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Research Article

Keywords: talent, randomness, chance, success, sport, fencing, agent-based model

Posted Date: January 11th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-698207/v2

The Role of Chance in Fencing Tournaments: an Agent-Based Approach

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ABSTRACT

It is a widespread belief that success is mainly due to innate qualities, rather than to external forces. This is particularly true in sport competitions, where individual talent is usually considered the main, if not the only, ingredient in order to reach success. In this study, with the help of both real data and agent-based simulations, we explore the limits of this belief by quantifying the relative weight of talent and chance in fencing, a combat sport involving a weapon. Fencing competitions are structured as direct elimination tournaments, where randomness is explicitly present in some rules. Our dataset covers the last decade of international events and consists of both single competition results and annual rankings for male and female fencers under 20 years old (Junior category). Our model is calibrated on the dataset and parametrized by just one free variable a describing the importance of talent – and, consequently, of chance – in competitions ($a = 1$ indicates the ideal scenario where only talent matters, $a = 0$ the complete random one). Our agent-based approach is able to reproduce the main stylized facts observed in real data, at the level of both single fencing tournaments and entire careers of a given community of fencers. We find that simulations approximate very well the real data for both Junior Men and Women when talent weights slightly less than chance, i.e. when a is around 0.45. We conclude that the role of chance in fencing is unusually high and it probably represents an extreme case for individual sports. Our results shed light on the importance of external factors in both athletes' results in single tournaments and their entire career, making even more unfair the "winner-takes-all" disparities in remuneration which often occur among the winner and the other classified.

Introduction

Thinking about successful careers in general, and in sports in particular, the common belief is that they are just the result only of hard work, endurance and effort. One could naively think they develop from a long series of successes in several competitions, after endless training sessions and great sacrifices, and that only predestined champions with innate uncommon abilities can get them. Inner talent, in this view, makes the main difference in career development. However, many recent studies have shown that talent does not vary so widely among people and that its probability distribution is limited and concentrated around a well defined mean value.[1–4]

Although individual talents do not differ that much from one person to another one, only a few reach the top. We experience it in many fields[4–7] and clearly in sports[3,8], where we observe how people suddenly get notoriety and often huge amounts of money when they win.

Even with comparable talents, usually very high, athletes could end up with totally different rewards in a competition. One event after another, small fortuitous differences might give rise to a cumulative advantage,[9,10] which generate a consistent increasing gap between individuals with similar talents. We tend to admire those who reach the top, disregarding others in the ranking. We are so used to this common "winner-takes-all" logic that it sounds inevitable, in sports, arts, even in science. And most often the rationale given for such selectivity is formulated in terms of innate talent, coupled with effort: "they are the best".

We still lack a clear understanding of the processes which lead to vastly different rewards in sports and many other settings. In reality many small and unpredictable circumstances often play a role, yet as much often we tend to ignore these influences, preferring to believe in truly exceptional or gifted people. This talent oriented view persists despite being rejected by a wealth of evidence.[2–7,11,12]

In this paper, we investigate these effects in the context of an individual sport. In this kind of sport disciplines it is usually easier to analyse results and then assess the consequences, since many of them have simple rules and athletes act in controlled environments. Thus, they are suitable for testing an agent-based approach which could reproduce, in a virtual environment, athletes' performance and tournaments' structure, capturing the role of individual abilities versus external circumstances in achieving success. We will specifically consider *fencing* as case study, since it is an individual sport made up of face-to-face matches (called bouts) that directly compare athletes, underlining their similarities in contrast with huge differences in their outcome. Moreover, it contains rules where randomness is involved (for example, in case of tie). Fencing is a combat sport involving a weapon, which can be of three different kinds, identifying three separate disciplines: épée, foil and sabre. We focus on épée because there is no right of way rule regarding attacks, which means that any hit is counted.[13,14]

Fencing is a perfect example of how unpredictable factors can strongly condition careers. First of all, randomness is explicitly present in its rules: in case of a tie, one extra minute is given, assigning a priority at random to one fencer. Secondly, competitions are arranged as tournaments with two distinct phases (pools and direct elimination), characterised by intensive bouts performed in short periods of time, with irregular breaks in between.[15] Thus, a competition can last an entire day and it is difficult to keep both physical and psychological energies under control; such a delicate aspect has been examined in several studies,[15–20] highlighting the unique cognitive processes enhanced by fencing.[21] For our purposes, we point out that the organization of tournaments itself exposes fencers to the influence of random events, which may globally condition the day of the competition (e.g. sudden injuries, fever, irregular schedule of bouts) and/or simply affect the match outcome. Finally, at the end of a fixed competition, each placement in the final classification corresponds to a certain number of points, which scales non-linearly from top to bottom. The total points earned become part of the so-called ranking, which collect all results of every athlete who participates in at least one event during the current season.[13] The following season, each corresponding result of the previous year in the ranking is deleted and consequently updated by the new one. This updating rule probably gives more importance to a single outstanding result, less regarding a constant and pretty decent career during the years.

We present an agent-based model able to reproduce young fencer career progression and compare the results of the simulations with real data, in both official ranking and single tournaments. Our dataset consists of 100 tournament classifications of Junior Men and Women World Cups across thirteen years. Out of them, eight are supported by their respective seasonal ranking. We observe how results of a specific competition cannot be representative of athlete's talent, either their corresponding points, while their sum in the official ranking provide a better understanding of talent distribution, but only if we assume that all competitions have the same weight. In this case, we find a good agreement between simulation and data in reproducing the fat tailed scale of total points for Junior Men and Women (namely, fencers under 20 years old). Such a behaviour shows that only a few people collect most of the prizes, leaving other opponents the crumbs at the end of the season. Yet, single tournament results remain unpredictable and, occasionally, surprising.

We conclude that the role of chance is significant, affecting any single point in a non-linear way. One point at a time, it influences even the final winner, thus creating a disparity between two fencers whose talent is very close in principle.

The paper is organized as follows: in Model we present our agent-based model for fencing tournaments; then we discuss the Simulation Results and Comparison with Real Data; finally, we close with some Conclusive Remarks. In Methods, we explain fencing sport more in detail and add information about our dataset and the calibration of the model itself.

Model

In this section we present an agent-based model realized in NetLogo environment[22] in order to reproduce the dynamics of several international competitive seasons in fencing.

In section Fencing rules of Methods we introduce the main notions of the chosen fencing discipline (épée), whose combat features make it particularly suitable for our main goal, that is the evaluation of the relative role of talent and chance in determining successful careers of athletes (fencers) belonging to a certain community.

To this aim, in every simulation run we consider a given number N_S of seasons/years, each made of a certain number N_T of tournaments (also called events or competitions). At the beginning of each run, all the agents are randomly listed in an initial ranking. Then, every season, each athlete of the community can "choose" the number of events ($\leq N_T$) he or she wants to participate during that year, with a probability related to the ranking order updated at the end of the previous season. Thus, each tournament is characterized by a different number N of participants (see Methods for more details).

Let us now describe how we model each single tournament, following the rules explained in Fencing rules. The structure of a standard fencing competition, which takes place over a single day, includes two subsequent phases, the round of pools and the direct elimination table, each consisting of several matches.

Round of pools

For sake of simplicity, in our model every pool is built with 6 athletes, on the basis of the scheme reported in Table 2. Thus, there will be a total of $N/6$ pools (N should be a multiple of 6). Inside a certain pool, a given competitor fences, in turn, with each one of all the other 5 competitors into single matches, or 'bouts', up to five touches within three minutes, keeping track of the victories, of the hits scored and of the hits received.

The sequence of touches occurring during a single match is realized by randomly choosing subsequent time intervals between 2 and 60 seconds. At the end of each interval, both athletes have the possibility to perform a valid touch (hit) according to the following quantity:

$$P_k = a\,T_k + (1-a)\,L_k, \quad k = 1,2 \tag{1}$$

For each fencer, P_k depends both on his/her talent T_k and on the chance parameter L_k. In analogy with previous studies[2,3,5,6], we represent talent with a real variable $T_k \in (0,1]$, randomly extracted from a Gaussian distribution with mean $\mu = 0.6$ and standard deviation $\sigma = 0.1$; thus, T_k envelopes all the inner qualities of an athlete (intelligence, skills, ability, training, motivation, etc.). Being an intrinsic feature of each agent, we assume that talent remains constant during an entire simulation run (made of several seasons). On the other hand, the chance parameter L_k is randomly extracted for each single touch in the interval $[\bar{L}_k - 0.3, \bar{L}_k + 0.3]$. This choice takes into account two different sources of randomness, acting on different temporal scales. The mean chance parameter $\bar{L}_k$ affects the average performance of the corresponding athlete during a tournament, due to external unpredictable factors that may influence that performance in the day of the competition (the organization of tournaments itself could be responsible of this, as mentioned in Introduction). Therefore, $\bar{L}_k$ it is randomly extracted in the interval $[0.3, 0.7]$ at the beginning of each competition and remain fixed for the entire tournament. Of course, during each single match, other unpredictable factors can influence the athlete's performance on a shorter time scale, thus we allow the chance parameter L_k to randomly fluctuate around its mean value $\bar{L}_k$ for every touch. For example, a fit agent could have $\bar{L}_k \sim 0.7$ and a chance parameter extracted from a uniform distribution in the interval $[0.4, 1]$, clearly in his/her favour; on the other hand, an out of condition fencer could have $\bar{L}_k \sim 0.3$ with a consequent interval for L_k limited to $[0, 0.6]$ while performing a valid touch. The common parameter $a \in [0,1]$ in Equation (1) represents the so called *talent strength*, i.e. the weight of talent in making the hit; as a consequence, $(1-a)$ weights the importance of chance. If $P_1 > P_2$ we assign a valid touch to the first fencer and his/her score is increased of 1; if $P_2 > P_1$ the opposite happens.

It is worth noting that the talent strength a is the only free global parameter in our model, which allow us to estimate - through the comparison with real data - the relative importance of talent and chance in the fencing discipline.

In a real fencing match, there is also the possibility to have one or more double-hits (i.e. simultaneous touches). This has been implemented in the model by allowing a given fraction F_d of touches to be considered a double-hit. On the basis of our experience[1], F_d can be defined as:

$$F_d = 0.4\left[\frac{1}{2}\left(1 - \langle T \rangle\right) + \frac{1}{2}\left(1 - \frac{\tilde{r}}{N}\right)\right] \tag{2}$$

and is the result of two different contributions of equal weight: $(1 - \langle T \rangle)$, where $\langle T \rangle$ represents the mean talent of the two opponents; $\left(1 - \frac{\tilde{r}}{N}\right)$, being $\tilde{r} = |r_1 - r_2|$ the difference between the initial ranking of the fencers considered. Thus, the possibility of a double-hit increases when both mean talent and ranking difference decrease. Notice that F_d is confined in the interval $[0, 0.4]$ thanks to the prefactor in Equation (2): in correspondence of each new touch, a random variable h is extracted in the interval $[0,1]$ and, if $F_d > h$, a double-hit occurs. In this case both the competitors increase their scores of 1.

