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Cutting Force and Nonlinear Chatter Stability of Ball-end Milling Cutter

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Abstract

Ball-end milling cutters are one of the most widely used cutters in the automotive, aerospace, die and machine parts industries. In addition, milling chatter will reduce the surface quality and production efficiency, resulting in noise. It is particularly important to model the cutting force and analyze the flutter stability of ball-end milling cutters. In this paper, a simplified milling force model of ball-end milling cutter with three degrees of freedom was established based on Merchant bevel cutting theory. The model simplified the milling force coefficient. The expressions of instantaneous milling area considering the vibration displacements in X , Y and Z directions were derived. The nonlinear dynamic cutting force model of ball-end milling cutter with three degrees of freedom was established. The nonlinear chatter vibration mechanical model of ball-end milling cutter with three degrees of freedom was established by introducing the time delay term, the stability analysis is carried out by time domain simulation. The proposed models were experimentally verified.

Key words: ball-end milling cutter; milling force; dynamics model; chatter test; chatter stability analysis

1. Introduction

Ball-end mills are widely used to produce complex surfaces. Because the cutter head is spherical, any concave surface with a radius not less than the radius of the ball can be polished, so they can be used for finishing curved surfaces [1]. In addition, the more complex the surface geometry of the workpiece, the higher the machining accuracy required, which poses a challenge to the working performance of the ball-end milling cutter [2]. In the process of milling, the geometry of the flute changes continuously along the complex tool path, which brings difficulties to the calculation of the milling force. Milling force is one of the important physical parameters in CNC machining. The change of milling force directly or indirectly affects tool wear, machining error, surface topography and cutting heat [3-5], and determines

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the power consumed in the cutting process of the CNC milling machine [6], and tool wear significantly affects milling efficiency and cost [3,2]. Milling force is an important basis for designing processing technology and optimizing cutting parameters [7-9], and it is of great significance to carry out research on the prediction of milling force.

Because the ball-end curve and milling process of ball-end milling cutters are much more complicated than that of flat-end milling cutters, it was not until 1991 that Yang and Park [10] of South Korea first published the research results on the modeling of the ball-end milling force. They proposed a functional expression of the normal front angle in spherical coordinates to represent the cutting edge curve of the tool, and based on the relationship between the shear angle and the friction angle, they established a milling force model for the ball-end mill. Since then, on this basis, scholars have made many research results on the modeling and prediction of the milling force of the ball-end milling cutter [7,11,12]

Hossein et al. [13] analyzed and fitted the milling depth, feed rate, axial and tangential milling depth, and gave the relative input parameters. With the help of the statistical software package Minitab, the model analysis was performed to predict the milling force. Milfelner et al. [14] used the measured cutting force and genetic programming to establish the genetic equation of the cutting force for ball-end milling. The system is used to conduct an experimental study on the cutting force monitoring in the process of ball-end milling. This type of research is based on experimental empirical methods to establish milling force models, and empirical formulas are subject to limited milling parameters and milling force experimental data.

Yucesan and Altintas [15] proposed a semi-mechanical model that predicts the shear and friction load distribution of the rake face and flank face of the spiral groove of a ball-end mill. They predicted the cutting force very accurately, but at the cost of a large number of ball-end milling experiments and the use of complex mathematical models. Azeem et al. [11] proposed a simple and effective method to determine the cutting force coefficient of the ball-end milling model. The unique feature of this new method is that it can calibrate the effective empirical force coefficient under a wide range of cutting conditions by performing only one half groove cutting. Similarly, Jeong et al. [16] used instantaneous cutting force coefficients that have nothing to do with the cutting conditions, and proposed a mechanical cutting force

model for 3D ball-end milling. Sim and Yang [17] assumed that the geometry of the ball-end milling cutter can be represented by sections of orthogonal cutting surfaces. They obtained basic cutting parameters from orthogonal cutting experiments and considered the chip load changes caused by static tool deflection. Merchant [18] proposed the oblique cutting theory based on orthogonal cutting and predicted the milling force. Lazoglu [19] proposed a mechanical model to predict the cutting force during ball-end milling. The program developed based on this model is modular and can easily incorporate different tools and workpiece geometries into the model. The model has the advantage of automatically calculating the intersection area of the workpiece and the tool for the given tool position, tool and workpiece geometry. Tuysuz et al. [20] proposed a mechanical model that predicts cutting force by modeling chip thickness distribution and indentation mechanics. The distribution of chip thickness and the geometry of the indentation area are evaluated by considering the five-axis movement of the tool along the tool path. Roushan et al. [21] combined the basic analytical method with orthogonal cutting finite element simulation to establish a micro-end milling force prediction model. The model takes into account the number of tool slots, milling volume and the material properties of the workpiece. This type of research does not require the support of milling parameters and milling force databases, and the established model only needs to be verified by experiments.

In 1946, British scholar Arnold[22] used experiments to prove for the first time that the cutting force within a certain range of cutting speed decreases with the increase of cutting speed. Regarding regenerative chatter, in recent years, many scholars have carried out modeling, simulation, and experimental studies on various cutting chatter problems, as well as chatter monitoring and control on the basis of considering different processing types and influencing factors of chatter [23-29]. Zhang and Liu [30] established a multi-degree-of-freedom chatter model of the tool-workpiece system in modal coordinates to decouple the delay differential equation. Solve the stable lobe diagram of ball-end milling by means of full discretization. The cutting tests conducted on the machining center verified the accuracy of the prediction model and observed the chatter phenomenon caused by different dominant modes. Kim et al. [31] proposed a dynamic ball-end milling force model, and developed software for calculation of cutting shapes and predicting dynamic cutting force in

general CNC machining and single-track machining. The frequency spectrum of chatter during ball-end milling can be detected. Somkiat et al. [32] used Daubechies wavelet to develop a method in the ball-end milling process, which can detect chatter in the time domain and frequency domain at the same time. Experiments showed that under any cutting conditions, the chatter can be easily detected by using the proposed algorithm and the obtained parameters. Sun and Altintas [33] proposed a method to automatically adjust the direction of the tool axis to avoid chattering along the tool path.

This paper proposes a simplified three-degree-of-freedom ball-end milling force model. The method is based on Merchant's bevel cutting theory and the geometric model of the ball-end milling cutter, which simplifies the complex milling force coefficients. Based on the regenerative chatter theory, an expression of the instantaneous dynamic milling area generated by the vibration displacement of the tool tip point in the X , Y and Z directions is proposed, and the change of the milling force during unstable cutting is considered. It is verified by experiments that the model and its solution method proposed in this paper can accurately predict the milling force and milling stability. It is simple and versatile and can be used to guide actual engineering needs.

2. Force model of ball-end milling cutter

This topic is based on the Merchant chamfering theory, which correlates the main section coordinate system with the orthogonal cutting edge curve, and obtains the micro-element cutting force model from the perspective of theoretical analysis.

2.1 Micro-milling force model

Since the geometry of the cutting edge of the ball-end mill is a complicated spatial spiral, when studying the cutting force of a ball-end milling cutter, it is usually based on the assumption of differential dispersion. The cutting edge is divided into infinitely small units, so that the complex milling process can be considered as many tiny oblique cutting along the cutting edge. The tiny cutting edge unit is abbreviated as a cutting micro element or a micro cutting edge, and the force acting on the micro cutting edge is called a micro cutting force.

