A Novel Mixed Method Using MM-SE with Change in Routhain Array for Model Order Reduction

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Research Article

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Posted Date: October 29th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-705686/v1>

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A Noval Mixed Method Using MM-SE with Change in Routhain Array for Model Order Reduction

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Abstract: The system modelling leads towards the higher-order differential equations. These systems are difficult to analyse. Therefore, for ease and understanding, the conversion of higher to lower order is required. The model order reduction(MOR) is a systematic procedure to tackle these kinds of situations. This paper offers a mixed method for MOR using the modified moment matching (MM) and stability equation (SE). The modification is applied in the routhain array of MM. The approach has been verified by examining the error between the original, proposed and compared with reduced order available in the literature. The obtained result has been compared on the basis of step response characteristics and the response indices error.

Keywords: Model order reduction, mixed-method, stability equation, moment matching, response indices.

1. Introduction

The physical machine systems are of higher order .The mathematical complexity increases as the order of the system increases. The design analysis of a complex system is confusing and time-consuming.To remove mathematical complexity and make it simple, model reduction is the solution for higher-order differential equations. Model order reduction (MOR) is a systematic mathematical procedure to reduce the higher-order system(HOS). The reduced-order presents a replica of useful characteristics of the original system. The MOR have extensive application in computational physics, MEMS devices, chemical process, control systems, power systems, circuit designing. Numerous methods are advocated by researchers as per their thoughts and practical situations for solving the HOMS' in the frequency and time domain.

Traditional techniques follows systematic mathematical processes to reduced HOS.The Pade approximation is the old technique to reduce the HOS. It gives the reduced order but due to the mathematical uncertainty of the procedural steps, reduced-order may become unstable. The algorithm introduce in [1], equate the time-moment and uses the continued fraction synthesis to make reduced order stable if higher is stable. The computer -oriented algorithm uses the evaluateing the moment of impulse reponse using the MM in [2].A combination of its Markov paparameter and moments given in [3] . The instability occurs because of unpredictable matrix output in the case of the time MM in [2, 4] making the reduced-order stable. The unstable reduced model has to regain stability using the pole placement method in [5]. The SE used the pole-zero patterns to obtain the reduced order model(ROM) in [6] with the guarantee of stability in ROM. The other method Routh Approximation[7], the continued fraction method [8] and its inversion is used to obtain the ROM for preserving the stability. The result obtained from reduced-order has an error of approximation. The error of approximation is improved by combining the result of two methods termed the mixed method.

The mixed-method consists of two methods, one method to obtain the numerator parameter and the other method is used to obtain the denominator parameter of ROM. The continued fraction third form where the $s=0$ to ∞ is proposed in [9]. The problem of instability of ROM is solved using the combination of SE and modified Cauer continued fraction [10, 11]. Pade approximation using the Routh-Hurwitz array for stable reduction [12, 13]. Dominant Eigen-value concept with continued fraction approach [14, 15]. Pade approximation using S.E. proposed by Chen *et al* in [16] and its improvement in [17]. Factor division with Routh Hurwitz array [18], Eigen spectrum analysis and Cauer Second form [19], Cauer application[20].Eigen spectrum and Pade approximation [21] factor division method with SE in [22]. Stability array with SE method in [23]. Differential equation method used to derive the denominator parameter .Routhain-array method for obtaining the numerator parametd[24] are used to obtain the ROM .

In recent times, nature-inspired techniques based on natural phenomena such as genes, swarm, physics and human behaviour based are being used for MOR. These all algorithms are used to minimize the approximation error

(response indices especially Integral Square Error) which result out from traditional methods of frequency or time domain. The well-established approach in MOR is a genetic algorithm(G.A.) with Routh-approximation by Li et al. in [25] in which the denominator parameter of reduced-order is calculated by Routh approximation and numerator parameter is calculated by the time delay are identified by G.A. Particle swarm optimization by Z.W. Wang *et-al* in [26]. A trend is followed by researchers for the implementation of optimization techniques in the MOR field [27-43] to find the unknown ROM. The numerator or denominator parameter in case of mixed-method [44] or to minimize the error between the original system response and reduced system response concluding a new reduced order transfer function.

The MM techniques have been used with Markova parameters for mixed-method in [3, 45]. However, its failed analysis of MM with the SE method is in [46]. This manuscript makes an effort to propose a mixed method with using the MMT and SE with changes in the Routhain-array of the MM technique. The proposed mixed method is tested on the selected higher-order systems and the ROM step response characteristics and response indices error are compared to justify the result. The conclusion shows the effectiveness of the novel mixed method.

The paper is well-written and divided into six sections and their subsections. Starting from the Introduction which consists of the literature review, the second section is of followed by problem formulation, the third section is of Methodology chosen for mixed-method with subsection 3.1 for Moment Matching and 3.2 Stability Equation followed by the numerical examples and results with discussion with the last part of conclusion effectively shows the resulting outcome of the proposed methods.

2. Problem Formulation

Consider the LTI higher-order system in transfer function form given in Equation (1)

$$G(s) = \frac{N^a(s)}{D^b(s)} = \frac{\sum_{i=0}^{n-1} b_i s^i}{\sum_{i=0}^n a_i s^i} \quad (1)$$

Here, $N^a(s)$ and $D^b(s)$ are the numerator and denominator polynomial of a higher-order system. The reduced-order system represented in Equation (2)

$$R_r(s) = \frac{N^u(s)}{D^u(s)} = \frac{\sum_{i=0}^{u-1} P_i s^i}{\sum_{i=0}^u Q_i s^i} \quad (2)$$

In the above Equation (2) $N^u(s)$ and $D^u(s)$ are the unknown numerator and denominator parameter of the reduced-order system.

3. Methodologies

The moment matching and stability equation are used to develop the novel method. The moment matching method is used to determine the denominator parameter and the stability equation used to determine the numerator parameter of the ROM.

