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Validation of a two-layer depth-averaged model by comparison with an experimental dilute stratified pyroclastic density current

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Conflicts of interest/Competing interests

Not applicable.

Availability of data and material

Not applicable.

Code availability

The complete code of the present two-layer PDC model (referred to as "SKS-1D:1.0"), including post-processing scripts (to plot the solution variables and create animations), is available at <https://github.com/HiroyukiShimizu/SKS-1D> (last access: 30 June 2021).

Abstract

Numerical results of a two-layer depth-averaged model of pyroclastic density currents (PDCs) were compared with an experimental PDC generated at the international eruption simulator facility (the Pyroclastic flow Eruption Large-scale Experiment; PELE) to establish a minimal dynamical model of PDCs with stratification of particle concentrations. In the present two-layer model, the stratification in PDCs is modeled as a voluminous dilute turbulent suspension layer with low particle volume fractions ($\lesssim 10^{-3}$) and a thin basal bedload layer with high particle volume fractions ($\sim 10^{-2}$) on the basis of the source condition in the experiment. Numerical results for the dilute layer quantitatively reproduce the time evolutions of the front position and body thickness of the dilute part in the experimental PDC. The numerical results of the bedload thickness and deposit mass depend on an assumed value of mean deposition speed at the bottom of the bedload (D). We show that the thicknesses of bedload and deposit in the simulations agree well with the experimental data, when D is set to about 3.5×10^{-4} m/s. This value of the deposition speed is two orders of magnitude smaller than that predicted by a hindered-settling model. The small value of D suggests that the erosion process accompanied by saltating/rolling of particles plays a role in the sedimentation in the bedload.

Keywords: Pyroclastic density current; Two-layer model; Experimental validation; Pyroclastic surge; Bedload; Fluidized granular flow

1 Introduction

Pyroclastic density currents (PDCs) are a frequent and hazardous process during volcanic eruptions. They occur when a hot mixture of volcanic particles and gas is ejected from the vent, but fails to become buoyant and instead propagates outwards as a ground-hugging gravity current (see the reviews by Branney and Kokelaar 2002; Sulpizio et al. 2014; Dufek 2016; Lube et al. 2020). The flow dynamics of PDCs depends on various factors: eruption conditions such as magma discharge rate (e.g., Bursik and Woods 1996; Dufek and Bergantz 2007; Shimizu et al. 2019), physical processes of PDCs such as ambient air entrainment, particle sedimentation, and basal friction (e.g., Roche et al. 2008; Andrews and Manga 2012; Lube et al. 2019), and topography (e.g., Esposti Ongaro et al. 2008; Andrews and Manga 2011; Kelfoun 2017). Because of the interplay between these different factors, the fluid dynamical features of PDCs are highly variable and form a wide range of pyroclastic deposit characteristics (e.g., Fisher and Schmincke 1984; Cas and Wright 1987; Branney and Kokelaar 2002).

One major reason behind the wide range of PDC dynamics arises due to strong vertical stratification

of PDCs with respect to particle concentration (Valentine 1987; Branney and Kokelaar 2002; Burgisser and Bergantz 2002; Breard et al. 2016). PDCs are composed mainly of an upper voluminous dilute turbulent suspension region with low particle volume fractions ($\lesssim 10^{-3}$) and a thinner basal region with high particle volume fractions ($\sim 10^{-2}$ – 10^{-1}). As the particle volume fractions change, the controlling factors of the flow dynamics also change. The flow dynamics of the upper dilute part is controlled mainly by the settling of particles in the current, entrainment of ambient air into the current, thermal expansion of the entrained air, and resistance of ambient air at the flow front (e.g., Sparks et al. 1993; Andrews and Manga 2012; Benage et al. 2016). On the other hand, the flow dynamics of the basal part is controlled mainly by friction and deposition at the bottom of the current (e.g., Roche et al. 2008; Girolami et al. 2010; Lube et al. 2019; Brosch and Lube 2020). The behavior of the whole stratified current is determined by the dynamics of both the dilute and basal parts, and the interactions between them (i.e., transfers of mass, momentum, and energy from one to the other). The effects of these physical processes on the flow dynamics depend on the source conditions and topography. In particular, the basal part has various characteristics depending on the source conditions (for instance the particle concentration at the source; Lube et al. 2015; Breard et al. 2018); the basal current flows as a dense gas-pore pressure-modified (i.e., fluidized) granular flow with very high particle volume fractions (~ 0.4 ; referred to as a “dense underflow”) or as a flow of saltating/rolling particles with relatively low particle volume fractions ($\sim 10^{-2}$; referred to as a “bedload”).