In case of a tie at the end of the three minutes, an extra minute of priority is given, as explained in Fencing rules. In the model, priority is implemented as follows: with a coin flip, priority is assigned to one of the two opponents; then, the extra minute starts and only a single touch is allowed; if a double-hit occurs the score is not updated; if the minute ends without any single valid hit, whoever owns priority wins the bout. Notice that the coin flip for the priority assignment is a third source of randomness, acting on an intermediate time scale, independent of the first two and intrinsic to the fencing rules.

When all the pools are completed, a summary classification is established on the basis of several indices described in Fencing rules; as a consequence, the first 70% of the athletes after the round of pools can access to the direct elimination table.

Direct elimination table

Direct elimination table is built according to the classification after pools and can be complete or incomplete: in the former case, the number of competitors is an exact power of 2 and all bouts of that round must be held; in the latter, the number of athletes, equal to the vacancies in the table, can advance without facing any opponent.

During this phase, the bout has its canonical structure, three periods of three minutes each and a maximum score of 15 points, each touch being assigned with the same procedure implemented for the bouts in the round of pools (Equations (1)

[1] One of the authors - C.Z. - is also a fencer and fencing instructor.

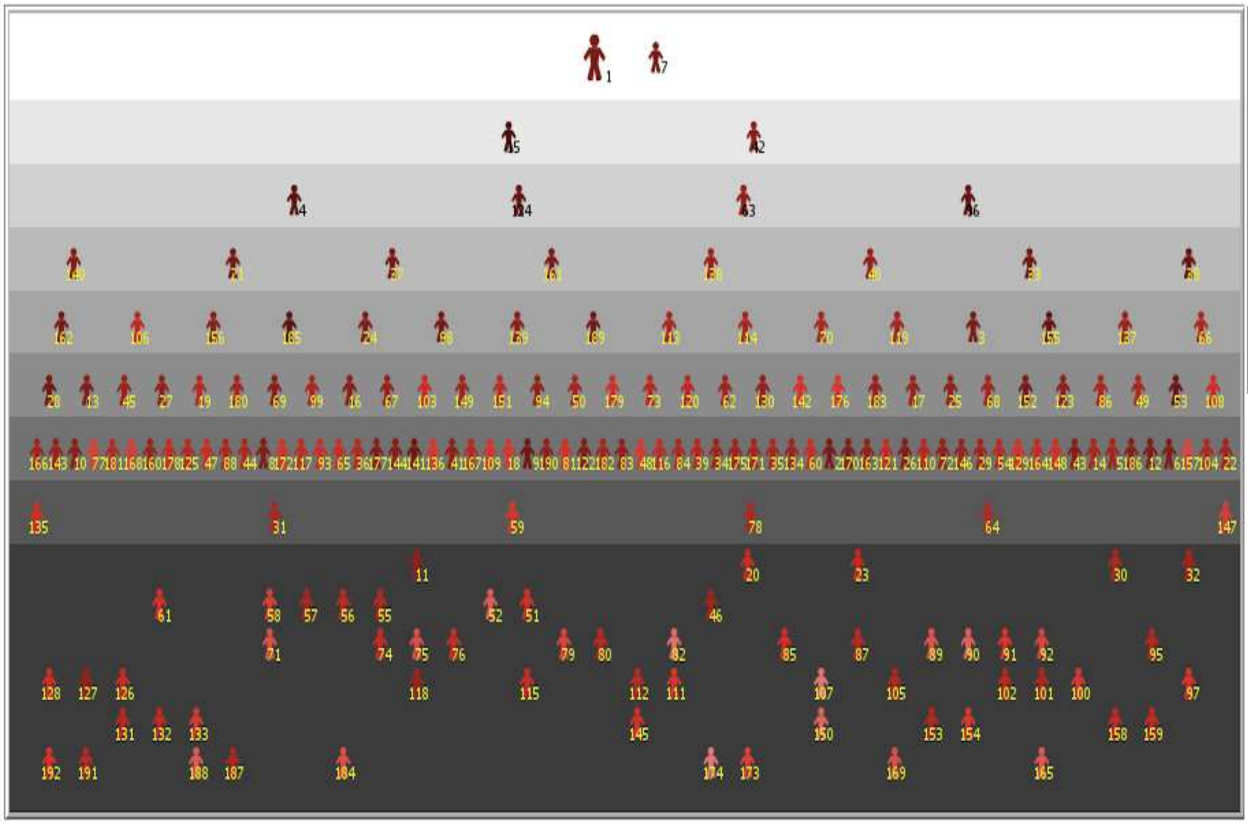


Figure 1. An example of athletes' classification at the end of a single simulated tournament. Subscripts display the initial ranking of the fencers, while their pyramidal arrangement indicates their final placements. Agents in the bottom, dark-gray, part of the figure are the 30% of athletes who did not access to the direct elimination table.

and (2)). Again, in case of a tie, an extra minute of priority is given, as already mentioned in Round of pools and further explained in Fencing rules. The fencer who wins the bout advances in the table, while the loser ends his/her competition. This selective mechanism is the same in every round of the table and, at the end of the tournament, produces a pyramidal arrangement similar to that one shown in Figure 1: at the top level we found the first and the second classified; at the bottom level, the 30% of athletes who did not pass the round of pool; all the other fencers lie in the middle levels. Each agent is labelled with his/her position in the initial ranking: in the example considered, the winner of the competition started from the first position in the ranking, while the second classified started from position 7$^{\text{th}}$, and so on.

At the end of the tournament, fencers receive an amount of points according to their classification, following Table 3 of Fencing rules. Notice that we do not distinguish between Championships and World Cups in our model, thus all events weight equally in simulation rankings. As explained more in detail in Fencing rules, ranking is not cumulative over the years. On the contrary, it rolls during a new season: each new result cancels out the result obtained in the corresponding competition of the previous year. The simulation stops when the last tournament of the last year ends. At this point it is possible to look at several output parameters, such as the final ranking of athletes, the relationship between their initial and final placement (calculated either for each season or for each single competition) and even the interplay between talent and rank, all of them as a function of the selected value of talent strength a.

Simulation Results and Comparison with Real Data

In the previous section we briefly showed the necessary features for simulating the careers of young fencers and how we inserted them in the structure of our model. Our aim is that of investigating fencing dynamics for different values of the global talent strength, comparing the simulation results with real data in order to evaluate the relative role of talent and chance in this sporting discipline.

As explained in section Dataset and model insights of Methods, for this comparison we chose a dataset containing the official rankings of both Junior Men and Junior Women from 2011 to 2019, with participants coming from all the countries of the International Fencing Federation (FIE). Since fencers who can access to Junior competitions must be between 14 and

20 years old, the longest possible career lasts six years. Therefore, we set to $N_S = 7$ the number of seasons/years to simulate, considering the first as a trial stage for the following ones. According to the average length of the official FIE rankings, we consider a community of $N_M = 600$ fencers to simulate the Junior Men seasons and a smaller community of $N_F = 500$ fencers for the Junior Women ones.

For every season, we simulate $N_T = 8$ distinct tournaments with a variable number N of participants, following effective data results considered in official ranking. In fact, during a given season, each athlete of the community can "choose" the number of events ($\leq N_T$) she wants to attend, with a probability related to the ranking order for that year. Those conditional probabilities have been extracted from the real dataset for both men and women, as explained in Dataset and model insights (see Figure 10), and the model has been calibrated accordingly.

In Table 1 we summarize the setup of the fixed parameters that we adopt in our simulations, included mean and standard deviation of the normal distribution of athletes' talent introduced in the previous section. In order to have statistically significant results, we always average the outputs over 10 simulation runs, each starting from a different realization of the talent distribution among agents. There is no need to add runs since we already observe quite stable results and lower errors than those found in the data.

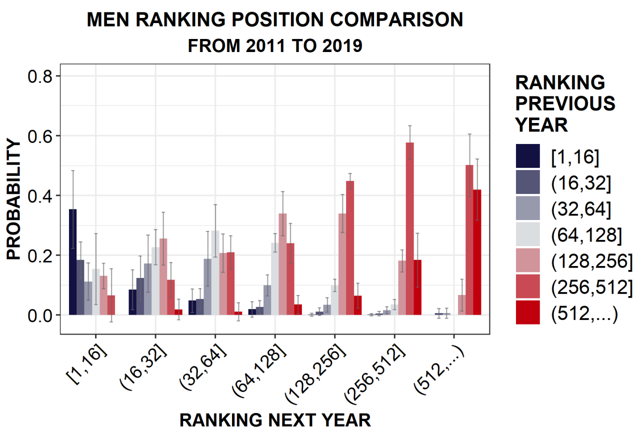
Parameters	Simulations	
	Men	Women
N-Years-to-simulate	7	7
N-Tournaments-per-year	8	8
Talent-gaussian-mean	0.6	0.6
Talent-standard-deviation	0.1	0.1
Total-athletes	600	500

Table 1. The fixed set of parameters for each simulation run.

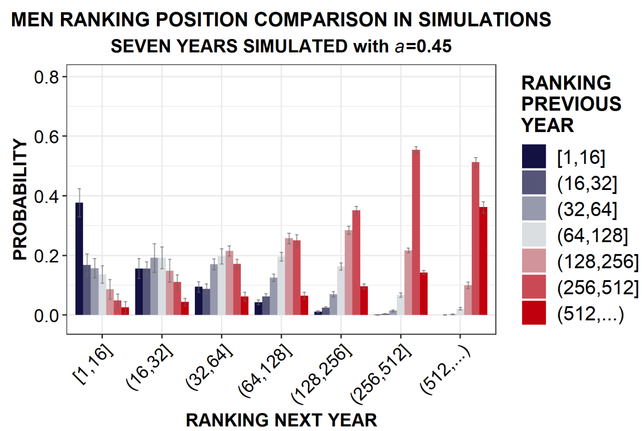
Our goal is to find the optimal value of the talent strength a able to produce the best agreement between simulations and real data. To do so, we first consider the probability of improving, maintaining or worsening the ranking placement obtained the previous year (see Methods for details). In fact, if the official ranking was a perfect mirror of athletes' talent, those probabilities should be peaked in the corresponding placements, season after season, with very small fluctuations. Instead, observing the real data analysis results in panels (a) of both Figure 2, for men, and Figure 3, for women, there is only a weak correlation between previous and following ranking positions: for example, athletes who conclude a certain season in the first 16 positions in the ranking, at the beginning of that season were in the same first 16 positions only with a probability between 0.3 and 0.4, while are slightly less likely to have started from lower positions in the ranking, and have still a not negligible probability to have started below position 500$^{\text{th}}$. This effect is even more pronounced for the other positions, thus suggesting that talent explains only a part of the story: evidently, the influence of external factors cannot be neglected and a certain role of chance should be also taken into account.

