Time-optimal Feedrate Scheduling with Actuator Constraints for 5-axis Machining

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Time-optimal Feedrate Scheduling with Actuator Constraints for 5-axis Machining

Woraphrut Kornmaneesang and Shyh-Leh Chen

Abstract— Cycle time minimization is one of the major goals that many manufacturers are eager to achieve. Maximizing feedrate is the direct solution, however, physical motions need to be under the specified motion limits to avoid high-frequency vibration, causing machining error. In this paper, a time-optimal feedrate scheduling approach for 5-axis G1 toolpath is presented for 5-axis machining. A quintic B-spline corner smoothing method is utilized to smoothen sharp corners in the toolpath. Then, the S-shape feedrate profile of each block is optimized under the actuator motion constraints, with the objective of minimizing the cycle time. Particle swarm optimization (PSO) is used to provide the optimized solution. Experiments are conducted to validate the proposed approach and the results are compared with two other existing approaches. It is found that the proposed method can achieve shorter cycle time and less contour errors, showing the effectiveness of the proposed approach.

Keywords— feedrate scheduling, corner smoothing, particle swarm optimization (PSO), robotic machining

I. INTRODUCTION

In manufacturing, the cycle time is considered as one of the most important factors that every manufacturer desire to minimize. However, reducing the cycle time without degrading the machining accuracy is challenging. Smooth motion planning that plays a key role in accomplishing the objective is composed of two fundamental elements: smoothness (continuity) in toolpath and smooth feedrate scheduling. Given a toolpath, infinite possible feedrate profiles can be designed along the same toolpath. Different feedrate profiles will lead to different machining results. In particular, nonsmooth feedrate profile will result in motion discontinuities which degrade the machining accuracy. Besides, maximizing the feedrate implies the cycle time reduction. Hence, the time-optimal smooth feedrate scheduling is one of the essential components to achieve the next-generation high-speed machining.

In machining, toolpaths generated from computer-aided manufacturing (CAM) software are mostly defined by a set of G1 command points. In other words, they are approximately represented using a series of short line segments. Sharp corners between every two line segments are the major cause of toolpath and motion discontinuities that excite high-frequency vibration to machining systems, degrading the machining

quality [1]. Thus, to remedy the problem, corner smoothing should be considered to achieve the smoothness in the toolpath.

To smoothen the sharp corners and control the trajectory geometry, many corner smoothing algorithms have been introduced thus far. The most commonly-used corner smoothing method is the circular arc method [2]. Although a lot of commercial computer numerical control (CNC) controllers have integrated this user-friendly method in their products as a machining strategy, only the tangential continuity is guaranteed. Then, numerous spline corner smoothing methods with different degrees and types, e.g. quartic/quintic Bezier splines [3, 4], cubic/quintic B-spline [5, 6], Pythagorean-hodograph (PH) spline [7], and clothoid splines [8], were presented. However, none of them considers 5-axis toolpath, in which the corner smoothing should handle the sharp corners in the orientation toolpath as well as those in the position toolpath, since the discontinuities in the orientation toolpath also lead to the vibration and machining error. In addition, synchronizing the position and orientation toolpaths is another difficulty.

For the 5-axis corner smoothing, Tulsyan and Altintas proposed a novel cornering method inserting quintic and septic B-splines for smoothing corners in the position and orientation toolpaths, respectively [9]. Nonetheless, in their proposed method, calculation of the control points to synchronize the two toolpaths is too complicated. Thus, the Newton-Raphson optimization algorithm is deployed to numerically determine the control points. To avoid using the numerical analysis tool, Yang et al. presented to smoothen the position and orientation toolpaths in the workpiece coordinate system (WCS) and the machine coordinate system (MCS), respectively [10, 11]. As a result, the synchronization can be achieved analytically without considering the constraint of the unit orientation vector. However, the above methods can only be implemented with the conventional 5-axis machine tools which simply embody 3 translational and 2 rotational drives. Meanwhile, there has recently been an increasing trend of using alternative machining technologies such as robotic machining systems in industry due to the satisfactory cost efficiency, high flexibility, and multi-functionality of industrial robots [12]. The above corner smoothing approaches [9-11] cannot directly work with such robotic machining systems because of nonlinear coupling kinematics of the robot.

On the other hand, one of the key factors to deliver the smooth feedrate profile is to confine jerk under a specific limit. Thus, many jerk-limited feedrate designs, such as time-optimal [13, 14], sine-shape [15], and S-shape [16-18], were introduced. Among others, the S-shape feedrate design is commonly adopted due to its simplicity and smoothness. Du et al. investigated a complete S-shape feedrate scheduling approach (CSFA) [17], while analytical feedrate scheduling methods

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with various modified S-shape designs were presented in [18]. Nonetheless, the aforementioned works were dedicated to only bi- or tri-axis translational machining. For 5-axis machining, it is necessary to plan the feedrate profile for the orientation toolpath as well as for the position toolpath, and furthermore, time synchronization is also required to merge both feedrate profiles. Huang et al. proposed a simultaneous linear and angular trajectory planning (SLATP) method to plan the time-synchronized feedrate profile with considering linear and angular motion constraints [19]. However, in addition to the linear and angular motion constraints, the actuator motion constraints should be considered since the actuator motion commands directly affect the system dynamics.

In most of the above feedrate scheduling methods, the feedrate profiles are planned separately for each block so that velocity at the junction may be discontinuous. Even though bi-directional scanning is used to make them continuous, it is required to analyze and establish the feedrate profile for the entire toolpath in one shot [17-19]. Due to memory limit of the CNC controller, the methods incorporating with the bi-directional scanning technique are able to work with only a limit number of blocks. Moreover, most of the studies on the G1 toolpaths implemented only the circular arc cornering method to smoothen the toolpath prior to scheduling the feedrate profile. Consequently, the acceleration and jerk may not be smooth since the curvature at the junction of line and corner segments is not continuous.

The objective of this paper is to propose a time-optimal S-shape feedrate scheduling approach for 5-degree-of-freedom (DOF) machining. The proposed method not only can be applied to the conventional 5-axis CNC machine tools, but also can be applied to other 5-axis machining systems, such as robotic machining systems, etc. It is assumed that the given command path is composed of G1 line segments with prescribed orientation. The position tool path with orientation will be regarded as a path in 5-dimensional (5-D) space. The 5-D space consists of the 3-D task space for position tool path and 2-D spherical coordinate system (SCS) for orientation. In other words, the command path can be considered as a piecewise linear 5-D curve. The feedrate scheduling and corner smoothing will be conducted on this 5-D path. In this way, the feedrate scheduling for both position and orientation toolpaths is integrated in one platform, and no synchronization is needed.

The PSO method proposed in [16] is utilized to obtain optimal parameters for the S-shape feedrate and for the smooth corner under various motion constraints. To achieve jerk-limited feedrate, a quintic B-spline corner smoothing method, instead of circular arc cornering used in [16], is employed here. The proposed method can smoothen the corners of the 5-D G1 toolpath according to user-defined error tolerances. Then, concerning each block, the feedrate profile is planned by PSO with respect to the actuator motion constraints separately and in succession. Finally, compared with two existing methods [17, 19], experiments will be carried out to highlight benefits of the high-order corner smoothing and effectiveness of the feedrate scheduling with considering the actuator motion constraints, instead of the tangential (linear and angular) motion constraints.

