

Wavelets for Long Waves: Was Kondratieff's Intuition Right?

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Research

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Abstract

The validity of the Kondratieff's theory of long-waves is still a controversial issue among historians, economic, social and political science scholars. In this work, both original Kondratieff's data (never processed by wavelet analysis in their whole dataset) jointly with additional among the longest and most representative economic indicators are investigated. By the application of this statistical technique, the main purpose of the paper is to furnish an additional information to the literature. We trace evidence on the presence of coherent periodicities in the case of original Kondratieff time series and for other relevant and up-dated economic indicators as well.

Keywords: Long waves, Kondratieff cycle, Wavelets, Multiresolution Decomposition

(*JEL* C22, E32, E37)

1. Introduction

When economies experience deep crises and/or resurge in new periods of growth, the hypothesis of the endogenous existence of periodical cycles assumes a renewed and stronger plausibility. Following this idea, history should repeat itself passing through the relevant changes in institutional arrangements (for example a change in international monetary system), technological dynamics and wars (Ghalayini, 2018). Even if economists generally do not agree on the genesis of economic crises, there is a consensus with historians on the fact that capitalistic systems swing between alternate phases of prosperity and depressions (Adelman, 1965). On this point, it is with the Minsky's cycle (1992) that financially driven factors affecting economic systems have been embedded in long run theoretical frameworks (Bernard et al., 2014). Reijnders (2009) lists the main cycle classifications following their average duration as pointed out by literature:

- Kitchin cycles (3-5 years);
- Juglar cycles (7-12 years);
- Kuznets cycles (15-25 years);
- Kondratieff cycles (40-60 years)
- Hegemonial cycles (over 60 years).

Among these archetypes, the Kondratieff theory of long waves remains a highly debated and controversial issue. The presence of potential long waves about 50 years long has been investigated by early twentieth-century pioneering works starting (probably) with Tugan-Baranowsky (1901). Nevertheless, as Reijnders (1988) reports, van Gelderen (publishing under the pseudonym of J. Fedder in 1913), De Wolff (1924 and 1929) and mainly Kondratieff (1926, with an English widespread translated version in 1935) followed by Schumpeter (1939) were the most renowned authors in this field (Growiec et al., 2018).

Despite such an interest, the potential empirical regular periodicity in economic patterns seemed obtain proper dignity with the viewpoint fostering the theory of the existence of a general business cycle (Aftalion 1909; Pigou 1927; Frisch 1933). Generally, economists are skeptical about the validity of the long waves (or Kondratieff waves, K-waves) hypothesis (Hilmola 2007; Devezas and Corredine 2001). Nevertheless, the fascinating idea of a determinist model with a (constant) time-period able to depict the mechanics of the subsequent long swings in economy has involved several aspects of theoretical research.

The classical and initial methods to explore long waves in economic time series were based on the decomposition approach, where the data are considered as deriving from the sum of four different unobserved components: trends, cyclical, seasonal and irregular components. Hence, the isolation of major fluctuations in the deviation of a variable around its trend was obtained by combining detrending techniques, smoothing procedures and specific assumptions. As an example among the others in the trend component estimation, the pre-definition of a particular form (linear, quadratic, exponential, etc.) is required. Obviously, all these drawbacks can be easily pointed out as crucial binding constraints. Essentially, Kondratieff suffered such shortcomings when his moving-average indicator was criticized considering the Slutsky (1927) and Yule (1927) induced effect on cycle-free data (Garvy, 1943).

Since the 1980s a wide range of more advanced statistical and mathematical tools have been applied to analyze various economic indicators and empirically detect the presence of long waves. Among these methods, it is possible to list: correlation analysis (Goldstein, 1988), filter design (Metz, 1992; Metz and Stier, 1992 and Kriedel, 2009), fractional integrated long memory processes (Diebolt, 2005), log-linear trends (Bieshaar and Kleinknecht, 1984), outlier identification and trend breaks in stochastic models (Darné and Diebolt, 2004 and Metz, 2011), polynomial regression by the best fitting function (Taylor, 1988), spectral analysis (Kuczynsky, 1978; Bossier and Hugé, 1981; van Ewijk 1982; Hausteijn and Neuwirth, 1982; Mosekilde and Rasmussen, 1989; Reijnders, 1990; Gerster, 1992; Metz, 1992 and 2006; Berry, 2006; Diebolt and Doliger, 2006 and 2008; Korotayev and Tsirel, 2010; Focacci, 2017), and structural time series models (Goldstein, 1999). In such an overall picture, spectral methods and filter design have exerted the dominant role. Nevertheless, despite the adoption of different approaches, basic issues remain still controversial. Crucial disagreements regard both long waves existence and their periodization. An appealing remedy has been recently proposed through the adoption of wavelets analysis (WA) to overcoming main concerns about non stationary features of data and the requirement of commitments to particular class of model (Gallegati et al., 2017).

Considering this last interesting approach, the first goal of this work lies in the application of the WA to explore original data presented in the Kondratieff's article (1926). Such an empirical analysis of the dynamics of the long-term movements allows to investigate the plausible existence of periodical cycles coherent with the well-known Russian economist's intuition. To the best of our knowledge, it is the very first WA application for all original series. This fills a gap existing in literature overcoming the limits encountered in processing Kondratieff's original dataset by Harmonic Analysis (HA). In these specific series the spectral detection and the existence of waves is generally disputed on the basis that samples contain too few observations for rigorous testing of

about 50-65 years long supposed cycles. The sample size is a very crucial aspect for long-wave periodicity detections through spectral analysis (Adelman 1965; Harkness 1968; Berry, 2000). On this aspect, literature proposes different suggestions: at least the repetition of three (hypothesized) sequences (thus a sample length in the range of $150 \leq N \leq 195$) for Klotz and Neal (1973), seven cycles ($350 \leq N \leq 455$) for Granger and Hatanaka (1964), while Soper (1975) recommends a minimum of ten repetitions ($500 \leq N \leq 650$).

Moreover, the second goal of this contribution is to furnish additional information about present consistency of the K-waves hypothesis by the investigation of additional and representative leading economic indicators.

The paper is organized as follows. The next section presents WA and its relative properties in processing non stationary data when specifically compared with HA. Section 3 introduces the whole datasets object of investigation with dedicated unit root and linearity analyses jointly with related considerations about the selection of the variables. Section 4 exhibits and discusses main findings. Finally, Conclusions are in Section 5.

2. Preliminary Considerations and the Application of the Methodology

In the present Section a brief overview of the WA related to HA is proposed. WA is a powerful mathematical tool for signal processing in the time-frequency domain able to overcome main drawbacks of HA (Kaiser, 2011).

HA -also known as Fourier Analysis (FA)- is a filtering approach of decomposition of a series $y(t)$ into a sum of sinusoidal (with different frequency) components (Bloomfield, 2000). To convert the time domain into the frequency domain, the Discrete Fourier Transform (DFT) is adopted:

$$Z_n = \left(\frac{1}{T}\right) \times \sum_{t=0}^{T-1} y_t \times e^{\frac{-iz\pi tn}{T}} \quad (1)$$

$$\text{for } n = 0, 1, 2, \dots, \frac{(T-1)}{2}$$

wherein:

- Z_n = the complex number resulting from the DFT formed by a real (a) and an imaginary part identified by a lower-case i (ib);

- T = the last term of the discrete series;

- e = Euler's number (also known as Nepier's constant equal to 2.71...);

- i = is the conventional $\sqrt{-1}$ for imaginary part;

- $\frac{2\pi tn}{T}$ = is the radians representation of the frequency ($f_n t$).

Generally, to speed iteration in elaboration, the Fast Fourier Transform (FFT) dedicated class of algorithm is employed. Through this procedure, then it is possible to perform a spectral analysis of time series data to detect meaningful cyclical periods by implementing the power spectrum (Warner, 1998). Periodogram Intensities (I_k) can be mathematically derived as:

$$I_k = \left(\frac{T}{2} \right) (a_k^2 + b_k^2) \quad (2)$$

where :

- a_k and b_k are the coefficients of the numbers Z_n for $k = 1, 2, \dots, K$ (K the last time period until the Nyquist-Shannon frequency; i.e. the minimum sampling period needed in order to identify a possible periodicity, usually represented as : $0 \leq f \leq 0.5 f$).

The sum of Periodogram Intensities represents all the variance of the time-series (Box et al., 2016):

$$\sum_{k=1}^T I_k = \text{var} (y_t) \quad (3)$$

Nevertheless, such an approach has two inherent disadvantages: the need of a stationary behavior of the variable, and the absence of a correct positioning of the cyclic components over time. For these reasons, data are usually pre-treated with proper detrending procedures (first differencing for example). However, the elimination of trend can create spurious cycles (Beaudry et al., 2020). Detrending diminishes the original information content of data resulting as not neutral to final findings (Freeman and Louça, 2001). The same holds true for band-pass filtering applications like for example the Baxter and King (1999) and/or Christiano and Fitzgerald (2003) common time domain filters.

As far as the correct positioning of the periodic component is concerned, it must be pointed out that in FA methods, a single disturbance affects all frequencies along the whole datasets through the sum of sine and cosine functions. For the Heisenberg uncertainty principle: the more certainty is related to the measurement of one dimension (for example frequency), the less certainty can be related to the other dimension (time location in time series analysis).

