cls*	*Tr*	*UR*	*Cls*	*Tr*	*UR*	*Cls*	*Tr*	*UR*	*Cls*	*Tr*
1	*A*	*1*	52.1	0	0	59.2	0	10	63	0	0	0	5	4	0	5	2
		2	68.4	0		57.5	0		65.1	1		73.3	0		73.5	0	
	B	*3*	47.7	0	0	0	5	16	33.3	2	0	73.3	0	15	73.4	0	16
		4	32.8	1		74.2	0		54	0		73.8	0		73.5	0	
2	*A*	*1*	60	0	0	76.1	0	14	71.9	0	0	76.2	0	15	76.2	0	4
		2	76.7	0		75.6	0		58.8	0		74.4	0		74.4	0	
	B	*3*	51.7	0	0	73.3	0	10	39.8	0	0	74.4	0	8	74.4	0	1
		4	44.8	0		0	5		56.7	0		0	5		0	5	
3	*A*	*1*	47.1	0	0	73.9	0	11	59	0	0	0	5	2	0	5	0
		2	70.2	0		0	5		52.3	0		74.8	0		74.8	0	
	B	*3*	59.6	1	0	59.6	0	20	25.3	1	0	74.8	0	18	74.8	0	10
		4	44.8	1		54	0		63.3	0		74.8	0		74.8	0	
4	*A*	*1*	51.4	0	0	64.4	0	6	68.8	1	1	63.9	0	7	64	0	1
		2	64.2	0		46.2	0		75.4	0		55.8	0		69.8	1	
	B	*3*	39.1	1	0	0	5	11	43.8	4	0	56	0	10	55.8	0	1
		4	44	1		64.8	0		51.3	1		0	5		0	5	
5	*A*	*1*	44.3	0	0	75.3	0	11	63.1	0	0	72.4	0	7	72.4	0	3
		2	83.3	0		67.7	0		55.2	0		0	5		68.5	0	
	B	*3*	45	0	0	65.4	0	12	48.1	0	0	69	0	18	0	5	1
		4	51.3	0		0	5		48.1	0		68.5	0		69	0	
6	*A*	*1*	55.4	0	0	72.8	0	3	62.4	0	1	71.1	0	6	71.1	0	2
		2	65	0		0	5		73.5	0		0	5		66.7	0	
	B	*3*	45.6	1	0	66.5	0	15	55.2	4	0	66.5	0	16	66.9	0	0
		4	55.6	0		64.6	0		62.1	0		67.1	0		0	5	
7	*A*	*1*	47.4	1	0	70.5	0	10	58	0	1	66.8	0	12	66.9	0	2
		2	99.7	1		60.4	0		72.1	0		75.3	1		60.2	0	
	B	*3*	50.5	1	0	0	5	8	31.3	3	0	60.4	0	10	0	5	1
		4	51	0		54.6	0		61.5	1		0	5		60.2	0	
8	*A*	*1*	54.7	0	0	70.5	0	10	58	0	1	66.8	0	12	66.9	0	2
		2	60.1	0		60.4	0		72.1	0		75.3	1		60.2	0	
	B	*3*	46.1	1	0	0	5	8	31.3	3	0	60.4	0	10	0	5	1
		4	52.1	1		54.6	0		61.5	1		0	5		60.2	0	
9	*A*	*1*	57.9	0	0	79.9	0	12	53.6	0	0	79.4	0	12	79.4	0	2
		2	65.3	1		78.5	0		58.8	0		79.1	0		0	5	
	B	*3*	28.1	2	0	0	5	0	41.7	4	0	0	5	0	79.2	0	4
		4	42.3	0		0	5		63.5	1		0	5		0	5	
10	*A*	*1*	55.6	0	0	72.8	0	13	69.9	0	1	71.7	0	8	71.7	0	3
		2	55.6	0		64.8	0		51.5	0		0	5		67.3	0	
	B	*3*	30.6	0	0	0	5	6	44.8	0	0	67.3	0	15	0	5	0
		4	72.5	0		68.1	0		41	0		67.3	0		67.3	0	
11	*A*	*1*	63.1	0	0	72.6	0	10	66.4	0	0	74	0	5	74.2	0	1
		2	78.8	0		69.4	0		73.8	0		0	5		0	5	
	B	*3*	49.7	1	0	75.2	0	9	54.9	2	0	71	0	17	71	0	9
		4	50.5	1		0	5		47.3	0		71.5	0		71.2	0	
12	*A*	*1*	56.3	0	0	72.8	0	10	61.8	0	0	0	5	3	0	5	0
		2	69.2	0		61.2	0		60.8	0		71.7	0		71.9	0	
	B	*3*	29.9	2	0	55.5	1	7	41.7	3	0	71.7	0	18	71.5	0	13
		4	54.2	1		0	5		55.7	1		71.5	0		71.5	0	

Note. *UR*: week average utilization rate in the open days, *Cls*: total number of inactive days of the week, *Tr*: number of transferred patients between the two SUs.