To show feasibility of the proposed approach for more general 5-axis machining systems, the target system of this

study is a 5-DOF dual-arm robotic machining system presented in [20]. In the system, two 5-DOF serial robot manipulators are used to hold a workpiece and work with a cutting tool fixed on a stationary base, forming 5-DOF machining configuration. Its structural kinematics is much more nonlinear and coupling than the conventional 5-axis CNC machine tool. The kinematics complexity leads to complicated relationship between the tangential motion and the actuator motion. As a consequence, the corner smoothing and the feedrate scheduling with actuator constraints for the 5-axis robotic machining are much more challenging than the conventional one. Moreover, the approaches of corner smoothing and feedrate scheduling presented in this paper can equally be applied to the conventional 5-axis CNC machine tool and other 5-axis robotic machining systems [12].

The contributions of the paper include

- The feedrate scheduling and corner smoothing for both position and orientation toolpaths are integrated in a 5-D space, and no synchronization is needed.
- The orientation toolpath is described in the SCS, instead of the commonly used unit sphere.
- The high-order corner smoothing, i.e. quintic B-spline corner smoothing, is proposed to achieve better corner smoothing.
- A robotic machining system with actuator motion constraints is studied for feedrate scheduling.

The rest of the paper is structured as follows. In Section II, the quintic B-spline is introduced to establish the corner smoothing for 5-D G1 toolpath, while Section III presents a time-optimal S-shape feedrate scheduling method based on PSO. Simulation and experimental results on the 5-DOF dual-arm robotic platform are shown in Section IV, and conclusion remarks are provided in Section V.

II. CORNER SMOOTHING FOR 5-AXIS G1 TOOLPATH

In 5-axis machining, a toolpath is in general composed of the tool position $\mathbf{P} = [P_x \ P_y \ P_z]^T$ and the tool orientation $\mathbf{O} = [O_i \ O_j \ O_k]^T$, which must be a unit vector. These two components are represented in the WCS (also called task coordinate system in the field of robotics). On the other hand, displacements of revolute joints denoted as $\mathbf{q} \in \mathbb{R}^5$ that are driven by actuators are defined in the MCS. Kinematic chains, i.e. forward kinematics and inverse kinematics, are used to transfer the coordinates between these two coordinate systems.

A G1 toolpath generated from CAM software can be represented as a series of line segments, in which end points are defined by G1 command points. For 5-axis G1 toolpath, each command point usually contains cutting position $\mathbf{P}$ and tool orientation $\mathbf{O}$, as demonstrated in Fig. 1(a). Between every two line segments, there exists a corner that lacks continuity of the tangent vector. The discontinuity excites high-frequency vibration to machining systems, degrading the machining accuracy [1]. Hence, in this section, the corners in the toolpath are smoothened satisfying the C^3 continuity under user-defined corner tolerances ε_{pw} and ε_{ow} , as depicted in Fig. 1(a), prior to scheduling feedrate profile.

The orientation toolpath is usually considered on a unit sphere surface representing the collection of tool orientation unit vectors. Although spline-based corner segments can be inserted into the toolpath, analysis of the control point location is difficult due to the constraint that magnitude of the tool orientation vector must be unity [9]. Hence, the orientation toolpath can be represented in the SCS as:

$$\mathbf{Z} = \begin{bmatrix} \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \cos^{-1}(O_k) \\ \tan^{-1}(O_j/O_i) \end{bmatrix} \quad (1)$$

In other words, the 5-axis G1 toolpath can be represented in the 5-D coordinates $\xi = [P_x \ P_y \ P_z \ \theta \ \phi]^T$, $\xi \in \mathbb{R}^5 \times [0, \pi] \times [-\pi, \pi]$.

The 5-D toolpath can still be treated as a piecewise linear toolpath, in which the corners need to be smoothened. Thus, the B-spline segments are inserted into the toolpath as illustrated in Fig. 1(b). For the sake of the C^3 continuity, location of the B-spline control points has to be determined properly. On the other hand, to control the toolpath geometry satisfying the user-defined corner tolerances ε_{pw} and ε_{ow} , analysis of the control points is considered separately as discussed below.

A. Corner Smoothing of Position Toolpath

Endpoints of the position toolpath are denoted as P_i , $i = 0, 1, L, N$, where N is the number of line segments. In order to avoid the discontinuity at the corner, a B-spline corner with $(n+1)$ control points will be adopted and is defined as:

$$\mathbf{Q}(u) = \sum_{i=0}^n B_{i,k}(u) \mathbf{p}_i; \quad 0 \leq u \leq 1 \quad (2)$$

where $\mathbf{p}_i = [p_{xi} \ p_{yi} \ p_{zi}]^T$, $i = 0, 1, L, n$ are the B-spline control points, and $B_{i,k}(u)$ is the k th-degree B-spline basis function of the spline parameter u defined on the nonperiodic knot vector $\mathbf{U}$ as introduced in [21].

To achieve the C^3 continuity, the degree and number of control points are chosen as 5 and 7, respectively. For the sake of symmetry across the angular bisector of the corner angle, the knot vector is designed as:

$$\mathbf{U} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.5 \ 1 \ 1 \ 1 \ 1 \ 1] \quad (3)$$

The C^3 continuity along the B-spline corner is guaranteed by the choice of degree. However, it is also required the C^3 continuity at the junctions between the line segments and the B-spline corner, which can be achieved by properly locating the control points. Considering the control points of the B-spline corner between two line segments $\overline{P_0P_1}$ and $\overline{P_1P_2}$, firstly, $\mathbf{p}_0$ and $\mathbf{p}_6$ must be the starting ($u = 0$) and ending ($u = 1$) points of the corner, respectively, as illustrated in Fig. 2. For C^3 continuity, the tangent vectors of the B-spline curve at two end points of the corner ($u = 0, 1$) must be parallel to the line segments. Thus, it is required that the control points be such that

Fig. 1. Representation of position and orientation toolpath in: (a) workpiece coordinate system, (b) 5-D coordinate system

Fig. 2. Corner smoothing of the position toolpath in the WCS

Fig. 3. Corner smoothing of the orientation toolpath in the SCS

$$\left. \frac{dQ}{ds} \right|_{u=0} = \frac{\overline{P_0 P_0}}{\|P_0 P_0\|}; \quad \left. \frac{dQ}{ds} \right|_{u=1} = \frac{\overline{P_6 P_2}}{\|P_6 P_2\|} \quad (4)$$

$$\left. \frac{d^2 Q}{ds^2} \right|_{u=0,1} = 0; \quad \left. \frac{d^3 Q}{ds^3} \right|_{u=0,1} = 0 \quad (5)$$