Numerical two-layer depth-averaged models have been developed as a minimal dynamical model to describe global features of stratified PDCs (e.g., Doyle et al. 2008; Kelfoun 2017; Shimizu et al. 2019). In the two-layer models, the stratification of particle concentration and density in PDCs is modeled as dilute and basal layers on the basis of the idea that the dilute- and basal-part flows in stratified PDCs are controlled by different physical processes. In dilute layers, the effects of particle settling, air entrainment, thermal expansion, and frontal air resistance on flow dynamics are mainly taken into account on the basis of experiments of particle-water dilute turbulent suspension flows (e.g., Parker et al. 1987; Bonnetaze et al. 1993; Shimizu et al. 2019). In basal layers, the effects of basal friction and deposition on flow dynamics are mainly considered on the basis of experiments of particle-air dense fluidized granular flows (e.g., Girolami et al. 2008, 2010; Roche et al. 2008; Shimizu et al. 2019). The two layers are coupled through mass and momentum exchanges such as inter-layer particle transfer. Although the concept of two-layer model is useful for systematically assessing the effects of the various physical processes on flow dynamics and the resulting deposits of stratified PDCs, the quantitative agreement of its numerical results with experimental observations needs to be tested.

Currently, a community-driven effort is underway to compare numerical PDC models with experimental data for the purposes of validation (assessing how well a numerical model represents the

physical problem) and benchmarking (comparison of different numerical models with one another) (Valentine 2019; Esposti Ongaro et al. 2020). As a part of the effort, a large-scale experiment was conducted at the international eruption simulator facility (the Pyroclastic flow Eruption Large-scale Experiment, PELE; Lube et al. 2015). This experiment involved the controlled gravitational collapse of a hot suspension of natural volcanic particles and air into an instrumented run-out section. The resulting stratified density current simulated a fully dilute, fully turbulent PDC (i.e., a pyroclastic surge) comprising a thick upper turbulent region and a thin basal bedload region. The detailed conditions of the experiment and the characteristics of the spatially and temporally evolving flow structure and deposit are described in Brosch and Lube (2020). The benchmark conditions are described in Supplementary Information S1 in Electronic Supplementary Material (ESM) 1. By comparison of numerical PDC models with the experimental PDC at PELE, we can further advance our understanding of the physical processes controlling PDCs and the resulting deposits.

This paper compares a numerical two-layer PDC model (Shimizu et al. 2019) with the experimental data from PELE for the community-driven benchmarking and validation in order to establish a minimal dynamical model of stratified PDCs. We assess how well the two-layer model reproduces the experimental stratified PDC to clarify its limitations. To reproduce the features of the basal bedload observed at PELE, we modify the model for the basal layer in the original two-layer model of Shimizu et al. (2019). We discuss the sedimentation process in the experimental bedload on the basis of numerical simulations of the modified two-layer model.

2 Method

We conducted a series of numerical simulations of a two-layer PDC model (see Appendix 1 for the basic equations) under the conditions defined in the benchmark. A two-layer PDC flows into run-out sections comprising proximal 6° inclined and distal horizontal channels at $x = 0$ – 9.68 m and $x > 9.68$ m, respectively, where x represents the distance in a direction parallel to the basal surface (Fig. 1). We set the source conditions using the experimental data at $x = 0$ (Profile 1), and compare the numerical results with the experimental data at the distal areas ($x > 0$) particularly at $x = 2.65$ and 7.78 m (Profiles 2 and 3, respectively). The source conditions in the simulations and the input parameters for the dilute and basal layers are as follows.

2.1 Dilute layer

The source of the upper dilute layer in the simulations is modeled as a supply of homogeneous dilute current at a constant mass flow rate from the inlet boundary $x = 0$ (Fig. 1). The values of the inlet boundary conditions for the dilute layer (i.e., thickness h_1 , velocity u_1 , solid mass

fraction n_{s1} , volcanic gas mass fraction n_{v1} , temperature T_1 , and mean solid density ρ_s) are obtained from the experimental data at Profile 1 ($x = 0$) (see Table 1). The values of h_1 , u_1 , n_{s1} , n_{v1} , and T_1 are estimated by depth- and time-averaging experimental data of flow velocity, temperature, and particle volume fraction as a function of time t and height z at Profile 1 (see Supplementary Information S1 in ESM 1). As the densities of solid particles in the experiment depend on their particle sizes, the value of ρ_s is estimated by $(\sum(n_{s,i}/\rho_{s,i}))^{-1}$, where $n_{s,i}$ and $\rho_{s,i}$ are the depth- and time-averaged solid mass fraction and solid density of the i -th particle class obtained from the experimental data at Profile 1 (see ESM 2 for details of the estimation). The inlet mass flow rate of the dilute current is given as $\rho_1 u_1 h_1$ for time $t = 0-4$ s and as 0 for $t > 4$ s (Fig. 1c), where ρ_1 represents the density of the dilute current at $x = 0$ and is estimated by the equation of state (Eq. (6)).