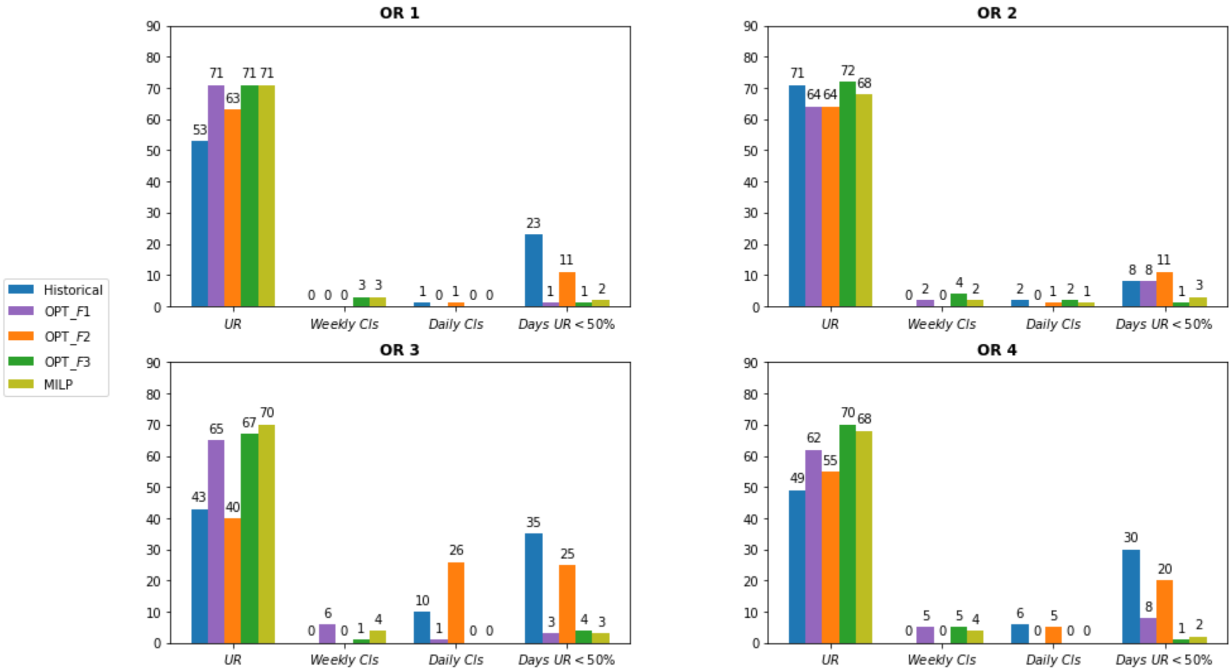

Fig. 6 Bar histograms of the ORs aggregated performances over the twelve weeks when minimizing the single goals individually and the weighted scalarized multi-objective function.

ter KPIs or if it is convenient to close one OR. To this aim, we performed some additional computational experiments, presented below in Section 5.2, in which we analyze different scenarios with a decreasing number of weekly working hours.

5.2 Opening time Scenarios

We test the MILP model for different solutions of ORs opening times in order to optimize the weekly opening hours. Table 7 reports different combinations for the ORs workload. We remark that at least one of the ORs must work for at least 10 hours because of some long surgeries time. For each of the scenarios, it was possible to obtain a feasible schedule for all the weeks. Thus we are able to reduce the total amount of working hours from 36 to 28. The performances obtained show analogies with the results of the previous experiments and make a more efficient use of the clinical resources. In particular, we analyse the results of the last scenario (#5), in which the closure of OR-4 in the SU-B is required. Indeed, closing an OR without reducing the level of service represents a large saving in terms of cost and medical staff. This appears to be the best choice given the three months of data collected. In Figure 7 we report four histograms, one for each OR, where we represent the average weekly ORs workload of Scenarios #0 and #5. When a bar chart is not reported, it means that in that week the OR is closed. It appears that the weekly workload in Scenario #5 is better distributed among the ORs. From the performance metrics in Table 8, it is clear that there is an overall improvement in every single objective. Indeed, the closure of OR-4 allows 34 transfers less than the Scenario #0 while ensuring a very low daily deviation from the ideal *UR* and a lower number of days with underused ORs (16). Lastly, in the scenario #5 all the ORs perform at least a surgery on each day (*Daily ORs Closures* equal 0).