WA is well suited to approach all above mentioned issues. As a matter of fact, instead and differently from FA, its transform is localized both in time and in its functional components (Rhif et al., 2019). Such a feature allows appropriate investigation of non-stationary signals (Torrence and Compo, 1998; Houtveen and Molenaar, 2001; Aristizabal and Glavinovic, 2003; Cazelles et al., 2008 and Sleziak et al., 2015). The approximations generated by the procedure are robust to small variations (Gallegati et al., 2017). WA provides an efficient way to deal with variables lasting for a finite time, or showing markedly different behavior in their time-sequence (Crowley, 2007), hence its adoption can be appropriated to analyze time series containing non stationary power at many different frequencies (Magazzino and Mutascu, 2019 and Daubechies, 1990). Due to such a flexibility, many disciplines (among the others astronomy, climatology, engineering, medicine) have extensively applied WA, and without doubt, this approach is useful also for research in economics. Pioneering contributions in the analysis of the frequency nature of economic relationships have been proposed by Ramsey and Lampard (1998a,b).

Without entering into excessive mathematical details retrievable in specific references also for economic and finance applications (see for example Percival and Walden, 2000; Gençay et al., 2001; Ramsey, 2002; Schleicher, 2002 as well as Gallegati and Semmler, 2014), wavelets are small waves (or wave packets) representing the varying duration of the components of a time series (Walker, 2008). This allows to obtain an alternative representation in the timescale domain of an original time domain represented function. Several types of wavelet functions exist with proper characteristics: *Daubechies*, *Haar*, *Mexican Hat* and *Morlet* among the most widespread. In general, we can identify “father” (ϕ) and “mother” (φ) wavelets. The first integrates to 1 and the second to 0:

$$\int \phi(t) dt = 1 \quad (4)$$

and

$$\int \varphi(t) dt = 0 \quad (5).$$

Substantially, the “father” (or scaling function) represents the low-frequency part of the series in the transform calculation, while the “mother” wavelets stand for the high-frequencies. The zero mean and decaying property of the φ represent the typical oscillations on the t -axis of the function behaving like a small wave losing its strength as it moves away from the center (Anguiar-Conraria and Soares, 2011). WA allows to a simultaneous estimation of several cyclical components, and its main characteristic lies in the possibility to separate out a variable into inner constituent (multi-

resolution) components (Crowley, 2007). Thus, a multi-resolution decomposition (MRD) also termed as multi-resolution analysis (MRA) of $y(t)$ can be represented as:

$$y(t) = \sum_k s_{j,k} \phi_{j,k}(t) + \sum_k d_{j,k} \varphi_{j,k}(t) + \sum_k d_{j-1,k} \varphi_{j-1,k}(t) + \dots + \sum_k d_{1,k} \varphi_{1,k}(t) \quad (6)$$

whereas:

-the basis functions $\phi_{j,k}(t)$ and $\varphi_{j,k}(t)$ are assumed to be orthogonal and represented as:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi\left(\frac{t-2^j k}{2^j}\right) \quad (7)$$

$$\varphi_{j,k}(t) = 2^{-\frac{j}{2}} \varphi\left(\frac{t-2^j k}{2^j}\right) \quad (8)$$

In sum,

- the functions ϕ and φ satisfy conditions (4) and (5).
- $j = 1, 2, \dots, J$ indexes the maximum scale sustainable with the data to process (each scale represents a fixed interval of frequencies);
- k indexes the translation parameter;
- $s_{j,k}$ are the trend smooth coefficients in the wavelet transform capturing the underlying behavior of the data at the coarsest scale;
- $d_{j,k}$ are the detail wavelet coefficients representing deviations from the smooth behavior.

Also termed as atoms or scale crystals for each scale (the higher the scale, the lower the frequency and/or inversely higher the period length) approximately they are given by the integrals of the following (Bruce and Gao, 1996):

$$s_{j,k} \approx \int y_t \phi_{j,k}(t) dt \quad (9)$$

$$d_{j,k} \approx \int y_t \varphi_{j,k}(t) dt \quad (10)$$

In a simpler manner a MRD of the variable y_t is given by:

$$\{S_J, D_J, D_{J-1}, \dots, D_1\} \quad (11)$$

wherein, S_j represents the first term on the right side of equation (6), D_j is the second term and so on. Transforms can be seen both in their continuous version in signal processing (CWT) and in their discrete one (DWT). Acting as a filtering approach to extract cycles at various frequencies from the data, DWT uses a given discrete function passed through the series and “convolved” with the data to yield the coefficients labeled as crystals. Convolution is a mathematical procedure to obtain a modified version of the original functions processing the signal (Crowley and Hallet, 2014). The case of discrete signal is very common in economics given that, generally, datasets consist of values sampled at evenly spaced points in time. In sum, with MRA it is possible to decompose any individual time series into its different timescale components (each corresponding to a specific frequency band) to properly isolate the stochastic periodical component of interest. The subsequent representation of the variability of the process on a scale-by-scale basis is then obtained by the energy decomposition analysis. Considering that orthogonal wavelets are energy (thus variance) preserving, and letting E expressing the total energy of a variable y_t for j from 1 to J , the corresponding total energy decomposition can be represented as:

$$E = E_J + \sum_{j=1}^J E_j \quad (12)$$

with:

$$E_J = \sum_{k=1}^{\frac{n}{2^J}} S_{J,k}^2 \quad (13)$$

as the energy power of the scaling coefficients (sum of the j_{th} level approximation signal)

and

$$E_j = \sum_{k=1}^{\frac{n}{2^j}} d_{j,k}^2 \quad (14)$$

representing the energy power of scalar crystals (sum of all detail level signals from the first to j_{th}). Considering that energy decomposition allows to detect the most contributing scale component, the frequency (or inversely period) having the leading role in approximating the original signal can be derived (for frequency interpretation of MRD and corresponding scale levels see Table 1).

Table 1 – Frequency interpretation of MRD levels ($J = 5$)

Scale level J	Scale crystals (detail level d_j)	Annual frequency resolution
1	d_1	2 – 4
2	d_2	4 – 8
3	d_3	8 – 16
4	d_4	16 – 32
5	d_5	32 – 64
6	s_5	> 64

Source: Personal elaboration from Crowley (2007) and Gallegati et al. (2017)

An ineludible aspect in practical implementation of WA through DWT to real datasets is sample length. Series have to be dyadic (a sample size divisible by 2^J). Such a requirement is a prerequisite for a reliable application of the technique. Considering both non dyadic feature of all gathered series and the drawback of non-shift invariant property of the DWT, we adopt the maximal-overlap DWT (MODWT). Originally introduced by Shensa (1992) with a subsequent phase-corrected version proposed by Walden and Crisan (1998), unlike DWT, MODWT skips the downsampling after the filtering of the data producing a more asymptotically wavelet variance estimator (all else being equal). The basic principle of MODWT consists in passing the wavelet function down the series by data observation, rather than moving the wavelet function down the series to cover a completely new data span (Crowley and Hallet, 2014). Using matrix notation, and let \mathbf{x} as a vector of N elements, the MODWT wavelet coefficients resulting from the transform represented by a vector $(J+1)N$ is given by:

$$\tilde{\mathbf{w}} = \tilde{\mathbf{W}}\mathbf{x} \quad (15)$$

with $\tilde{\mathbf{W}}$ representing the $(J+1)N \times N$ matrix defining the MODWT (Crowley, 2007).

In order to perform and implement any analysis with WA, some preliminary choices are essential. Firstly, the family of wavelets must be defined. Secondly, the boundary conditions at the end of the series must be handled. In relation to the first aspect, several families of wavelet filters with different characteristics of the transfer function for what concerns both the filtering and filter length are available (Haar discrete, symmlets, and so on). On these points, the present choice is to apply the Daubechies least asymmetric (LA) having a length $L = 8$ (wherein eight are the non zero

coefficients). In wavelet notation this filter is commonly expressed as LA(8). The LA(8) is the most widely adopted filter for economic applications both because it can be applied to a wide variety of data types and because of its ability to balance the most common desirable features for wavelets basis functions like smoothness, (a)symmetry and length (Gençay et al., 2001). As far as the boundary conditions are concerned, the choice is for a reflecting method (the original signal is reflected about its end points to obtain a series of length $2N$ having the same statistical properties in mean and variance).

3. Datasets and related considerations

Since we are interested in assessing the plausibility of Kondratieff hypothesis and the potential current existence of long-waves in economics through the analysis of appropriate indicators, we use two different datasets.

The first group encompasses all the original data presented by Kondratieff (1926) as they can be retrieved in Gattei (1981). The full list is:

- England-Index number of commodity prices 1780-1922 (labeled by a capital letter in this work to simplify proper identification, and starting by A);
- France-Index number of commodity prices 1858-1922 (B);
- USA- Index number of commodity prices 1791-1922 (C);
- England-Quotations of interest-bearing securities 1816-1922 (D);
- France-Quotations of interest-bearing securities 1814-1922 (E);
- England-Index of Weekly wages in agriculture 1789-1913 (F) and Cotton Textiles 1807-1913 (G);
- France-Foreign trade 1827-1913 in per capita francs (H);
- England-Coal production 1855-1917 in t /1,000 inhabitant (I);
- France-Coal consumption 1827-1913 in t/1,000 inhabitant (J);
- England-Pig iron production 1840-1914 in t/1,000 inhabitant (K);
- England-Lead production 1855-1920 in t/1,000 inhabitant (L).