In order to quantitatively estimate this role, we report in the other panels of the same figures the analogous results obtained with our simulations for different values of the talent strength a. It is possible to see that the simulation outputs yielding the best agreement (inside the error bars) with real data are those obtained in correspondence of $a = 0.45$, as shown in panels (b) of both the figures. On the other hand, results observed for smaller ($a = 0.2$) or greater ($a = 1$) values of the talent strength, reported in panels (c) and (d) of both the figures, are clearly not compatible with real data. The selection of the optimal value $a = 0.45$ is supported by the mean squared error estimation, as shown in detail in Dataset and model insights (see Figure 12, top panels). This first finding suggests that the role of chance in fencing competitions, estimated by the factor $(1-a)$ in Equation (1), is absolutely not negligible if compared with talent: actually, it seems to be quite consistent, even slightly above 50% for both men and women.

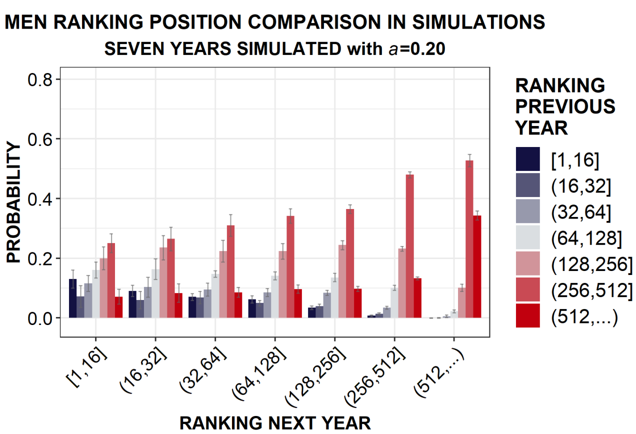
To further support the choice of $a = 0.45$ as the best candidate for talent strength, we can move forward with other comparisons between data and simulations. In particular, it is interesting to look at the normalised points, cumulated by real athletes at the end of the season, as a function of their final ranking and compare them with the analogous ones obtained through simulations as a function of a. A very good agreement emerges for values included in a narrow range between $a = 0.4$ and $a = 0.6$ but, again, the lowest mean square error is obtained for $a = 0.45$ (the complete analysis can be found in Dataset and model insights, see again Figure 12, bottom panels). The corresponding curves, obtained averaging over all the considered seasons and normalized to their maximum value, are reported in Figure 4 for both men (a) and women (b). One can notice significant overlaps, which are almost indistinguishable for the first 200 placements. For ranking placements higher than 200, data and simulation points act differently: the former decrease more rapidly, which can be due to a finite size effect more pronounced in data since ranking lengths vary over time, other than to an update of the rules for the point scale, as mentioned in section Fencing rules of Methods.



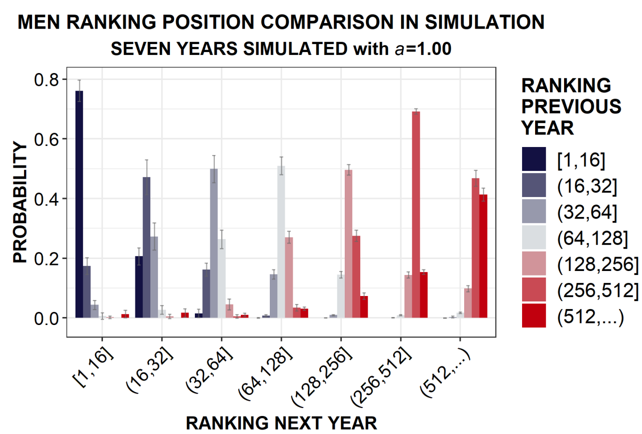
(a) Data



(b) Simulation

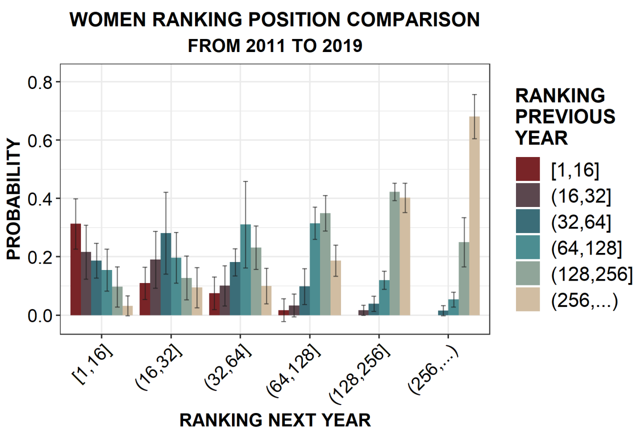


(c) Simulation

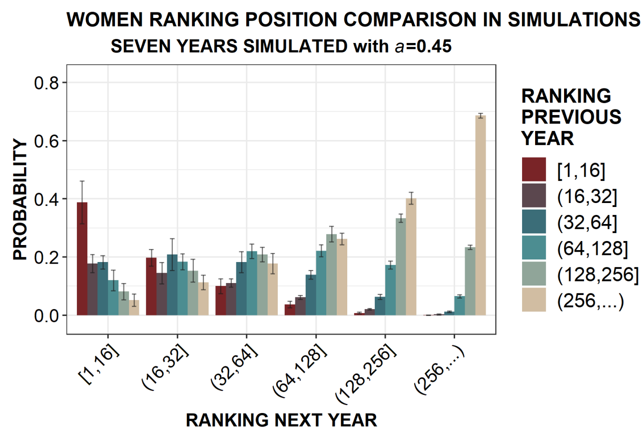


(d) Simulation

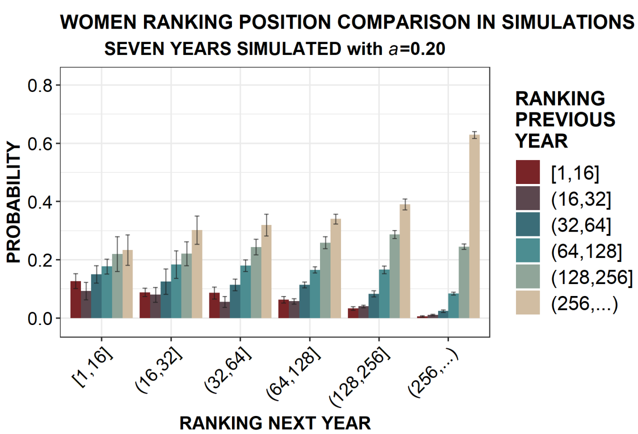
Figure 2. Comparison between data and simulations (averaged over 10 runs). Probability of having the same or a different placement the following year in Junior Men, given the associated ranking placement in the previous year. Ranking positions are arranged in groups of different sizes, to enhance visualization. For each mean outcome, the corresponding standard deviation is also reported as error bar.



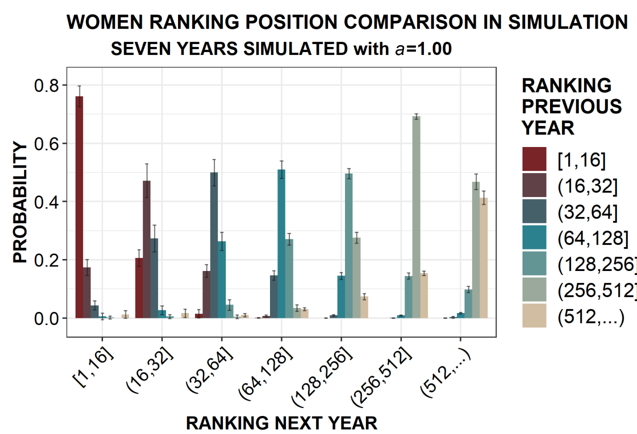
(a) Data



(b) Simulation

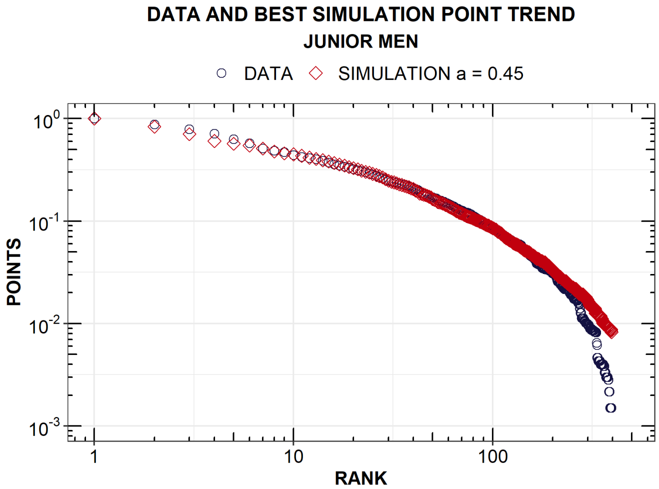


(c) Simulation

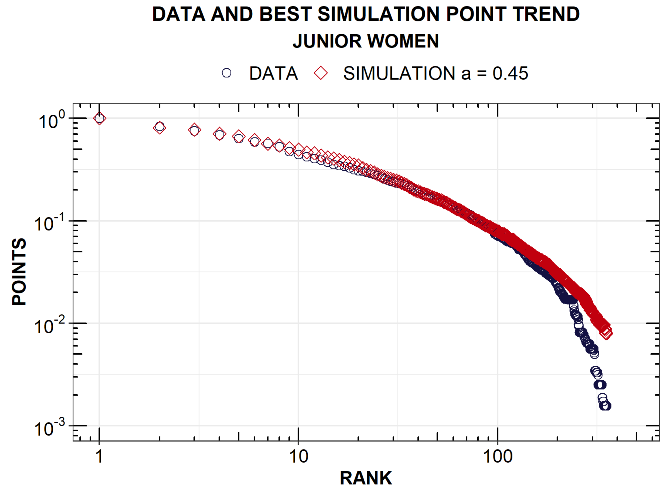


(d) Simulation

Figure 3. Comparison between data and simulations (averaged over 10 runs). Probability of having the same or a different placement the following year in Junior Women, given the associated ranking placement in the previous year. Ranking positions are organized in groups of different sizes, to enhance visualization. For each mean outcome, the corresponding standard deviation is also reported as error bar.



(a) Men



(b) Women

Figure 4. Trend of the average total points in Junior rankings, normalised to their maximum value, compared to simulations for male (left panels) and female (right panels) fencers.

Summarizing this first set of results, obtained analyzing fencing competitions along several seasons, we should conclude that: (i) real data analysis already shows an evident role of external factors (chance) in determining the athletes' placements at the end of each season; (ii) the comparison with the simulation outputs of our fencing model allow us to quantify this role by tuning the talent strength parameter and finding the value which minimizes the error. In the remaining part of the paper we fix this parameter at its optimal value $a = 0.45$, which turned out to be the same for both men and women, to further explore the potential of our model focusing on single tournaments.

Specifically, we observe the relationship between initial ranking and final placement at the end of a given competition. This has been done averaging over 80 events for both men and women in simulation and over 52 events for men and 48 events for women in our dataset. We monitor the top sixteen agents in the ranking or in the final classification. For those athletes, we would like to ask the following two questions:

(1) What is the conditional probability of obtaining a certain final placement in the tournament provided that one starts from a certain initial ranking position?

(2) What is the conditional probability of having started from a certain position provided that one reaches a certain final placement in the tournament?