The flow dynamics of the dilute current is dependent on three factors: the mean settling speed of solid particles at the bottom of the current (W_s), the imposed frontal Froude number (Fr_N), and the basal drag coefficient (C_{dc}). The values of W_s , Fr_N , and C_{dc} are estimated from existing models (see Table 1). The value of W_s is estimated by $\sum(n_{s,i}W_{s,i})$, where the settling speed of each i -th particle class ($W_{s,i}$) is formulated by a terminal-velocity model (Eq. (A.4) of Shimizu et al. 2019; see ESM 2 for details of the estimation). The value of Fr_N is based on the theoretical model for steady-state inviscid gravity currents (Benjamin 1968). The value of C_{dc} is estimated on the basis of the empirical formula of Hager (1988) (cf. Hogg and Pritchard 2004). The values of W_s , Fr_N , and C_{dc} are assessed by comparison of the numerical results of the dilute layer with the experimental data at $x > 0$.

2.2 Basal layer

To reproduce the experimentally observed fluid dynamical features of the basal-part flow with relatively low particle volume fractions $\sim 10^{-2}$ (i.e., the bedload), we modified the model for the basal layer in the original two-layer model of Shimizu et al. (2019), which assumed that the solid volume fraction in the basal layer (ϕ_{sH}) is equal to that in the deposit (ϕ_{sD}). The present two-layer model, on the other hand, considers a more generalized situation where $\phi_{sH} < \phi_{sD}(= 0.6)$.

In the simulations, a basal current (i.e., bedload) is supplied at the inlet boundary $x = 0$ at a constant mass flow rate (Fig. 1). The inlet boundary conditions of the basal current (i.e., thickness h_{H1} , velocity u_{H1} , and mean solid volume fraction ϕ_{sH}) are obtained from the experimental data (see Table 1 and Supplementary Information S1 in ESM 1). The value of h_{H1} is based on the time-series of the bedload thickness at Profile 1 ($x = 0$). The value of u_{H1}

is estimated by depth averaging the time-averaged height-dependent fitting function of flow velocity at Profile 1 between heights 0 and h_{H1} . The value of ϕ_{SH} is based on the observation that the bedload has solid volume fractions of $\sim 10^{-2}$ (Brosch and Lube 2020). The inlet mass flow rate of the basal current is given as $\rho_H u_{H1} h_{H1}$ for time $t = 0-4$ s and as 0 for $t > 4$ s (Fig. 1c), where ρ_H represents the mean density of the basal current (see Appendix 1 for the estimation of ρ_H).

The flow dynamics of the basal current is controlled by two major factors: the basal drag coefficient (C_{db}) and the mean deposition speed (D). The value of C_{db} (see Table 1) is based on the empirical formula of Hager (1988) (i.e., $C_{db} = 0.025 Re_{H1}^{-0.2}$; cf. Hogg and Pritchard 2004), where $Re_{H1} (\equiv \rho_H u_{H1} h_{H1} / \eta_H)$ represents the Reynolds number of the basal layer at $x = 0$, and the bulk dynamic viscosity of the basal layer (η_H) is set to 10^{-5} Pa s. The mean deposition speed D , on the other hand, depends strongly on unknown physical processes in the basal current. For this reason, we set D as a tuning parameter and estimate its value on the basis of fitting the numerical results to the experimental data at $x > 0$ (see Table 1) and discuss the sedimentation process in the experimental bedload.

3 Results

3.1 Dilute layer

In the numerical simulations for a wide range of D , a dilute current is generated from the inlet boundary ($x = 0$) and flows into the run-out sections ($x > 0$) (see Supplementary Movie 1 (ESM 3) for $D = 3.5 \times 10^{-4}$ m/s). The results of the dilute currents in the simulations are almost unaffected by the characteristics of the basal current, because the basal layers have a negligible effect on the dynamics of the dilute layer (e.g., the interfacial drag between the two layers). Here we describe the flow dynamical features of the dilute current in the results reproducing the behavior of the experimental bedload (i.e., Table 1; Supplementary Movie 1 (ESM 3)).

The numerical results successfully reproduce the qualitative features of the dilute-part flow observed in the experiment. They develop a typical gravity current structure comprising a leading thick gravity current “head” and a trailing gravity current “body” (see Supplementary Movie 1 (ESM 3); cf. Brosch and Lube 2020). The results also agree well quantitatively with the main components of the experimental data. They reproduce the time evolution of the front position in the experiment (Fig. 2a). The flow thickness in the simulations ($h(x, t)$) is consistent with the time evolution of the body thickness at Profiles 2 and 3 ($x = 2.65$ and

7.78 m) in the experiment (Fig. 2b and c).