To satisfy the tangency continuity condition (4), let us locate p_0 , p_1 , and p_2 on the line segment $\overline{P_0 P_1}$ and p_4 , p_5 , and p_6 on the line segment $\overline{P_1 P_2}$, respectively. For symmetry of the corner, the p_3 is placed at the corner point P_1 , as shown in Fig. 2. Then, to smoothen the corner under the user-defined corner error ε_{pw} and satisfy the C^3 continuity (4) and (5), following [9], the location of the control points can be obtained as:

$$\begin{aligned} p_3 &= P_1; \quad p_2 = p_3 + l_p \overline{l_{pa}}; \quad p_4 = p_3 + l_p \overline{l_{pb}} \\ p_1 &= 2p_2 - p_3; \quad p_0 = 2.5p_2 - 1.5p_3 \\ p_5 &= 2p_4 - p_3; \quad p_6 = 2.5p_4 - 1.5p_3 \end{aligned} \quad (6)$$

with

$$l_p = \min \left\{ \frac{4\varepsilon_{pw}}{3\cos(\alpha/2)}, \frac{L_{pa}}{5}, \frac{L_{pb}}{5} \right\} \quad (7)$$

where $\overline{l_{pa}} = \overline{P_1 P_0} / \|\overline{P_1 P_0}\|$, $\overline{l_{pb}} = \overline{P_1 P_2} / \|\overline{P_1 P_2}\|$, $L_{pa} = \|\overline{P_1 P_0}\|$, $L_{pb} = \|\overline{P_1 P_2}\|$; α is the corner angle. The derivation can be found in Appendix II. Here, l_p is a tuning parameter to determine the control points p_2 and p_4 . They will be closer to the corner point P_1 (i.e., p_3) for smaller l_p and farther for larger l_p . The first term in the minimum function in (7) is for the requirement of corner error. The second and third terms are to avoid the situation that the inserted corner occupies over half length of the original line segment.

B. Corner Smoothing of Orientation Toolpath

Similar to the position toolpath, endpoints of the orientation toolpath are denoted as Z_i , $i=0,1,L,N$. Following the same line in the position toolpath corner smoothing, the corner is inserted by the quintic B-spline corner expressed as:

$$R(u) = \sum_{i=0}^6 B_{i,5}(u)z_i; \quad 0 \leq u \leq 1 \quad (8)$$

where $z_i = [z_{\theta_i} \ z_{\dot{\theta}_i}]^T$, $i=0,1,L,6$ are the B-spline control points. The knot vector (3) is also implemented here. The spline parameter u is shared with the position spline function in order to synchronize the orientation toolpath with the position toolpath. As a result, it is no need to schedule the feedrate profiles for the two toolpaths, separately. Only the feedrate scheduling for only the position toolpath is required. Similar to the position corner smoothing, the C^3 continuity along the toolpath can be achieved simply by the proper choice of the degree of the B-spline basis function.

On the other hand, to achieve the C^3 continuity at the junctions between the corner and line segments, the control points need to be chosen properly. They also need to be chosen to meet the corner tolerance for orientation ε_{ow} . As a result, the design of control points for the orientation toolpath is more involved than that for the position toolpath. This is because that ε_{ow} depends not only on the corner error in SCS ($\|\Delta Z\|$ in Fig. 3), but also on the position toolpath. Let us consider a corner

formed by two line segments $\overline{Z_0 Z_1}$ and $\overline{Z_1 Z_2}$ as depicted in Fig. 3. Similar to (4) and (5), the C^3 continuity of the toolpath requires the control point to satisfy:

$$\left. \frac{dR}{ds} \right|_{u=0} = \frac{\overline{Z_0 Z_0}}{\|P_0 P_0\|}; \quad \left. \frac{dR}{ds} \right|_{u=1} = \frac{\overline{Z_6 Z_2}}{\|P_6 P_2\|} \quad (9)$$

$$\left. \frac{d^2 R}{ds^2} \right|_{u=0,1} = 0; \quad \left. \frac{d^3 R}{ds^3} \right|_{u=0,1} = 0 \quad (10)$$

Through the derivation process provided in Appendix III, the control points to achieve the condition (10) can be obtained as:

$$\begin{aligned} z_3 &= Z_1; \quad z_2 = z_3 + l_{oa} \overline{l_A}; \quad z_4 = z_3 + l_{ob} \overline{l_B} \\ z_1 &= 2z_2 - z_3; \quad z_0 = 2.5z_2 - 1.5z_3 \end{aligned} \quad (11)$$

where $\overline{l_A} = \overline{Z_1 Z_0} / \|\overline{Z_1 Z_0}\|$ and $\overline{l_B} = \overline{Z_1 Z_2} / \|\overline{Z_1 Z_2}\|$, while the tuning parameters l_{oa} and l_{ob} are to be determined to meet the condition (9) and the corner tolerance ε_{ow} .

Firstly, by the chain rule, (9) can be written as:

$$\left. \frac{dR}{ds} \right|_{u=0,1} = \left(\frac{dR}{du} \frac{du}{ds} \right) \Big|_{u=0,1} \quad (12)$$

where $(du/ds)|_{u=0,1} = 1/\|dQ/du\|_{u=0,1} = 1/5l_p$. Secondly, from (9) and (12), we can obtain

$$\left. \frac{dR}{du} \right|_{u=0} = 5l_p \frac{\overline{Z_0 Z_0}}{\|P_0 P_0\|} = 10z_0 z_1; \quad \left. \frac{dR}{du} \right|_{u=1} = 5l_p \frac{\overline{Z_6 Z_2}}{\|P_6 P_2\|} = 10z_5 z_6 \quad (13)$$

Note that the vector $\overline{Z_0 Z_0}$ and $\overline{Z_6 Z_2}$ are the same direction with $\overline{z_0 z_1}$ and $\overline{z_5 z_6}$, respectively. From (11) to (13), we get

$$\frac{l_{oa}}{l_p} = \frac{\|Z_0 Z_1\|}{\|P_0 P_1\|} = \frac{\|z_0 z_1\|}{\|p_0 p_1\|} = \frac{\|Z_0 Z_0\| + \|z_0 z_1\|}{\|P_0 P_0\| + \|p_0 p_1\|} = \frac{\|Z_0 Z_1\|}{\|P_0 P_1\|} \quad (14)$$

$$\frac{l_{ob}}{l_p} = \frac{\|Z_1 Z_2\|}{\|P_1 P_2\|} = \frac{\|z_1 z_2\|}{\|p_1 p_2\|} = \frac{\|Z_1 Z_6\| + \|z_1 z_2\|}{\|P_1 P_6\| + \|p_1 p_2\|} = \frac{\|Z_1 Z_2\|}{\|P_1 P_2\|} \quad (15)$$

Hence, to ensure the condition (9), the parameters l_{oa} and l_{ob} are proportional to the tuning parameter l_p for the corner smoothing of the position toolpath. The proportional ratio is related to the lengths of line segments:

$$l_{oa} = \frac{\|Z_0 Z_1\|}{\|P_0 P_1\|} l_p = k_1 l_p; \quad l_{ob} = \frac{\|Z_1 Z_2\|}{\|P_1 P_2\|} l_p = k_2 l_p \quad (16)$$

Thus, once l_p is given, the control points of the orientation corner segment can be determined by (11) and (16).