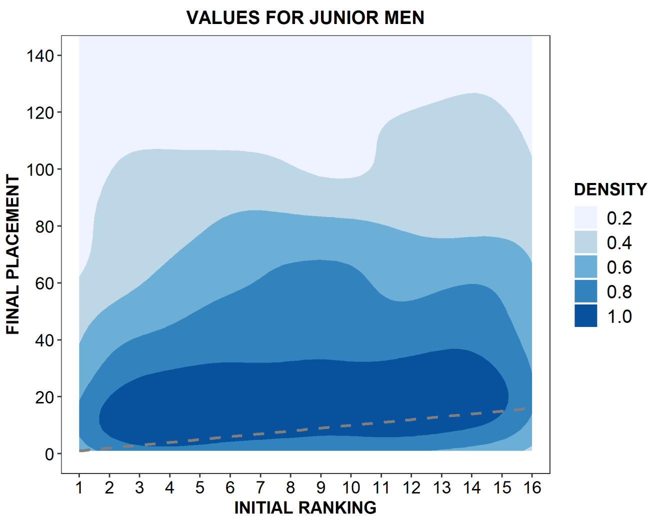
In the ideal case in which talent would be the only variable influencing both ranking placements and tournaments' results, these two variables should be strongly correlated and such a correlation should be highlighted by an opportune density kernel analysis[23]. Thus, since we already found that the role of chance in fencing competitions matters at least as much as that of talent, one should expect important deviations from this ideal behaviour. Moreover, one could also expect that our model should be able to numerically reproduce these deviations. Actually, this is precisely what we observe in Figure 5, for male fencers, and in Figure 6, for female ones, where the (normalised) density kernel plots are reported both for real data, panels (a) and (c), and for simulated ones, panels (b) and (d). Looking at these figures one can draw the following conclusions:

(1) In all the panels the observed behaviour is very far from the ideal case, represented by a gray dashed line.

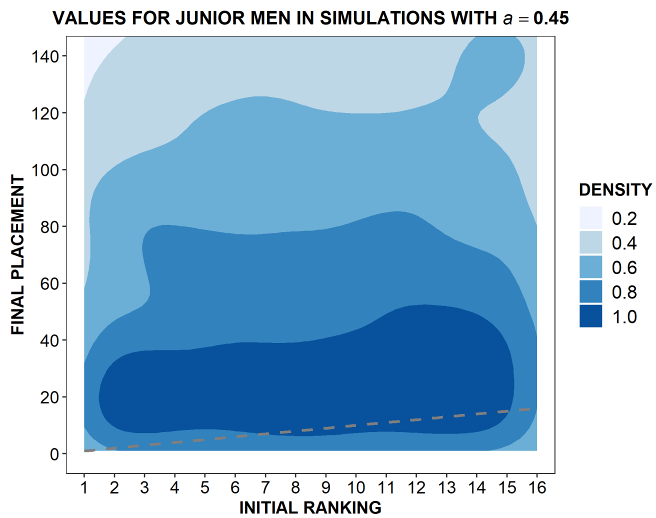
(2) The comparison between panels (a) and (b) shows that our model is able to capture the fact that the first 16 (male and female) athletes in the ranking have a quite high probability to reach a final placement included in approximately the first 30 positions, but - evidently due to the consistent role of chance - have also a decreasing, not negligible, probability to close the tournament in lower (and sometimes much lower) positions.

(3) At the same time, looking at panels (c) and (d), simulation results essentially reproduce the analogous effect observed in real data, where (male and female) athletes placed in the first 16 positions at the end of a tournament came from the first 30 or 40 positions in the initial ranking, but with a not negligible decreasing probability to come also from lower positions.

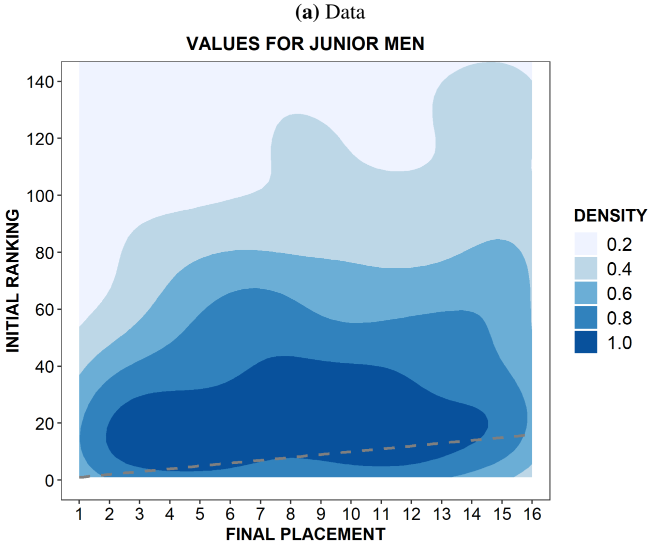
One last consideration about talent. In general, from the analysis of real data about sport disciplines it is not possible to directly extract the distribution of talent, being the latter an hidden individual variable. However, one could think that a strong correlation between ranking and talent should be, in some way, preserved by the competitive selection of tournaments. Thus, athletes in the top ranking positions are usually assumed, by definition, to be the most talented and, vice-versa, those in the bottom positions the less gifted. Unfortunately, the latter is exactly the kind of assumption which should be questioned in



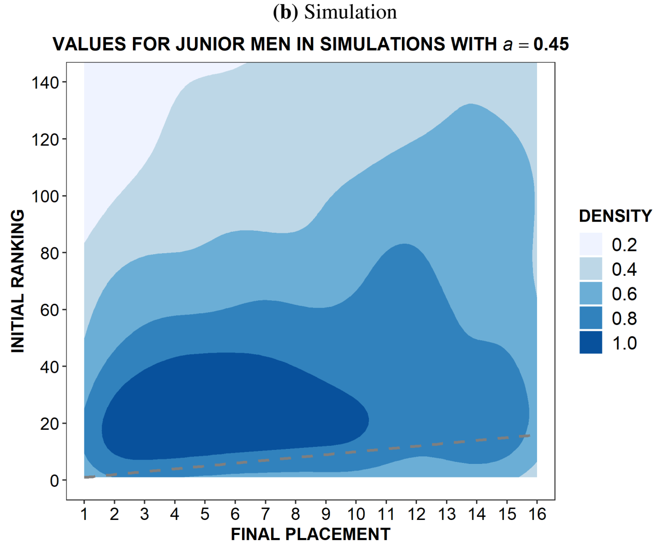
(a) Data



(b) Simulation

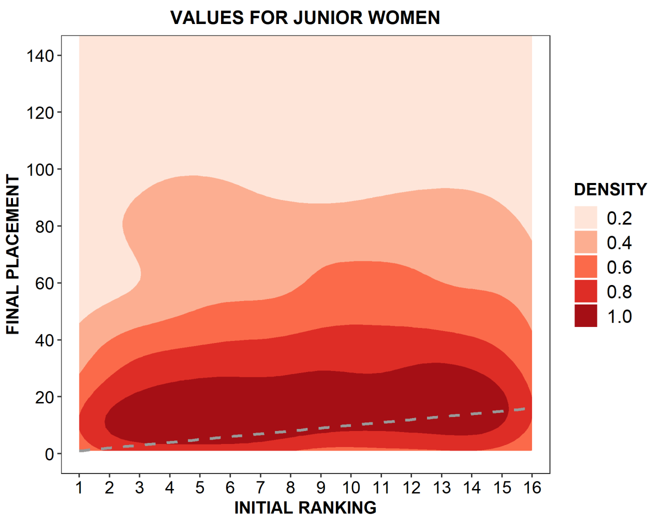


(c) Data

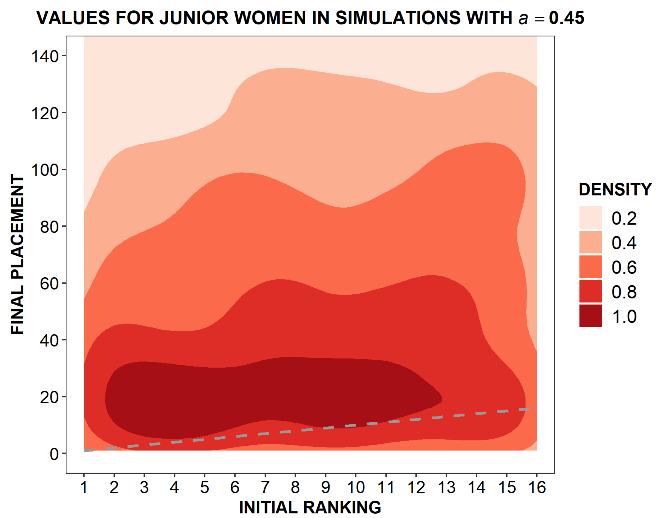


(d) Simulation

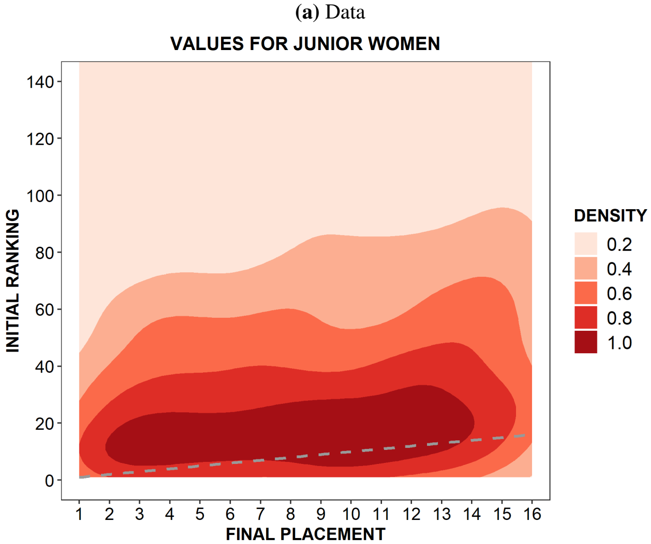
Figure 5. Single tournaments: kernel density plots for initial ranking position versus final classification (top panels) and vice-versa (bottom panels) in both World Cups (a-c) and simulations (b-d) for male fencers. In all the panels we also report a gray dashed line representing the ideal case in which talent would be the only variable influencing both ranking placements and tournaments' results.