3.1.Moment Matching

The moment matching method is given in[2], according to the method consider a system having transfer function in the ratio of two polynomial

$$G_{ij}(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0 s^0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 s^0}, (m \leq n) \quad (3)$$

The $G_{ij}(s)$ represents the original transfer function of the higher-order system which may also be represented as $G_{11}(s)$ or $G(s)$. The $b_m, b_{m-1}, \dots, b_1, b_0$ are the numerator parameter and denominator parameter is represented by $a_n, a_{n-1}, \dots, a_1, a_0$. The diminished order of $G_{ij,r}(s)$ is represented as in Equation (4)

$$G_{ij,r}(s) = \frac{P_0 s^{m_r-1} + P_1 s^{m_r-2} + P_2 s^{m_r-3} + \dots + P_{m_r-1} s + P}{Q_0 s^{n_r} + Q_1 s^{n_r-1} + Q_2 s^{n_r-2} + \dots + Q_n s + Q}, \quad (m_r \leq n_r) \quad (4)$$

The reduced-order numerator parameter is represented by $P_0, P_1, P_2, \dots$ and denominator parameter is represented as $Q, Q_0, Q_1, \dots$. The system is considered for time moment matching with the transfer function represented in Equation (5)

$$G(s) = \int_0^{\infty} y_{\delta}(t) e^{-st} dt \quad (5)$$

$y_{\delta}(t)$ is the impulse response of $G(s)$. The power series expansion e^{-st} is written as in Equation (6)

$$G(s) = \int_0^{\infty} y_{\delta}(t) \left\{ 1 - st + \frac{s^2 t^2}{2!} - \frac{s^3 t^3}{3!} + \dots \right\} dt \quad (6)$$

$$= \int_0^{\infty} y_{\delta}(t) dt - s \int_0^{\infty} t y_{\delta}(t) dt + s^2 \int_0^{\infty} \frac{t^2}{2!} y_{\delta}(t) dt - \dots$$

$$G(s) = c_0 + c_1 s + c_2 s^2 + \dots \quad (7)$$

Where coefficient c_i

$$c_i = \frac{(-1)^i}{i!} \int_0^{\infty} t^i y_{\delta}(t) dt, \quad i = 0, 1, 2, 3, \dots \quad (8)$$

The $\int_0^{\infty} t^i y_{\delta}(t) dt$ is the i^{th} moment of $y_{\delta}(t)$.

3.2. Stability Equation

The Stability Equation (SE) method is selected from the [6]. The traditional method gives the guarantee of stability of the reduced-order which is calculated as mentioned. Let the transfer function of a HOS given in Equation (3) be written as in Equation (9)

$$G(s) = \frac{G_N(s)}{G_D(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0 s^0}{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0 s^0} \quad (9)$$

Where $m \geq n$, $G_N(s)$ and $G_D(s)$ are the numerator and denominator of the $G(s)$. Splitting $G_N(s)$ and $G_D(s)$ into their even and odd parts which is generally represented in Equation (10).

$$G(s) = \frac{G_{Ne}(s) + G_{No}(s)}{G_{De}(s) + G_{Do}(s)} \quad (10)$$

Where,

$$\begin{cases} G_{Ne}(s) = \sum_{i=0,2,4,\dots}^n b_i s^i \\ G_{No}(s) = \sum_{i=1,3,5,\dots}^n b_i s^i \end{cases} \quad (11)$$

And

$$\begin{cases} G_{De}(s) = \sum_{i=0,2,4,..}^m b_i s^i \\ G_{Do}(s) = \sum_{i=1,3,5,..}^m b_i s^i \end{cases} \quad (12)$$

Equation (11) and Equation (12) are S.E. of the numerator $[G_N(s)]$ and the denominator $[G_D(s)]$, respectively. The roots of G_{Ne} and G_{De} are known as zeros (z_i) and those of G_{No} and G_{Do} are known as poles (p_i). The poles and zeros obtained by the conventional method. Then, for a stable system Equation (11) and (12) can be factored as.

$$\begin{cases} G_{Ne}(s) = \prod_{i=1}^{n'} (s^2 + z_{Ni}^2) \\ G_{No}(s) = s \prod_{i=1}^{n'} (s^2 + p_{Ni}^2) \end{cases} \quad (13)$$

$$\begin{cases} G_{De}(s) = \prod_{i=1}^{m'} (s^2 + z_{Ni}^2) \\ G_{Do}(s) = s \prod_{i=1}^{m'} (s^2 + p_{Ni}^2) \end{cases} \quad (14)$$

Where,

$$\begin{aligned} m' &= \frac{m}{2} \quad \text{if } m \text{ is even,} \\ &= \frac{(m-1)}{2} \quad \text{if } m \text{ is odd;} \\ m' &= \frac{n}{2} \quad \text{if } n \text{ is even,} \\ &= \frac{(n-1)}{2} \quad \text{if } n \text{ is odd;} \end{aligned} \quad (15)$$

$p_1^2 < p_2^2 < p_3^2, \dots$, are poles and $z_1^2 < z_2^2 < z_3^2, \dots$. zeros having lesser magnitudes are more dominant with larger magnitudes, so, discarding the higher magnitude is a systematic method for dropping the order using S.E. The diminished models of the polynomial $G_N(s)$ and $G_D(s)$ can be built, and finally, the diminished model of $G(s)$ can be achieved. To make the steady-state answer of the ROM is identical to that of the original system.