Next, the conditions on l_p will be studied to limit the maximum orientation corner error under the user-defined orientation corner error ε_{ow} . Let us define $\|\Delta O\|$ as the maximum deviation of the tool orientation vectors that meet the tolerance ε_{ow} . Mathematically, it can be written as:

$$\|\Delta O\| \leq \left| 2 \sin \left(\frac{\varepsilon_{ow}}{2} \right) \right| \quad (17)$$

Then, $\|\Delta O\|$ can be approximated by corner error in SCS as:

Fig. 4. The S-shape feedrate profile

Fig. 5. Feedrate profile for multiple blocks

$$\|\Delta \mathbf{O}\| = \|\mathbf{J} \Delta \mathbf{Z}\|; \text{ with } \mathbf{J} = \begin{bmatrix} \cos(\theta) \cos(\phi) & -\sin(\theta) \sin(\phi) \\ \cos(\theta) \sin(\phi) & \sin(\theta) \cos(\phi) \\ -\sin(\theta) & 0 \end{bmatrix} \quad (18)$$

where the Jacobian function $\mathbf{J}$ should be evaluated at θ and ϕ of the corner point $\mathbf{Z}_1$. The corner error in the SCS, i.e. $\Delta \mathbf{Z}$, can be obtained as:

$$\begin{aligned} \Delta \mathbf{Z} &= \mathbf{z}_3 - \mathbf{R}(0.5) \\ &= \frac{1}{16} \mathbf{z}_3 \mathbf{z}_1 + \frac{1}{4} \mathbf{z}_3 \mathbf{z}_2 + \frac{1}{4} \mathbf{z}_3 \mathbf{z}_4 + \frac{1}{16} \mathbf{z}_3 \mathbf{z}_5 \\ &= \frac{3}{8} (\mathbf{l}_{oa} \mathbf{l}_A + \mathbf{l}_{ob} \mathbf{l}_B) = \frac{3}{8} \mathbf{l}_p (k_1 \mathbf{l}_A + k_2 \mathbf{l}_B) \end{aligned} \quad (19)$$

From (17) to (19), we can obtain

$$l_p \leq \frac{16}{3} \frac{\sin(\varepsilon_{ow}/2)}{\|\mathbf{J}(k_1 \mathbf{l}_A + k_2 \mathbf{l}_B)\|} \quad (20)$$

Finally, combining with the conditions imposed for the corner smoothing of the position toolpath (7), l_p should be updated as:

$$l_p = \min \left\{ \frac{4\varepsilon_{pw}}{3\cos(\alpha/2)}, \frac{L_{pa}}{5}, \frac{L_{pb}}{5}, \frac{16}{3} \frac{\sin(\varepsilon_{ow}/2)}{\|\mathbf{J}(k_1 \mathbf{l}_A + k_2 \mathbf{l}_B)\|} \right\} \quad (21)$$

Here, the sharp corners in the 5-axis G1 toolpath are inserted by the B-spline corner segments under the user-defined corner errors ε_{pw} and ε_{ow} . As a result, the smoothed 5-axis G1 toolpath possesses the C^3 continuity. Since the position and orientation toolpaths can be regarded as being smoothed in a 5-D coordinate system with the same spline parameter u , they

Table 1. Formulas for the S-shape feedrate profile

τ	$J(t)$	$a(t)$	$v(t)$	$d(t)$
τ_1	J_1	$J_1 \tau_1$	$v_s + \frac{1}{2} J_1 \tau_1^2$	$v_s \tau_1 + \frac{1}{6} J_1 \tau_1^3$
τ_2	$-J_2$	$A_A - J_2 \tau_2$	$v_2 + A_A \tau_2 - \frac{1}{2} J_2 \tau_2^2$	$s_2 + v_2 \tau_2 + \frac{1}{2} A_A \tau_2^2 - \frac{1}{6} J_2 \tau_2^3$
τ_3	0	0	v_c	$s_3 + v_c \tau_3$
τ_4	$-J_4$	$-J_4 \tau_4$	$v_c - \frac{1}{2} J_4 \tau_4^2$	$s_4 + v_c \tau_4 - \frac{1}{6} J_4 \tau_4^3$
τ_5	J_5	$-A_D + J_5 \tau_5$	$v_5 - A_D \tau_5 + \frac{1}{2} J_5 \tau_5^2$	$s_5 + v_5 \tau_5 - \frac{1}{2} A_D \tau_5^2 + \frac{1}{6} J_5 \tau_5^3$

are automatically synchronized. Thus, only feedrate scheduling for the position toolpath is required. Moreover, it can be seen that no structural kinematics is involved in the corner smoothing approach. Hence, this approach is applicable to any machining system.

III. TIME-OPTIMAL S-SHAPE FEEDRATE SCHEDULING FOR THE SMOOTHENED TOOLPATH

In this section, the time-optimal feedrate profile is proposed and scheduled after the 5-axis G1 toolpath was smoothed with the B-spline corner presented in Section II. The S-shape feedrate profile as shown in Fig. 4 is adopted for each line segment. Since the orientation toolpath is synchronized with the position toolpath using the same spline parameter, the feedrate scheduling is designed based on the position toolpath. It is to establish the time-optimal feedrate scheduling under various motion constraints. In this study, the actuator (joint) motion constraints are taken into account instead of the tangential motion constraints considered in most previous studies. For the robotic machining system in this study, satisfaction of the joint motion constraints is crucial to the smooth motion due to the nonlinear kinematics and constraints. It is clear that the feedrate scheduling can be formulated as a nonlinear constrained optimization problem, in which parameters of the feedrate profile are optimized to minimize the cycle time. PSO, one of the most popular evolutionary algorithms, is utilized to solve the optimization problem with respect to the joint motion constraints.

A. Fundamental Principle of S-shape Feedrate Profile

The S-shape feedrate profile is mainly composed of 3 zones: acceleration (ACC), constant velocity (CON), and deceleration (DEC), in which they can be divided into 5 time segments, i.e. $T_i, i=1,2,L,5$, as illustrated in Fig. 4. In the figure, the feedrate profile is scheduled according to one toolpath segment with the length L and given parameters, i.e. starting velocity v_s , and ending velocity v_e . In the acceleration profile, a_A and a_D are the maximum acceleration and deceleration in the ACC zone and DEC zone, respectively. Meanwhile, in the jerk profile, J_1, J_2, J_4 , and J_5 are the jerk during T_1, T_2, T_4 , and T_5 , respectively.

Given the toolpath length L , the starting velocity v_s , the ending velocity v_e , and the time segments $T_i, i=1,2,L,5$, the S-shape feedrate profile can be generated through formulas listed in Table 1, in which $\tau_i, i=1,2,L,5$ denote the relative

time variables. The acceleration/deceleration and jerks can be calculated using the following formulas [16]:

$$a_A = \frac{[6L - 2(3T_1 + 3T_2 + 3T_3 + 2T_4 + T_5)v_s - 2(T_4 + 2T_5)v_e]}{(T_1 + T_2)(T_1 + 2T_2 + 3T_3 + 2T_4 + T_5)} \quad (22)$$

$$a_D = \frac{[6L - 2(2T_1 + T_2)v_s - 2(T_1 + 2T_2 + 3T_3 + 3T_4 + 3T_5)v_e]}{(T_4 + T_5)(T_1 + 2T_2 + 3T_3 + 2T_4 + T_5)} \quad (23)$$

$$a_A = J_1T_1 = J_2T_2; \quad a_D = J_4T_4 = J_5T_5 \quad (24)$$

In general, the toolpath length L and the starting velocity v_s are given as initial conditions, while the ending velocity v_e , and the time segments $T_i, i=1,2,L,5$, are taken to be the design parameters. However, it can be seen in (22) and (23) that arbitrary choice of the design parameters may yield negative a_A and a_D , resulting in the false feedrate profile. Hence, it is required to include two additional constraints

$$a_A \geq 0; \quad a_D \geq 0 \quad (25)$$

as well as the joint motion constraints, i.e. joint velocity limits $\dot{\phi}_{\max}$, acceleration limits $\ddot{\phi}_{\max}$, and jerk limits $\dddot{\phi}_{\max}$.