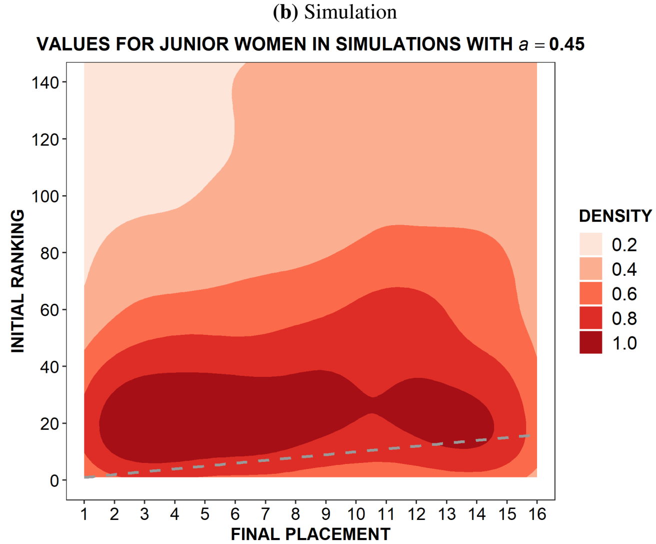
(a) Data



(b) Simulation

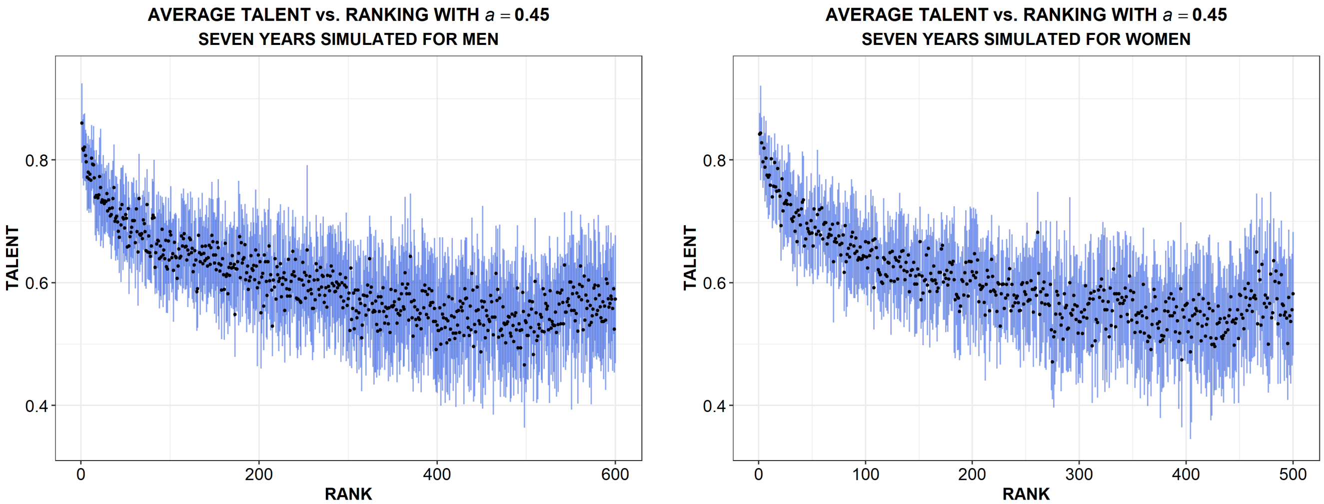


(c) Data



(d) Simulation

Figure 6. Single tournaments: kernel density plots for initial ranking position versus final classification (top panels) and vice-versa (bottom panels) in both World Cups (a-c) and simulations (b-d) for female fencers. In all the panels we also report a gray dashed line representing the ideal case in which talent would be the only variable influencing both ranking positions and tournaments' results.



(a) (b)

Figure 7. Distribution of talent as a function of fencers' ranking in simulations. Mean values from the seventh year of ten runs are shown, error bars (blue) representing their standard deviation.

disciplines where success is noticeably influenced by chance, as we have shown to happen for fencing. Therefore, we can intuitively expect some violations of this assumption in fencing competitions.

Simulations help us to confirm such an intuition, since we assign a fixed talent to all the fencers at the beginning of a given simulation run, thus we are also able to report their talent as a function of their final ranking. This has been done in Figure 7 for men (a) and women (b), respectively. Black points represent the mean talent of athletes for each position in the ranking at the end of seven seasons/years, averaged over 10 simulation runs. Error bars are the corresponding standard deviations. In both panels we observe an initial rapid decreasing trend of talent, but with strong fluctuations which progressively increase as ranking gets worse. Such a behaviour implies that, if it is true that, in average, no athletes with talent below the mean (0.6) can be found in the best 50 positions, it is also true that very talented athletes (above one standard deviation from the mean) can be found at any position in the ranking. Moreover, the trend starts to slowly bend upward approaching the last positions, an effect that likely takes into account the possibility that certain pretty talented agents could attend very few competitions during the considered interval of time. In the real world, they could represent new fencers at the beginning of their career, maybe the youngest ones, who start competing without any ranking points even if they are as talented as other older participants.

Conclusive Remarks

Summarizing, our agent-based model, calibrated on data of real competitions, allow us to estimate the relative weight of chance (external factors, random lucky or unlucky events) with respect to talent in fencing competitions: actually, we found that this weight is quite high, around 50% for both men and women. Following the line of reasoning expressed in the conclusive remarks of Ref.[3], we could probably claim that the peculiar rules of fencing make this discipline the best candidate to provide the upper limits of chance contribution in sport competitions with individual scores, whereas the Olympic 100-meter dash studied in Ref.[3], with its 4% for men and 6% for women, would give the lower limits. Thus, in an hypothetical spectrum, these two disciplines would probably represent the extremes, with all the other sports - high jump, long jump, tennis, golf, car racing, motorcycle racing, etc. - in the middle. If this were true, it would seriously question the methods of awarding cash prizes which, typically, follow an exponentially decreasing trend going from the first classified to the last one. Those methods are based on the implicit assumption of a one-to-one correspondence between talent of athletes and their performance in competitions. But we have shown that such a perfect correspondence probably does not exist, since in all these sports chance and randomness could heavily influence the performance of any athlete.

Let us close the paper with some economics considerations, which will be extended in future works.

Sport competitions are innately motivated to foster physical improvement of mankind and establish new frontiers in the ability against natural forces or among athletes. Difficulties often descend from the designed complexity of the discipline task, tremendous effort needed to reach the topmost levels, and compliance.

Every person needs great dedication and self-control to become a good athlete in any discipline, but the economic conditions surrounding the individual talent at work in different sports may create incentives in very differentiated ways. In many countries, for example, the military services sponsor young athletes, often in "non-military" sports. This provides economic support to young people, but at the same time serves as effective PR for the military services.

That is frequent for the vast majority of disciplines without market-related environments. On the contrary, in other cases, rewards for winners do not seem to be conceived to remunerate the superiority of an athlete against others and, instead, they appear to serve for the creation of the highest incentive to compete, which may induce fierce competitiveness while inducing the most spectacular entertainment. This aspect is particularly interesting, from a first point of view, in order to focus on the nature of involvement in sports, specifically when comparing more or less "profitable" disciplines.

The lack of sponsors, a reduced number of fans, a smaller involvement of press, advertising and notoriety effects, all reduce the amount of money related to sport careers and championships. A football player can earn more than a tennis player who, in turn, earns much more than an archer or a fencer, despite - for example - fencing is the most medal discipline among Italian ones.

A second aspect worth noticing is related to the proportionality of differences in rewards with respect to corresponding differences in sport rankings. The possibility to measure the true impact of personal talent in sport disciplines is essential to discover the part of the success of an athlete that should be remunerated, i.e., the part measuring the "size" of the merit. Most likely, tastes of the audience cannot be guided by equative reasons, thus it is highly improbable that the Champions League final can have the same public of the final of a fencing tournament. Nonetheless, by analysing sport competitions for different disciplines, we are looking to focus on the specific components of those sports, seeking for the role of chance.

Once a reasonable range will be established, at least for individual sports, from the most aleatory ones - where randomness counts as much as (or even more) than the athlete's talent in reaching victory - to the most genuine ones - where individual talent does it almost all -, a proportionate reward scheme can be designed. In turn, this will open possible policy implications related to financing conditions, in favour of the widest opportunities for athletes in all disciplines. The economic exploitation of market-oriented implications of sport competitions should not cause distortionary effects against pluralism of disciplines. "Profitable" disciplines could be helpful for other ones, e.g., in helping young athletes starting their career, by draining resources in excess from the formers to the latter.

Methods

In this section we first describe fencing technical rules, competition formula and point assignment in ranking during an entire season (Fencing rules); then, we present our dataset more in detail and we add some considerations on our agent-based model (Dataset and model insights).

Fencing rules[2]

Fencing is a combat sport involving a weapon, which can be of three different kinds, identifying three separate disciplines: foil, sabre, épée.

Each weapon has its own peculiarities, but they also have common characteristics: the two opponents compete on a piste, 14 metres long and 1.5 metres wide; the goal is to score a valid touch on your opponent, which counts as a point; the first fencer that achieves 15 points, in a bout (individual match) composed at most of three rounds (called periods) of three minutes each, wins; touches and time are controlled by a referee, according to an electrical recording apparatus.

In case of a tie at the end of the third period, one extra minute is given, assigning a priority at random to one fencer: if no one scores a single touch during this time, the opponent with priority wins. Therefore, the role of randomness is explicitly present in fencing rules, making this sport an interesting candidate in studying the effects of chance (good or bad luck, other external factors) in competitions.

In this work, we focus on épée because there are no right of way rules regarding attacks, which means that any hit is counted and the point is assigned to the fencer who makes the hit first. As a consequence, referee discretion is strongly limited and we can neglect human error contribution.

The main features of épée are the following:

- the attack is possible only with the point of the weapon;
- the target area is the entire body;
- since any touch is counted, double-hits are allowed if they occur within 40 milliseconds.

Those very essential rules allow épée tournaments to be simulated by an agent-based model, as described in Model. But first, we need to explain the organization of fencing competitions.

Usually, a competition (tournament) takes place over a single day and consists of two main phases, one round of eliminating pools (Table 2) followed by a direct elimination table (Figure 8).

The pools comprise 6 or 7 fencers, depending on the number of participants. They are composed taking into account the latest official FIE ranking, which collects the points obtained in the previous events of the current season or in the previous season on the basis of athletes' placements.

The allocation of fencers in the pools follows the method shown in Table 2.

Pool		A		B		C	
Fencer ranked		1	→	2	→	3	↓
	↓	6	←	5	←	4	←
	→	7	→	8	→	9	↓
	↓	12	←	11	←	10	←
	→	13	→	14	→	15	↓
	↓	18	←	17	←	16	←

Table 2. Allocation method explained for three different pools (A, B and C), based on ranking placements of 18 competitors. Each column represents a complete pool with 6 fencers.

In pool rounds, each competitor fences a bout against all of the other members of their pool, up to five hits in only one period of three minutes and an extra priority minute in case of tie.

After the pools, a single general ranking of all the athletes is established, on the basis of the following indices: first, $\frac{V}{M}$ is considered, where $V =$ is the number of victories and $M =$ the number of bouts; then, in case of equality, the difference $HS - HR$ between the hits scored (HS) and the hits received (HR) is taken into account; finally, in case of further equality in both $\frac{V}{M}$ and $HS - HR$, the fencer who has scored most hits (highest HS) is seeded highest. In the special case of absolute equality, the order is decided by drawing lots.