We performed a parametric study for a wide range of input parameters (i.e., $W_s = 0.3\text{--}3$ m/s, $Fr_N = 1\text{--}\sqrt{2}$, and $C_{dc} = 10^{-4}\text{--}10^{-2}$) to assess the values of W_s , Fr_N , and C_{dc} in Table 1. The results of the parametric study indicate that the dynamical features of the dilute current are insensitive to C_{dc} . On the other hand, they sensitively depend on W_s and Fr_N . The evolution of the body thickness is primarily affected by the flow density, which in turn depends on W_s . The front position strongly depends not only on W_s but also on Fr_N . This is because the time evolution of the flow front, as well as the head and body structure, is controlled primarily by the resistance pressure caused by the acceleration of the ambient air at the flow front (i.e., the front condition (Eq. (7)); Shimizu et al. 2017). The above good agreement of the body thickness between the numerical and experimental results (Fig. 2b and c) implies that the mass weighted average of terminal velocities (i.e., $\sum(n_{s,i}W_{s,i})$) used in the present model can explain the effective value of W_s in the experiment. The agreement for the front position (Fig. 2a) implies that the theoretical model of Fr_N for steady-state inviscid gravity currents (Benjamin 1968) used in the present model explains the mechanical balance at the flow front in the experiment.

3.2 Basal layer and deposit

In the simulations, a basal current (i.e., bedload) is generated from the inlet boundary ($x = 0$) and flows into the run-out sections ($x > 0$) (see Supplementary Movie 1 (ESM 3) for the values of the input parameters in Table 1). The basal current obtains the mass and momentum of particles settling from the upper dilute current. The basal current evolves downstream until its frontal parallel mass flux becomes zero owing to basal deposition (cf. Shimizu et al. 2019). The deposits progressively aggrade upward from the bottom of the basal current in the proximal area and directly from the bottom of the dilute current in the distal area where the basal current is absent (cf. Regime 2a of Shimizu et al. 2019).

When the mean deposition speed at the bottom of the basal current (D) is set to 3.5×10^{-4} m/s (Table 1), the numerical results agree quantitatively with the experimental data of the bedload and deposit. They reproduce the time-series of the bedload thickness at Profiles 2 and 3 ($x = 2.65$ and 7.78 m) in the experiment (Fig. 3a and b). The deposit mass in the simulation is consistent with the profile of the final deposit mass per unit area in the experiment (Fig. 3c). The fact that the numerical results do not reproduce the exponential decay of the experimental deposit mass with distance (see Fig. 3c) is due to that the deposition speed at the bottom of the basal current is given by a constant value (i.e., $D \cos \theta$) in the present model (see Eq. (10)).

The results of the numerical simulations for a wide range of D indicate that, as D increases, the slope of the bedload thickness decreases, and the deposit mass derived from the bedload increases. These results allow us to estimate the value of D from the following experimental observations: (1) the bedload thickness has almost the same value (i.e., 0.005–0.02 m) at Profiles 1–3 (see Fig. 3a and b; see Fig. S1a in Supplementary Information S1 in ESM 1) and (2) the deposit derived from the bedload has a mass of 0.8–5 kg/m² at Profiles 1–3 (see Fig. 3c). The estimated value of D depends on the uncertainties of other parameters for the bedload such as h_{H1} , u_{H1} , ϕ_{sH} , and C_{db} ; these uncertainties are caused mainly by depth- and/or time-averaging procedures for the experimental data at Profile 1 (see Supplementary Information S1 in ESM 1 for details of the estimations of these uncertainties). To assess the effects of these uncertainties, we performed sensitivity analyses for $h_{H1} = 0.005\text{--}0.02$ m, $u_{H1} = 2.36\text{--}3.27$ m/s, $\phi_{sH} = 0.01\text{--}0.05$, and $C_{db} = 1.0 \times 10^{-3}\text{--}4.0 \times 10^{-4}$.

The results of the sensitivity analyses indicate that the bedload thickness is dependent on these parameters, whereas the final deposit mass is insensitive to them. When the uncertainties of h_{H1} , u_{H1} , ϕ_{sH} , and C_{db} are taken into account, the results of our model with $D = 0\text{--}9.1 \times 10^{-4}$ m/s agrees with the experimental data of the bedload thickness at Profiles 1–3 (i.e., 0.005–0.02 m). On the other hand, our two-layer model indicates that the final deposit mass derived from the basal layer is given by $\phi_{sD}\rho_s D \Delta t \cos \theta$ (see Eq. (10)), where Δt is the time interval within which the bedload exists at x (i.e., $\Delta t(x = 0\text{--}7.78) = 3.30\text{--}3.77$ s; see Figs. 3a, 3b, and S1a). Accordingly, the experimental deposit mass at $x = 0\text{--}7.78$ m (i.e., $\phi_{sD}\rho_s D \Delta t \cos \theta = 0.8\text{--}5$ kg/m²) is explained by $D = 1.9 \times 10^{-4}\text{--}1.0 \times 10^{-3}$ m/s. We conclude that the range of D best explaining the experimental data of both the bedload thickness and deposit mass is $1.9 \times 10^{-4}\text{--}9.1 \times 10^{-4}$ m/s.