The reduced-order achieved using SE of $G_N(s)$ can be written as in Equation (16)

$$\begin{cases} G'_{Ne}(s) = z_n^2 \prod_{i=1}^{n'-1} (s^2 + z_i^2) \\ G'_{No}(s) = s p_n^2 \prod_{i=1}^{n'-1} (s^2 + p_i^2) \end{cases} \quad (16)$$

Then the reduced system represented as in Equation (17)

$$F_{n-1}(s) = \frac{F'_N(s)}{F'_D(s)} = \frac{h_{12}s + h_{11}}{v_{23}s + v_{22}s + v_{21}} \quad (17)$$

3.3. Routhain Approach

The moment matching has the Routhain array approach for obtaining a reduced order. This can be clarified from [2] and mentioning it as case-1 in our experiment analysis.

3.3.1. Case-1

The higher-order physical system may be represented in transfer function form as in Equation. (3).

Here, $b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0$ are numerator parameter and $a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0$ is the denominator parameter of the higher-order system. Divide the higher-order transfer function a_0 to make it unity.

$$G_{ij}(s) = \frac{b_m s^m / a_0 + b_{m-1} s^{m-1} / a_0 + \dots + b_1 s / a_0 + b_0 / a_0}{a_n s^n / a_0 + a_{n-1} s^{n-1} / a_0 + \dots + a_1 s / a_0 + a_0 / a_0}, (m \leq n) \quad (18)$$

The new coefficient of “s” is formed as $b_m s^m / a_0 = B_m$, $b_{m-1} s^{m-1} / a_0 = B_{m-1}$, $b_0 / a_0 = B_0$ for the numerator and $a_n s^n / a_0 = A_n$, $a_{n-1} s^{n-1} / a_0 = A_{n-1}$, $a_0 / a_0 = 1$ for denominator parameters. Equation (3) changes as Equation (19)

$$G_{ij}(s) = \frac{B_m s^m + B_{m-1} s^{m-1} + \dots + B_1 s + B_0}{A_n s^n + A_{n-1} s^{n-1} + \dots + A_1 s + 1}, (m \leq n) \quad (19)$$

The denominator 1, $A_1, A_2, \dots, A_n$ and numerator $B_0, B_1, B_2, \dots, B_m$ parameter are used to create the routhain array as mentioned in Table 1.

Table 1: Routhain Table for Time Moment Matching

	1	$A_1 = \frac{a_1}{a_0}$	$A_2 = \frac{a_2}{a_0}$	L
c_0	$B_0 = \frac{b_0}{a_0}$	$B_1 = \frac{b_1}{a_0}$	L	
c_1	$C_1 = ((1 \times B_1) - B_0 \times A_1)$			
c_2				
N				

The routhain parameter used to find the lower order using [2] the following matrix Equation. (20).

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ M \\ c_m \\ K \\ c_{m+1} \\ c_{m+2} \\ M \\ c_{m+n} \end{bmatrix} = \begin{bmatrix} 0 & 0 & K & 0 & 0 & M & 0 & 0 & 0 & 0 \\ -c_0 & 0 & K & 0 & 0 & M & 0 & 0 & 0 & 0 \\ -c_1 & -c_0 & K & 0 & 0 & M & 0 & 0 & 0 & 0 \\ M & M & M & M & M & M & M & M & M & M \\ -c_{m-1} & -c_{m-2} & K & -c_0 & 0 & M & 0 & 0 & 0 & 0 \\ K & K & K & K & K & K & K & K & K & K \\ -c_m & -c_{m-1} & K & -c_1 & -c_0 & M & L & L & L & 0 \\ -c_{m+1} & -c_m & K & & -c_1 & M & -c_0 & L & L & 0 \\ & & K & & & M & L & -c_0 & 0 & 0 \\ -c_{m+n-1} & -c_{m+n-2} & K & & & M & L & -c_1 & -c_0 & 0 \end{bmatrix} \begin{bmatrix} c_{21} \\ c_{31} \\ c_{41} \\ M \\ c_{n,1} \\ K \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c_{31} \\ c_{33} \\ c_{43} \\ M \\ c_{2,m+1} \\ K \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

The c_{11}, c_{12}, c_{21} and c_{22} represents the upper left hand, upper right hand lower left hand and lower right-hand sub-matrices of the coefficient matrix. The reduced second order from the above Equation (21) is given by

$$\hat{a}_1 = C_{21}^{-1} \hat{c}_2 = \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} u_{12} \\ u_{13} \end{bmatrix} \quad (21)$$

$$\hat{a}_2 = \hat{c}_1 - C_{11} \hat{a}_1 = \begin{bmatrix} 0 & 0 \\ -c_0 & 0 \end{bmatrix} \times \begin{bmatrix} u_{12} \\ u_{13} \end{bmatrix} = \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} \quad (22)$$

Above Equation. (21) gives the numerator and Equation (22) gives the denominator part of the reduced second-order and represented as in Equation. (23):

$$R_1(s) = \frac{u_{21} + u_{22}s}{1 + u_{12}s + u_{13}s^2} \quad (23)$$

3.3.2. Case-2 Proposed approach

In the experimental case, we reduced-order obtained the following steps as given in Equation (24) supposing that

$$G_{ij}(s) = \frac{\frac{b_m s^m}{a_n s^n} + \frac{b_{m-1} s^{m-1}}{a_n s^n} + L \quad L + \frac{b_1 s}{a_n s^n} + \frac{b_0}{a_n s^n}}{\frac{a_n s^n}{a_n s^n} + \frac{a_{n-1} s^{n-1}}{a_n s^n} + L \quad L + \frac{a_1 s}{a_n s^n} + \frac{a_0}{a_n s^n}}, (m \leq n) \quad (24)$$