All these datasets have been object of exploration through a FA with spectral analysis for the very first time by Focacci (2017). Metz (2011) analyzed just one Kodratieff data series adopting spectral methods, while outliers identification, modelling and bandwidth of filtering process for successfully detection were the main focus by van Ewijk (1982).

In Table 2 the Shapiro-Wilk and Jarque-Bera tests for normality, and four among the most widespread and well-known unit root tests (Augmented Dickey-Fuller: ADF, Augmented Dickey-Fuller Generalized Least Squares Regression: ADF-GLS, Kwiatkowsky-Phillips-Schmidt-Shin: KPSS and Phillips-Perron: PP) are applied. Additionally, since WA is one of the most powerful tool for investigating complex nonlinear data, linearity properties are explored by the Keenan and the Brock-Dechert-Scheinkman (BDS) tests.

Table 2-Normality, Unit root and Linearity tests for original Kondratieff series (A-F) (to be continued)

Series	A	B	C	D	E	F
<i>N</i>	143	65	132	107	109	125
Normality test						
Shapiro-Wilk	0.92	0.68	0.89	0.87	0.97	0.98
p < 0.05	0.00	0.00	0.00	0.00	0.03	0.04
Jarque-Bera	28.59	353.98	32.90	51.61	2.05*	0.97*
<i>p-value</i> ($\alpha = 0.05$)	p < 0.05	p < 0.05	p < 0.05	p < 0.05	0.36	0.61
Unit root test						
ADF with const	-2.58	-3.33*	-2.29	-1.61	-2.41	-2.34
p-value ($\alpha = 0.05$)	0.10	0.01	0.18	0.47	0.14	0.16
ADF with const and trend	-3.24	3.12	-2.40	-1.99	-2.16	-3.43*
p-value ($\alpha = 0.05$)	0.08	1.00	0.38	0.60	0.51	0.05
ADF_GLS τ	-2.15	-0.54	-2.31	-0.83	-1.52	-1.70
Critical value ($\alpha = 0.05$)	-2.93	-3.03	-2.93	-2.93	-2.93	-2.93
KPSS test	0.73	0.30*	0.83	0.37*	0.75	1.64
Critical value($\alpha = 0.05$)	0.46	0.46	0.46	0.46	0.46	0.46
PP Test Z τ	-2.45	-2.32	-2.37	-1.66	-2.16	-2.11
p-value ($\alpha = 0.05$)	0.13	0.17	0.15	0.45	0.22	0.24
Linearity test						
Keenan test	14.37	30.68	10.56	2.23	3.81	4.80
p-value ($\alpha = 0.05$)	0.00**	0.00**	0.00**	0.13	0.05**	0.03**
BDS test p-value ($\alpha = 0.05$)	0.00**	0.00**	0.00**	0.00**	0.00**	0.00**

Source: Gattei (1981)

BIC Criterion for ADF and ADF_GLS

*indicates normality and or stationarity at 5% level

**indicates nonlinearity properties at 5% level

Table 2-Normality, Unit root and Linearity tests for original Kondratieff series (G-L)

Series	G	H	I	J	K	L
<i>N</i>	107	87	63	87	75	66
Normality test						
Shapiro-Wilk	0.90	0.92	0.96	0.93	0.89	0.88
p < 0.05	0.00	0.00	0.03	0.00	0.00	0.00
Jarque-Bera	9.68	2.55*	3.03*	6.17	8.97	7.00
<i>p</i> -value ($\alpha = 0.05$)	p < 0.05	0.28	0.22	p < 0.05	p < 0.05	p < 0.05
Unit root test						
ADF with const	0.97	1.58	-1.51	2.32	-2.03	-0.73
p-value ($\alpha = 0.05$)	1.00	1.00	0.53	1.00	0.27	0.83
ADF with const and trend	-2.70	-0.67	-3.85*	-1.12	-1.83	-2.02
p-value ($\alpha = 0.05$)	0.24	0.97	0.02	0.92	0.68	0.58
ADF_GLS τ	-0.43	-0.89	-2.44	-0.60	-1.72	-2.02
Critical value ($\alpha = 0.05$)	-2.93	-3.03	-3.03	-3.03	-3.03	-3.03
KPSS test	1.94	2.10	1.62	2.22	1.61	1.70
Critical value ($\alpha = 0.05$)	0.46	0.46	0.46	0.46	0.46	0.46
PP Test Z τ	0.85	1.47	-1.75	2.85	-2.04	-0.69
p-value ($\alpha = 0.05$)	0.99	1.00	0.40	1.00	0.27	0.84
Linearity test						
Keenan test	0.25	0.29	5.15	0.22	9.85	0.91
p-value ($\alpha = 0.05$)	0.62	0.59	0.03**	0.64	0.00**	0.34
BDS test p-value ($\alpha = 0.05$)	0.00**	0.00**	0.00**	0.00**	0.00**	0.00**

Source: Gattei (1981)

BIC Criterion for ADF and ADF_GLS

*indicates normality and or stationarity at 5% level

**indicates nonlinearity properties at 5% level

The second dataset encompasses:

- United States of America real 2012 US\$ per-capita GDP 1790-2019 (M) (Williamson, 2020);
- United Kingdom real 2013 pounds per-capita GDP 1700- 2018 (N) (Thomas and Williamson, 2020);
- Australia real 2010 AUSS\$ per-capita GDP 1828-2018 (O) (Hutchinson and Ploeckl, 2020);
- United States of America Annual Consumer Price Index CPI 1774-2018 (P) (Officer and Williamson, 2020).

Their statistical properties are summarized in Table 3.

The selection of this second group of data is accounted for different reasons. As far as GDP measure is concerned, it must be pointed out it remains the widespread and leading measure

internationally employed when approaching economic themes (despite several critics and adverse theoretical opinions arguing against its real effectiveness in representing people's material well-being). Series pertain three industrialized Countries for which the longest and homogeneous continuous datasets are retrievable. The dynamic of GDP growth is not re-proposed here; Korotayev and Tsirel (2010) have already investigated the GDP growth rate joining different sources by employing spectral methods. For what concerns the US CPI, its selection is in line with long-waves research tradition studying price series.

Table 3- Normality, Unit root and Linearity tests for the longest available series (M-P)

Series	M	N	O	P
<i>N</i>	229	319	191	245
Normality test				
Shapiro-Wilk	0.76	0.67	0.80	0.57
p < 0.05	0.00	0.00	0.00	0.00
Jarque-Bera	77.42	320.50	61.15	311.94
p-value ($\alpha = 0.05$)	p < 0.05	p < 0.05	p < 0.05	p < 0.05
Unit root test				
ADF with const	4.21	5.42	5.24	5.42
p-value ($\alpha = 0.05$)	1.00	1.00	1.00	1.00
ADF with const and trend	1.09	2.65	1.68	3.89
p-value ($\alpha = 0.05$)	1.00	1.00	1.00	1.00
ADF_GLS τ	0.84	1.38	0.89	-0.45
Critical value ($\alpha = 0.05$)	-2.89	-2.89	-2.93	-2.89
KPSS test	3.64	3.49	3.09	2.22
Critical value ($\alpha = 0.05$)	0.46	0.46	0.46	0.46
PP Test Z τ	5.86	7.58	5.41	9.55
p-value ($\alpha = 0.05$)	1.00	1.00	1.00	1.00
Linearity test				
Keenan test	5.50	4.69	0.01	47.60
p-value ($\alpha = 0.05$)	0.02**	0.03**	0.90	0.00**
BDS test p-value ($\alpha = 0.05$)	0.00**	0.00**	0.00**	0.00**

Source: Williamson (2020), Thomas and Williamson (2020), Hutchinson and Ploeckl (2020), Officer and Williamson (2020)

BIC Criterion for ADF and ADF_GLS

*indicates normality and or stationarity at 5% level

**indicates nonlinearity properties at 5% level

4. WA estimation of long waves in the series

In the present Section all findings are proposed. Considering the presence of unit roots and nonlinearity depicted in Table 2 and 3, the adoption of WA is an attractive method to pursue the aim of the research and, contrary to statistical intuition, to strongly relax length issues (observations

for original Kondratieff Series from A to L are shorter than for Series from M to P). WA transform proceeds by processing pieces of fixed length for a given period. The overall average power is not related to the length of the series. The shorter the series, the lesser the pieces having the same characteristics (Rösch and Schmidbauer, 2018).

A further aspect to highlight concerns potential outliers. A formal theoretical evaluation can be conducted both to detect and model them. No specific treatment is here applied for three reasons. Firstly, original data are not normally distributed (as Shapiro-Wilk and Jarque-Bera tests within Tables 2 and 3 show). In such conditions, both formal outliers detection tests and non-formal ones could be questionable. Secondly, no processed data is the result of an input error or a demonstrable proof of a truly aberrant origin (Hair et al. 2014). Whatsoever modeling approach could be considered by skeptics like an artificial adjustment to induce or emphasize a specific result. However, short-lived transient episodes (like abrupt changes, volatility and clustering typical results of extraordinary and unexpected events as wars or crisis) are expression of a not homogenous behavior of time series data. Additionally, it is possible to point out that series have not dyadic properties and, thus, the MODWT can treat any sample size properly. For what concerns this last aspect, sample length dictates the meaningful number of crystals that can be produced ($N \geq 2^J$). However, considering that the highest scale (lower frequency) can only just be resolved, for practical applications it is usually recommended to diminish of an additional unit the number of crystals to produce (Crowley, 2007). Taking this advice into account, crystal determination is reported for each series (Table 4).