From the geometric relationship in Fig. 1, when it is in orthogonal cutting, the resultant force F_r acting on the rake face and the resultant force F_{rs} acting on the shear plane are a pair of interaction forces [34]. The shear force F_s can be expressed as

$$F_s = \tau_s A = \frac{\tau_s a_c a_w}{\sin \phi_n} \quad (1)$$

where ϕ_n is the shear angle of the shear plane; A is the area of the shear plane; τ_s is the shear stress, and a_c is the chip thickness; a_w is the chip width. And the expression of F_r is

$$F_r = \frac{F_{s(n)}}{\cos(\phi_n + \beta_n - \gamma_n)} = \frac{\tau_s a_c a_w}{\cos(\phi_n + \beta_n - \gamma_n) \sin \phi_n} \quad (2)$$

where γ_n is the rake angle on the normal surface, β_n is the friction angle on the normal surface. In orthogonal cutting, the expression of the friction force F_f during orthogonal cutting can be expressed as

$$F_f = F_{rs} \sin \beta_n = \frac{\tau_s a_c a_w}{\cos(\phi_n + \beta_n - \gamma_n) \sin \phi_n} \sin \beta_n \quad (3)$$

Fig. 1 Single shear plane cutting model for single edge orthogonal cutting

Based on the principle of power conservation in cutting, the cutting power P , the shear power P_s and the friction power P_f satisfied the following relationship

$$P = P_s + P_f \quad (4)$$

where the cutting power P , the plastic shear power P_s and friction power P_f are as

$$\begin{cases} P = F_t V \\ P_s = F_s V_s \\ P_f = F_f V \end{cases} \quad (5)$$

where V is the cutting speed; V_s is the shear speed; V_c is the chip flow speed; F_t is the tangential force on the effective plane[18].

Fig. 2 shows the Merchant oblique cutting model, and points out the geometric relationship between cutting speed V , shear speed V_s and chip flow speed V_c on the effective plane (PBCDEF). From Fig. 2, the expression of the shear area A can be given by

$$A = \frac{a_c}{\cos \phi_\lambda \sin \phi_e} \frac{a_w}{\cos \lambda_s} \quad (6)$$

where ϕ_e is the shear angle on the effective plane; γ_e is the rake angle on the effective plane.

The relationship between the cutting speed V , the shear speed V_s and the chip flow speed V_c can be listed

$$\begin{cases} V_s = \frac{\cos \gamma_e}{\cos(\phi_e - \gamma_e)} V \\ V_c = \frac{\sin \phi_e}{\cos(\phi_e - \gamma_e)} V \end{cases} \quad (7)$$

In single-edge bevel cutting, the resultant force on the effective plane is $F_{rs(e)}$ and the frictional force is $F_{f(e)}$, and the frictional force $F_{f(e)}$ is expressed as

$$F_{f(e)} = \frac{\tau_s \sin \beta_e}{\sin \phi_e \cos(\phi_e + \beta_e - \gamma_e)} \frac{a_c}{\cos \phi_\lambda} \frac{a_w}{\cos \lambda_s} \quad (8)$$

where β_e is the effective friction angle. And the tangential force F_t on the effective plane is expressed as

$$F_t = \frac{\tau_s a_c a_w}{\cos(\varphi_e - \gamma_e) \cos \phi_\lambda \cos \lambda_s} \left[\frac{\cos \gamma_e}{\sin \varphi_e} + \frac{\sin \beta_e}{\cos(\varphi_e + \beta_e - \gamma_e)} \right] \quad (9)$$

Fig. 2 Merchant Oblique cutting model

In the direction of the cutting speed V , the components of $F_{f(e)}$ and F_n should be equal to the principal tangential force F_t , and the normal force F_n acts on the rake face in the normal direction. The expression of the normal force F_n can be derived as

$$F_n = \frac{F_t - F_f \sin \gamma_e}{\cos \gamma_n \cos \lambda_s} \quad (10)$$

the sum of tangential force F_t , axial force F_a , and radial force F_r is equal to the sum of friction F_f and normal force F_n on the rake face of the tool. Then the relationship of the five components of the cutting force on the rake face can be expressed as

$$F_t + F_a + F_r = F_f + F_n \quad (11)$$

then, the axial force F_a and radial force F_r can be determined by

$$F_a = F_f (\sin \phi_\lambda \cos \lambda_s - \cos \phi_\lambda \sin \lambda_s + \sin \gamma_e \tan \lambda_s) - F_t \tan \lambda_s \quad (12)$$

$$F_r = F_f (\cos \phi_\lambda \cos \gamma_n + \frac{\sin \gamma_e \tan \gamma_n}{\cos \lambda_s}) - \frac{F_t \tan \gamma_n}{\cos \lambda_s} \quad (13)$$

2.2 The geometric model of the ball-end milling cutter

Due to the excellent cutting performance of tools with orthogonal cutting edge curves, the cutting edge curves of most spiral tools in actual production are orthogonal cutting edge curves. The cutting edge curve equation is expressed by the lead P_s and the axial height z as

$$\varphi_{(z)} = \frac{z}{P_s} \quad (14)$$

where $\varphi_{(z)}$ is the lag angle between the axial position z and $z = 0$ on the flute.

Associate the main section coordinate system with the orthogonal cutting edge curve, and the transformation process is shown in Fig. 3.

Fig. 3 Transformation of coordinate system

As shown in Fig. 4, the detailed geometry of the flute of the ball-end mill is obtained. As shown in Fig. 4(a) and (b), the point $P_{(z)}$ on the flute is at the axial location z , the angular position θ_i is the angle between the positive direction of the Y axis and the $P_{(z)}$ point in X - Y plane, it can reference the position of the point $P_{(z)}$ on the flute i in X - Y plane. In Fig. 4(c), the

geometry of the rake surface of the flute is obtained, and the radial rake angle and relief angle are defined in X - Y plane. As shown in Fig. 4(d), the height angle q is the angle between the positive direction of the X axis and the point $P_{(z)}$ in X - Z plane, and q can be expressed as

$$q = \arcsin(z/R_0) \quad (15)$$

where R_0 is the sphere radius of the tool measured at O .

Fig. 4 Detailed geometric model of ball-end milling cutter

As shown in Fig. 4, the local radius R_z in X - Y plane at the point $P_{(z)}$ is given by

$$R_z^2 + z^2 = R_0^2 \quad (16)$$

According to the Eq. (14), the angular position of the point $P_{(z)}$ can be expressed as

$$\theta_i = \chi_i + \varphi_{(z)i} = \left[(i-1) \frac{2\pi}{n_g} + \omega t \right] + \frac{z}{P_s} \quad (17)$$

where χ_i is the rotation angle of the flute i at height $z = 0$; ω is the angular velocity of the spindle ; $\varphi_{(z)i}$ is the lag angle of the point $P_{(z)}$ on the flute i ; n_g is the number of teeth.