6 Conclusions

The aim of the present work is to make the AOU-Policlinico ORs utilization more efficient exploiting a mathematical formulation for the weekly surgery schedule. We formulate a multi-criteria integer linear model as a support tool for health direction and, more in general, for all the medical staff involved in the surgery scheduling phase of two surgical units at AOU-Policlinico.

We first gave an overview of the data at hand and analyzed their main features then, in collaboration with the hospital team, we identified the most critical optimization criteria. Extensive tests were carried out considering not only the contribution given by each individual goal (alone and within the multi-objective function) but also by varying the weekly opening hours of the four ORs under analysis. The results obtained by

Table 7 Scenario for the weekly ORs opening hours O_s. Scenario *#0* is the AOU-Policlinico actual opening time.

# Scenario	Opening time O_s				Total opening time
	OR-1	**OR-2**	**OR-3**	**OR-4**	$\sum_{s \in S} O_s$
0	12	8	8	8	36
1	12	6	10	6	34
2	12	6	8	6	32
3	10	6	8	6	30
4	10	6	6	6	28
5	12	8	8	0	28

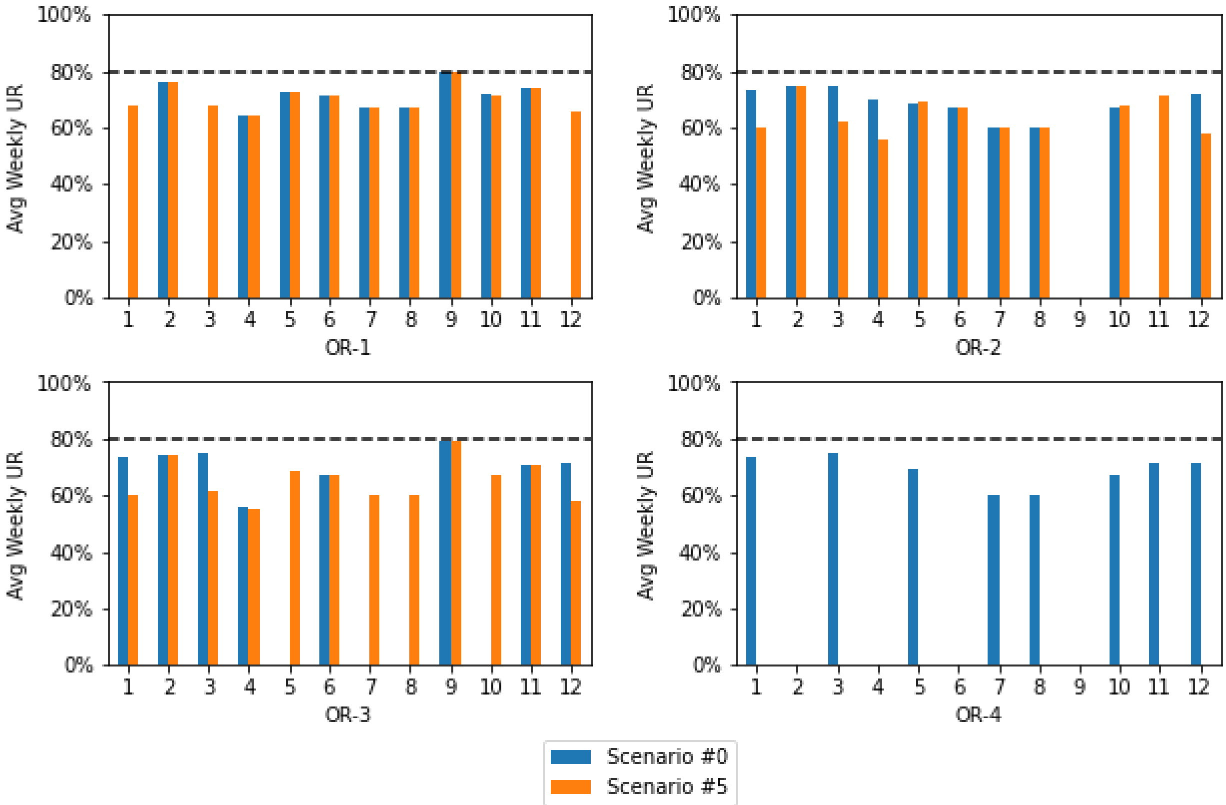

Fig. 7 Histogram showing the weekly workload obtained for each week by Scenario *#0* and Scenario *#5* ORs opening hours. We report on the x axis the number of the week and on the y axis the average weekly *UR*. Note that, in the Scenario #5 the OR-4 is unavailable.

Table 8 Comparison between the Scenario *#0* and the Scenario *#5* of ORs opening hours.

	Scenario #0	**Scenario #5**
Transferred Patients	79	45
Weekly ORs Closures	13	1
Daily ORs Closures	1	0
#Days UR < 50%	31	16

running the model on 12 weeks real data showed that AOU-Policlinico has multiple options to improve the efficiency of its surgical units and to greatly benefit from clinical resources. Indeed, we highlighted some unnecessary opening of ORs throughout the weeks, so that the closure of one OR can be considered. This OR can be assigned to another division or it can be dedicated to emergency cases or it can be used to increase the number of elective patients scheduled. Of course last option must take into account the availability of elective beds in the wards and might open to a change of distribution policy among emergency and elective beds.