[2] fie.org

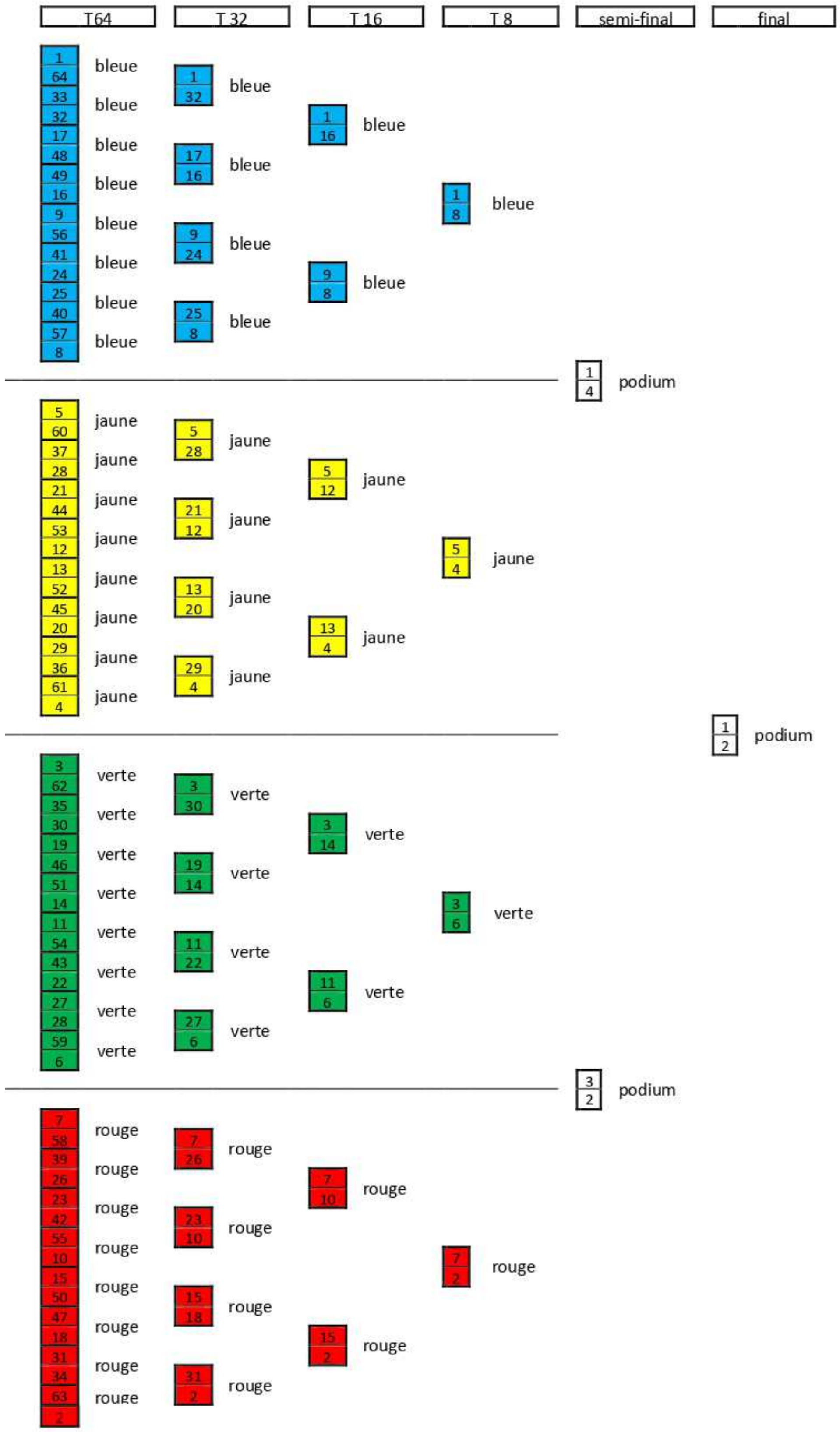


Figure 8. Sketch of the direct elimination table of a typical competition round, from 64 up to the final, scaling with decreasing powers of 2. Different colours identify different pistes. For clarity of the picture, larger tables (128, 256) have been omitted.

From the round of pools, only 70% of the fencers is qualified for the direct elimination phase, depending on the classification after pools.

Direct elimination table consists of many rounds that scale with decreasing powers of 2 (usually 256, 128, 64, 32, 16, 8, 4 and final) as shown in Figure 8: as we can see, the first classified after pools is coupled with the 64th, the second one with the 63rd and so on.

For example, if the competition starts with 100 participants, after pools there are 70 athletes qualified to the direct elimination round, which is an incomplete table of 128 (since they are more than 64). Therefore, the last 12 athletes, from number 59 up to 70 after pools, have to win one more match to access the table of 64, while the first 58 fencers automatically advance. In detail, the couples are: 59-70, 60-69, 61-68, 62-67, 63-66, 64-65. Once these matches are completed, table of 64 starts for all the participants left, following the bouts indicated in Figure 8.

Intuitively, in each round the fencer who wins his bout have access to the next one, while the loser ends his competition and obtains a placement coincident with the reached round, according to the ranking after pools. For example, if fencer A loses against fencer B in the table of 16, fencer A can be placed in the classification among the 9th and the 16th place.

The general classification is compiled from the winner of the final bout, who is also the winner of the competition, followed by the second, the one who loses the bout for the first place; then, there is an *ex aequo* for the third place, assigned to the two fencers defeated at round of semi-finals; the other placements are given as explained above.

The goal of attending competitions is to rise in the official FIE ranking. In fact, at the end of each tournament, all the participants gain a certain amount of points, fixed by the following scale:

1st place	32 points
2nd place	26 points
3rd place *ex aequo*	20 points
5th - 8th	14 points
9th - 16th	8 points
17th - 32nd	4 points
33rd - 64th	2 points
65th - 96th	1 point*
97th - 128th	0.5 point**
129th - 256th	0.25 point**
beyond 256th	0.1 point**

Table 3. Scale points in the official FIE ranking.

We can see that the scale decreases in a non-linear way, following a power law from the 9th place on. Notice that some values were added later: * was introduced in season 2015/2016; ** were introduced only in 2019.

Ranking is not cumulative over years, instead it rolls during the season: the new result cancels out the previous year result in the corresponding competition. Moreover, the official Junior ranking of the FIE considers only the best six results of the World Cup events in which the fencer has participated, other than the Zonal and World Championships, for a total of eight results collected. As a consequence, only those athletes who actually attended at least one event in a season are listed in the ranking.

We need to precise that different kinds of competitions weight differently in ranking: points obtained in World Cups events are multiplied by a factor of 1; Zonal Championship points in our dataset are multiplied by a factor of 1.5 (this rule has been updated in season 2019/2020, the factor being reduced to 1); points obtained in World Championships are multiplied by a factor of 2.5.

Dataset and model insights

In the previous subsection we introduced the main rules of fencing, with particular regard to épée discipline, as well as the competition formula adopted by the type of events which we are interested in. Those ingredients are all important to understand the construction of the official ranking, whose analysis is our main purpose.

We collected the official Junior Men and Women rankings from 2011 to 2019, excluding previous years because there were very different criteria in point assignment and year 2020 since it is incomplete (World Championship has been cancelled due to COVID-19 pandemic). All participants are in between 14 and 20 years old and they are from all the countries that are FIE members. The official rankings in our dataset come from FIE website[13]. Since the longest possible career in Junior category lasts six years (as already mentioned in Simulation Results and Comparison with Real Data) and we have nine years available, the group of athletes involved is not constant over time. Instead, every season there are some fencers who become too “old” and must change their category; on the other hand, the “youngest” fencers who can participate in Junior Events for the first time come into play. For this reason, official ranking does not have a fixed number of competitors; we find that the average length for Junior Men is about 600 participants, while for women is about 500 in our dataset, as reported in Simulation Results and Comparison with Real Data.

We also collected the initial rankings and final classifications of 100 World Cups, from 2008 to 2020, to perform the analysis of single tournaments shown in Simulation Results and Comparison with Real Data. Data availability was a non trivial issue in this case, several sources were used in the attempt of extending the dataset as much as possible.[13,24–30] In detail, our data comprise 52 results for Junior Men and 48 for Junior Women, considering only events with at least 100 competitors. We observed that the number of participants in a competition can reach a maximum value of 280 for Men and 230 for Women, with an average around 170 and 150 respectively, as visible in Figure 9.

From the variety of participation in tournaments, it follows that the list of participants in single competitions cannot have

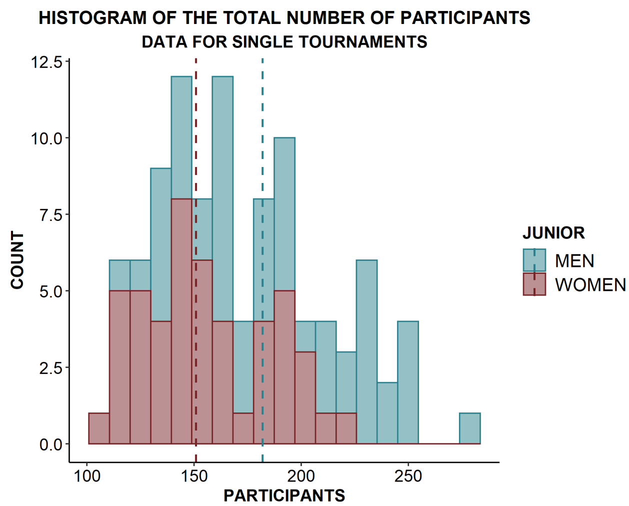
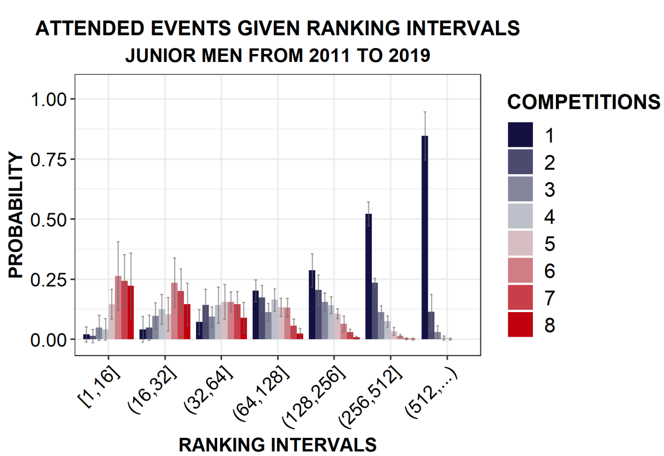
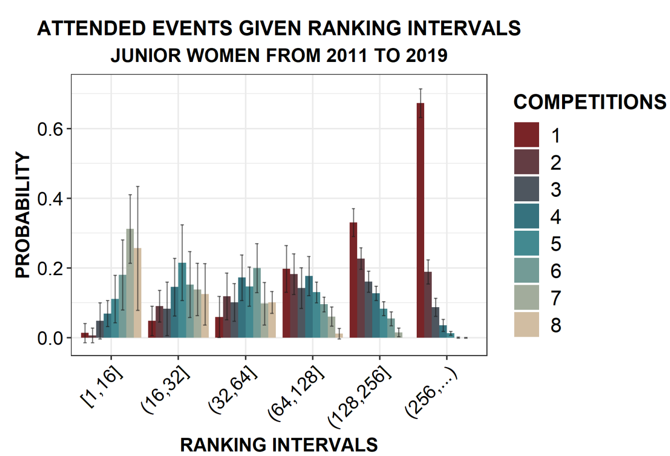


Figure 9. Histograms showing the total number of participants in every event, i.e. Junior Men and Women World Cups, from 2008 to 2020.

a fixed length either. That is why we let the number of participants N vary in simulation runs (see Simulation Results and Comparison with Real Data). It is quite natural that athletes attend only a certain number of events during a given season, according to their position in the ranking and to their country's regulations. One could think that the higher the position in ranking, the more events the athlete took part in. Data analysis for both men and women shows that this is generically correct, but we find a non-trivial probability distribution of ranking placements conditional to the actual attended events (see Figure 10) which allowed us to calibrate the model.