Table 4 – Scale crystals for each series as dictated by sample length

Series	A	B	C	D	E	F	G
N	143	65	132	107	109	125	107
Scale crystal d_j	6	5	6	5	5	5	5
Annual frequency resolution	64 – 128	32 - 64	64 - 128	32 – 64	32 - 64	32 - 64	32 - 64
Series	H	I	J	K	L	M	N
N	87	63	87	75	66	229	319
Scale crystal d_j	5	4	5	5	5	6	7
Annual frequency resolution	32 – 64	16 - 32	32 – 64	32 – 64	32 - 64	64 - 128	128 - 256
Series	O	P					
N	191	245					
Scale crystal d_j	6	6					
Annual frequency resolution	64 – 128	64 - 128					

Source: Personal elaboration on data (values are rounded down to the nearest integer).

At this point, all Kondratieff series except one (Series I) can be investigated by WA producing at least the scale crystal d_5 . The d_5 detail component covers an annual frequency resolution range between 32 and 64 years, with an average cycle length around 48 years long. Since no assumption has been made on the underlying nature of the data generating process (signal) and a criterion of the same kind of a locally adaptive bandwidth has been adopted, the crystal component d_5 represents a nonparametric estimation of the Kondratieff long waves.

The application of the MODWT allows the investigation of the energy distribution within the signal. Corresponding different frequency bands and the relative importance of all cyclical components can be extracted. In the present work, the long term component (S_5) has been removed before the application of the MODWT energy decomposition analysis in order to better assessing the relative importance of the different cyclical components. The five crystals energy-related importance in percentage terms (net of the S_5 component) for all Series except Series I are resumed in Table 5.

Table 5 – Energy decomposition analysis by scale crystals (net of the S_5 component)

Energy distribution	A	B	C	D
d_1	0,11%	0,24%	0,25%	0,04%
d_2	0,27%	1,38%	0,35%	0,06%
d_3	0,51%	2,55%	0,63%	0,06%
d_4	0,80%	0,91%	1,01%	0,10%
d_5	4,69%	6,33%	2,77%	0,76%
Energy distribution	E	F	G	H
d_1	0,06%	0,02%	0,03%	0,06%
d_2	0,13%	0,05%	0,05%	0,11%
d_3	0,19%	0,16%	0,04%	0,22%
d_4	0,35%	0,28%	0,07%	0,83%
d_5	2,57%	1,02%	0,36%	2,10%
Energy distribution	J	K	L	M
d_1	0,03%	0,12%	0,09%	0,01%
d_2	0,07%	0,18%	0,10%	0,02%
d_3	0,09%	0,20%	0,16%	0,06%
d_4	0,15%	0,17%	0,36%	0,07%
d_5	1,06%	0,86%	1,11%	0,17%
Energy distribution	N	O	P	
d_1	0,00%	0,01%	0,00%	
d_2	0,01%	0,02%	0,01%	
d_3	0,05%	0,03%	0,02%	
d_4	0,04%	0,06%	0,11%	
d_5	0,18%	0,29%	0,67%	

Source: Personal elaboration on Gattei (1981), Williamson (2020), Thomas and Williamson (2020), Hutchinson and Ploeckl (2020), Officer and Williamson (2020).

Two main findings emerge from results in Table 5. First of all, residual energy at each scale tends to diminish with the corresponding level. Secondly, the long wave d_5 component concentrates the most of this energy. Substantially, even if the total energy residual appears modest (the S_5 components have the predominance), it is possible to conclude that the long waves component carries the most meaningful contribution in terms of the overall variance also considering classical business cycles shorter term components.

A visual representation of raw data series from A to P (dotted lines) along with the corresponding D_5 detail components (solid lines) are displayed in Figures from 1 to 15.