4 Discussion

In the foregoing section, we have obtained the result that our numerical model can quantitatively reproduce the evolutions of the basal bedload and deposit in the experiment when the mean deposition speed at the bottom of the bedload is set to $D = 1.9 \times 10^{-4}\text{--}9.1 \times 10^{-4}$ m/s. We discuss below the sedimentation process in the experimental bedload on the basis of these estimated values of D .

Previously, the sedimentation process in basal parts of PDCs has been explained by the hindered-settling model on the basis of experiments of initially fluidized granular flows (Girolami et al. 2010); we have confirmed in Appendix 2 that the numerical results of a basal-layer model with the hindered-

settling model agree well with the experimental data of an initially fluidized granular flow reported by Girolami et al. (2008). On the other hand, the estimated values of D for the bedload (i.e., the circle with error bars in Fig. 4) deviate significantly from those predicted by the hindered-settling model (i.e., the gray region in Fig. 4; Eqs. (11) and (12)). The circle with error bars in Fig. 4 shows the relationship between the ratio of the estimated values of D for the bedload to the value of W_s (i.e., $D/W_s = 2.6 \times 10^{-4} - 1.2 \times 10^{-3}$) and the experimental observation of the solid volume fractions in the bedload (i.e., $\phi_{sH} = 0.01 - 0.05$). The estimated values of D/W_s for the bedload are two orders of magnitude smaller than that predicted by the hindered-settling model for $\phi_{sH} = 0.01 - 0.05$ (i.e., $D/W_s = 1.5 \times 10^{-2} - 8.2 \times 10^{-2}$; see Fig. 4). Setting D to a value predicted by the hindered-settling model ($D = 1.2 \times 10^{-2}$ m/s) gives a run-out distance of the basal current that is too short (~ 0.1 m) to explain the experimental observations for the bedload (see Supplementary Movie 2 (ESM 4)). These results suggest that the sedimentation process in the experimental bedload is governed by a mechanism other than hindered settling.

The small estimated values of the mean deposition speed D for the bedload can be explained by the combination of deposition and erosion processes (cf. Parker et al. 1986; Cao et al. 2004; Trinh et al. 2017; Rauter and Köhler 2020). The experimental observations showed that saltating/rolling of particles occurred in the bedload (Brosch and Lube 2020); the saltating/rolling of particles accompany a complicated combination of deposition and erosion processes. These processes may reduce the effective value of the deposition speed (i.e., D).

In this paper, we have focused on the dynamics of stratified PDCs with a bedload generated at the base. As mentioned in the introduction section, the basal current in a stratified PDC flows as either bedload ($\phi_{sH} \sim 10^{-2}$) or dense underflow ($\phi_{sH} \sim 0.4$), and this difference in the basal current changes the flow and sedimentation of PDCs (e.g., Lube et al. 2015; Breard et al. 2018; Lube et al. 2020). Our two-layer model can predict the behavior of stratified PDCs only when the inlet source conditions of the basal current (i.e., h_{H1} , u_{H1} , and ϕ_{sH}) as well as the effective vertical mass fluxes of particles (i.e., W_s and D) and the effective basal friction (i.e., C_{db}) are provided. Future works will attempt to develop additional models to determine these parameters for cases where dense underflow or bedload develops at the source (e.g., Breard et al. 2018; Lube et al. 2019; Valentine 2020).

5 Summary

Numerical results of a two-layer depth-averaged model of PDCs with stratification of particle concentrations were compared with an experimental dilute PDC generated at PELE. In numerical simulations of the present two-layer model, the stratification in PDCs is modeled as a voluminous

dilute turbulent suspension layer with low particle volume fractions ($\lesssim 10^{-3}$) and a thin basal bedload layer with high particle volume fractions ($\sim 10^{-2}$) on the basis of the experimental source conditions. By fitting the numerical results to the experimental data for the bedload, the mean deposition speed at the bottom of the bedload (D) has been estimated to be about 3.5×10^{-4} m/s. This value is two orders of magnitude smaller than that predicted by the hindered-settling model. The small value of D suggests that the erosion process accompanied by saltating/rolling of particles plays a role in the sedimentation in the bedload. Further understanding of PDC dynamics based on the two-layer model would require similar comparisons under various source conditions (e.g., those where a dense underflow develops).

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Appendices

Appendix 1. Basic equations of the two-layer model for pyroclastic density currents

This appendix provides details of a two-layer depth-averaged model for pyroclastic density currents (PDCs) with stratification of particle concentrations. The present two-layer model is based on that of Shimizu et al. (2019); the two layers (i.e., the dilute and basal currents) are coupled through mass and momentum exchanges as suspended particles in the dilute current settle into the basal current, and a deposit progressively aggrades upward from the bottom of the basal (or dilute) current (see Fig. 1b). Shimizu et al. (2019) assumed an axisymmetric PDC spreading radially from the source along the horizontal ground surface, whereas this paper designs a PDC flowing into an inclined one-dimensional channel on the basis of the experimental setting of PELE (see Fig. 1a). The basic equations of the dilute and basal currents and the deposit are described below (see Shimizu et al. 2019 for the numerical procedures).