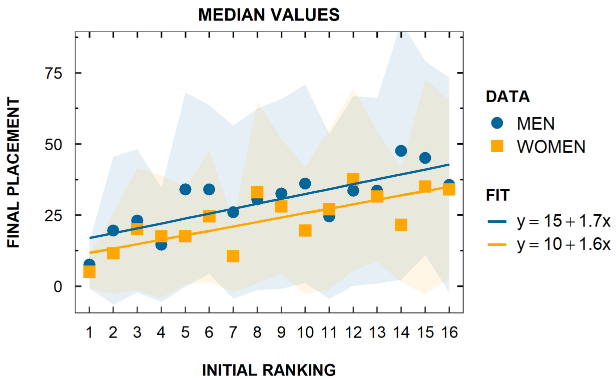
(a) Mean values with their sigma for male fencers.



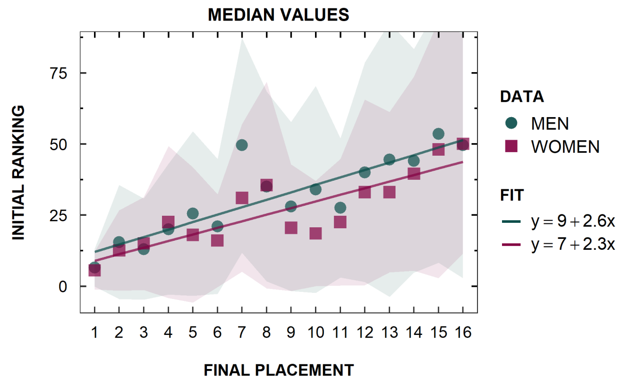
(b) Mean values with their sigma for male fencers.

Figure 10. Real data: probability of having attended a certain number of competitions in Junior category, given the associated ranking placement in the same year.

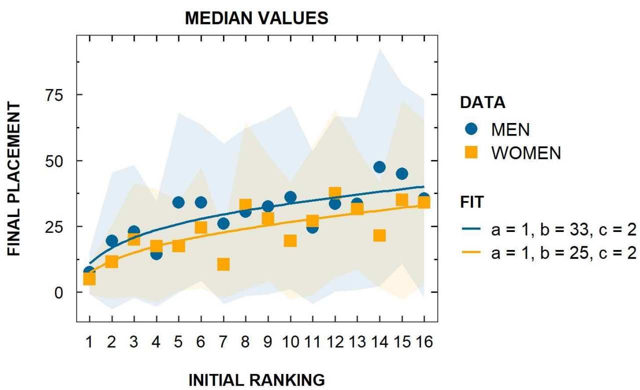
If we focus on the first sixteen athletes, who are usually considered the top performers, we can examine the relationship between ranking positions before a certain competition and the final results of the same event, and vice-versa, for both Men and Women. We evaluate the median value of the initial/final ranking for the top sixteen fencers, rather than the mean value, because of the asymmetric nature of data themselves. In Figure 11 we show the results. The shadows indicate the median absolute deviation, a measure of spread which is suitable for median values.



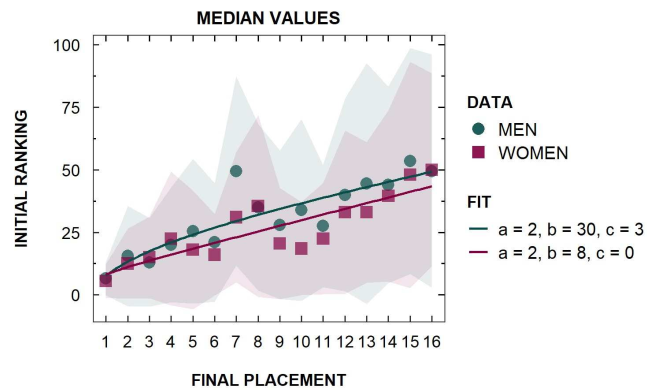
(a) Relationship between initial ranking position and final classification in World Cups (from 2008 to 2020) with linear fits.



(b) Relationship between final classification and initial ranking position in World Cups (from 2008 to 2020) with linear fits.



(c) Relationship between initial ranking position and final classification in World Cups (from 2008 to 2020) with non-linear (quadratic) fits.



(d) Relationship between final classification and initial ranking position in World Cups (from 2008 to 2020) with non-linear (quadratic) fits.

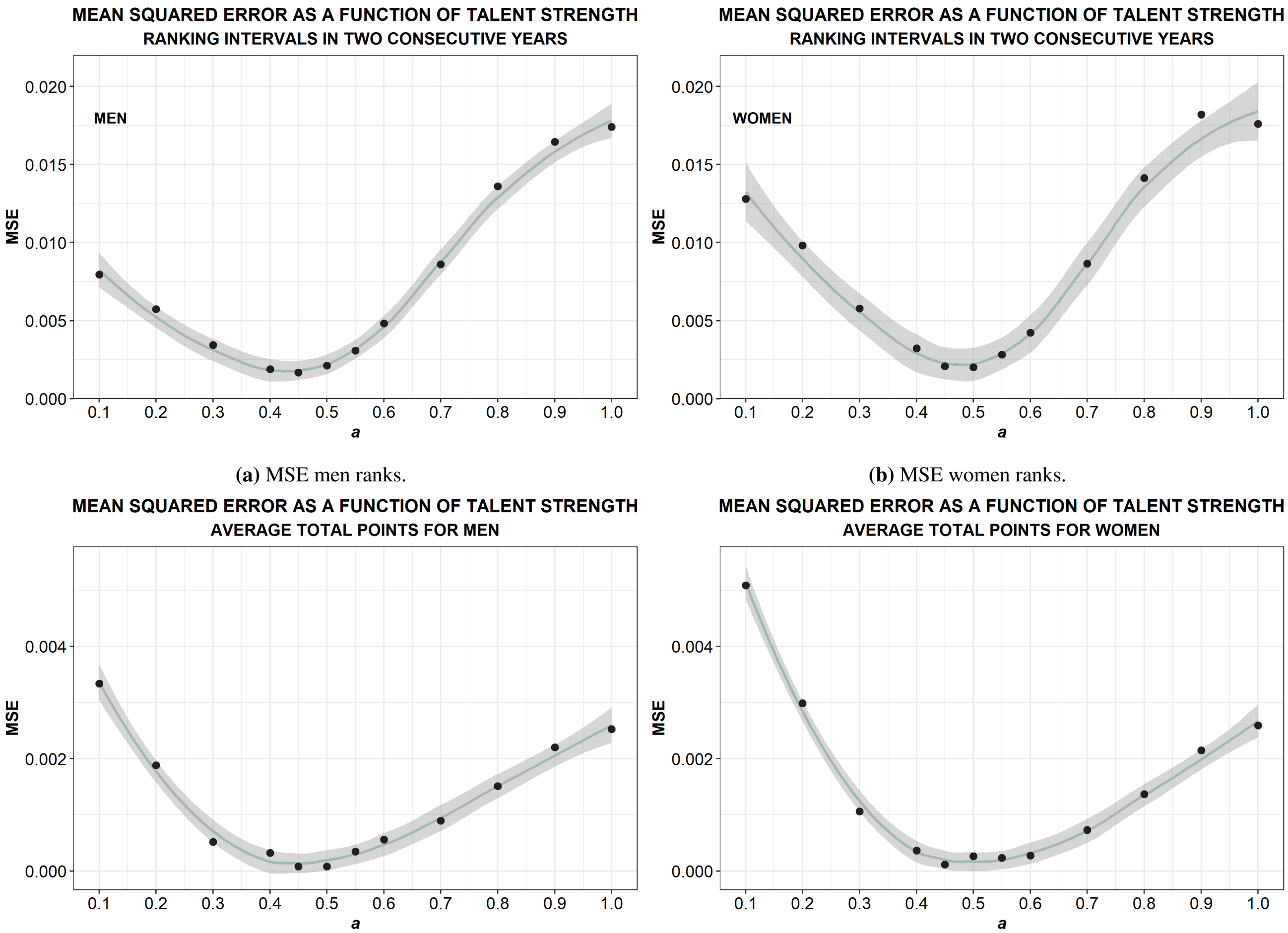
Figure 11. Results for the first sixteen Junior fencers, shown with their fits over their median values. Shadows correspond to the median absolute deviation, for a measure of spread.

In the upper panels Men and Women data are fitted with linear functions, while in the lower panels are fitted with a non linear (quadratic) function. In Table 4 the fitting parameters for both functions are reported with the corresponding errors. Those are quite high, reflecting the wide uncertainty of data.

Data	Linear Fit $y = q + mx$			Non Linear Fit $y = (ax^2 + bx)/(x + c)$			
	$q \pm \sigma_q$	$m \pm \sigma_m$	RSE	$a \pm \sigma_a$	$b \pm \sigma_b$	$c \pm \sigma_c$	RSE
Men IN-FIN	15 ± 3	1.7 ± 0.4	6.5	0.8 ± 1.1	33 ± 21	2 ± 2	6.2
Men FIN-IN	9 ± 4	2.6 ± 0.4	7.3	1.8 ± 1.6	30 ± 46	3 ± 7	7.3
Women IN-FIN	10 ± 3	1.6 ± 0.3	6.3	0.8 ± 1.2	25 ± 26	2 ± 5	6.3
Women FIN-IN	7 ± 3	2.3 ± 0.3	6.7	2.2 ± 0.6	8 ± 13	0.3 ± 2.4	6.9

Table 4. Parameters and residual standard error (RSE) of both linear and non linear fit, which are modelled over the median values of initial versus final rankings in single tournaments and vice-versa.

Taking into account those information available from data, expressed by the fixed parameters of our model (see Table 1 in Model), we ran several simulations varying only talent strength a (Equation (1)). For every value of a, we collected 10 simulation runs and we derived the mean squared error for the two main properties we compared with real data: the probability of having the same or a different ranking position in two consecutive years; the trend of average total points as a function of ranking order. Results are plotted in Figure 12 and suggest that $a = 0.45$ is the optimal value for reproducing data, with an uncertainty of 0.5.



(a) MSE men ranks.

(b) MSE women ranks.

(c) MSE points men.

(d) MSE points women.

Figure 12. Evaluation of the mean squared error to find the optimal value of a.