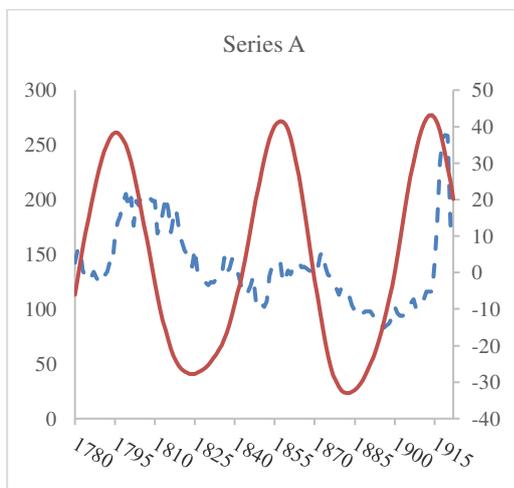


Figure 1. Series A raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

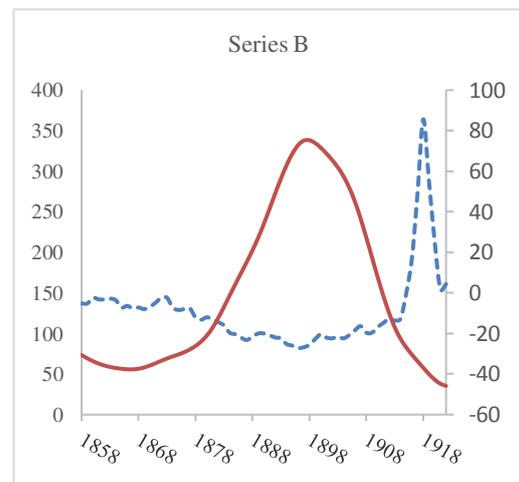


Figure 2. Series B raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

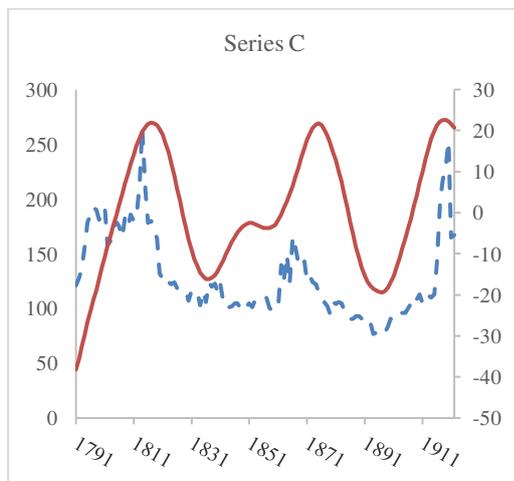


Figure 3. Series C raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

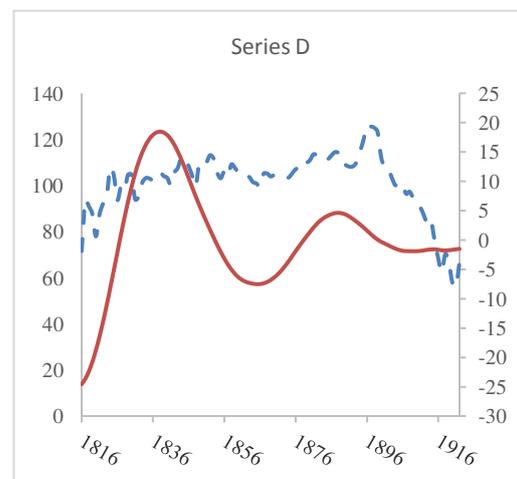


Figure 4. Series D raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

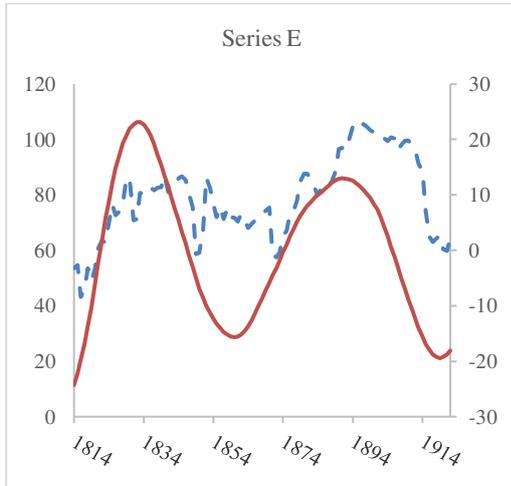


Figure 5. Series E raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

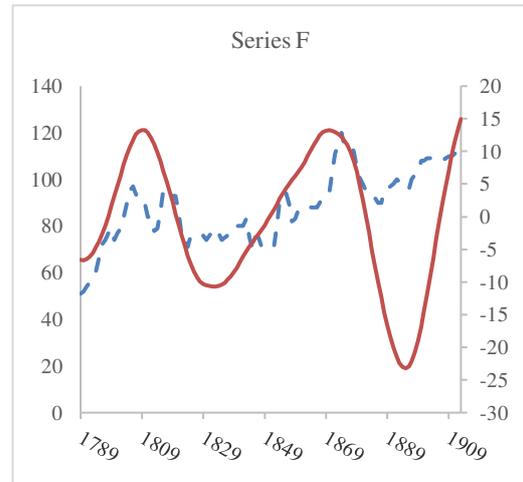


Figure 6. Series F raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

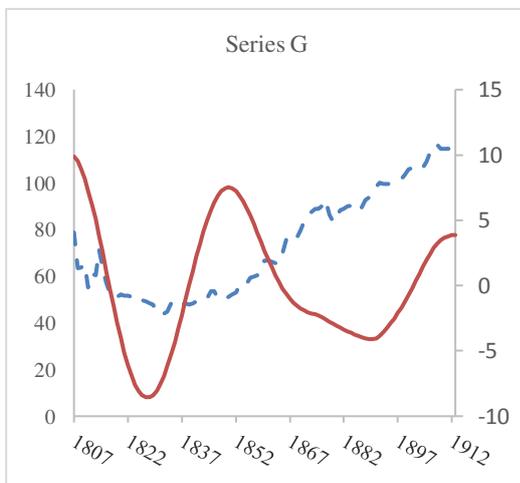


Figure 7. Series G raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

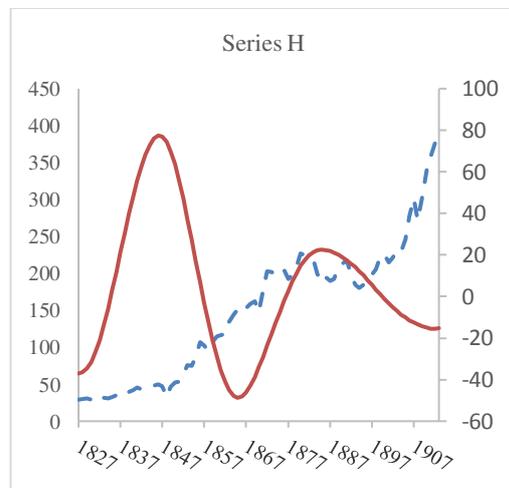


Figure 8. Series H raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

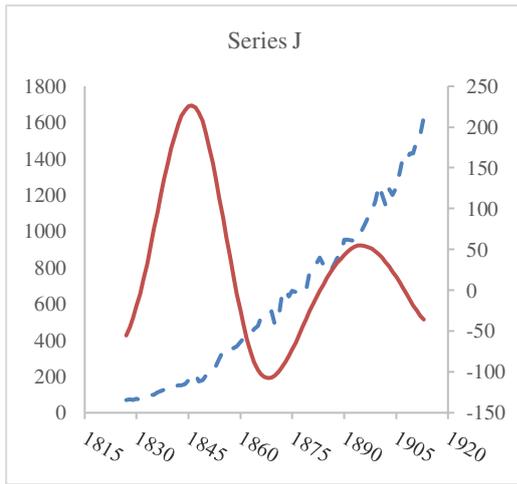


Figure 9. Series J raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

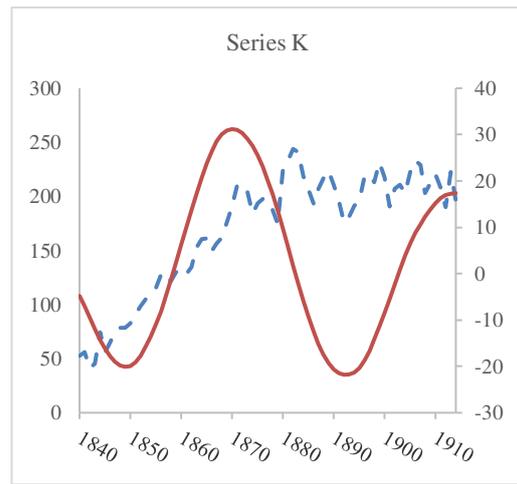


Figure 10. Series K raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

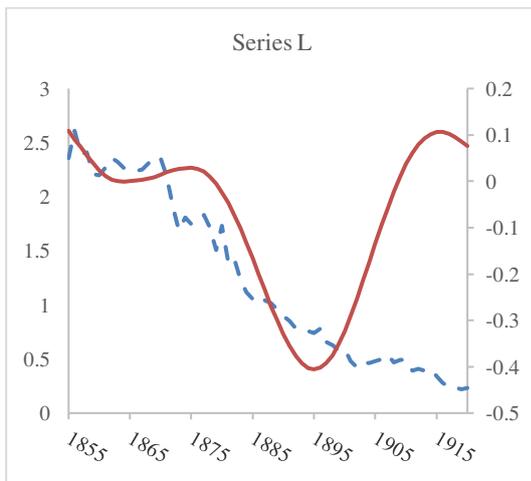


Figure 11. Series L raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Gattei (1981)

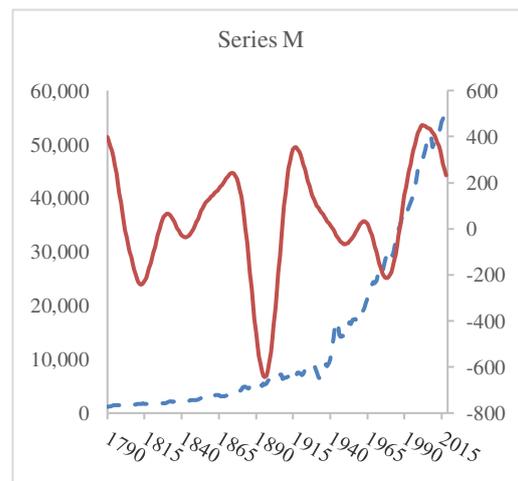


Figure 12. Series M raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Williamson (2020)

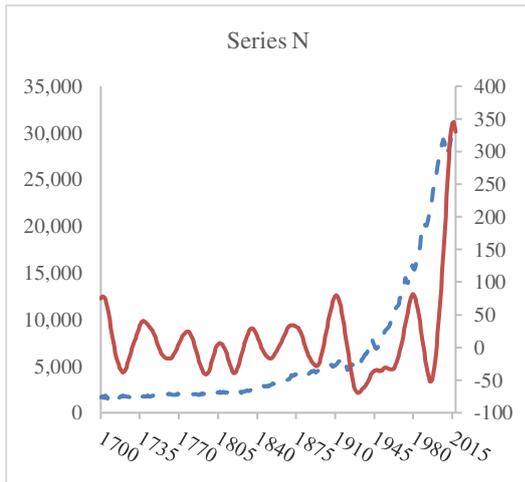


Figure 13. Series N raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Thomas and Williamson (2020)

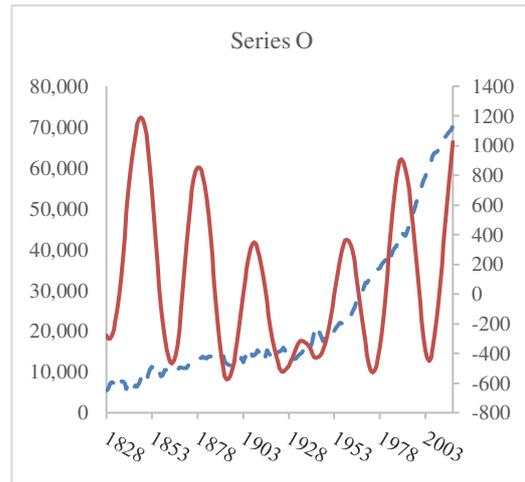


Figure 14. Series O raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Hutchinson and Ploeckl (2020)

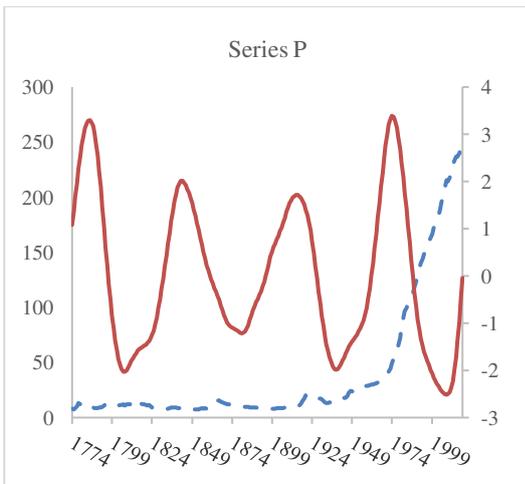


Figure 15. Series P raw data (dotted lines) along with its corresponding wavelet detail component D_5 (solid line)
Source: Personal elaboration on Officer and Williamson (2020a)

As detected by the D_5 components, the whole chronology of subsequent upswings (phase A) and downswings (Phase B) is summarized in Table 6. Additionally, the average duration (years) of each phase is reported in Table 7 (Kondratieff Series A-L are separated from Series M-P in both Tables).