Fig. 5 shows the geometry of the micro-element flute segment and chip in a two-dimensional coordinate system, and in Fig. 5(d) defines the normal rake angle γ_n and the normal shear angle ϕ_n on the rake face of the tool. From the geometry of the chip, the expressions of the chip thickness a_c and the differential chip width a_w are given by

$$\begin{cases} d_c = a_f \sin \theta_i \cos q_i \\ da_w = dz / \cos q_i \end{cases} \quad (18)$$

where a_f is the feed per tooth, and the expressions of a_f is

$$a_f = V_f / n_s n_g \quad (19)$$

Fig. 5 Geometry of micro element cutting edge in two-dimensional coordinate system

2.3 Milling force model of ball-end milling cutter

The differential cutting force expression is the product of the cutting force coefficient

182 and the cutting area on the effective plane, the differential cutting force can be expressed as

$$\begin{cases} dF_t = K_t a_c a_w \\ dF_r = K_r a_c a_w \\ dF_a = K_a a_c a_w \end{cases} \quad (20)$$

183 In the main section coordinate system, the cutting characteristics of micro-segmentation are
 184 similar to oblique cutting. The cutting coefficient K needs to be measured by multiple
 185 experiments. Here, the simplified cutting force coefficient B is substituted, and the expression
 186 is

$$\begin{cases} dF_t = B_t \sin \theta_i dz \\ dF_r = B_r \sin \theta_i dz \\ dF_a = B_a \sin \theta_i dz \end{cases} \quad (21)$$

187 where the simplified cutting force coefficients B_t , B_a , and B_r are expressed as

$$\begin{cases} B_t = \frac{dF_t}{\sin \theta_i dz} \\ B_r = \frac{dF_r}{\sin \theta_i dz} \\ B_a = \frac{dF_a}{\sin \theta_i dz} \end{cases} \quad (22)$$

188 Substituting Eqs. (9), (12), (13), (18), and (19) into (22), the simplified expressions of the
 189 cutting force coefficients B_r , B_a and B_t are given by

$$B_r = \frac{\tau_s V_f}{n_s n_g \cos \phi_\lambda \cos \lambda_s} \left[\frac{\sin \beta_e}{\cos(\varphi_e + \beta_e - \gamma_e)} \left(\cos \phi_\lambda \cos \gamma_n + \frac{\sin \gamma_e \tan \gamma_n}{\cos \lambda_s} \right) - \frac{\tan \gamma_n}{\cos \lambda_s \cos(\varphi_e - \gamma_e)} \left(\frac{\cos \gamma_e}{\sin \varphi_e} + \frac{\sin \beta_e}{\cos(\varphi_e + \beta_e - \gamma_e)} \right) \right] \quad (24)$$

$$B_a = \frac{\tau_s V_f}{n_s n_g \cos \phi_\lambda \cos \lambda_s} \left[\frac{\sin \beta_e}{\sin \varphi_e \cos(\varphi_e + \beta_e - \gamma_e)} (\sin \phi_\lambda \cos \lambda_s - \cos \phi_\lambda \sin \lambda_s + \sin \gamma_e \tan \lambda_s) - \frac{\tan \lambda_s}{\cos(\varphi_e - \gamma_e)} \left(\frac{\cos \gamma_e}{\sin \varphi_e} + \frac{\sin \beta_e}{\cos(\varphi_e + \beta_e - \gamma_e)} \right) \right] \quad (25)$$

$$B_t = \frac{\tau_s V_f}{n_s n_g \cos(\varphi_e - \gamma_e) \cos \phi_\lambda \cos \lambda_s} \left[\frac{\cos \gamma_e}{\sin \varphi_e} + \frac{\sin \beta_e}{\cos(\varphi_e + \beta_e - \gamma_e)} \right] \quad (23)$$

190 Transform the main profile coordinate system (X_2, Y_2, Z_2) into the global coordinate
 191 system (X, Y, Z) to obtain the equation of the differential cutting force of the ball-end milling
 192 cutter, they can be expressed as

$$\begin{cases} dF_{xi} = (-B_r \sin \theta_i \cos q_i - B_t \cos \theta_i + B_a \sin \theta_i \sin q_i) \sin \theta_i dz \\ dF_{yi} = (-B_r \cos \theta_i \cos q_i + B_t \sin \theta_i + B_a \cos \theta_i \sin q_i) \sin \theta_i dz \\ dF_{zi} = (-B_r \sin q_i - B_a \cos q_i) \sin \theta_i dz \end{cases} \quad (26)$$

193 Substituting Eqs. (15) and (17) into (26), taking z as the limit of integration, the model of the
 194 ball-end milling force is obtained by integration. The expressions of the cutting forces F_{xi}, F_{yi} ,
 195 and F_{zi} of the flute i in the X, Y and Z directions are given by

$$\begin{aligned} F_{xi} = & \left\{ \frac{-1}{16384} \left(\frac{75B_r}{R_0} \left(-3P_s^2 \left(P_s^2 - 2z^2 + \frac{44z}{9} - \frac{258}{25} \right) \cos^2 \left(\chi_i + \frac{z}{P_s} \right) \right. \right. \right. \\ & - 3P_s \left(\left(z - \frac{11}{9} \right) P_s^2 - \frac{2z^3}{3} + \frac{22z^2}{9} - \frac{258z}{25} + \frac{898}{9} \right) \sin \left(2\chi_i + \frac{2z}{P_s} \right) + \\ & \left. \left. \left(\chi_i^4 - 3\chi_i^2 + 3 \right) P_s^4 + P_s^3 \left(\frac{44\chi_i^3}{9} - \frac{22\chi_i}{3} \right) + P_s^2 \left(\frac{774\chi_i^2}{25} - \frac{774}{25} + \frac{22z}{3} - 3z^2 \right) \right. \right. \\ & \left. \left. + \frac{1796\chi_i P_s}{3} - z^4 + \frac{44z^3}{9} - \frac{774z^2}{25} + \frac{1796z}{3} \right) \right\} + \frac{B_t P_s}{4} \cos \left(2\chi_i + \frac{2z}{P_s} \right) \\ & - \frac{B_a}{4R_0} \left(P_s z \sin \left(2\chi_i + \frac{2z}{P_s} \right) - P_s^2 \sin^2 \left(\chi_i + \frac{z}{P_s} \right) + \chi_i^2 P_s^2 - z^2 \right) \Bigg|_{z_{i1}}^{z_{i2}} \end{aligned} \quad (27)$$

$$\begin{aligned} F_{yi} = & \left\{ \frac{75P_s B_r}{8192R_0} \left(\left(\left(\frac{3z}{2} - \frac{11}{6} \right) P_s^2 - z^3 + \frac{11z^2}{3} - \frac{387z}{25} + \frac{449}{3} \right) \cos \left(2\chi_i + \frac{2z}{P_s} \right) \right. \right. \\ & - \frac{3P_s}{4} \left(P_s^2 - 2z^2 + \frac{44z}{9} - \frac{258z}{25} \right) \sin \left(2\chi_i + \frac{2z}{P_s} \right) + \frac{449}{3} + P_s^3 \left(\chi_i^3 + \frac{3}{2} \chi_i \right) \\ & \left. \left. + P_s^2 \left(\frac{11}{3} \chi_i^2 + \frac{11}{6} \right) + \frac{387}{25} \chi_i P_s \right) \right\} + \frac{B_t P_s}{2} \left(-\sin \left(\chi_i + \frac{z}{P_s} \right) + \chi_i + \frac{z}{P_s} \right) + \\ & \frac{B_a P_s}{4R_0} \left(P_s \sin \left(\chi_i + \frac{z}{P_s} \right) \cos \left(\chi_i + \frac{z}{P_s} \right) + \chi_i P_s + z - 2z \cos^2 \left(\chi_i + \frac{z}{P_s} \right) \right) \Bigg|_{z_{i1}}^{z_{i2}} \end{aligned} \quad (28)$$

$$F_{zi} = \left\{ \frac{B_r P_s}{R_0} \left(P_s \sin \left(\chi_i + \frac{z}{P_s} \right) - z \cos \left(\chi_i + \frac{z}{P_s} \right) \right) - \frac{225 P_s B_a}{1024 R_0} \left(\left(-\frac{449}{18} + \left(-z + \frac{11}{9} \right) P_s^2 + \frac{z^3}{6} - \frac{11z^2}{18} + \frac{129z}{50} \right) \cos \left(\chi_i + \frac{z}{P_s} \right) + P_s \sin \left(\chi_i + \frac{z}{P_s} \right) \left(P_s^2 - \frac{z^2}{2} + \frac{11z}{9} - \frac{129}{50} \right) \right) \right\} \Big|_{z_{i1}}^{z_{i2}} \quad (29)$$