Regardless of the chosen objective function, the computational tests proved that the use of mathematical tools in the surgeries scheduling allows for a more efficient use of resources. Our research project not only addresses a real-life scheduling problem, but also intends to simplify the overall scheduling process from a practical perspective. The approach presented can be significantly helpful for the administration also to organize more efficiently the surgeon group shifts. We remark that in the current implementation of the model the effective surgery times have been used as deterministic values but instead surgery times are stochastic. As a future work, we plan to use a robust optimization approach to tackle possible variations in the standard times defined by clinicians. Moreover, we want to formulate a more complex model that considers preference-related measures, for both the patients and surgeons. Doctors may want to give different emphasis to weekly and daily priorities; this can be done by inserting priority goals, with different coefficients and orders of magnitude, in the objective function.

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References

Agnetis A, Coppi A, Corsini M, Dellino G, Meloni C, Pranzo M (2013) A decomposition approach for the combined master surgical schedule and surgical case assignment problems. Health care management science DOI 10.1007/s10729-013-9244-0

Agnetis A, Bianciardi C, Iasparra N (2019) Integrating lean thinking and mathematical optimization: A case study in appointment scheduling of hematological treatments. Operations Research Perspectives 6:100110, DOI 10.1016/j.orp.2019.100110

AOU-Policlinico (2017) Piano di efficientamento 2017-2019, delibera n.269 del 21 marzo 2017 p 144, URL https://www.policlinicoumberto1.it/media/924288/delibera_n._269_del_21.3.2017.pdf

AOU-Policlinico (2018) Carta dei servizi Azienda Ospedaliera - Unversitaria Policlinico Umberto I. http://www.policlinicoumberto1.it/media/1005553/carta_2018.pdf

Aringhieri R, Landa P, Soriano P, Tanfani E, Testi A (2015) A two level metaheuristic for the operating room scheduling and assignment problem. Computers & Operations Research 54:21–34, DOI 10.1016/j.cor.2014.08.014

Cardoen B, Demeulemeester E, Beliën J (2009) Optimizing a multiple objective surgical case sequencing problem. International Journal of Production Economics 119:354–366, DOI 10.1016/j.ijpe.2009.03.009

Cardoen B, Demeulemeester E, Beliën J (2010) Operating room planning and scheduling: A literature review. European Journal of Operational Research 201:921–932, DOI 10.1016/j.ejor.2009.04.011

Davila MP (2013) A methodology for scheduling operating rooms under uncertainty. University of South Florida, Graduate Theses and Dissertations URL https://scholarcommons.usf.edu/etd/4463/

Day R, Garfinkel R, Thompson S (2012) Integrated block sharing: A win–win strategy for hospitals and surgeons. Manufacturing & Service Operations Management 14:567–583, DOI 10.1287/msom.1110.0372