Table 6 – Periodization of upswings and downswings

Phases	A	B	C	D
ascending phase A1	-1795	-	-1817	-
falling phase B1	1795-1824	-	1817-1837	-
A2	1824-1857	-	1837-1876	-1838
B2	1857-1882	-1867	1876-1897	1838-1866
A3	1882-1914	1867-1898	1897-1920	1866-1888
B3	1914-	1898-	1920-	1888-1909
Phases	E	F	G	H
A1	-	1790-1809	-	-
B1	-	1809-1833	-1828	-
A2	-1832	1833-1870	1828-1850	1827-1846
B2	1832-1860	1870-1895	1850-1891	1846-1865
A3	1860-1891	1895-	1891-	1865-1885
B3	1891-1919	-	-	1885-1912
A4	1919-	-	-	-
Phases	J	K	L	
A1	-	-	-	
B1	-	-	-1864	
A2	1827-1846	1849-1870	1864-1875	
B2	1846-1868	1870-1892	1875-1895	
A3	1868-1895	1892-	1895-1916	
B3	1895-	-		
A4	-	-		
Phases	M	N	O	P
A1	-	-1702	-	
B1	-	1702-1720	-	
A2	-	1720-1738	-	
B2	-	1738-1762	-	
A3	-	1762-1778	-	
B3	-	1778-1794	-	
A4	-	1794-1807	-	-1785
B4	-1813	1807-1820	-1830	1785-1807
A5	1813-1830	1820-1836	1830-1847	1807-1842
B5	1830-1842	1836-1852	1847-1864	1842-1880
A6	1842-1874	1852-1871	1864-1879	1880-1914
B6	1874-1896	1871-1893	1879-1894	1914-1939
A7	1896-1917	1893-1911	1894-1910	1939-1974
B7	1917-1950	1911-1931	1910-1925	1974-2008
A8	1950-1963	1931-1980	1925-1960	2008-
B8	1963-1978	1980-1996	1960-1975	
A9	1978-2002	1996-2015	1975-1990	
B9	2002-	2015-	1990-2005	
A10	-	-	2005-	

Source: Personal elaboration from diagrams

Table 7- Average duration of each phase (years)

Phases	A	B	C	D
A1				
B1	29		20	
A2	33		39	
B2	25		21	28
A3	32	31	23	22
B3				21
average	30	30	26	25
Phases	E	F	G	H
A1		19		
B1		24		
A2		37	22	19
B2	28	25	41	19
A3	31			20
B3	28			27
average	29	26	32	21
Phases	J	K	L	
A1				
B1				
A2	19	21	11	
B2	22	22	20	
A3	27		21	
B3				
A4				
average	23	22	17	
Phases	M	N	O	P
A1				
B1		18		
A2		18		
B2		24		
A3		16		
B3		16		
A4		13		
B4		13		22
A5	17	16	17	35
B5	12	16	17	38
A6	32	19	15	34
B6	22	22	15	25
A7	21	18	16	35
B7	33	20	15	34
A8	13	49	35	
B8	15	16	25	
A9	24	19	15	
B9			15	
average	21	20	19	32

Source: Personal elaboration from Table 6

Note: Two phases equal one Kondratieff long wave

Looking at graphs and Tables, it is possible to note the absence of regularities both in terms of length and amplitude in long wave patterns. Obviously, the different sample sizes must be taken into account. More in detail, for what concerns Kondratieff series a similar synchronization is present in Series A and C (England and USA commodity price index respectively). This can be considered as an expected result considering the mutual relationship between these economies already highlighted in literature (Goldstein, 1988). The d_5 vector is fitting quite well the pattern in raw data, and three distinct waves about 60 years long can be clearly detected. On the contrary, Series B (France commodity price index) is out-of-phase with England and USA commodity-price index series. Moreover, the amplitude of swings is wider. Also this result confirms previous literature findings. The ascending phase A3 is clearly detectable and is about 30 years long (half of a whole cycle). In this case, the length of the sample for France starts in 1858. Table 8 resumes turning points in empirical literature focused on wholesale prices series jointly with the dates detected by the detailed d_5 component for series A, B and C (considering such Series as the most homogeneous with previous works).

Table 8 – Turns of the d_5 component in the Series A, B and C and comparison with literature on wholesale price series in the pre-World War II period

Turning points	d_5	Gallegati et al. (2017)	Kondratieff (1935)	Burns and Mitchell (1946)	Imbert (1959)	Bosserelle (1991)
P _{UK}	1795	1812	1814	1813	1810	1810
P _{USA}	1838	1813	1814	1814	1814	1814
P _{FRA}	-	-	-	1820	1817	-
T _{UK}	1824	1835	1849	1849	1849	1849
T _{USA}	1866	1847	1849	1843	1849	1849
T _{FRA}	1867	1833	-	1851	1851	1851
P _{UK}	1857	1871	1873	1873	1873	1873
P _{USA}	1876	1870	1866	1864	1865	1865
P _{FRA}	1898	1871	1873	1872-1873	1872	1872
T _{UK}	1882	1898	1896	1896	1896	1896
T _{USA}	1897	1895	1896	1896-1897	1896	1896
T _{FRA}	1898	1901	1896	1896	1896	1896
P _{UK}	1914	1919	1920	1920	1920	1920
P _{USA}	1920	1919	1920	1920	1920	1920
P _{FRA}	-	1919	1920	1926	1926	1926
T _{UK}	-	1937	-	1933	1933	1933
T _{USA}	-	1937	-	1932	1932	1932
T _{FRA}	-	-	-	1935	1935	1935

Source: Personal elaboration with references cited in head columns

Legend: P for peak, T for Trough with indexed Countries (UK-A, France-B and USA-C)

Financial series (D and E, England and France -Quotations of interest-bearing securities) show mutual and similar in-phase patterns as produced by D_5 component. We can observe at least two complete cycles. Chronology is not exactly overlapping in the descending phase B3.

Series pertaining England Weekly wages in agriculture (F) and Cotton Textiles (G) have different properties. In the F sample the long wave vector better depicts the pattern on a visual basis than in the case of the G series, wherein a trending behavior of raw data -particularly evident since the year 1850 onwards- makes the oscillations hard to be detected. The same trend influence holds for series H, J, K and (substantially) L. In the case of the L Series, the trend has a specific negative slope. However, at least two distinct long waves can be identified in the cases H and J with very similar periodization. In summary, also the application of WA to original Kondratieff series seems to foster the Russian's economist intuition of the presence of long waves. The similar oscillation paths of raw data and corresponding d_5 components are better distinguishable in pure monetary variables. As can be appreciated, Kondratieff's original price Series analyzed by WA do substantially confirm previous findings for UK and USA. Some discrepancies emerge for France.

Analyzing non- Kondratieff Series from M to P, first of all the presence of strong positive trends must be highlighted. As previously pointed out, this does not allow a simple detection of long waves. However, four and a half waves are detected in Series M, seven for series N and, finally, five for Series O. Synchronization is quite similar for pre-WWII data in the cases of UK and USA (Series M and N). The last segments of d_5 vectors show both UK and USA Series in a downward trajectory. Totally out-of-phases behavior can be recorded in the case of Australian GDP per capita (Series O). As far as Series P (US CPI) is concerned, at least three waves can be clearly traced by the detail component d_5 . Also in this case, the synchronization with per-capita GDP is totally absent. Especially, this feature is emphasized and argued for the period subsequent to the WWII (Goldstein, 1988 and Reijnders, 2006). According to some authors, positing a long expansion period from 1945 to 1973 followed by a subsequent long phase of economic difficulty (Gallegati et al., 2017), our results confirm such a pattern. Indeed, our detections date the ascending phase A7 for P from 1939 to 1974, while the subsequent descending period B7 (about 34 years long) records its trough in 2008.

Overall, the analysis of the whole results does confirm the existence of cycles having an about 40-60 years coherent periodicity Especially, the WA applied to the original Kondratieff's series corroborates the idea of the Russian economist on "his own" data. The same conclusions hold when the GDP-per capita and CPI selected series are processed.

5. Conclusions

In this paper, the WA approach is applied to original Kondratieff's datasets. Further representative economic up-dated time series are analyzed. The main aim is to explore the plausibility of long waves having an about 40-60 years period according to the Russian economist original conclusions. Spectral analysis methods are generally adopted to investigate the presence of cycles in time series. In economic terms, WA provides the same information content in studying the dynamics of long-term movements. However, non-stationarity and non-linearity properties of data can be better approached by WA in empirical economic and financial applications. Unlike, spectral methods, wavelets have the ability to detect irregularly spaced cyclical components. A relevant fundamental benefit of WA in contrast with Fourier methods (or also spline regression models) lies in the possibility to approach random occurring shocks distorting a dynamical system where statistical properties change among periods. A further distinguishing point of this approach is in the fact that WA does not require to process time series having to respect length pre-requisites in sequence repetitions. Under this point of view, the WA transform is a suitable methodology to investigate historical data that cannot be up-dated. In such cases HA procedures encounter important and crucial binding constraints. In this sense, WA enhances the possibility for researchers to develop their work overcoming several difficulties faced in HA empirical implementation considering that no specific assumption on the characteristics of underlying data generation process is required.

As far as the findings are concerned, the present attempt confirms the presence of the coherent periodical cycles in economic data as long-waves theory would suggest. Curiously, Kondratieff original series seem to give a helping hand to Kondratieff's theory when a WA approach is applied. Such results can be considered quite interesting, because they acknowledge the important influence Kondratieff's intuitions have exerted (not only) in economic studies (van Duijn, 2006). This work fills a gap present in current literature on the topic. As a matter of fact, WA has not been previously applied to the Russian economist's dataset from which the whole strand of literature started. A wealth of subsequent and related literature is available, but a very limited focus has been proposed on this specific aspect. Well-defined and Kondratieff-coherent patterns depicting endogenous and periodical fluctuations of macroeconomic variables appear plausible in a biological perspective (also for modeling economic activity) as pointed out by Devezas and Corredine (2001). Confirming and similar interpretations can be found in Dalgaard and Strulik (2015) wherein economic growth is related to physiological factors.