196 The milling force is the sum of the milling forces of multiple teeth, the overall milling force
 197 of the ball-end milling cutter can be expressed as

$$\begin{cases} F_X = \sum_i^{n_g} F_{Xi} \\ F_Y = \sum_i^{n_g} F_{Yi} \\ F_Z = \sum_i^{n_g} F_{Zi} \end{cases} \quad (30)$$

198 .4 Dynamic integration interval of ball-end milling

199 According to the position change relationship of the ball-end milling cutter during the
 200 machining process, the relationship between the in-cut angle and the out-cut angle and the
 201 axial position are established. and the integration interval under the milling state is further
 202 obtained. Express the integral limits z_l and z_2 by the lag angle, the overall milling force model
 203 of the tool was derived.

204 When the axial depth of cut is h_a , the expression of the axial height z_a and z_b is

$$\begin{cases} z_a = R_0 - h_a \\ z_b = R_0 \end{cases} \quad (31)$$

205 Then the lag angles φ_a and φ_b of points $P_{(a)}$ and $P_{(b)}$ can be expressed as

$$\begin{cases} \varphi_a = \frac{R - h_a}{P_s} \\ \varphi_b = \frac{R}{P_s} \end{cases} \quad (32)$$

206 In the milling process, the expression of the angle α_0 corresponding to the length of the flute

involved in milling is

$$\alpha_0 = \frac{R_0}{P_s} - \frac{R_0 - h_a}{P_s} \quad (33)$$

In Fig. 6, it is shown that the length of the flute involved in milling is dynamically changing when the tool is in the machining state after cutting into the sample. In this case, the cutting area on the X - Y plane is a semicircular sector, and the cutting edge involved in milling gradually increases from the point $P_{(b)}$ to $P_{(a)}$. The whole action can be divided into four stages.

Fig. 6 Dynamic changes of cutting edge length involved in cutting under milling state

In the first stage, the cutting edge of the tool does not participate in the meshing, and the resultant force on the cutter is zero. In the second stage, the cutter-sample contact length increases from 0 to the maximum, so the force gradually increases. In the third stage, the cutting process is relatively stable, because the length of the cutting edge reaches its maximum and remains unchanged. The fourth stage is opposite to the second stage. The contact length of the cutter-sample is reduced from the maximum to 0, so the cutting force is gradually reduced.

Based on Fig. 6 and Eqs. (38) and (39), the integration interval of the ball-end milling force in the milling state can be derived. As shown in Table 1, the equations of the upper and lower limits of the integral are obtained.

Table 1 Upper and lower limits of integration

Stages	Integration limits	Time	Time limits
Phase I:	$[\varphi_b, \varphi_b]$	$\Delta_1 \in [0, t_1]$	$t_1 = (\pi - \alpha_0)/\omega$
Phase II:	$[\varphi_b, \varphi_b - \omega(t - t_1)]$	$\Delta_2 \in [t_1, t_2]$	$t_2 = \alpha_0/\omega$
Phase III:	$[\varphi_b, \varphi_a]$	$\Delta_3 \in [t_2, t_3]$	$t_3 = (\pi - \alpha_0)/\omega$
Phase IV:	$[\varphi_b - \omega(t - t_3), \varphi_a]$	$\Delta_4 \in [t_3, t_4]$	$t_4 = \alpha_0/\omega$

2.5 Milling parameters of cutting force model

It is necessary to determine the milling parameters included in the ball-end milling force model before the simulation. In the ball-end milling force model, the basic milling parameters mainly include friction angle β , normal shear angle ϕ_n , and shear stress τ , the geometric parameters and cutting conditions of the cutter. According to [35] and the cutting datas, through computer processing, we can get the shear stress τ and the friction angle β when the cemented carbide tool cuts the 45 steel workpiece, and the friction angle β is expressed as

$$\beta_e = 0.9201\gamma_n^{0.5533}a_c^{-0.1695}V_f^{-0.03358} \quad (34)$$

According to the Merchant's first equation, the expression of the normal shear angle ϕ_n can be expressed as

$$\phi_n = \frac{\pi}{4} + \frac{\gamma_e - \beta}{2} \quad (35)$$

The milling parameters of the ball-end milling force model selected in this paper are shown in Table 2.

Table 2 Selection of milling parameters

Parameter	Value
$R_0(\text{mm})$	5
P_s	$5\sqrt{3}$
n_g	2
$\gamma_n(^{\circ})$	10.5

3 Dynamic milling force model of ball-end milling cutter

The chatter in the machining process is mainly caused by the change of the cutting thickness with time, and this chatter is also called regenerative chatter. The main mechanism is shown in Fig. 7.

Fig. 7 Unstable milling caused by regeneration effect

In the machining process, small disturbances of external factors cause dynamic cutting force produce small changes. The small changes act on the processing system, and causing the system to produce the vibration displacement, then the flute i leaves chatter marks on the surface of the workpiece. After a time lag τ , the milling edge $i+1$ will be milled again on the the workpiece's surface with chatter marks. However, due to the presence of chatter marks, the instantaneous milling thickness of the flute $i+1$ will change, then the dynamic cutting force will change. Finally, the milling surface will also have corrugated chatter marks.

According to the mechanism of the regeneration effect, when chatter occurs in the milling process, the vibration displacement of cutter promotes the instantaneous dynamic milling thickness change. Fig. 8 has shown the instantaneous chip shape during stable cutting

254 is expressed, and the instantaneous dynamic milling thickness is expressed as

$$t_n = a_f \sin \theta_i \quad (36)$$

256 Fig. 8 Milling area during stable milling

257 Fig. 9 shows the changes of the instantaneous dynamic milling area when there is
258 vibration displacement in the X directions.