The design parameters, i.e. $T_i, i=1,2,L,5$ and v_e , must be nonnegative. Nonetheless, there are possibilities that some of the time segments are zero, causing the feedrate profile to lose the acceleration phase or the deceleration phase. By (24), in case T_1 (or T_2) is zero, T_2 (or T_1) must be set to zero. The same rule also applies to T_4 or T_5 . When the choice of the design parameters falls to these situations, the ending velocity v_e is no longer a design variable and will be determined as follows instead:

a) *Without ACC Zone:* T_1 and T_2 equal to zero and the starting velocity v_s is nonzero ($T_1 = T_2 = 0$ and $v_s \neq 0$)

$$v_e = \frac{3L - v_s(3T_3 + 2T_4 + T_5)}{T_4 + 2T_5} \quad (26)$$

b) *Without DEC Zone:* T_4 and T_5 equal to zero ($T_4 = T_5 = 0$)

$$v_e = \frac{3L - v_s(2T_1 + T_2)}{T_1 + 2T_2 + 3T_3} \quad (27)$$

c) *Without both ACC and DEC Zones:* T_1, T_2, T_4 , and T_5 equal to zero and the starting velocity v_s is nonzero ($T_1 = T_2 = T_4 = T_5 = 0$ and $v_s \neq 0$). T_3 also needs to be modified as

$$T_3 = L/v_s \quad (28)$$

$$v_e = v_s \quad (29)$$

B. Feedrate Profile for Multiple Line Segments

A G1 toolpath composed of multiple line segments is usually defined by G1 command points P_i . In another perspective, the point $P_i, i=1,2,L,N-1$, also serves as corner points. After smoothening the corners, let us define the middle point of the inserted corner segment as the new corner point, i.e. $c_i, i=1,2,L,N-1$, as shown in Fig. 5. For these multiple segments, i.e. $\langle P_0c_1 \rangle, \langle c_1c_2 \rangle, L, \langle c_{N-1}P_N \rangle$, (each called a block), the local S-shape feedrate profile is individually generated, sequentially. In this configuration, the starting

velocity of the first block and the ending velocity of the last block must equal to zero ($v_s^1 = 0$ and $v_e^N = 0$). Note that the superscript i stands for the block $i, i=1,2,L,N$. The ending velocity of the current block will be the starting velocity of the next block, i.e. $v_s^{i+1} = v_e^i, i=1,2,L,N-1$.

In the feedrate scheduling of each block, the 6 design parameters, i.e. $T_i, i=1,2,L,5$ and v_e , need to be determined under the constraint (25) and the joint motion constraints. This can be formulated as the nonlinear constrained optimization problem. Thus, PSO is adopted to optimize the design parameters in order to achieve the objective of the minimum machining time, to be presented in Section III-C below. Note that other optimization tools can also be implemented to solve the constrained optimization problem.

C. PSO for Time-optimal Feedrate Scheduling

PSO is one of the evolutionary optimization tools that iteratively updates the position of M optimizing particles (design parameters) $X_m = (x_{1m}, x_{2m}, L, x_{Dm}), m=1,2,L,M$ in a D -dimensional search space, i.e. space of design parameters, in order to approach the optimal solution while satisfying all constraints. In each iteration k , the optimizing particles are driven by the velocity vector $V_m = (v_{1m}, v_{2m}, L, v_{Dm})$. The position and velocity of the particle m can be updated corresponding to an update mechanism [16]:

$$V_m[k+1] = w[k]V_m[k] + c_1 \text{rand}(\cdot)(Y_m[k] - X_m[k]) + c_2 \text{rand}(\cdot)(Y_g[k] - X_m[k]) \quad (30)$$

$$X_m[k+1] = X_m[k] + V_m[k+1] \quad (31)$$

where w, c_1 , and c_2 are the inertia weight, the individual learning gain, and the global learning gain, respectively; $\text{rand}(\cdot)$ stands for a uniformly random number in range of $[0, 1]$; $Y_m = (y_{1m}, y_{2m}, L, y_{Dm})$ and $Y_g = (y_{g1}, y_{g2}, L, y_{gD})$ represent the individual best position of the particle m and the global best position of the swarm, respectively, which are evaluated through comparisons of the user-defined cost function $f(X_m)$. For the constrained optimization, penalty rules are necessary to deprioritize any particles that do not satisfy the constraints; as a result, the optimized designed parameters will be prevented from violating the constraints. Through iterations, the PSO will

Table 2. Pseudocode of standard PSO

Algorithm 1 Particle swarm optimization

Initialize particle randomly;

repeat

for particle $m = 1$ to M **do**

 calculate the cost function $f(X_m)$

 penalize $f(X_m)$ if violates the constraints

 update the individual best particles Y_m

end

 update the global best particles Y_g

for particle $m = 1$ to M **do**

 update the velocity vector (30)

 update the position vector (31)

end

$k = k+1$;

until stopping criterion ($k == k_{\max}$)

Table 3. Proposed time-optimal feedrate scheduling

Algorithm 2 Proposed time-optimal feedrate scheduling	
Input:	User-defined position corner error ε_{pw} and orientation corner error ε_{ow} Joint motion constraints $\dot{\theta}_{\max}^1, \dot{\theta}_{\max}^2, \dot{\theta}_{\max}^3$ G-01 command points $P_i, O_i, i = 0, 1, L, N$
Output:	Feedrate profiles and interpolation points
1:	Set $v_s^1 = 0$;
2:	for $i = 1$ to N do
3:	Determine control points for corner smoothing according to (6) and (11)
4:	Determine length of block including line and corner segments
5:	Optimize the designed parameters $T_1^i, T_2^i, T_3^i, T_4^i, T_5^i$, and v_e by Table 2 with the cost function (32) and the constraints (25) and (33)
6:	Generate a feedrate profile and interpolation points corresponding to the block i
7:	$v_s^{i+1} = v_e^i$;
8:	end

cease when the maximum number of iterations $k_{\max}$ is met. Pseudocode of the standard PSO is presented in Table 2.