The new coefficient of “s” is formed as $\frac{b_m s^m}{a_n s^n} = E_m, \frac{b_{m-1} s^{m-1}}{a_n s^n} = E_{m-1}, L \quad L, \frac{b_1 s}{a_n s^n} = E_1$ and $\frac{b_0}{a_n s^n} = E_0$ for the numerator and $\frac{a_n s^n}{a_n s^n} = 1, \frac{a_{n-1} s^{n-1}}{a_n s^n} = F_{n-1}, L \quad L, \frac{a_1 s}{a_n s^n} = F_1, \frac{a_0}{a_n s^n} = F_0$ for the denominator parameter. The new transfer function form as in Equation (25)

$$R_{2ij}(s) = \frac{E_m + E_{m-1}s^{m-1} + L \quad L + E_1s + E_0s^0}{1 + F_{n-1}s^{n-1} + L \quad L + F_1s + F_0s^0}, (m \leq n) \quad (25)$$

The coefficient's of “s” denominator and numerator put in routhain array as mentioned in Table 2

Table 2:Routhain Array for Moment Matching

The	1	$F_{n-1} = \frac{a_{n-1}s^{n-1}}{a_n s^n}$	$F_{n-2} = \frac{a_{n-2}s^{n-1}}{a_n s^n}$	L
c_0	$E_m = \frac{b_m s^m}{a_n s^n}$	$E_{m-1} = \frac{b_{m-1} s^{m-1}}{a_n s^n}$	L	
c_1	$C_1 = (1 \times E_{m-1} - E_m \times F_{n-1})$			
c_2				
N				

value of $c_0, c_1, c_2, c_3, K \quad K, c_n$ is obtained from Table 1 and Table 2 different in both cases.

The steps from Equation (20) to Equation (22) are repeated and a new reduced order formed using the MMT as given in Equation (26)

$$R_2(s) = \frac{e_{21} + e_{22}s}{1 + f_{12}s + f_{13}s^2} \quad (26)$$

The experimental transfer function from Equation (25) is reduced and reduced order using the stability equation mentioned in Equation (27)

$$R_3(s) = \frac{l_{21} + l_{22}s}{1 + k_{12}s + k_{13}s^2} \quad (27)$$

4. Proposed Mixed-Mixed Method

The numerator parameter of Equation (24) and denominator parameter in Equation (17) are mixed in order to create a mixed method given in Equation (28)

$$R_{MS}(s) = \frac{u_{21} + u_{22}s}{v_{23}s + v_{22}s + v_{21}} \quad (28)$$

The mixed-method in Equation (28) is not resulted in satisfactory using the MM and SE. Therefore, experimental case 2 is implemented. The reduced-order numerator parameter used from the SE from Equation (17) and denominator parameter from the reduced moment matching in transfer function Equation (27), resulted in mixed-method mentioned in Equation (29)

$$R_{SM}(s) = \frac{h_{12}s + h_{11}}{u_{13}s^2 + u_{22}s + 1} \quad (29)$$

The proposed mixed-method approach is justified using the implementation of a higher-order system as mentioned in section 5.

5. Implementation and Result analysis

The method is applied on the LTI SISO system given as Example 1 and Example 2

5.1. Example 1: . Consider a sixth-order system from [39] as mentioned in transfer function format given in Equation (30)

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1} \quad (30)$$

The higher-order mentioned in Equation (30) is reduced according to the steps from Equation (19-25). The routhain array for obtaining the reduced-order given in Table 3

Table 3: Routhain Array with Case-1

	1	18.3	102.42	209.46	155.94	33.6	2
$c_0 = 1$	1	8	20	16	3	2	0
$c_1 = -10.3$	-10.3	-82.42	-193.46	-152.94	-31.6	-2	0
$c_2 = 106.07$	106.07	861.466	2004.498	1574.582	344.08	20.6	
$c_3 = -1079.62$	-1079.62	-8859.19	-20642.8	-16196.5	-3543.35		
	10897.76	89931.33	209939.7	164811.8			
	-109498	-906209	-2117834				
	1097599	9096925					
	-1.1E+07						

The denominator and numerator coefficient of ROM are obtained in Equation (32) and Equation (31) respectively

$$\begin{aligned} \hat{a}_1 &= C_{21}^{-1} \hat{c}_2 = \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} \\ &= \begin{bmatrix} -(-10.3) & -(1) \\ -(106.07) & -(-10.3) \end{bmatrix}^{-1} \times \begin{bmatrix} 106.07 \\ -1079.62 \end{bmatrix} \\ &= 1.0e+03 \begin{bmatrix} 0.5150 & 0.0500 \\ 5.3035 & 0.5150 \end{bmatrix} \times \begin{bmatrix} 106.07 \\ -1079.62 \end{bmatrix} = 1.0e+03 \begin{bmatrix} 0.6450 \\ 6.5379 \end{bmatrix} = \begin{bmatrix} 645 \\ 6537.9 \end{bmatrix} \end{aligned} \quad (31)$$

$$\begin{aligned} \hat{a}_2 &= \hat{c}_1 - C_{11} \hat{a}_1 = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -c_0 & 0 \end{bmatrix} \times \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -10.3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 645 \\ 6537.9 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 634.7500 \end{bmatrix} \end{aligned} \quad (32)$$

The reduced order using the MM case-1 is given in Equation (33)

$$R_1(s) = \frac{634.75s + 1}{6537.9s^2 + 645s + 1} \quad (33)$$

The ROM using S.E. is given in Equation (34)

$$R_2(s) = \frac{8s+1}{102.42s^2 + 18.3s+1} \quad (34)$$

The reduced mixed method using the MM and SE as mentioned in Equation (30) given in Equation (35)

$$R_1(s) = \frac{634.75s+1}{102.42s^2 + 18.3s+1} \quad (35)$$

The step response characteristics of the Mixed method not follows the original system and reduced Therefore the experimental case-2 is applied to the original system in Equation (31) to obtain a well-mixed method.