De Santis M, Grani G, Palagi L (2020) Branching with hyperplanes in the criterion space: The frontier partitioner algorithm for biobjective integer programming. European Journal of Operational Research 283(1):57–69, DOI https://doi.org/10.1016/j.ejor.2019.10.034

Ehrgott M (2005) Multicriteria optimization, vol 491. Springer Science & Business Media

Freeman N, Melouk S, Mittenthal J (2015) A scenario-based approach for operating theater scheduling under uncertainty. Manufacturing & Service Operations Management 18, DOI 10.1287/msom.2015.0557

Fügener A, Schiffels S, Kolisch R (2017) Overutilization and underutilization of operating rooms - insights from behavioral health care operations management. Health Care Management Science 20:115–128

Guerriero F, Guido R (2011) Operational research in the management of the operating theatre: A survey. Health care management science 14:89–114, DOI 10.1007/s10729-010-9143-6

Guido R, Conforti D (2016) A hybrid genetic approach for solving an integrated multi-objective operating room planning and scheduling problem. Computers & Operations Research 87, DOI 10.1016/j.cor.2016.11.009

Gul S, Denton B, Fowler J, Huschka T (2011) Bi-criteria scheduling of surgical services for an outpatient procedure center. Production and Operations Management 20:406 – 417, DOI 10.1111/j.1937-5956.2011.01232.x

Gupta D (2007) Surgical suites' operations management. Production and Operations Management 16:689 – 700, DOI 10.1111/j.1937-5956.2007.tb00289.x

Gür S, Eren T (2018) Application of operational research techniques in operating room scheduling problems: Literature overview. Journal of Healthcare Engineering 2018:1–15, DOI 10.1155/2018/5341394

Hans E, Vanberkel P (2012) Operating Theatre Planning and Scheduling, vol 168, pp 105–130. DOI 10.1007/978-1-4614-1734-7_5

Koppka L, Wiesche L, Schacht M, Werners B (2018) Optimal distribution of operating hours over operating rooms using probabilities. European Journal of Operational Research 267(3):1156–1171, DOI 10.1016/j.ejor.2017.12.02

Mannino C, Nilssen EJ, Nordlander TE (2012) A pattern based, robust approach to cyclic master surgery scheduling. Journal of Scheduling 15(5):553–563

Marques I, Captivo M (2015) Bicriteria elective surgery scheduling using an evolutionary algorithm. Operations Research for Health Care 7, DOI 10.1016/j.orhc.2015.07.004

Meskens N, Duvivier D, Arnauld H (2012) Multi-objective operating room scheduling considering desiderata of the surgical team. Decision Support Systems 55:650–659, DOI 10.1016/j.dss.2012.10.019

Penn M, Potts C, Harper P (2017) Multiple criteria mixed-integer programming for incorporating multiple factors into the development of master operating theatre timetables. European Journal of Operational Research 262, DOI 10.1016/j.ejor.2017.03.065

Rachuba S (2017) A fuzzy multi-criteria approach for robust operating room schedules. Annals of Operations Research 251, DOI 10.1007/s10479-015-1926-1

Rahimi I, Gandomi A (2020) A comprehensive review and analysis of operating room and surgery scheduling. Archives of Computational Methods in Engineering DOI 10.1007/s11831-020-09432-2

Riet C, Demeulemeester E (2014) Trade-offs in operating room planning for electives and emergencies: A review. SSRN Electronic Journal DOI 10.2139/ssrn.2553849

Samudra M, Riet C, Demeulemeester E, Cardoen B, Vansteenkiste N, Rademakers F (2016) Scheduling operating rooms: achievements, challenges and pitfalls. Journal of Scheduling 19, DOI 10.1007/s10951-016-0489-6

Wullink G, Houdenhoven M, Hans E, Oostrum J, Lans M, Kazemier G (2008) Closing emergency operating rooms improves efficiency. Journal of medical systems 31:543–6, DOI 10.1007/s10916-007-9096-6

Yang XS (2014) Chapter 14 - multi-objective optimization. In: Yang XS (ed) Nature-Inspired Optimization Algorithms, Elsevier, Oxford, pp 197 – 211, DOI https://doi.org/10.1016/B978-0-12-416743-8.00014-2, URL `http://www.sciencedirect.com/science/article/pii/B9780124167438000142`

Zhu S, Fan W, Yang S, Pei J, Pardalos P (2019) Operating room planning and surgical case scheduling: a review of literature. Journal of Combinatorial Optimization 37, DOI 10.1007/s10878-018-0322-6