To achieve the time-optimal feedrate scheduling in each block using PSO, the particle X is used to denote the design parameters T_1, T_2, T_3, T_4, T_5 , and v_e , i.e. $X = (T_1, T_2, T_3, T_4, T_5, v_e)$. For time optimization in the feedrate scheduling, the cost function is defined as:

$$f(X) = T_1 + T_2 + T_3 + T_4 + T_5 \quad (32)$$

Besides, constraints of the optimization problem include (25) and the joint motion constraints, i.e. joint velocity, acceleration, and jerk limits:

$$|\dot{\theta}(t)| \leq \dot{\theta}_{\max}^1, |\ddot{\theta}(t)| \leq \ddot{\theta}_{\max}^1, |\dddot{\theta}(t)| \leq \dddot{\theta}_{\max}^1 \quad (33)$$

Although v_e is not involved in the cost function (32), it is confined by the motion constraints (33). Recall that violating the constraints (25) can fail to schedule the feedrate profile, while the constraints (33) can be violated with low preference. Let us define (25) and (33) as the *hard constraint* and *soft constraint*, respectively. Hence, the penalty rules can be set that the cost function will be penalized to be infinity when the hard constraint is violated, and will be multiplied by a large positive constant when the soft constraint is violated, i.e.

$$f(X) = \begin{cases} \infty; & \text{hard constraint violated} \\ f(X) \cdot c_p; & \text{soft constraint violated} \end{cases} \quad (34)$$

where $c_p \gg 1$. Note that the tangential motion constraints can also be added in the constraints of the optimization problem. However, the issue is not explored further here for simplicity.

Given a 5-axis G1 toolpath, in which corners are smoothened according to Section II, the time-optimal feedrate profile for each block will be established using PSO with the cost function (32) and the penalty rules (34). Then, the smooth toolpath can be interpolated corresponding to the optimized feedrate profile. As a consequence of considering the joint motion constraints, the joint motion corresponding to the generated feedrate is expected to be bounded under the joint motion constraint (33). The whole procedure of the proposed feedrate scheduling is summarized in Table 3.

The proposed feedrate scheduling can generate the feedrate profile and interpolate the command toolpath for each block,

Table 4. Motion constraints of the experimental platform

Motion constraints	Velocity	Acceleration	Jerk
Linear motion constraints (m/s m/s ² m/s ³)	0.1	5.0	50
Angular motion constraints (rad/s rad/s ² rad/s ³)	0.12	10.0	100
1	0.3125	36.0	1,800
2	0.3125	36.0	1,800
Joint motion constraints (rad/s rad/s ² rad/s ³)	3	61.5	3,080
4	0.625	119.5	6,000
5	1.0	153.6	7,680

Fig. 6. The dual-arm robotic machining platform

Fig. 7. Chevron arrow toolpaths with and without smoothing

separately and in succession, unlike the bi-directional scanning based feedrate scheduling methods, which need to analyze and establish the feedrate profile for the entire toolpath in one shot [17, 19]. For real-time feedrate scheduling, the bi-directional scanning-based approaches are only able to work with a limit number of blocks due to the memory limit.

IV. SIMULATION AND EXPERIMENTAL RESULTS

In order to validate and highlight the performance of the proposed feedrate scheduling approach, simulation and experimental results were carried out. Two cases, i.e. planar G1 toolpath and 5-axis G1 toolpath, are studied. Results of the

Fig. 8. Feedrate profiles with respect to arclength

Fig. 9. Tangential motion profiles with respect to time

Fig. 10. Joint motion profiles: (left) left arm, (right) right arm

Fig. 11. Actual joint motion profiles: (left) left arm, (right) right arm

planar and 5-axis cases with the proposed approach will be compared with the CSFA [17] and the SLATP [19], respectively, which are the feedrate scheduling methods using the circular arc corner smoothing and considering only the tangential motion constraints.

The simulations and experiments were conducted by a 5-DOF dual-arm robotic machining platform which consists of a host PC, dSPACE modules, and an in-house-developed dual-arm robot machining system, as illustrated in Fig. 6. The robot is comprised of two identical 5-DOF serial robot arms that are used to hold a workpiece and cooperate with a stationary cutting tool. This is equivalent to a 5-axis machining

Fig. 12. Contour error for the planar toolpath

Fig. 13. 5-axis G1 toolpath with and without smoothing: (a) position in the WCS, (b) orientation in the SCS

Fig. 14. Feedrate profiles with respect to arclength: (top) linear, (bottom) angular

Fig. 15. Motion profiles with respect to time: (a) linear, (b) angular

Fig. 16. Joint motion profiles: (left) left arm, (right) right arm

system. Each arm embodies 5 revolute joints; therefore, the motion constraints of all the 10 joints will be considered.

In the corner smoothing, the user-defined corner errors were set to $\varepsilon_{pw} = 50 \mu\text{m}$ and $\varepsilon_{ow} = 50 \mu\text{rad}$, respectively. The sampling time is $T_s = 1 \text{ ms}$. Based on the servomotors' specifications, the tangential and joint motion constraints are specified and listed in Table 4. In the table, the joint motion constraints belong to both left and right arms due to their identical design. For the PSO, the number of particles and the

maximum iterations are set to $M = 30$, and $k_{\max} = 20$, respectively; both c_1 and c_2 are chosen as 2.5; c_p is set as 10^6 . For fast convergence, following [22], the inertia weight w can be adaptive according to the iteration k as:

$$w[k+1] = w[k] + \frac{(w_{\min} - w_{\max})}{k_{\max}} \quad (35)$$

where $w[1]$, $w_{\min}$, and $w_{\max}$ are 1.0, 0.4, and 1.0, respectively.

Fig. 17. Actual joint motion profiles: (left) left arm, (right) right arm

Fig. 18. FFT comparison of actual linear acceleration

A. Planar G1 toolpath

For planar case, a chevron arrow toolpath plotted on the XY-plane is studied and shown in Fig. 7. Since the tool orientation is fixed, the corner smoothing for the orientation toolpath and the angular motion constraints are not considered in this case.

Fig. 7 shows the chevron toolpath before and after the corners smoothed by the circular arc method and the B-spline method introduced in Section II. With the feedrate scheduling by the CSFA and the proposed approach, Fig. 8 shows the feedrate profile with respect to arc length, and Fig. 9 plots the tangential motion profiles, including velocity, acceleration, and jerk, with respect to time. Through kinematics chain of the machining system found in [23], the joint motion profiles are obtained and presented in Fig. 10. After implementing the planned feedrate commands for machining, the resulting joint motion profiles and contour error are depicted in Fig. 11 and Fig. 12, respectively. Moreover, the cycle time, and root-mean-square (RMS) and peak values of the contour error are summarized in Table 5.

As observed in Fig. 8, the feedrate planned by the proposed approach is apparently superior to the CSFA, especially during the CON phases. The area difference between the two feedrate profiles can determine amount of the cycle time that the proposed approach can reduce. This is expressed in Fig. 9. It can be seen that the proposed approach can achieve 17.9% shorter cycle time than the CSFA. The main reason behind this is that the feedrate scheduling with the CSFA considers the

Fig. 19. Error comparisons: (a) contour error, (b) tool orientation error

tangential motion limits, while the proposed approach deals with the joint motion limits. As a result, the maximum tangential velocity and jerk with the CSFA are bounded by the limits, whereas those of the proposed approach surpass them by approximately 50% at most.