Table 4:Routhain array Case-2

	1	16.8	77.97	104.73	51.21	9.15	0.05
$c_0 = 1$	1	1.5	8	10	4	0.5	
$c_1 = -15.3$	-15.3	-69.97	-94.73	-47.21	-8.65		
$c_2 = 187.07$	187.07	1098.211	1555.159	774.863			
$c_3 = -2044.57$	-2044.57	-13030.7	-18817				
$c_4 = 21318$	21318	140597.8					
	-217545						

Reduced-Order Modelling with case-2 follows the steps from Equation (26,27), Routhain Table 2 and from Equation (18-23).

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1} \quad (36)$$

$$= \frac{s^5 + 1.5s^4 + 8s^3 + 10s^2 + 4s + 0.5}{s^6 + 16.8s^5 + 77.97s^4 + 104.73s^3 + 51.21s^2 + 9.15s + 0.5}$$

$$\hat{a}_1 = C_{21}^{-1} \hat{c}_2 = \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} -(-15.3) & -(1) \\ -(187.07) & -(-15.3) \end{bmatrix}^{-1} \times \begin{bmatrix} 187.07 \\ -2044.57 \end{bmatrix} \quad (37)$$

$$= \begin{bmatrix} 0.3254 & 0.0213 \\ 3.9785 & 0.3254 \end{bmatrix} \times \begin{bmatrix} 187.07 \\ -2044.57 \end{bmatrix} = \begin{bmatrix} 17.3884 \\ 78.9720 \end{bmatrix}$$

Equation (39) gives the reduced order denominator parameter of MM with experimental case-2

$$\hat{a}_2 = \hat{c}_1 - C_{11} \hat{a}_1 = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -c_0 & 0 \end{bmatrix} \times \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} \quad (38)$$

$$= \begin{bmatrix} 1 \\ -15.3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 17.3884 \\ 78.9720 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 2.0884 \end{bmatrix}$$

Equation (38) gives the numerator parameter and denominator parameter in Equation (37) of the MM with experimental case-2. The obtained reduced order represented in Equation (39)

$$R_{11}(s) = \frac{2.0884s+1}{78.9720s^2 + 17.3884s+1} \quad (39)$$

The obtained reduced order of experimental case-2 is given in Equation (40)

$$R_{22}(s) = \frac{4s+0.5}{51.21s^2 + 9.15s+0.5} \quad (40)$$

The proposed reduced-order follows using the SE and MM is given in Equation (43). The reduced-order follows the step response of the original system approximately

$$R_{22}(s) = \frac{8s+1}{78.9720s^2 + 17.3884s+1} \quad (41)$$

The obtained reduced order from case-1 and using case-2 all are compared in the table and compared with the proposed mixed method. The obtained reduced order of HOS as in Equation (32) from case -1 and case-2 are given in step response characteristics in Figure 1.

Figure 1: Step response characteristics of HOS with proposed reduced system

The response characteristics of the system is given in Table 5 along with the reduced order. The ST min, ST max and peak is approximately same as that of the original system

Table 5: Step response characteristics of obtained reduced order from Case-1, Case-2 and mixed methods with proposed mixed method

Reduced Order	Step response characteristics					
	RT	ST	ST min	ST amx	Peak	PT
HOS Eq. (32)	22.7109	40.0475	0.9001	0.9994	0.9994	75.4235
Eq.(38)	22.7109	40.0475	0.9001	0.9994	0.9994	75.4235
Eq. (35)	22.6239	40.2598	0.9001	1.0000	1.0000	108.6304
Eq. (36)	21.7088	33.7630	0.9020	1.0036	1.0036	53.6096
Eq. (41)	28.3754	47.3095	0.9007	0.9991	0.9991	74.8759
Eq. (42)	21.7088	33.7630	0.9020	1.0036	1.0036	53.6096
$R(s) = \frac{634.75s+1}{102.42s^2+18.3s+1}$	0.1351	58.6403	0.9690	24.9314	0.24.9314	10.8250
$R(s) = \frac{8s+1}{6537.9s^2+645s+1}$	1.3945e+03	2.4853e+03	0.9041	1.000	1.000	6.6934e+03
$R(s) = \frac{2.0884s+1}{51.21s^2+9.15s+0.5}$	28.8871	45.6568	1.8023	2.0027	2.0027	71.6513
$R(s) = \frac{4s+0.5}{78.9720s^2+17.3884s+1}$	20.3957	35.1523	0.4518	0.4999	0.4999	66.5099
$R(s) = \frac{8s+1}{78.9720s^2+17.3884s+1}$	20.3957	35.1523	0.9036	0.9997	0.9997	66.5099

Figure 2: Pole zero mapping of the original system , reduced order using MM and SE , improved reduced order using MM and SE, proposed reduced order.

The selection of the proposed reduced order obtained from the MM in traditional format and with SE is represented in the pole –zero plot in Figure 2. The pole of the higher order are -10,-5,-1,-0.5,-0.2,-0.1 and zeros are $0.0711+2.6929i$, $-0.0711-2.6929i$, $-0.7415+0i$, $-0.3530+0i$, $-0.3530+0i$, $-0.2632+0.0i$, reduced second order using the MM poles are -0.9711 , -0.0061 and zero are -0.0016 , second order using SE is $-0.0893+0.0422i$, $-0.0893-0.0422i$ and zeros are -0.1250 and the proposed second order using the SE and MM have poles $-0.1101+0.0233i$; $-0.1101-0.233i$, zeros are -0.1250 . The pole of improved MMT is $-0.1101+0.0233i$, $-0.1101-0.0233i$ and zero is -0.4788 and SE is $-0.0893+0.0422i$, $-0.0893-0.0422i$ and -0.1250 . The pole zero placement of the proposed and higher order is clearly given in Figure 2 shows that the proposed order retains stability and its step response characteristics shows that response is similar to HOS. Table 5 concludes that the proposed reduced-order approximately replicates the original higher-order system. The justification of the reduced order is verified by comparing it with the reduced order available in the literature. Figure 3 shows the step reponse comparison of the proposed and reduced order available in the literature.