The dilute current is modeled as a highly turbulent suspension flow consisting of solid particles, volcanic gas, and entrained ambient air. When the current flows into a one-dimensional channel with a slope angle θ (Fig. 1), the basic equations of the dilute current with thickness $h(x, t)$, velocity $u(x, t)$, density $\rho(x, t)$, solid mass fraction $n_s(x, t)$, volcanic gas mass fraction $n_v(x, t)$, air mass fraction $n_a(x, t)$, temperature $T(x, t)$, and specific heat at constant pressure $C_p(x, t)$ are as follows.

Conservation of flow mass:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x}(\rho u h) = \rho_a E |u| - n_s \rho W_s \cos \theta, \quad (1)$$

Conservation of entrained air mass:

$$\frac{\partial}{\partial t}(n_a \rho h) + \frac{\partial}{\partial x}(n_a \rho u h) = \rho_a E |u|, \quad (2)$$

Conservation of solid particle mass:

$$\frac{\partial}{\partial t}(n_s \rho h) + \frac{\partial}{\partial x}(n_s \rho u h) = -n_s \rho W_s \cos \theta, \quad (3)$$

Conservation of flow momentum:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho u h) + \frac{\partial}{\partial x} \left(\rho u^2 h + \frac{\rho - \rho_a}{2} g h^2 \cos \theta \right) \\ = (\rho - \rho_a) g h \sin \theta - (\rho - \rho_a) g h \cos \theta \frac{\partial z_c}{\partial x} \\ - n_s \rho u W_s \cos \theta - \tau_c, \end{aligned} \quad (4)$$

Conservation of flow enthalpy:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho C_p T h) + \frac{\partial}{\partial x}(\rho C_p T u h) \\ = \rho_a E |u| \left(C_{pa} T_a + \frac{u^2}{2} + \frac{g h}{2} \cos \theta \right) \\ - n_s \rho W_s \cos \theta \left(C_s T - \frac{g h}{2} \cos \theta \right), \end{aligned} \quad (5)$$

Equation of state:

$$\frac{1}{\rho} = \frac{n_s}{\rho_s} + \frac{T}{p} (n_a R_a + n_v R_v), \quad (6)$$

where t is the time, x is the distance in the direction parallel to the basal surface, $\rho_a (\equiv p/(R_a T_a))$ is the density of ambient air, E is the entrainment coefficient (see Eq. (A.1) of Shimizu et al. 2019), W_s is the mean settling speed of solid particles at the bottom of the dilute current, g is the gravitational acceleration, z_c is the height of the basal contact, τ_c is the basal drag (see Eq. (A.2) of Shimizu et al. 2019), C_{pa} is the specific heat of air at constant pressure, T_a is the temperature of ambient air, C_s is the specific heat of solid particles, ρ_s is the mean density of solid particles, R_a is the gas constant of air, R_v is the gas constant of volcanic gas, and p is the pressure. The mass fractions satisfy the condition of $n_s + n_v + n_a = 1$. The specific heat of the dilute current at constant pressure is given by $C_p = n_s C_s + n_a C_{pa} + n_v C_{pv}$. To describe realistic dilute current dynamics, a balance between the buoyancy pressure driving the flow front and the resistance pressure caused by the acceleration of the ambient air at the front (i.e., the front condition):

$$\frac{dx_N}{dt} = u_N = Fr_N \sqrt{\frac{\rho_N - \rho_a}{\rho_a} g h_N \cos \theta} \quad \text{at } x = x_N(t) \quad (7)$$

is taken into account, where the subscript N denotes the front, and Fr_N is the imposed frontal Froude

number.

The basal current is modeled as a homogeneous high particle concentration flow consisting of solid particles and air. The basic equations of the one-dimensional channelized basal current with thickness $h_H(x, t)$ and velocity $u_H(x, t)$ are as follows.