In Fig. 7, the B-spline corner is compared to the circular arc corner which is fixed curvature. Thus, there may exist an abrupt change of curvature at the joints between the inserted corner and the line segments, resulting in motion discontinuities in the toolpath. The discontinuities are reflected as spikes in the jerk profile scheduled by the CSFA as seen in Fig. 10, even though all the joint motions including velocity, acceleration, and jerk are strictly within the limits. In Fig. 11, all the actual motion

Table 5. Cycle time and contour error of the experiments

Methods		Cycle time (sec)	Contour error		Tool orientation error	
			Mean (μm)	Peak (μm)	Mean (mrad)	Peak (mrad)
2-axis	CSFA [18]	4.002	11.9006	48.0571	-	-
	Proposed	3.284	11.8203	36.2364	-	-
5-axis	SLATP [20]	4.921	15.1934	66.9147	0.0498	0.2838
	Proposed	4.596	13.1110	48.1065	0.0535	0.1856

Table 6. Command points of the 5-axis G1 toolpath

No.	P_x (mm)	P_y (mm)	P_z (mm)	O_i	O_j	O_k
1	70.3931	64.5236	-2.8115	0.1845	0.2188	0.9582
2	74.6972	45.8383	-1.5763	0.2103	0.2112	0.9545
3	72.3351	21.9796	0.1774	0.2360	0.1919	0.9526
4	60.9183	0.9482	2.0505	0.2495	0.1625	0.9546
5	52.0579	-7.1712	3.4713	0.2490	0.1441	0.9577
6	36.9951	-8.3081	6.4293	0.2237	0.1180	0.9675
7	22.8180	2.1220	5.5408	0.1869	0.0979	0.9775
8	11.0843	17.2202	4.3290	0.1411	0.0794	0.9868
9	-5.1288	33.6768	2.9339	0.0807	0.0501	0.9955
10	-11.4887	38.0770	2.4154	0.0596	0.0376	0.9975
11	-20.8299	40.3019	1.3052	0.0359	0.0201	0.9992
12	-24.3979	38.8504	-0.3435	0.0322	0.0155	0.9994
13	-26.4046	29.7055	-3.1760	0.0517	0.0172	0.9985
14	-16.1765	-10.9980	-6.1725	0.1732	0.0411	0.9840
15	-15.0870	-29.8597	-6.8073	0.2200	0.0450	0.9745
16	-21.3546	-46.7830	-5.0447	0.2601	0.0407	0.9647
17	-37.0114	-57.3913	-2.6463	0.2868	0.0223	0.9577
18	-55.8292	-61.2004	-1.4745	0.2988	-0.0015	0.9543
19	-74.9840	-59.5357	-0.0877	0.3028	-0.0270	0.9527
20	-92.6066	-51.9961	1.4873	0.2968	-0.0520	0.9535

profiles behave as planned, but the jerk spikes become so severe that they exceed the limits. Furthermore, the jerk spikes violating the limit lead to peaks in the contour error, as seen in Fig. 12.

B. 5-axis G1 toolpath

In this case, a fan-shaped toolpath with varying tool orientation, which contains 20 G1 command points ($N = 19$), is taken into account and illustrated in Fig. 13. Locations of the command points are listed in Table 6.

Fig. 13 demonstrates the fan-shape toolpath before and after corners smoothed by the B-spline method. The position and orientation toolpaths are smoothed under the user-defined corner tolerances (ε_{pw} and ε_{ow}). Owing to (21), the corner tolerance is ensured smaller than defined. Also, the toolpaths are synchronized with the same spline parameter. Then, the feedrate profiles for the position and orientation toolpath are simultaneously planned by the proposed approach with the joint motion constraints. Fig. 14 plots linear and angular velocity profiles with respect to arc length scheduled by the proposed approach and the SLATP. In addition, the linear and angular motion profiles and the joint motion profiles with respect to time are depicted in Fig. 15 and Fig. 16, respectively. As to the results of the feedrate scheduling, the actual joint motion profiles are shown in Fig. 16 and 10. To underline smoothness of the machining motion, frequency analysis of the measured linear acceleration and the contour error are plotted in

Fig. 18 and 12, respectively. The resulting cycle time and contour error are summarized in Table 5.

In most of the blocks, the maximum feedrate planned by the proposed approach is noticeably higher than those of the SLATP, as observed in Fig. 14. Thus, the motion duration of the proposed approach is shorter than that of the SLATP, as summarized in Table 5. On the other hand, not only the linear and angular velocities but also the jerks scheduled by the proposed approach is mostly larger than the SLATP, as seen in Fig. 15. Considering the joint motions, the joint velocity and acceleration profiles of both approaches are very similar and all the joint motion profiles satisfy the constraints. Nonetheless, many spikes can be found in the jerk profiles planned by the SLATP, as shown in Fig. 16.

In perspectives of the experimental results, the joint velocity and acceleration of both the approaches behave the same as scheduled and they are strictly under the motion limits as plotted in Fig. 17. For the jerk profiles, although the actual jerks tend to be larger than scheduled, those of the proposed approach still satisfy the constraints. However, in case of the SLATP, the jerk spikes become much harsher so that the constraints are violated. The spikes are due to the existing jerk discontinuity in the toolpath, which the circular arc corner smoothing cannot handle. They cause the high-frequency vibration shown in Fig. 18. It can be clearly seen that the SLATP contains more high-frequency components than the proposed approach does. Consequently, the feedrate command from the proposed approach can yield better contouring results than the SLATP, as presented in Fig. 12 and summarized in Table 5.

V. CONCLUSIONS

For 5-axis machining systems, a PSO-based feedrate scheduling approach has been proposed to achieve a time-optimal feedrate profile for 5-axis G1 toolpaths. The proposed method includes two stages. Firstly, the sharp corners between line segments are smoothed by quintic B-spline corner segments. Due to the corner smoothing, the C^3 continuity is guaranteed throughout the toolpath. Furthermore, the spline functions of the position and orientation toolpaths share the same spline parameter, hence, the feedrate profiles can be scheduled with respect to only the position toolpath. With the corner smoothing, the second stage of the method is to schedule the feedrate profile for the whole toolpath based on PSO with the objective of minimizing the total machining time. The S-shape feedrate profile is adopted for each block. The actuator motion constraints are imposed during the optimization. The proposed method has been validated experimentally on a 5-DOF dual-arm robotic machining system. The results were compared with two existing methods, i.e. CSFA and SLATP. It is found that the proposed method can

achieve shorter cycle time and less contour errors, showing the effectiveness of the proposed approach.