Figure 3: Step response characteristics of Novelty Condition 1 (SE) with condition 2(MM) (SE/MM)

Table 6: Comparative Step response characteristics of the Original system with proposed and equivalent systems from the literature

Reference	Reduced-order	Step response characteristics					
		RT	ST	ST _{min}	ST _{max}	Peak	PT
Original	Eq. (32)	22.710 9	40.047 5	0.900 1	0.9994	0.999 4	75.4235
Proposed	$\frac{8s+1}{78.9720s^2+17.3884s+1}$	20.395 7	35.152 3	0.903 6	0.9997	0.999 7	66.5099
MCF	$\frac{s+1.53725}{s^2+16.8373s+1.53725}$	23.935 8	42.005 7	0.901 0	0.9991 0	0.999 0	74.7456

Sambariya [47]; 2016	$\frac{0.0910s + 0.0109}{s^2 + 0.2012s + 0.01099}$	21.594 7	36.245 8	0.901 6	1.0001	1.000 1	68.2078
Soloklo[39];201 4; MOHSA	$\frac{7.80016s + 0.81849}{87.58712s^2 + 12.43314s + 0.81657}$	14.299 7	52.986 0	0.905 7	1.0787	1.078 7	31.1441
Soloklo[39];201 4; Typical R-PA	$\frac{6.87815s + 1.09228}{89.54625s^2 + 12.96860s + 0.99732}$	15.057 4	50.376 1	0.987 7	1.1769	1.176 9	31.1621
Sambariya [48]; Bat Algo.	$\frac{2.0950s + 1.3677}{20.0736s^2 + 16.2644s + 1.3667}$	22.993 9	40.931 6	0.902 1	1.0007	1.000 7	115.587 9
Sambariya [24]; Routh array	$\frac{2535.69s + 357.14}{2458.08s^2 + 2196s + 360}$	1.3335	16.976 0	0.913 4	1.2037	1.203 7	4.2170
Lavania[49]; PA+MM	$\frac{s + 1.53}{s^2 + 16.83s + 1.53}$	24.039 3	42.186 8	0.901 0	0.9990	0.999 0	75.0688
Lavania[50]; PA+ES	$\frac{s + 0.307}{s^2 + 5.599s + 0.307}$	38.026 3	67.230 2	0.900 1	0.9981	0.998 1	109.637 8

In Table 6 it is clear that the proposed reduced-order system has the approximately same response as that of the original system and competitively competes with the response from literature. To justify the response of the proposed reduced order of the randomly chosen higher-order system from the literature have a minimum error between them so, the calculation of Integral time absolute error (ITAE), Integral absolute error (IAE) and Integral square error (ISE) is calculated and compared in Table 7. The response indices, *i.e* Integral Square Error (ISE) penalizes the larger errors more than the smaller error. This gives a more conservative response and the reduced system return faster to the set point. Integral Absolute Error (IAE) essentially takes the absolute value of the error. The negative area is accounted for when the IAE is used and diminishing the problem encountered with Integral error and Integral Time Absolute Error (ITAE) is the absolute value of error multiplied by the time, this results in error existing overtime being penalized even they may be small and result in more heavy damped response and these errors are given as

Integral Square Error (ISE) is given in Eq. (42)

$$ISE = \int_0^{\infty} [G(t) - G_r(t)]^2 dt \quad (42)$$

Integral Absolute Error (IAE) is given in Eq. (43)

$$IAE = \int_0^{\infty} |G(t) - G_r(t)| dt \quad (43)$$

Integral Time Absolute Error (ITAE) is given in Eq. (44)

$$ITAE = \int_0^{\infty} t |G(t) - G_r(t)| dt \quad (44)$$

In response indices, Eq. $G(t)$ is the step response of the higher-order and $G_r(t)$ is the response of the lower order.

Table 7: Comparative Response Indices of Proposed and from Literature

Reference	Reduced –Order	Response Indices Error		
		ITAE	IAE	ISE
Proposed	$\frac{8s + 1}{78.9720s^2 + 17.3884s + 1}$	0.7852	0.1151	0.00226

MCF	$\frac{s+1.53725}{s^2+16.8373s+1.53725}$	0.6773	0.1707	0.005015
Sambariya [47]; 2016	$\frac{0.0910s+0.0109}{s^2+0.2012s+0.01099}$	0.6824	0.1291	0.002222
Soloklo[39];2014; MOHSA	$\frac{7.80016s+0.81849}{87.58712s^2+12.43314s+0.81657}$	1.34	0.1853	0.5377
Soloklo[39];2014; Typical R-PA	$\frac{6.87815s+1.09228}{89.54625s^2+12.96860s+0.99732}$	1.365	0.221	2.8887
Sambariya [48]; Bat Algo.	$\frac{2.0950s+1.3677}{20.0736s^2+16.2644s+1.3667}$	3.2244	0.03557	4.85e-04
Sambariya [24]; Routh array	$\frac{2535.69s+357.14}{2458.08s^2+2196s+360}$	34.94	6.376	4.812
Lavania[49]; PA+MM	$\frac{s+1.53}{s^2+16.83s+1.53}$	0.6202	0.1621	0.0397
Lavania[50]; PA+ES	$\frac{s+0.307}{s^2+5.599s+0.307}$	2.556	0.5108	0.0063

5.2. Example 2: The fourth order system considered from [43] , represented in Equation (45)

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \quad (45)$$

The reduced order of the considered example 2 is reduced using case 1 and following the steps from Equation (18-24).

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} = \frac{0.0417s^3 + 0.2917s^2 + s + 1}{0.0417s^4 + 0.4167s^3 + 1.4583s^2 + 2.0833s + 1} \quad (46)$$

The routhain Table of Example 2 is given in Table 8.