259 Fig. 9 Cutting thickness with vibration displacement in the X direction

261 Then the expression of the changed instantaneous dynamic milling thickness is

$$t'_n = (a_f - \Delta x) \cdot \sin \theta \quad (37)$$

Fig. 10 shows the changes of the instantaneous dynamic milling area when there is vibration displacement in the Y directions. Then the expression of the changed instantaneous dynamic milling thickness is

$$t'_n = \frac{s_t}{\cos \Delta} \sin(\Delta + \theta) = s_t (\sin \theta + \cos \theta \tan \Delta) = a_f \sin \theta + \Delta y \cos \theta \quad (38)$$

Fig. 10 Cutting thickness with vibration displacement in the Y direction

Fig. 11 represents the changes of the instantaneous dynamic milling area when there is vibration displacement in the Z direction. Then the expression of the changed instantaneous dynamic milling thickness is

$$t'_n = \sqrt{R_0^2 - z^2} - \sqrt{R_0^2 - (z - \Delta z)^2} + a_f \cdot \sin \theta \quad (39)$$

The expression of the cut-in angle φ_a' of the changed milling force model is

$$\varphi_a' = \frac{R_0 - h_a + \Delta z}{P_s} \quad (40)$$

Fig. 11 Cutting thickness with vibration displacement in the Z direction

In Fig. 12, the instantaneous dynamic milling area change is obtained when there are vibration displacements in the X , Y , and Z directions at the same time. Then the changed instantaneous dynamic chip thickness is expressed as

$$t_n' = (a_f - \Delta x) \sin \theta + \Delta y \cos \theta + \sqrt{R_0^2 - z^2} - \sqrt{R_0^2 - (z - \Delta z)^2} \quad (41)$$

Fig. 12 Cutting thickness with vibration displacement in all three directions

279 Due to the occurrence of vibration displacement, the dynamic milling thickness and
 280 milling width change, then the instantaneous dynamic cutting force change. Substituting the
 281 instantaneous dynamic chip thickness in Eq. (41) into the ball-end milling force model (Eq.
 282 (30)), then nonlinear dynamic cutting force model of ball-end milling cutter with three
 283 degrees of freedom is obtained.

284 The dynamic cutting force of the ball-end milling cutter $F_x(t)$ in the X direction is
 285 expressed as

$$\begin{aligned}
 F_{xi}(t) = & \left\{ \frac{-(a_f - \Delta x)}{16384} \left(\frac{75K_r}{R_0} \left(-3P_s^2 \left(P_s^2 - 2z^2 + \frac{44z}{9} - \frac{258}{25} \right) \cos^2 \left(\chi_i + \frac{z}{P_s} \right) \right. \right. \right. \\
 & - 3P_s \left(\left(z - \frac{11}{9} \right) P_s^2 - \frac{2}{3} z^3 + \frac{22}{9} z^2 - \frac{258z}{25} + \frac{898}{9} \right) \sin \left(2\chi_i + \frac{2z}{P_s} \right) + (\chi_i^4 - \\
 & 3\chi_i^2 + 3)P_s^4 + P_s^3 \left(\frac{44\chi_i^3}{9} - \frac{22\chi_i}{3} \right) + P_s^2 \left(\frac{774\chi_i^2}{25} - \frac{774}{25} + \frac{22z}{3} - 3z^2 \right) - z^4 \\
 & \left. \left. \left. + \frac{1796\chi_i P_s}{3} + \frac{44z^3}{9} - \frac{774z^2}{25} + \frac{1796z}{3} \right) \right) \right\} + \frac{(a_f - \Delta x)K_t P_s}{4} \cos \left(2\chi_i + \frac{2z}{P_s} \right) \\
 & - \frac{(a_f - \Delta x)K_a}{4R_0} \left(P_s z \sin \left(2\chi_i + \frac{2z}{P_s} \right) - P_s^2 \sin^2 \left(\chi_i + \frac{z}{P_s} \right) + \chi_i^2 P_s^2 - z^2 \right) + \\
 & \frac{75P_s K_r \Delta y}{8192R_0} \left(\left(\left(\frac{3z}{2} - \frac{11}{6} \right) P_s^2 - z^3 + \frac{11z^2}{3} - \frac{387z}{25} + \frac{449}{3} \right) \cos \left(2\chi_i + \frac{2z}{P_s} \right) \right. \\
 & - \frac{3P_s}{4} \left(P_s^2 - 2z^2 + \frac{44z}{9} - \frac{258}{25} \right) \sin \left(2\chi_i + \frac{2z}{P_s} \right) + \frac{449}{3} + P_s^3 \left(\chi_i^3 + \frac{3}{2} \chi_i \right) \\
 & \left. \left. \left. + P_s^2 \left(\frac{11}{3} \chi_i^2 + \frac{11}{6} \right) + \frac{387}{25} \chi_i P_s \right) \right) \right) - \frac{\Delta y K_t P_s}{4} \left(\sin \left(2\chi_i + \frac{2z}{P_s} \right) + 2\chi_i + \frac{2z}{P_s} \right) \\
 & \left. - \frac{\Delta y K_a P_s}{4R_0} \left(\frac{P_s}{2} \sin \left(2\chi_i + \frac{2z}{P_s} \right) + \chi_i P_s + z - 2z \cos^2 \left(\chi_i + \frac{z}{P_s} \right) \right) \right) \Bigg|_{z_{i1} + \Delta z}^{z_{i2}}
 \end{aligned} \tag{42}$$

286 The dynamic cutting force of the ball-end milling cutter $F_y(t)$ in the Y direction is
 287 expressed as

$$\begin{aligned}
F_{yi}(t) = & \left\{ \frac{75(a_f - \Delta x)P_s K_r}{8192R_0} \left(\left(\left(\frac{3z}{2} - \frac{11}{6} \right) P_s^2 - z^3 + \frac{11z^2}{3} - \frac{387z}{25} + \frac{449}{3} \right) \cos \left(2\chi_i + \frac{2z}{P_s} \right) \right. \right. \\
& - \frac{3P_s}{4} \left(P_s^2 - 2z^2 + \frac{44z}{9} - \frac{258z}{25} \right) \sin \left(2\chi_i + \frac{2z}{P_s} \right) + \frac{449}{3} + P_s^3 \left(\chi_i^3 + \frac{3\chi_i}{2} \right) + P_s^2 \left(\frac{11\chi_i^2}{3} + \frac{11}{6} \right) \\
& + \frac{387}{25} \chi_i P_s \left. \right) + \frac{(a_f - \Delta x)K_i P_s}{2} \left(-\sin \left(\chi_i + \frac{z}{P_s} \right) + \chi_i + \frac{z}{P_s} \right) + \frac{(a_f - \Delta x)K_a P_s}{4R_0} \left(P_s \sin \left(\chi_i + \right. \right. \\
& \left. \left. \frac{z}{P_s} \right) \cos \left(\chi_i + \frac{z}{P_s} \right) + \chi_i P_s + z - 2z \cos^2 \left(\chi_i + \frac{z}{P_s} \right) \right) + \frac{-75\Delta y K_r}{16384R_0} \left(6P_s^2 \left(\chi_i P_s - z + \frac{22}{9} \right) \left(\chi_i P_s \right. \right. \\
& + z \left. \right) \cos^2 \left(\chi_i + \frac{z}{P_s} \right) + 3P \left(\left(z - \frac{11}{9} \right) P_s^2 - \frac{2z^3}{3} + \frac{22z^2}{9} - \frac{258z}{25} + \frac{898}{9} \right) \sin \left(2\chi_i + \frac{2z}{P_s} \right) - \\
& \left(P_s^4 \left(6\chi_i^2 - 3 \right) + \frac{44\chi_i P_s^3}{3} + \frac{774P_s^2}{25} \right) \sin^2 \left(\chi_i + \frac{z}{P_s} \right) + \left(\chi_i P_s + z \right) \left(P_s^3 \left(\chi_i^3 - 3\chi_i \right) + P_s^2 \left(+3z \right. \right. \\
& - z\chi_i^2 + \frac{44\chi_i^2}{9} - \frac{22}{3} \left. \right) + \chi_i P_s \left(z^2 - \frac{44z}{9} + \frac{774}{25} \right) - z^3 + \frac{44z^2}{9} - \frac{774z}{25} + \frac{1796}{3} \left. \right) + \left(\frac{z^2}{P_s^2} - \chi_i^2 + \right. \\
& \left. \frac{z}{P_s} \sin \left(2\chi_i + \frac{2z}{P_s} \right) - \sin^2 \left(\chi_i + \frac{z}{P_s} \right) \right) \frac{\Delta y K_a P_s^2}{4R_0} + \frac{\Delta y K_i P_s}{2} \sin^2 \left(\chi_i + \frac{z}{P_s} \right) \left. \right\} \Bigg|_{z_{i1}}^{z_{i2}} + \Delta z
\end{aligned} \tag{43}$$