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Availability of data and materials

If the paper were accepted for publication, I have no problem to furnish the whole dataset

Competing interests

The author(s) declare(s) that they have no competing interests”

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Figures

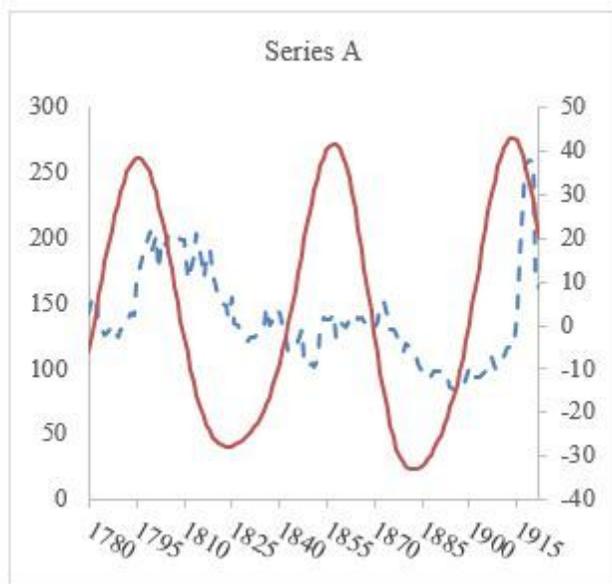


Figure 1

Series A raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
Source: Personal elaboration on Gattei (1981)

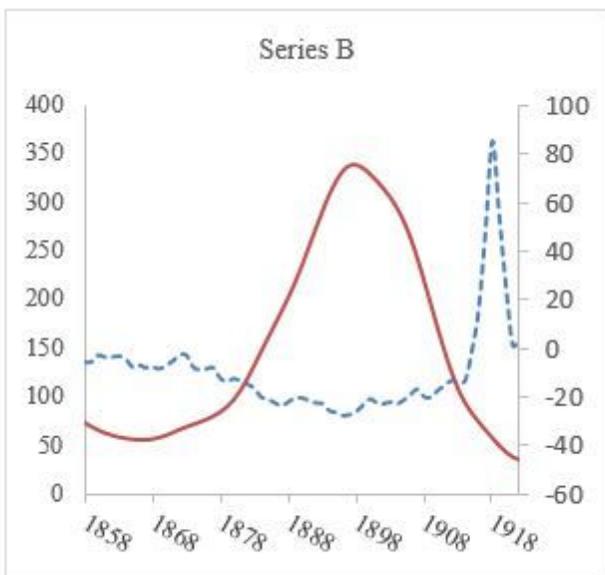


Figure 2

Series B raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
Source: Personal elaboration on Gattei (1981)