Table 8 Routhain Table for Higher-order system case-1

	1	2.0833	1.4583	0.4167	0.0417
$c_0 = 1$	1	1	0.2917	0.0417	0
$c_1 = -1.8033$	-1.0833	-1.1666	-0.375	2.9583	0
$c_2 = 1.090239$	1.090239	1.204776	3.409711	2.045174	0
$c_3 = -1.06652$	-1.06652	1.819816	1.590871	-0.04546	0
$c_4 = 4.041693$	4.041693	3.146175	0.398955	0.044474	
$c_5 = -5.27388$	-5.27388	-5.49505	-1.6397		
$c_6 = 5.492038$	5.492038	6.051207			
$c_7 = -5.39036$	-5.39036				

The values of c_0, c_1, c_2, c_3 obtained from the routhani Table 8 is used to obtain the second order reduced ofrmat of the considered higher order in Equation (47-50)

$$\begin{aligned}\hat{a}_1 = C_{21}^{-1} \hat{c}_2 &= \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} -(-1.0833) & -(1) \\ -(1.090239) & -(1.0833) \end{bmatrix}^{-1} \times \begin{bmatrix} 1.090239 \\ -1.0833 \end{bmatrix} \\ &= \begin{bmatrix} 13.0048 & 12.0048 \\ 13.0881 & 13.0048 \end{bmatrix} \times \begin{bmatrix} 1.090239 \\ -1.0833 \end{bmatrix} = \begin{bmatrix} 1.3750 \\ 0.3993 \end{bmatrix}\end{aligned}\quad (47)$$

$$\begin{aligned}\hat{a}_2 = \hat{c}_1 - C_{11} \hat{a}_1 &= \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -c_0 & 0 \end{bmatrix} \times \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1.0833 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 1.3750 \\ 0.3993 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 0.2917 \end{bmatrix}\end{aligned}\quad (48)$$

The obtained reduced order is given in Equation (49)

$$R_{MM}(s) = \frac{0.2917s + 1}{0.3993s^2 + 1.3750s + 1} \quad (49)$$

The obtained reduced-order using the SE is as follows in Equation (50)

$$R_{SE}(s) = \frac{24s + 24}{35s^2 + 50s + 24} \quad (50)$$

Considering the case-2

Considering the fourth order from Equation (46) To reduce the higher-order using the steps from Equation (26-27), offered routhain Table 2. Table 9 is the routhain array for case-2.

Table 9: Routhain Table case-2

	1	10	35	50	24
$c_0 = 1$	1	7	24	24	
$c_1 = -3$	-3	-11	-26		
$c_2 = 19$	19	79			
$c_3 = -111$	-111				

The values of c_0, c_1, c_2, c_3 L are used to obtain the reduced order

$$\begin{aligned}\hat{a}_1 = C_{21}^{-1} \hat{c}_2 &= \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} -(-3) & -(1) \\ -(19) & -(-3) \end{bmatrix}^{-1} \times \begin{bmatrix} 19 \\ -111 \end{bmatrix} \\ &= \begin{bmatrix} -0.3000 & -0.1000 \\ -1.9000 & -0.3000 \end{bmatrix} \times \begin{bmatrix} 19 \\ -111 \end{bmatrix} = \begin{bmatrix} 5.4000 \\ -2.8000 \end{bmatrix}\end{aligned}\quad (51)$$

Equation (53) gives the result that the reduced-order may be instable. For gaining the stable reduced order the we apply the pole placement technique[5] i.e. $-a_{12} + a_{13} = -1$

$$\begin{aligned}\hat{a}_1 = C_{21}^{-1} \hat{c}_2 &= \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} \\ &= \begin{bmatrix} -(-3) & -(1) \\ -1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 19 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.5000 & 0.5000 \\ -0.5000 & 1.5000 \end{bmatrix} \times \begin{bmatrix} 19 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}\end{aligned}\quad (52)$$

$$\begin{aligned}\hat{a}_2 = \hat{c}_1 - C_{11} \hat{a}_1 &= \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -c_0 & 0 \end{bmatrix} \times \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 9 \\ 8 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 6.0000 \end{bmatrix}\end{aligned}\quad (53)$$

The reduced-order obtained using the case-1 is given in Equation (54)

$$R_{MM}(s) = \frac{6s + 1}{8s^2 + 9s + 1} \quad (54)$$

The reduced-order obtained using the SE has given in Equation (55)

$$R_{SE}(s) = \frac{s+1}{1.4583s^2 + 2.0833s + 1} \quad (55)$$

The reduced-order using the proposed mixed method is obtained and compared in Table 10

Table 10: Step response characteristics of obtained reduced order of Example 2 from Case 1, Case 2 and mixed methods with proposed mixed method

Reduced 2 nd order	Step reponse characteristics					
	RT	ST	ST min	ST max	Peak	PT
HOS Eq. (47)	2.2603	3.9308	0.9019	0.9990	0.9990	6.8847
Eq.(51)	2.2608	3.9472	0.9019	1.0000	1.0000	10.3230
Eq. (52)	2.3001	3.3769	0.9081	1.0126	1.0126	5.1578
Eq. (57)	8.2678	21.2741	0.9008	0.9977	0.9977	38.5913
Eq. (58)	2.3000	3.3768	0.9081	1.0126	1.0126	5.1578
$R_{\frac{MM}{SE}}(s) = \frac{0.2917s+1}{35s^2+50s+24}$	3.2014	1.8650	0.0376	0.0419	0.419	7.1564
$R_{\frac{SE}{MM}}(s) = \frac{24s+24}{0.3993s^2+1.3750s+1}$	0.8365	1.3940	21.6275	24.0722	24.0722	2.5520
$R_{\frac{MM}{SE}C2}(s) = \frac{6s+1}{1.4583s^2+2.0833s+1}$	0.2322	6.7650	0.9010	2.3587	2.3587	1.5473
$R_{\frac{SE}{MM}C2}(s) = \frac{s+1}{8s^2+9s+1}$	14.8576	26.4306	0.9027	0.9993	0.9993	48.6617
$R_{\frac{SEC1}{MMC2}}(s) = \frac{s+1}{0.3993s^2+1.3750s+1}$	0.8365	1.3940	0.9011	1.3008	1.0030	2.5520

Figure 4 gives the step response characteristics of the proposed reduced-order and reduced-order using the given methodologies in the manuscript. The proposed reduced-order attains the rise time in the shortest times period, which is 0.8365 and settles early as compared with others, which are 1.3940 after that, the response line follows the original system path.