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The dynamic cutting force of the ball-end milling cutter $F_z(t)$ is expressed as

$$\begin{aligned}
F_{zi}(t) = & \left\{ \frac{(a_f - \Delta x)K_r P_s}{R_0} \left(P_s \sin \left(\chi_i + \frac{z}{P_s} \right) - z \cos \left(\chi_i + \frac{z}{P_s} \right) \right) - \frac{225(a_f - \Delta x)P_s K_a}{1024R_0} \left(\left(\frac{z^3}{6} \right. \right. \right. \\
& - \frac{449}{18} + \left(-z + \frac{11}{9} \right) P_s^2 - \frac{11z^2}{18} + \frac{129z}{50} \left. \right) \cos \left(\chi_i + \frac{z}{P_s} \right) + P_s \sin \left(\chi_i + \frac{z}{P_s} \right) \left(P_s^2 - \frac{z^2}{2} + \frac{11z}{9} \right. \\
& - \frac{129}{50} \left. \right) \left. \right) - \frac{\Delta y K_r P_s}{R_0} \left(z \sin \left(\chi_i + \frac{z}{P_s} \right) + P_s \cos \left(\chi_i + \frac{z}{P_s} \right) \right) - \frac{\Delta y P_s K_a}{2048R_0} \left(\left(6 \cos \left(\chi_i + \frac{z}{P_s} \right) P_s^2 \right. \right. \\
& - 3z^2 \cos \left(\chi_i + \frac{z}{P_s} \right) + 6z P_s \sin \left(\chi_i + \frac{z}{P_s} \right) - \frac{z^3}{P_s} \sin \left(\chi_i + \frac{z}{P_s} \right) \left. \right) 75P_s + \left(\frac{z}{P_s} \cos \left(\chi_i + \frac{z}{P_s} \right) - \right. \\
& \sin \left(\chi_i + \frac{z}{P_s} \right) + \frac{z^2}{2P_s^2} \sin \left(\chi_i + \frac{z}{P_s} \right) \left. \right) 550P_s^2 - 1161P_s \left(\cos \left(\chi_i + \frac{z}{P_s} \right) - \frac{z}{P_s} \sin \left(\chi_i + \frac{z}{P_s} \right) \right) \\
& + 11225 \sin \left(\chi_i + \frac{z}{P_s} \right) \left. \right) \left. \right\} \Bigg|_{z_{i1}}^{z_{i2}} + \Delta z
\end{aligned} \tag{44}$$

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4 Dynamic model of milling with three degrees of freedom

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Regarding the cutting regeneration effect, the general machine tool cutting system is

often expressed as a delay differential equation(DDE). As shown in Fig. 8, the milling area is determined by the Q' point and the Q point. The Q' point is the position point of the current milling edge i , the point Q is the position point of the previous milling edge $i-1$ at time $t-\tau$, where τ is the time lag of the milling edge i at time t .

According to the regenerative effect and Figs. 9–12, the chatter displacements in the X , Y , and Z directions are functions of time t and the time lag. Then the vibration displacement Δx , Δy and Δz in the X , Y and Z directions are expressed as

$$\begin{cases} \Delta x = x(t) - x(t - \tau_x) \\ \Delta y = y(t) - y(t - \tau_y) \\ \Delta z = z(t) - z(t - \tau_z) \end{cases} \quad (45)$$

where τ_x , τ_y , and τ_z are the time lags in the X , Y , and Z directions, respectively. According to the expressions of chatter displacements Δx , Δy and Δz , the dynamic milling forces $F_x(t)$, $F_y(t)$ and $F_z(t)$ are also functions of time t and time lag τ . According to the characteristics of milling chatter, the expression of the time lag τ_x , τ_y , and τ_z are given by

$$\begin{cases} \tau_x = \frac{1}{n_s \cdot n_g} \\ \tau_y = \frac{L}{V_f} \\ \tau_z = \tau_y \end{cases} \quad (46)$$

where L is the length of the workpiece in the direction of the feed speed.

Machining errors, installation eccentricity, and manufacturing errors will cause the eccentricity e of the spindle of the machine tool. The spindle will generate eccentric load in the X and Y directions during rotation. The eccentric load F_{0x} and F_{0y} can be expressed as

$$\begin{aligned} F_{0x} &= me\omega^2 \cos(\omega t) \\ F_{0y} &= me\omega^2 \sin(\omega t) \end{aligned} \quad (47)$$

where m is the mass, and the expression $m \cdot e$ can be identified through experiments.

Fig. 13 shows a schematic diagram of the dynamics of the milling process. During the milling process of the machine tool, the cutting tool will generate cutting vibration in the X , Y and Z directions.

Fig. 13 Schematic diagram of milling process

On the basis of further considering the influence of the tool's axial vibration on the dynamic cutting force, assuming that the damping forces of the tool subsystem and the workpiece subsystem are proportional to the vibration velocity v_x , v_y and v_z in their respective directions, the following coupled three degrees of freedom dynamic equations are established.

The expressions are

$$\begin{cases} m_x \ddot{x}(t) + c_x v_x + k_x \Delta x = F_x(t) + F_{0x} \\ m_y \ddot{y}(t) + c_y v_y + k_y \Delta y = F_y(t) + F_{0y} \\ m_z \ddot{z}(t) + c_z v_z + k_z \Delta z = F_z(t) \end{cases} \quad (48)$$

where m_x , m_y and m_z are the equivalent masses of the tool in the X , Y and Z directions

respectively; c_x , c_y and c_z represent the equivalent damping of the tool in the X , Y and Z directions respectively; k_x , k_y and k_z represent the equivalent stiffness of the tool in the X , Y , and Z directions, respectively.

5 Simulation and experiment

5.1 Experiment of the ball-end milling force

The experiment is shown in Fig. 14. The milling experiment in this study is carried out on the XK5032A CNC milling machine. A three-phase force sensor (CL-YD-3210) is used to measure the milling force of the three perpendicular directions (X , Y , and Z).

Fig. 14 Milling force measurement

The basic parameters of the cutter for milling force simulation and experiment are listed in Table 2. Three sets of milling test were carried out under the milling conditions shown in Table 3, they increase the diversity of test data.