We now focus on total points as a function of ranking order, highlighting their non-linear trend by testing four different fits with heavy-tailed behaviour: power law, log-normal, stretched exponential and power law with exponential cut-off (also known as truncated power law) curves. Following[31] formalism (except for the log-normal function that here is shown in a more general form), we rely on equations of Table 5, using a Levenberg-Marquardt algorithm[32] to actually perform the fits. As already mentioned in Simulation Results and Comparison with Real Data, total points are averaged over the seasons and normalised to their maximum. Results of the fits are summarised in Table 6 and plotted in Figure 13. Examining data and simulation fits in Table 6, we found very similar fitting parameters, as supported by the comparison between panels (a)-(b) for Men and (c)-(d) for Women.

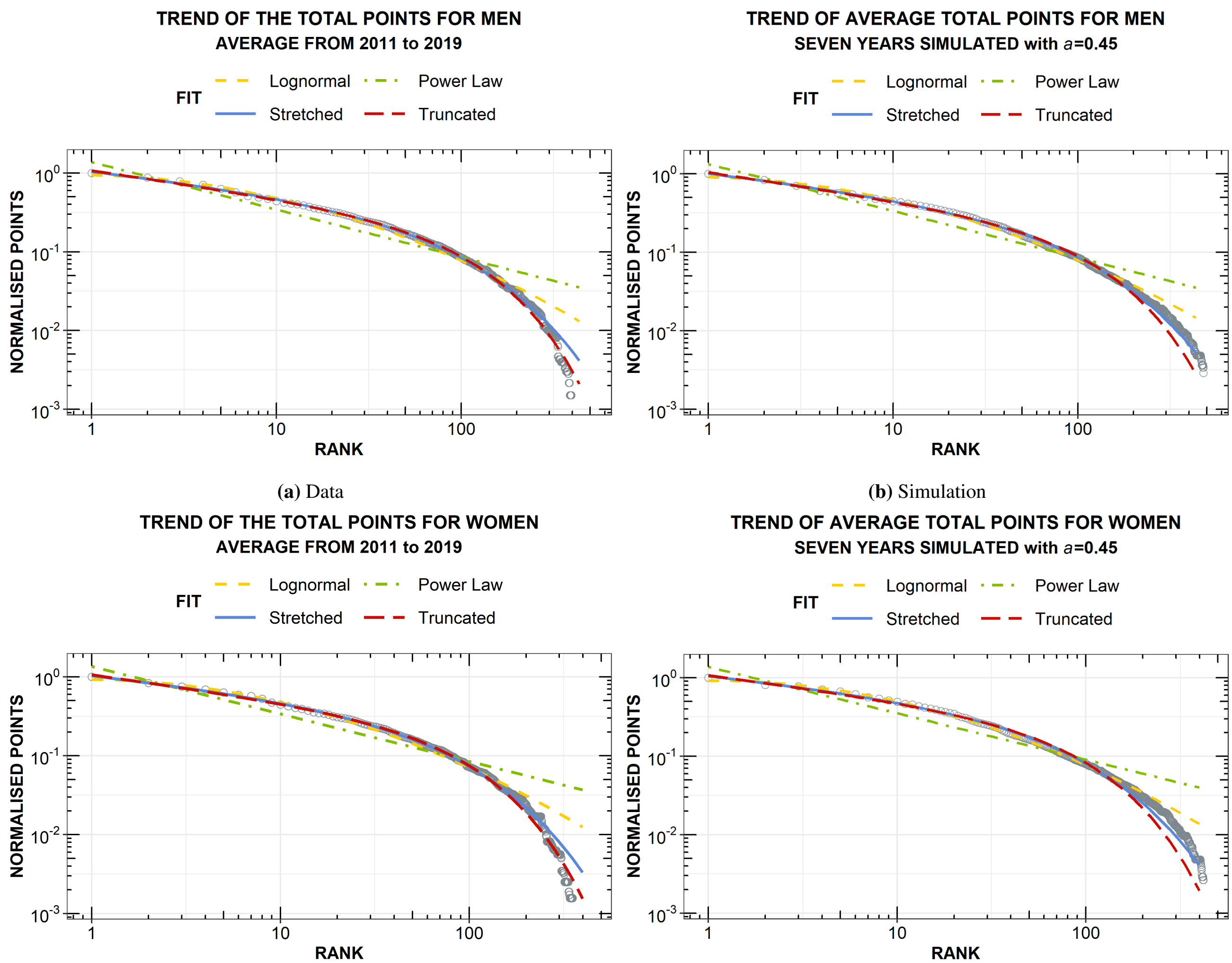
Preferring one fit to another is beyond our purposes, since we are mainly interested in underline a general "rich-get-richer" phenomenon in point accumulation as a function of ranking; nonetheless, we performed AIC model selection[33] for Women and Men data and simulations, observing that AIC is minimized by the stretched exponential function.

We precise that all the statistical analysis was performed using R software.[34]

Finally, we also checked that the conditional probabilities of having attended a certain number of competitions for simulated fencers of the Junior categories, given the associated ranking placement in the same year, are consistent with the data driven ones previously adopted for the calibration of the model (already shown in Figure 10). Looking at Figure 14, we can ascertain such a correspondence both for men (a) and women (b). Additionally, simulation results give back a variable number of participants to the tournaments during a given season that range from $N = 186$ to $N = 276$ for men and from $N = 156$ to $N = 216$ for women, both consistent with our dataset.

Function	Equation
Power law	$y \sim x^{-\alpha}$
Truncated power law	$y \sim x^{-\alpha} e^{-\lambda x}$
Stretched exponential	$y \sim x^{\beta-1} e^{-\lambda x^{\beta}}$
Log-normal	$y \sim \frac{1}{x} \exp\left[-\frac{1}{2}\left(\frac{\log x - b}{c}\right)^2\right]$

Table 5. Functional forms of the four different fits tested on average total points related to ranking order in both data and simulations.



(a) Data

(b) Simulation

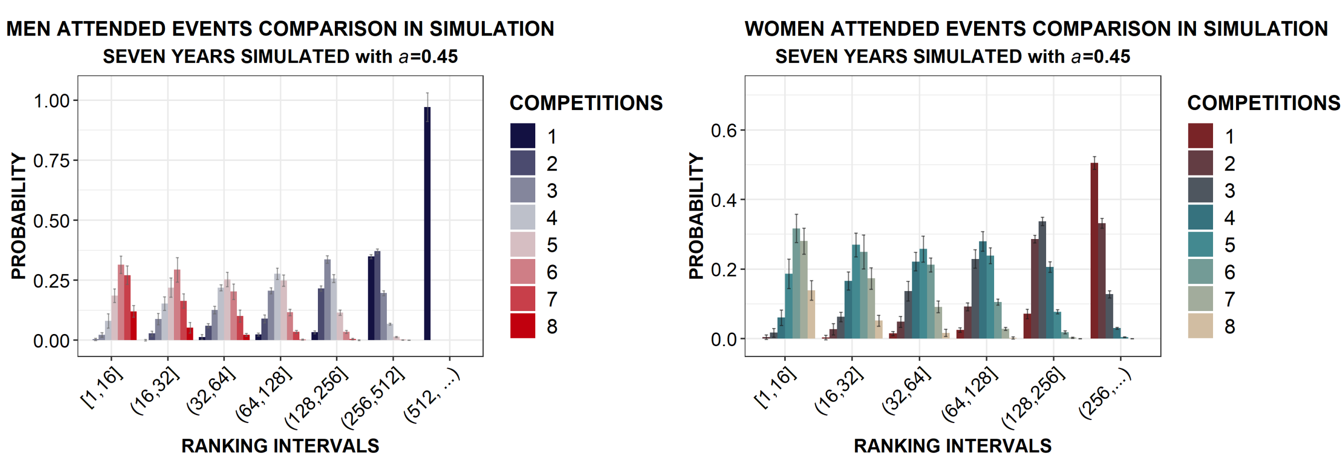
(c) Data

(d) Simulation

Figure 13. Comparison of the non-linear fits tested on both data and simulations. It is possible to see that the stretched exponential (blue, solid line) captures very well the trend of the normalised total points of Junior Men and Women (real and simulated). It is worth noting that the truncated power law has good agreement with data, less with simulation values, while log-normal curves are closer to simulation than to data points. In any case, power law fitting is the poorest, since it fails in reproducing the tail of those curves.

Non-linear Fit	Total Points	Parameter $\pm$ Standard Error		Residual Standard Error
Power law		$\alpha \pm \sigma_\alpha$	/	
	Data Men	0.603 ± 0.008		0.042
	Simulation Men	0.596 ± 0.007		0.038
	Data Women	0.601 ± 0.009		0.045
	Simulation Women	0.591 ± 0.009		0.045
Truncated power law		$\alpha \pm \sigma_\alpha$	$\lambda \pm \sigma_\lambda$	
	Data Men	0.341 ± 0.002	0.0096 ± 0.0001	0.0063
	Simulation Men	0.345 ± 0.002	0.0088 ± 0.0001	0.0062
	Data Women	0.329 ± 0.003	0.0115 ± 0.0001	0.0069
	Simulation Women	0.320 ± 0.004	0.0111 ± 0.0002	0.011
Stretched exponential		$\beta \pm \sigma_\beta$	$\lambda \pm \sigma_\lambda$	
	Data Men	0.733 ± 0.002	0.0458 ± 0.0003	0.0053
	Simulation Men	0.727 ± 0.001	0.0438 ± 0.0002	0.0039
	Data Women	0.745 ± 0.002	0.0493 ± 0.0003	0.0052
	Simulation Women	0.749 ± 0.002	0.0463 ± 0.0004	0.0072
Log-normal		$b \pm \sigma_b$	$c \pm \sigma_c$	
	Data Men	4.41 ± 0.04	2.14 ± 0.02	0.013
	Simulation Men	4.60 ± 0.04	2.20 ± 0.02	0.012
	Data Women	4.20 ± 0.04	2.07 ± 0.02	0.013
	Simulation Women	4.25 ± 0.02	2.05 ± 0.01	0.0089

Table 6. Summary of the results for several non linear fits of ranking points in Junior category, both for data and simulations with the chosen talent strength $a = 0.45$.



(a) Mean values with their sigma for male fencers.

(b) Mean values with their sigma for male fencers.

Figure 14. Simulations with $a = 0.45$: probability of having attended a certain number of competitions in Junior category, given the associated ranking placement in the same year.

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Acknowledgements

C.Z. would like to thank Tristan Müller (Fencing Vision) for his fundamental help and support in collecting data about International Competitions, other than Adriano Bernardini (Technical Sports Area Manager - FIS Federazione Italiana Scherma) and Natalia Bodrova (Sport Technology and Administration Manager - FIE), for having spent some of their time in an attempt of solving the issue of data availability and storage in fencing sport. A.P., A.E.B. and A.R. acknowledge partial financial support of PRIN 2017WZFTZP *Stochastic forecasting in complex systems*.

Author contributions statement

A.P. conceived the Netlogo model, C.Z. analysed real data and performed the numerical simulations. All authors analysed the results, wrote and reviewed the manuscript.

Additional information

Competing interests The authors declare no competing interests.