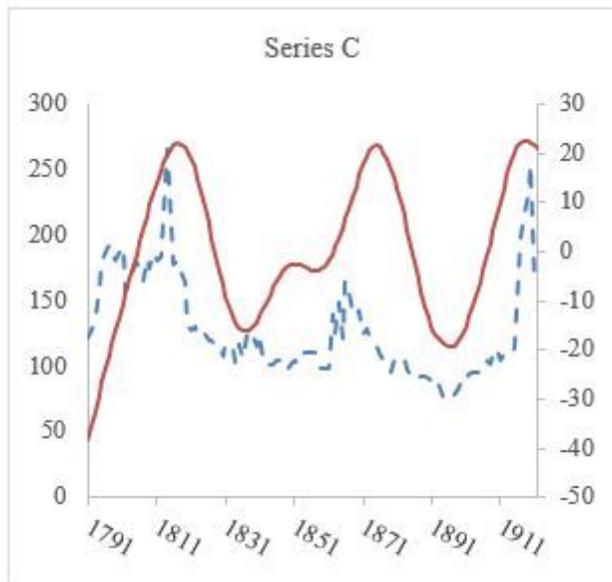


Figure 3

Series C raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

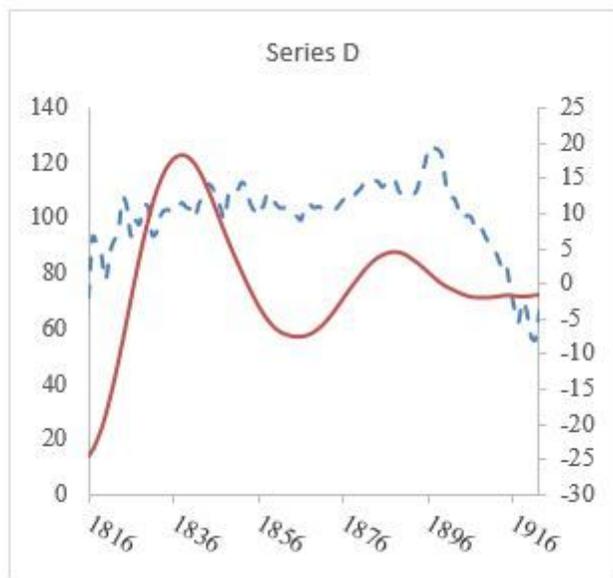


Figure 4

Series D raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

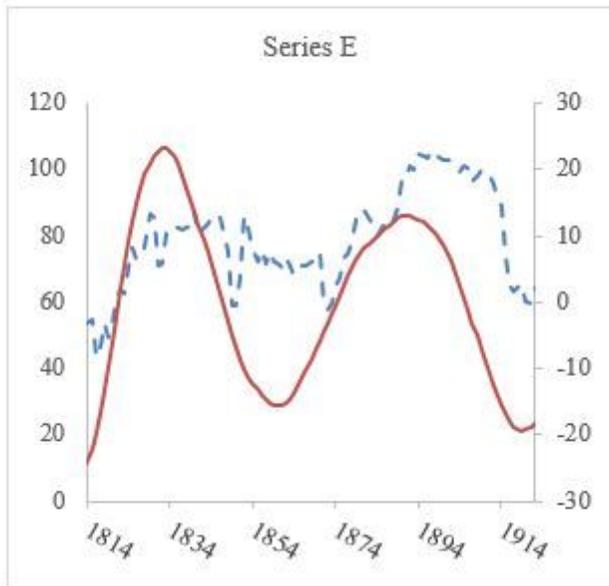


Figure 5

Series E raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

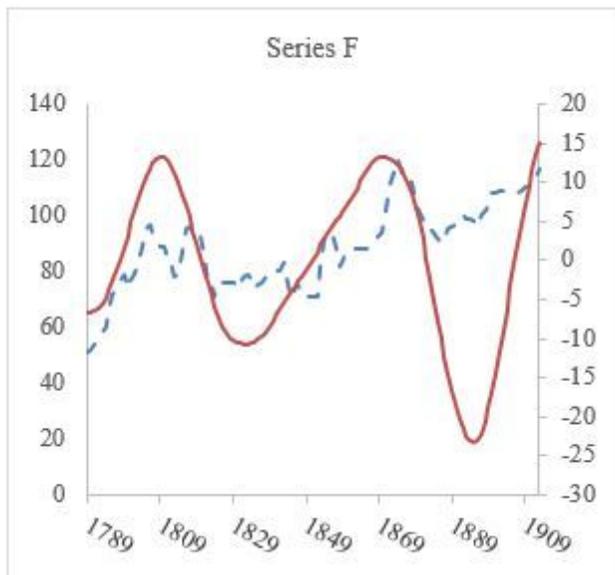


Figure 6

Series F raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

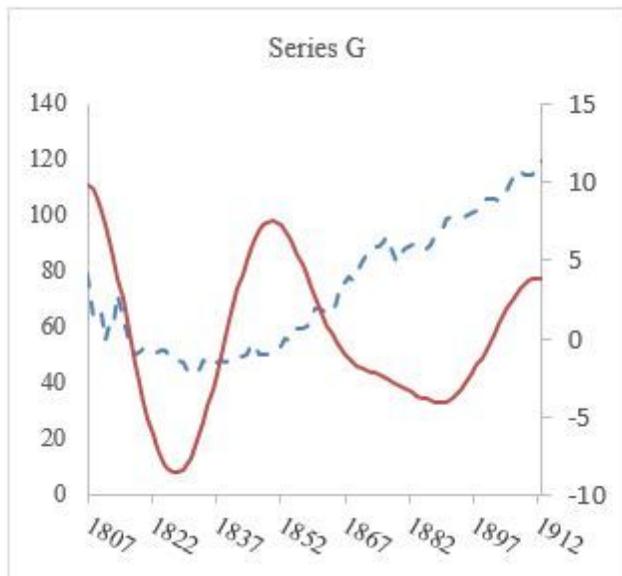


Figure 7

Series G raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

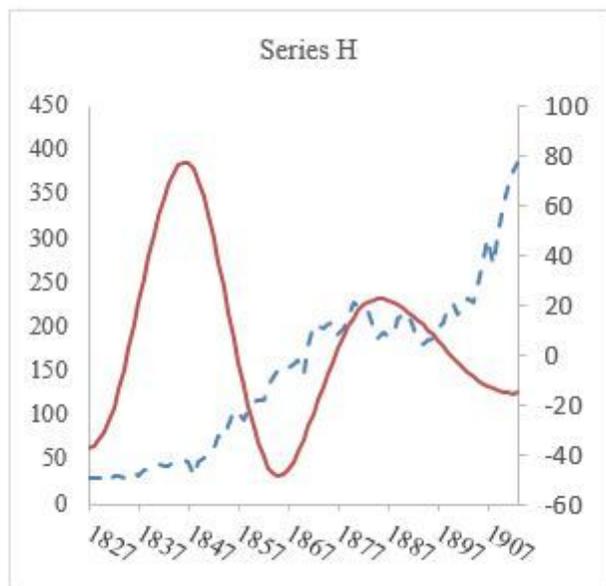


Figure 8

Series H raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

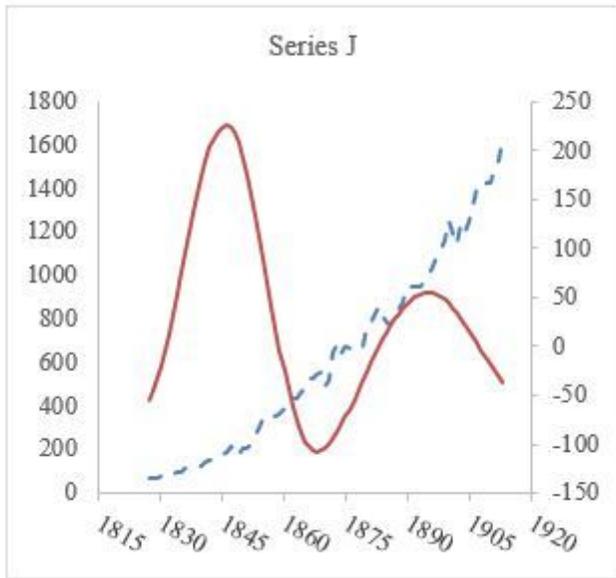


Figure 9

Series J raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

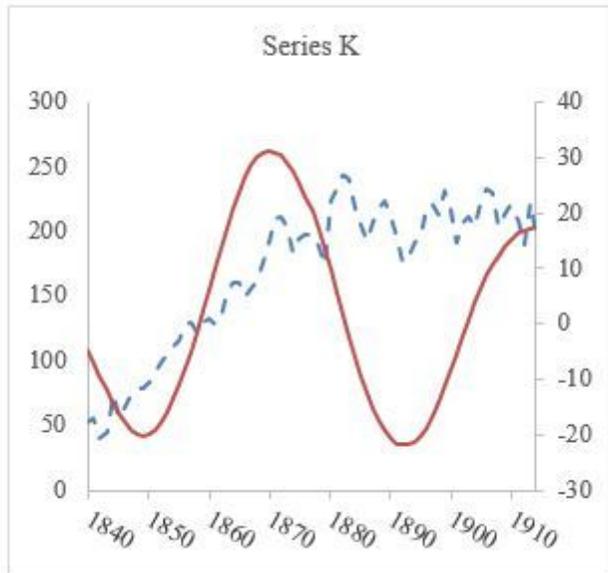


Figure 10

Series K raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

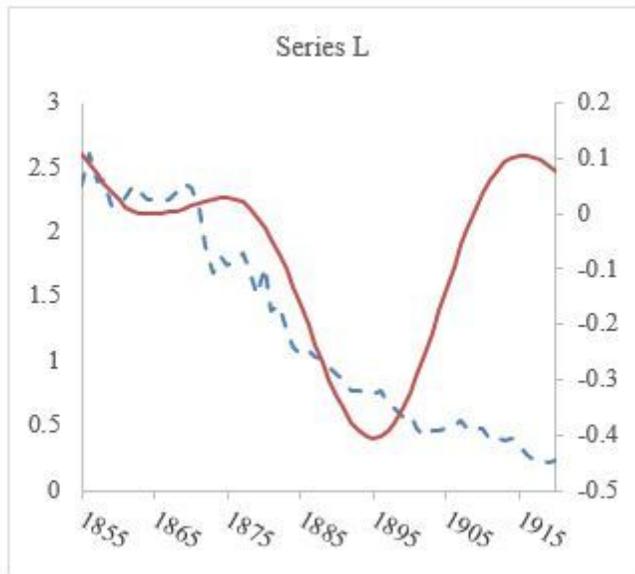


Figure 11

Series L raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

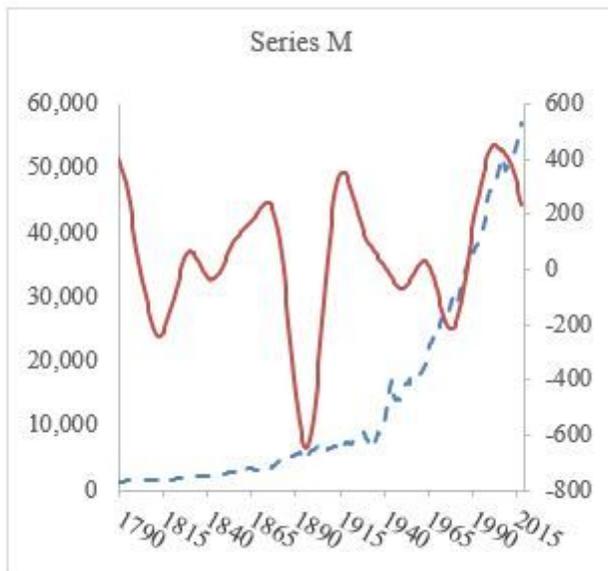


Figure 12

Series M raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

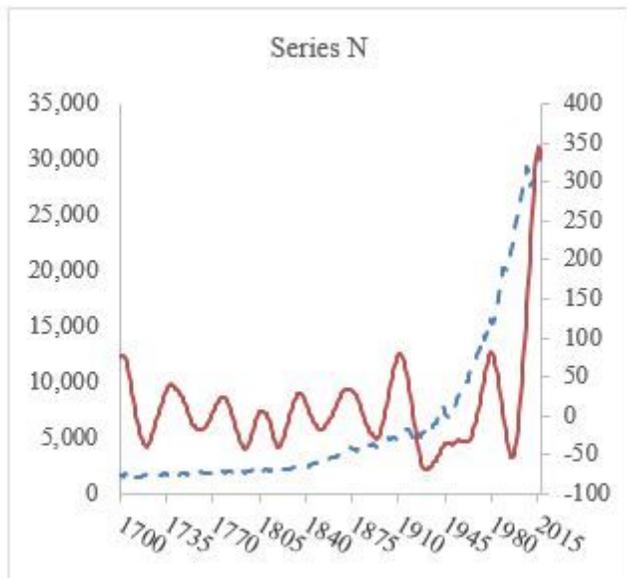


Figure 13

Series N raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

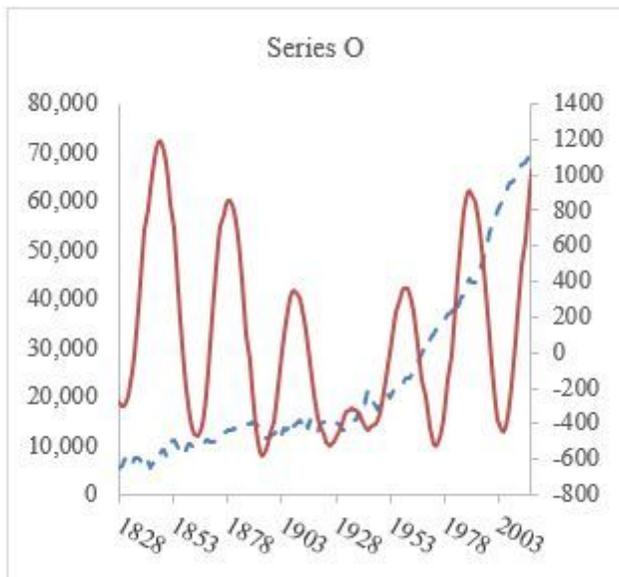


Figure 14

Series O raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
 Source: Personal elaboration on Gattei (1981)

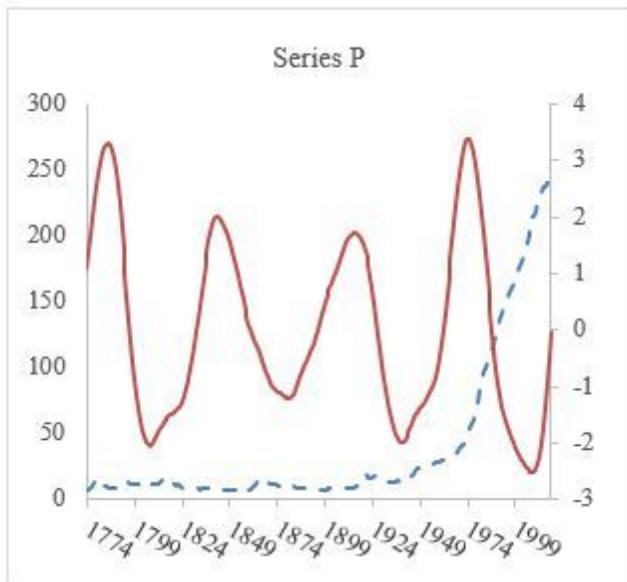


Figure 15

Series P raw data (dotted lines) along with its corresponding wavelet detail component D5 (solid line)
Source: Personal elaboration on Gattei (1981)