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Table 3 Selection of milling conditions

No.	Spindle speed n_s (r/min)	Feed speed V_f (mm/min)	Axial depth of cut h_a (mm)
1	600	30	0.5
2	800	30	0.5
3	1000	30	0.3

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Then the curves of test results were compared with the simulation curves of the ball-end milling force model (Ep.(30)), the results are shown in Fig. 15, Fig. 16 and Fig. 17, respectively. As shown in Figs.15–17, the curves obtained from the test results is basically the same as the simulation curves of the milling force model. Considering the errors of installation and clamping, it is within the error range. As shown in Fig.15 and Fig.16, as the spindle speed n_s increases, the ball-end milling forces F_x , F_y and F_z all decrease. It is worth noting that in Figs. 15–17, the cutting forces obtained from the experiments are different in the first half cycle and the second half cycle, which is affected by the chatter and the eccentric load of the spindle.

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Fig. 15 Curves of milling force of experiment 1

Fig. 16 Curves of milling force of experiment 2

Fig. 17 Curves of milling force of experiment 3

5.2 Experiment of dynamic milling force of the ball-end cutting

In order to verify the accuracy of the dynamic milling force model, the displacement sensor was used to measure the vibration displacement of the cutter during the machining process. Then the measured vibration displacement was substituted into the dynamic cutting

force model (Eqs. (42)–(44)), so that the dynamic cutting force was obtained through simulation. Finally, the simulated dynamic cutting force is compared with the experimentally measured dynamic milling force.

The experiment is shown in Fig. 18. The experiment of dynamic milling force used a laser displacement sensor (optoNCDT2300-20) to measure the vibration displacement of the cutter, and a three-phase force sensor (CL-YD-3210) measured the dynamic milling force during milling. It is worth noting that due to the limitation of the experimental equipments, it is impossible to measure the vibration displacement in the Z direction. The milling process also was carried out on the XK5032A CNC milling machine.

Fig. 18 Dynamic milling force measurement

The experiments were carried out under the same condition that spindle speed n_s is 600 r/min, feed speed V_f is 90 mm/min, and cutting depth h_a is 1 mm. The results are shown in Fig. 19, and Fig. 19 (a), (b) and (c) are the curves of the dynamic milling force $F_x(t)$, $F_y(t)$, and $F_z(t)$, respectively.

Fig. 19 Curves of dynamic milling force

The vibration displacements in the X and Y directions measured by the laser displacement sensor are substituted into the dynamic milling force Eqs. (3.7)–(3.9). At this time, the ball-end milling force changes from static to dynamic. As shown in Fig. 18, the dynamic milling force curve becomes no longer smooth, the curves obtained from the test data is basically the same as the simulation curves of the dynamic milling force model. Considering the errors of installation and clamping, it is within the error range.

5.3 Identification of the dynamic characteristic parameters

Before simulating and analyzing the dynamic model of the milling system, it is necessary to obtain the dynamic characteristic parameters of the milling system. In this paper, the modal tests of the system are designed to identify the equivalent stiffness and damping ratio of the milling system in the X , Y and Z directions. Fig. 20 shows the structure diagram of the test system. The test uses LC-01A impact hammer to provide excitation signal at the tool, and at the same time, the 4524-B-001 three-axis acceleration sensor is fixed on the tool handle at the

position opposite to the hammer head to measure the vibration signal.

Fig. 20 Structural block diagram of the test system

Fig. 21(a) and (b) are the detailed diagrams of the modal hammer test in the X and Z directions respectively, which mainly describe the placement of the impact hammer and acceleration sensor. In addition, the test methods in the Y direction and the X direction are basically the same.

Fig. 21 Modal hammer test

The obtained input and output signals in the X , Y , and Z directions are Fourier

391 transformed and expressed in Fig. 22, Fig. 23, and Fig. 24.

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Fig. 22 Frequency response function in X direction

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Fig. 23 Frequency response function in Y direction

Fig. 24 Frequency response function in Z direction

Through the modal hammer test in the X , Y and Z directions designed in this paper, the results of the natural frequency ω_n and damping ratio ζ in the X , Y and Z directions obtained by Fourier analysis are shown in Table 4.

Table 4 System modal parameters

Directions	Natural frequency ω_n (rad/s)	Damping ratio ζ
X	8796.46	3.394%
Y	8639.38	2.8%
Z	13615.67	1.643%

In this paper, a stiffness test is carried out to determine the stiffness of the system. The principle is that the tool is subjected to squeezing force and the deformation of the tool is measured. Fig. 25 shows the structure diagram of the test system., the loading force provided by the hydraulic jack is transmitted to the CL-YD-3210 force sensor, and the DH5956 dynamic signal test system is used to realize data collection; At the same time, the laser displacement sensor measures the deformation of the tool. Finally uses a computer to

visualize the force signal and displacement signal.

Fig. 25 System structure diagram for static stiffness measurement

As shown in Fig. 26, when the static stiffness is measured in the X and Z directions, the details of the test equipment such as hydraulic jacks, tools, force sensors, and displacement sensors are shown.

(a) Stiffness measurement in X direction

(b) Stiffness measurement in Y direction

Fig. 26 Device diagram for measuring static stiffness

Fig. 27 and Fig. 28 show the test datas and calculation results of the stiffness in the X direction and Z direction, respectively.

Fig. 27 Static stiffness in the X direction

Fig. 28 Static stiffness in the Z direction

Fig. 27(a) and Fig. 28(a) show the change curves of the loading force in the X direction

and Z direction; Fig. 27(b) and Fig. 28(b) show the change curves of the displacement, it can be seen that the amount of deformation is basically the same as the increase in force; Fig. 27(c) and Fig. 28(c) show the calculated stiffness curves. In addition, the stiffness in the X direction and Y direction is considered to be the same here.

According to the fitting curves in Fig. 27(c) and Fig. 28(c), the stiffness k_x , k_y and k_z in the X , Y and Z directions are determined, and the results are shown in Table 5. Through comparison, it can be found that the stiffness k_z in the Z direction is much larger than the stiffness k_x and k_y in the X and Y directions.

Table 5 The stiffness

k_x (N/mm)	K_y (N/mm)	k_z (N/mm)
1.3257×10^4	1.3257×10^4	1.9987×10^5

5.4. Experiment and simulation of dynamic model

Due to the eccentricity e of the spindle of the machine tool results in the existence of eccentric force F_0 . By collecting the displacement data of the ball-end milling cutter during milling for analysis and identification, the product of m and e in the Eq. (47) is obtained as $0.21 \text{ kg} \cdot \text{m}$.

During the milling process of the ball-end milling cutter, the laser displacement sensor measures the displacement of the cutter, and compares the test results with the vibration response obtained from the time domain simulation to verify the dynamic model. Because the laser displacement sensor cannot collect the vibration displacement in the Z direction, this paper uses the two sets of milling conditions in Table 6 to test and verify the X and Y directions. And select the milling parameters and modal parameters determined in Table 2, Table 4 and Table 5.

Table 6 Selection of milling conditions

Directions	Spindle speed n_s	Feed speed V_f	Axial depth of cut h_a
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	(r/min)	(mm/min)	(mm)
X	600	90	1
Y	600	120	1

Comparing the time domain diagram with the chatter displacement data measured by the displacement sensor, and Fig. 29 and Fig. 30 show the results in the X and Y directions, respectively.

As shown in Fig. 29 and Fig. 30, the response curves obtained from the simulation in the X and Y directions are basically consistent with the curves of the displacement data obtained in the experiment. Therefore, the accuracy of the dynamic model is verified based on the time-domain diagrams of the X and Y directions and the vibration displacement fitting results collected from the experiment.