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APPENDIX I

The equations (5) can be alternatively expressed as:

$$\begin{aligned} \left. \frac{d^2 \mathbf{Q}}{ds^2} \right|_{u=0,1} &= \left[\frac{d^2 \mathbf{Q}}{du^2} \left(\frac{du}{ds} \right)^2 + \frac{d\mathbf{Q}}{ds} \frac{d^2 u}{ds^2} \right]_{u=0,1} \\ \left. \frac{d^3 \mathbf{Q}}{ds^3} \right|_{u=0,1} &= \left[\frac{d^3 \mathbf{Q}}{du^3} \left(\frac{du}{ds} \right)^3 + 3 \frac{d^2 \mathbf{Q}}{du^2} \frac{du}{ds} \frac{d^2 u}{ds^2} + \frac{d\mathbf{Q}}{du} \frac{d^3 u}{ds^3} \right]_{u=0,1} \end{aligned} \quad (\text{I-1})$$

where

$$\frac{du}{ds} = 1 / \left\| \frac{d\mathbf{Q}}{du} \right\| = 1 / \sqrt{\left(\frac{d\mathbf{Q}}{du} \right)^T \left(\frac{d\mathbf{Q}}{du} \right)} = \frac{1}{f(u)}$$

$$\frac{d^2 u}{ds^2} = -\frac{f'(u)}{f^3(u)}$$

$$\frac{d^2 u}{ds^2} = \frac{3(f'(u))^2 - f(u)f''(u)}{f^5(u)}$$

with

$$\begin{aligned} f'(u) &= \left(\frac{d\mathbf{Q}}{du} \right)^T \left(\frac{d^2 \mathbf{Q}}{du^2} \right) / \left\| \frac{d\mathbf{Q}}{du} \right\| \\ f''(u) &= \frac{\left\{ \left\| \frac{d\mathbf{Q}}{du} \right\|^2 \left[\left(\frac{d\mathbf{Q}}{du} \right)^T \left(\frac{d^3 \mathbf{Q}}{du^3} \right) + \left(\frac{d^2 \mathbf{Q}}{du^2} \right)^T \left(\frac{d^2 \mathbf{Q}}{du^2} \right) \right] - \left[\left(\frac{d\mathbf{Q}}{du} \right)^T \left(\frac{d^3 \mathbf{Q}}{du^3} \right) \right]^2 \right\}}{\left\| \frac{d\mathbf{Q}}{du} \right\|^3} \end{aligned}$$

It is shown that the continuity conditions (5) are fulfilled when one holds

$$\left. \frac{d^2 \mathbf{Q}}{du^2} \right|_{u=0,1} = 0 \quad \text{and} \quad \left. \frac{d^3 \mathbf{Q}}{du^3} \right|_{u=0,1} = 0 \quad (\text{I-3})$$

With the B-spline function (2), the conditions (I-3) can be written as:

$$\begin{aligned} \left. \frac{d^2 \mathbf{Q}}{du^2} \right|_{u=0} &= 2\mathbf{p}_0 - 3\mathbf{p}_1 + \mathbf{p}_2 = 0; \quad \left. \frac{d^3 \mathbf{Q}}{du^3} \right|_{u=0} = -4\mathbf{p}_0 + 7\mathbf{p}_1 - 4\mathbf{p}_2 + \mathbf{p}_3 = 0 \\ \left. \frac{d^2 \mathbf{Q}}{du^2} \right|_{u=1} &= \mathbf{p}_4 - 3\mathbf{p}_5 + 2\mathbf{p}_6 = 0; \quad \left. \frac{d^3 \mathbf{Q}}{du^3} \right|_{u=1} = \mathbf{p}_3 - 4\mathbf{p}_4 + 7\mathbf{p}_5 - 4\mathbf{p}_6 = 0 \end{aligned} \quad (\text{I-4})$$

By arranging (I-4) and defining $\left\| \mathbf{p}_2 \mathbf{p}_3 \right\| = \left\| \mathbf{p}_3 \mathbf{p}_4 \right\| = l_p$, we finally obtain (6).

APPENDIX II

Equation (10) can be rewritten as:

$$\begin{aligned} \left. \frac{d^2 \mathbf{R}}{ds^2} \right|_{u=0,1} &= \left(\frac{d^2 \mathbf{R}}{du^2} \left(\frac{du}{ds} \right)^2 + \frac{d\mathbf{R}}{du} \frac{d^2 u}{ds^2} \right)_{u=0,1} \\ \left. \frac{d^3 \mathbf{R}}{ds^3} \right|_{u=0,1} &= \left(\frac{d^3 \mathbf{R}}{du^3} \left(\frac{du}{ds} \right)^3 + 3 \frac{d^2 \mathbf{R}}{du^2} \frac{du}{ds} \frac{d^2 u}{ds^2} + \frac{d\mathbf{R}}{du} \frac{d^3 u}{ds^3} \right)_{u=0,1} \end{aligned} \quad (\text{II-1})$$

From Appendix I, it is found that

$$\left\{ \left. \frac{d^2 \mathbf{Q}}{du^2} \right|_{u=0,1} = 0, \left. \frac{d^3 \mathbf{Q}}{du^3} \right|_{u=0,1} = 0 \right\} \rightarrow \left\{ \left. \frac{d^2 u}{ds^2} \right|_{u=0,1} = 0, \left. \frac{d^3 u}{ds^3} \right|_{u=0,1} = 0 \right\} \quad (\text{II-2})$$

which is achieved by implementing (6). Hence, the remaining condition to satisfy the continuity condition (10) is

$$\left. \frac{d^2 \mathbf{R}}{du^2} \right|_{u=0,1} = 0 \quad \text{and} \quad \left. \frac{d^3 \mathbf{R}}{du^3} \right|_{u=0,1} = 0 \quad (\text{I-3})$$

With the B-spline function (8), the conditions (II-3) can be obtained as:

$$\begin{aligned} \left. \frac{d^2 \mathbf{R}}{du^2} \right|_{u=0} &= 2\mathbf{z}_0 - 3\mathbf{z}_1 + \mathbf{z}_2 = 0; \quad \left. \frac{d^3 \mathbf{R}}{du^3} \right|_{u=0} = -4\mathbf{z}_0 + 7\mathbf{z}_1 - 4\mathbf{z}_2 + \mathbf{z}_3 = 0 \\ \left. \frac{d^2 \mathbf{R}}{du^2} \right|_{u=1} &= \mathbf{z}_4 - 3\mathbf{z}_5 + 2\mathbf{z}_6 = 0; \quad \left. \frac{d^3 \mathbf{R}}{du^3} \right|_{u=1} = \mathbf{z}_3 - 4\mathbf{z}_4 + 7\mathbf{z}_5 - 4\mathbf{z}_6 = 0 \end{aligned} \quad (\text{II-4})$$

Through arranging (II-4) and defining $\left\| \mathbf{z}_2 \mathbf{z}_3 \right\| = l_{oa}$ and $\left\| \mathbf{z}_3 \mathbf{z}_4 \right\| = l_{ob}$, we get (11).

DECLARATIONS

A. Funding

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B. Conflicts of Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

C. Availability of Data and Material

The datasets generated and analyzed during the study are available from the first author, i.e. WK, on reasonable request.

D. Code Availability

Codes for the corner smoothing and the proposed time-optimal feedrate scheduling in the MATLAB language are available from the first author, i.e. WK, on reasonable request.

E. Ethic Approval

Not applicable.

F. Consent to Participate

Not applicable.

G. Consent for Publication

Not applicable.

H. Authors' Contributions

SC conceived of the original idea. WK developed the theory and carried out the simulations and experiments, under SC's supervision. SC and WK participated in the analysis and interpretation of the data. The manuscript was originally drafted by WK, and reviewed and edited by SC. Both authors read and approved the final manuscript.

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