Conservation of solid particle mass:

$$\frac{\partial h_H}{\partial t} + \frac{\partial}{\partial x}(u_H h_H) = \frac{n_s \rho}{\phi_{sH} \rho_s} W_s \cos \theta - \frac{\phi_{sD}}{\phi_{sH}} D \cos \theta, \quad (8)$$

Conservation of solid particle momentum:

$$\begin{aligned} \frac{\partial}{\partial t}(u_H h_H) + \frac{\partial}{\partial x} \left(u_H^2 h_H + \frac{1}{2} \frac{\rho_H - \rho_a}{\rho_H} g h_H^2 \cos \theta \right) \\ = \frac{\rho_H - \rho_a}{\rho_H} g h_H \sin \theta - \frac{\rho_H - \rho_a}{\rho_H} g h_H \cos \theta \frac{\partial z_b}{\partial x} \\ - \frac{h_H}{\rho_H} \frac{\partial}{\partial x} ((\rho - \rho_a) g h \cos \theta) \\ + \frac{n_s \rho}{\phi_{sH} \rho_s} u W_s \cos \theta - \frac{\phi_{sD}}{\phi_{sH}} u_H D \cos \theta \\ + \frac{\tau_c}{\rho_H} - \frac{\tau_b}{\rho_H}, \end{aligned} \quad (9)$$

where the subscript H denotes the basal high particle concentration current, ϕ_{sH} is the mean volume fraction of solid particles in the basal current, ϕ_{sD} is the volume fraction of solid particles in the deposit, D is the mean deposition speed at the bottom of the basal current, z_b is the height of the contact between the basal current and the deposit, and τ_b is the basal drag (see Eq. (A.3) of Shimizu et al. 2019). The basal current is assumed to have a constant bulk density $\rho_H = \phi_{sH} \rho_s + (1 - \phi_{sH}) \rho_{gH}$, where $\rho_{gH} (\equiv p/(R_a T_1))$ is the density of the gas phase in the basal current, and T_1 is the initial temperature of the upper dilute current (see the method section).

The deposit progressively aggrades upward from the bottom of the basal or dilute current. The aggradation rate of material in the deposit is as follows.

$$\frac{\partial z_b}{\partial t} = \begin{cases} D \cos \theta & \text{(Aggradation from basal current),} \\ \frac{n_s \rho}{\phi_{sD} \rho_s} W_s \cos \theta & \text{(Aggradation from dilute current).} \end{cases} \quad (10)$$

The aggradation for the dilute current occurs when the particle-settling rate at the bottom of the dilute current is lower than the deposition rate of the basal current at the position where the basal current is absent (i.e., the two conditions that the right-hand side of Eq. (8) < 0 and $h_H = 0$ are satisfied). In this study, we determined the value of D for the bedload on the basis of comparison with the PELE experiment.

When the basal current flows as a dense fluidized granular flow (e.g., Girolami et al. 2008, 2010; Roche et

al. 2008), the mean deposition speed D can be obtained using the hindered-settling model (Druitt et al. 2007; Girolami et al. 2008, 2010):

$$D = \frac{\phi_{sH}}{\phi_{sD} - \phi_{sH}} W_{sH}, \quad (11)$$

$$W_{sH} = (1 - \phi_{sH})^m W_s, \quad (12)$$

where W_{sH} is the hindered settling speed of solid particles, and m is an empirical exponent (typically 2 to 6 for very fine and well sorted materials with volcanic particles up to 0.25 mm, and 7 to 12 for poorly sorted materials with particles up to 4 mm; Druitt et al. 2007). These formulae allow us to apply the hindered-settling model for the case of $\phi_{sH} < \phi_{sD}$ such as the bedloads ($\phi_{sH} \sim 10^{-2}$; see Fig. 4).

Appendix 2. Validation of the hindered-settling model for a dense underflow

In previous two-layer PDC models (Doyle et al. 2008; Kelfoun 2017; Shimizu et al. 2019), the basal layer of the stratified PDC is modeled as a dense underflow. The sedimentation process in dense underflow has been explained by the hindered-settling model on the basis of experiments of initially fluidized granular flows (Girolami et al. 2010). In this appendix, the results of a numerical simulation using the hindered-settling model (i.e., Eqs. (11) and (12)) are compared with the experimental results of an initially fluidized granular flow (Girolami et al. 2008).

In the numerical simulation, on the basis of the experimental setting, a stationary dense mixture in a rectangular-lock domain of length x_0 and height h_{H0} is released at time $t = 0$ and it flows into a one-dimensional horizontal channel (i.e., slope angle $\theta = 0^\circ$) at $t > 0$. The dynamics of the dense current with thickness $h_H(x, t)$ and velocity $u_H(x, t)$ is described by the conservation equation of solid particle mass (i.e., Eq. (8) where the first term on the right-hand side is set to 0) and that of solid particle momentum (i.e., Eq. (9) where the third, fourth, and sixth terms on the right-hand side are set to 0). The aggradation rate of material in the deposits is given by Eq. (10) where only the case of the aggradation from the basal current is taken into account.

The values of input parameters (i.e., x_0 , h_{H0} , ϕ_{sH} , ρ_s , C_{db} , and D) are determined by the experimental data at $t = 0$ and existing models (see Table 2). The length and thickness of the initial dense mixture (x_0 and h_{H0}) and the mean solid volume fraction in the dense current (ϕ_{sH}) are obtained from Table 2 of Girolami et al. (2015) under the additional assumption of $\phi_{sD} = 0.6$. The mean solid density (ρ_s) is estimated by $(\sum(n_{s,i}/\rho_{s,i}))^{-1}$, where $n_{s,i}$ and $\rho_{s,i}$ are the solid mass fraction and solid density for each i -th particle class obtained from the experimental data (Table 1 of Girolami et al. 2008 and Table 1 of Druitt et al. 2007). The basal drag coefficient (C_{db}) is estimated on the basis of the empirical formula of Hager (1988). The mean deposition speed (D) is estimated on the basis of the hindered-settling model (i.e., Eqs.