Fig. 29 Verification result of the X direction test of 90mm/min

Fig. 30 Verification result of the Y direction test of 120mm/min

6. Discussion of milling parameters

In the cutting process, the milling parameters will have different effects on the cutting force. Prior knowledge can help the process planners to select the more appropriate cutting conditions to reduce excessive cutter wear or damage and even ensure the machining accuracy of the parts. This requires simulations and analyses of cutting force models for different milling parameters. Since this paper mainly established the ball-end milling force model during slot milling, then taking the axial cutting depth h_a and feed rate V_f as the research objects to perform MATLAB simulation to analyze their influence on the milling force.

6.1 The influence of the axial cutting depth on milling force

A simulation analysis of the influence of the axial cutting depth h_a on the milling force is carried out. When the spindle speed n_s is 600 r/min and the feed speed V_f is 30 mm/min, the axial cutting depth h_a is 0.3 mm, 0.5 mm and 0.7 mm respectively. As shown in Fig. 31 the force of the workpiece under different h_a is simulated. As shown in Fig. 31(a), (b) and (c), when the axial cutting depth h_a increases, the values of the milling force in the X, Y and Z

directions all increase, and the curve characteristics will not change basically. According to the Eqs. (18) and (32), the increase of the axial cutting depth h_a increases the milling width, which ultimately leads to the increase in the milling forces.

Fig. 31 Influence of axial cutting depth on milling force

6.2 The influence of the feed speed on milling force

When the spindle speed n_s is 600 r/min and the axial cutting depth h_a is 0.3 mm, the ball-end milling forces under different conditions of the feed speed V_f of 30 mm/min, 45 mm/min and 60 mm/min are compared. As shown in Figs. 32(a), (b) and (c), the changes in the ball-end milling forces F_x , F_y and F_z in the X , Y and Z directions are described respectively. It can be seen that when the feed rate increases, the value of the milling force increases, but the characteristics of the curves have no change basically. According to Eqs. (18)–(20), the increase in the feed rate V_f causes the feed per tooth a_f to increase, so that the milling thickness a_c increases. As a result, the milling forces F_x , F_y and F_z increase.

Fig. 32 Influence of feed velocity on milling force

6.3 The influence of the axial cutting depth on dynamic model

The dynamic model of the proposed ball-end milling cutter is simulated. The milling parameters and system modal parameters are selected from the parameters determined in Table 2, Table 4 and Table 5. The spindle speed n_s is 800 r/min and the feed speed V_f is 60 mm/min, the axial milling depth h_a are 0.15 mm and 0.35 mm. The time domain diagram and phase diagram when the axial milling depth h_a is 0.15 mm are shown in Fig. 33.

Fig. 33 System response of $h_a = 0.15$ mm

The time domain diagram and phase diagram when the axial milling depth h_a is 0.35 mm are shown in Fig. 34.

Fig. 34 System response of $h_a = 0.35$ mm

As shown in Fig. 33 and Fig. 34, under the same simulation conditions, the vibration response of different axial milling depths h_a were different. In Fig. 33, when the axial milling depth h_a was 0.1 mm, the phase diagram was a closed circle. The vibration amplitude in the

three directions was very small, and the vibration amplitude changed periodically, so the machine is in a stable milling state. In Fig. 34, when the axial milling depth h_a was 0.35 mm, the phase diagram was a wide range of messy curves. The vibration amplitude changes with time t , the vibration amplitude becomes larger and the change becomes more and more severe, and the milling system was in a chatter state at this time. It can be seen that the axial milling depth h_a is negatively related to stable milling. In order to reduce tool vibration and improve the stability of milling, it is necessary to select a smaller axial milling depth h_a .

Fig. 35 shows the waterfall diagram in the X , Y and Z directions when the spindle speed n_s was 600 r/min. It can be seen that the amplitude increases with the axial cutting depth h_a . The amplitude peaked at a frequency of 20 Hz, which was twice the harmonic of the spindle speed n_s . This is because the ball-end milling cutter selected in this paper is a two-tooth. There were multiple frequency multiplications in the X , Y and Z directions.

Fig. 35 Waterfall diagram when $n_s = 600\text{r} / \text{min}$

Fig. 36 shows the waterfall diagram in the X , Y and Z directions when the spindle speed n_s was 800r/min. It can be seen that there were frequency doublings in the X , Y and Z directions, and the amplitude increases with the axial cutting depth h_a . The amplitude reached the peak value at the frequency of 26.667 Hz, which is twice the harmonic of the spindle speed n_s .

Fig. 36 Waterfall diagram when $n_s = 800\text{r} / \text{min}$

6.4 The influence of the stiffness on dynamic model

The stiffness of the system has a crucial influence on the stability of the milling system. However, there is no obvious explicit relationship between the chatter stability domain and the stiffness of the milling system in actual research. This paper uses simulation methods to study the relationship between stiffness and flutter stability domain. The time domain simulation is performed based on the machine stiffness in Table 7.

Table 7 The selection of stiffness

k_x (N/mm)	k_y (N/mm)	k_z (N/mm)
3.9771×10^4	3.9771×10^4	5.9961×10^5

Fig. 37 shows the time domain diagram, Poincare section, FFT and three-dimensional phase diagram under the milling condition with the spindle speed of 800 r/min and the axial cutting depth of 0.35 mm, which is the same as the selected milling condition in Fig. 34. However, it can be seen from Fig. 37 that the milling system was in the absolute stable zone, which is different from that in Fig. 34. The curve characteristics of the time domain diagram and the phase diagram are similar to those in Fig. 33. The amplitude of the chatter response is small and changes periodically. It can also be seen from the Poincare cross section and FFT that the milling system is in a single cycle of movement in the X , Y , and Z directions.

(g) FFT in Y direction

(h) Three-dimensional phase diagram in Y direction

(i) Time domain diagram in Z direction

(j) Poincare section in Z direction

(k) FFT in Z direction

(l) Three-dimensional phase diagram in Z direction

Fig. 37 System response of $h_a = 0.35$ mm

Fig. 38 shows the time domain diagram and phase diagram under the milling condition with the spindle speed n_s of 800 r/min and the axial cutting depth of 0.75 mm. The curve characteristics of the time domain diagram and the phase diagram were similar to those in Fig. 34. The amplitude of the vibration response in the X, Y, and Z directions all increased sharply, and with the change of time t , the amplitude changed more and more severely. At this time, the ball end milling lost stability and it was in a chatter state.

Fig. 38 System response of $h_a = 0.75$ mm

It can be seen from Fig. 33, 34, 37 and 38 that when the stiffness increases, the critical axial cutting depth corresponding to the spindle speed increases, and the stable range of milling processing also increases. Increasing the stiffness contributes to the stability of milling.

7. Conclusions

A new ball-end milling force model, which can predict three-dimensional cutting forces, is presented in this study. This method simplifies the complex milling force coefficient. Then based on the established ball-end milling force model and regenerative chatter theory, considering the influence of the cutter's vibration displacement in the X , Y and Z directions on the dynamic cutting force, a nonlinear cutting force model of ball-end milling cutter with three degrees of freedom is proposed. It can accurately predict the cutting force under stable and unstable milling conditions. The ball-end milling force model and the nonlinear dynamic cutting force model were tested separately. The test results shown that the models proposed in this paper can accurately predict the milling force, and can be used to guide actual engineering needs.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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