Figure 4: Step response Characteristics of Higher Order with proposed reduced-order

The proposed reduced-order pole-zero plotting with the original higher-order system, second-order reduced using the moment matching, 2nd order reduced using the SE, 2nd order using the Improved MM, 2nd order using the improved SE and proposed reduced-order using the SE and Moment matching is given in Figure 5.

Figure 5: Pole Zero plot of Original system 2, reduced-order using the MM, SE, improved MM, the improved SE, the proposed reduced-order system using SE/MM

Figure 6: Comparative Step response characteristics of HOS with proposed reduced-order and reduced-order from literature

The step response comparative analysis of the HOS 2 is in table 9. The result obtained is competitive with the response from literature and it can be observed from the step response characteristics Figure 6. The response indices of system 2 are available in Table 11 which is also competitive, however, the ITAE, IAE and ISE are calculated and comparison is given in Table 12.

Table 11: Step response characteristics of HOS with proposed reduced and from reduced from literature

References	Reduced 2 nd order	Step reponse characteristics					
		RT	ST	ST _{min}	ST _{max}	Peak	PT
Equation (47)	Original condition 1	2.260 3	3.930 8	0.90 19	0.99 90	0.999 0	6.884 7
Proposed	$\frac{s+1}{0.3993s^2+1.3750s+1}$	0.836 5	1.394 0	0.90 11	1.30 08	1.003 0	2.552 0
S. Mukherjee[51]	$\frac{0.800003s+3}{s^2+3s+2}$	2.484 2	4.290 8	1.35 11	1.49 85	1.498 5	7.322 2
Routh Approximation [Hutton and Friedland][51]	$\frac{0.7947(s+1)}{s^2+1.6557s+0.7947}$	2.320 7	3.710 5	0.90 55	1.00 22	1.002 2	6.174 7
Davison chansambara[51]	$\frac{2-s^2}{s^2+3s+2}$	1.696 4	3.256 9	0.80 65	0.99 83	1	0
Hsia[51]	$\frac{-0.0694s^2+0.8333s+2}{s^2+3s+2}$	2.217 9	3.937 5	0.89 55	0.99 93	0.999 3	7.322 2
Third Caue Form[51]	$\frac{s+2.3014}{s^2+5.7946s+2.3014}$	4.960 8	8.834 2	0.90 07	0.99 97	0.999 7	18.34 97
J. Pal	$\frac{16.0008s+24}{30s^2+42s+24}$	2.198 7	5.570 0	0.90 79	1.02 69	1.026 9	4.539 4
Prasad and Pal	$\frac{s+34.2465}{s^2+239.8082s+34.2465}$	15.37 60	27.35 30	0.90 04	1.00 0	1.000 0	73.80 23

Table 12: Response indices of proposed reduced and from reduced order from literature

Reference	Reduced –Order	Response Indices Error		
		ITAE	IAE	ISE
Proposed	$\frac{s+1}{0.3993s^2+1.3750s+1}$	1.681	0.7082	0.185
S. Mukherjee [51]	$\frac{0.800003s+2}{s^2+3s+2}$	23.05	3.735	1.713
Routh Approximation [51]	$\frac{0.7947(s+1)}{s^2+1.6557s+0.7947}$	0.2054	0.04649	0.0003783
Davison Chansambara [51]	$\frac{2-s^2}{s^2+3s+2}$	0.5764	0.4167	0.2202
Hsia[51]	$\frac{-0.0694s^2+0.8333s+2}{s^2+3s+2}$	0.08528	0.04164	0.001133
Third Caue Form[51]	$\frac{s+2.3014}{s^2+5.7946s+2.3014}$	4.187	0.9567	0.1472
Parmar et-al [52]	$\frac{0.6667s+4}{s^2+5s+4}$	0.06627	0.02612	2.64337e-04
J. Pal	$\frac{16.0008s+24}{30s^2+42s+24}$	0.9679	0.2534	0.1188
Prasad and Pal	$\frac{s+34.2465}{s^2+239.8082s+34.2465}$	20.84	3.963	1.893

The response indices from the proposed ITAE is 1.681, IAE is 0.7082 and ISE is 0.185 which is better than the 3rd Caue form, Prasad and Pal, S Mukherjee and competitive with other reduced orders from literature.

6. Conclusion

The rigorous study of a new method for MOR has been carried out in this article. The technique unifies of two strategies i.e time moment matching with the concept of modification in Routhain array and stability equation. The modifications in the routhain array from descending to ascending way of coefficient of “s” make systematic changes in the value of c_0, c_1, c_2, c_3L . The change in values c_0, c_1, c_2, c_3L results in the change in reduced order. The proposed strategy applied to SISO LTI systems of sixth and fourth order. The succession of the approach is justified by obtaining the approximation error and comparing it with the result available in the literature. The approach of the proposed mixed-method results successfully with conditions $b_0 = a_0$ or $b_0 = a_0 = 1$ and $b_m = a_n$ or $b_n = a_n = 1$. The settling min, settling-max and peak of the diminished system are very close to the original system.

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