(11) and (12)). The values of the other input parameters are the same as those used in the numerical simulations of the two-layer PDC model (see Table 1).

In the numerical simulation, a dense current flows into the horizontal channel, and it produces the deposits by progressive aggradation at a constant speed D during flowing (see Supplementary Movie 3 (ESM 5)); the dense current eventually stops its further propagation at the distal point where the horizontal mass flux of the current at the front becomes zero owing to basal deposition (cf. Shimizu et al. 2019). These features are consistent with the sedimentation process observed in the experiments (Girolami et al. 2008). The numerical results also quantitatively reproduce the experimental data of the time-series of the flow front position (Fig. 5a) and that of the profile of the final deposit height (Fig. 5b). Details of this experimental validation will be reported in future work.

Appendix 3. Notation

We provide the list of the mathematical symbols used in this paper (Table 3).

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Captions

Fig. 1 Schematic illustration of a series of numerical simulations of the two-layer PDC model. (a) A two-layer PDC is generated from the inlet boundary ($x = 0$) and flows into a proximal inclined channel with length 9.68 m (slope angle $\theta = 6^\circ$) and into a distal horizontal channel ($\theta = 0^\circ$). (b) There are three interfaces, between the dilute current with thickness h and the ambient air ($z = z_f$), between the basal current with thickness h_H and the dilute current ($z = z_c$), and between the deposit and the basal current ($z = z_b$). (c) The inlet mass flow rates of the dilute and basal currents are given as constant values for time $t = 0-4$ s and as 0 for $t > 4$ s

Table 1 Input parameters and constants for simulation of the two-layer PDC model

Fig. 2 Comparison of the numerical results for the dilute layer with the experimental data for the dilute-part flow at PELE. (a) Front position of the dilute current as a function of time t ($x_N(t)$). Solid curve represents the numerical result. Circles represent the experimental data. (b and c) Thicknesses of the dilute current at $x = 2.65$ and 7.78 m as a function of time t ($h(x = 2.65, t)$ and $h(x = 7.78, t)$). Black solid curves represent the numerical results, where the thick head passes initially and the body passes subsequently at $x = 2.65$ and 7.78 m. Gray solid lines represent the time-dependent fitting functions of body thickness of the experimental dilute-part flow ((b) $4.6(0.3215 - 0.01421t)$ m; (c) $4.6(0.2721 + 0.003674t)$ m)

Fig. 3 Comparison of the numerical results for the basal layer and deposit with the experimental data for the bedload and deposit at PELE. Solid curves represent the numerical results. Circles represent the experimental data. (a and b) Thicknesses of the bedload at $x = 2.65$ and 7.78 m as a function of time t ($h_H(x = 2.65, t)$ and $h_H(x = 7.78, t)$). (c) Final deposit mass per unit area as a function of distance x

$$(\phi_{sD}\rho_s z_b(x, t = \infty))$$

Fig. 4 Ratio of mean deposition speed (D) to terminal velocity (W_s) as a function of volume fraction of solid particles (ϕ_{sH}). Gray region represents the prediction by the hindered-settling model (i.e., Eqs. (11) and (12) with the empirical exponent $m = 2-12$ (Druitt et al. 2007)). Circle with error bars represents the relationship between the estimated values of D/W_s for the bedload and the experimental observation of ϕ_{sH} for the bedload

Table 2 Input parameters and constants for simulation of the basal-layer model for initially fluidized granular flow

Fig. 5 Comparison of numerical results for the basal-layer model based on the hindered-settling model (black curves) with experimental data for an initially fluidized granular flow (gray curves; Girolami et al. 2008). (a) Front position of the dense current as a function of time t ($x_{NH}(t) - x_0$). The magnitude of typical errors in the experimental data is expressed by the thickness of the gray curve. (b) Final deposit height as a function of distance $x - x_0$ ($z_b(x, t = \infty)$)

Table 3 List of symbols

Figures

Figure 1

Schematic illustration of a series of numerical simulations of the two-layer PDC model.

Figure 2

Comparison of the numerical results for the dilute layer with the experimental data for the dilute-part flow at PELE.

Figure 3

Comparison of the numerical results for the basal layer and deposit with the experimental data for the bedload and deposit at PELE. Solid curves represent the numerical results. Circles represent the experimental data

Figure 4

Ratio of mean deposition speed

Figure 5

Comparison of numerical results for the basal-layer model based

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