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Yongbin Zhang (✉ engmech1@sina.com)

Changzhou University

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Multiscale mixed hydrodynamics in line contacts

Yongbin Zhang*

College of Mechanical Engineering, Changzhou University, Changzhou, Jiangsu Province, China

Abstract:

In the hydrodynamic line contact, there is a very thin layer physically adsorbed to the solid surface. When the surface separation is sufficiently small, the Hertzian contact zone will be completely filled with the boundary layer, while in most of the inlet zone still occurs continuum hydrodynamics, which lies between the mated adsorbed layers. The present paper studies this mixed hydrodynamics by a multiscale analysis. The boundary layer flows are simulated from the flow factor approach model; The intermediate continuum fluid flow is simulated from the continuum fluid model. The flow equations are given respectively for the boundary layers and for the intermediate continuum fluid. The final governing equation has been obtained relating the surface separation to the solid surface speeds and the carried load. The calculation results show that for a high rolling speed the hydrodynamic behavior in the contact agrees with the classical hydrodynamic theory; However for a critically low rolling speed it gives the surface separation greatly higher than that calculated from the classical hydrodynamic theory, showing the significant adsorbed layer effect.

Keywords: Adsorbed layer; Fluid; Hydrodynamics; Multiscale; Surface separation

Introduction

Line contact is a popular concentrated contact mode between counter-formal solid surfaces such as occurring on the mechanical elements¹. Classical hydrodynamic theories²⁻⁴ ignore the effect of the boundary layer physically adsorbed to the solid surface. However, in a line contact the surface separation can be very small and may be on the same scale with the thickness of the adsorbed boundary layer^{5, 6}. It is still not clear that whether classical hydrodynamic theories stand for this line contact and when and how the hydrodynamics in this line contact deviate from classical hydrodynamics.

Chauveteau et al.⁷ experimentally found that for the attractive pore wall the viscosity of the adsorbed layer near the wall was higher than the bulk viscosity in the central region for polymer solutions in a very small pore the radius of which is on the same scale with the polymer molecule diameter. They suggested that the effective viscosity across the pore radius can be easily calculated by assuming both the adsorbed layer and the intermediate continuum fluid flowing respectively with different viscosities. Chan and Horn⁸ suggested the ordering molecule layers of octamethylcyclotetrasiloxane (OMCTS) adjacent to the solid mica surface the thickness of which should be no more than 2.5nm in their drainage experiment. Similar as Chauveteau et al.⁷, they showed that the effective viscosity across the surface separation can be easily calculated by assuming both the solidified layer on each solid surface and the continuum fluid flow between the two adsorbed layers when the surface separation was between 50 nm and 5nm. They found that by this way the drainage deviation from the Reynolds continuum lubrication theory can be satisfactorily explained. In the multiscale simulation for micro channel flows, Zhang⁹ showed that the adsorbed boundary layer with the thickness 2nm will have a considerable influence on the channel flow even when the channel height was 100nm, but the channel flow was actually mainly in the continuum flow.

* Email: engmech1@sina.com

Obviously, the presence of the adsorbed layer on the contact surface can have a considerable influence on the hydrodynamics even when the adsorbed layer thickness is far lower than the surface separation. Conventional hydrodynamic theories for line contacts ignored the effect of the adsorbed layer²⁻⁴. The author ever analyzed the performance of line contact hydrodynamics by assuming the solid layers respectively on the two contact surfaces, and obtained the results qualitatively agreeable with the experimental observations¹⁰. By assuming an equivalent solid layer on the contact surface and the equivalent intervening continuum fluid flow, the author also analyzed the performance of line contact hydrodynamics when the surface separation was very low so that the effect of the surface plastic deformation was involved¹¹. Later, the author further analyzed the performance of line contact hydrodynamics with very low surface separations by assuming both the solidified layer on the contact surface and the intervening flowing molecule layers between the two solid layers, when the effects of the surface plastic and thermal deformations were involved¹².

In the study of microchannel flows where the thickness of the adsorbed molecule layer on the wall is comparable with the channel height, the effect of the adsorbed layer was modeled by molecular dynamics simulation (MDS), and the fluid flow intermediate between the two adsorbed layers was simulated from the continuum fluid model¹³⁻¹⁸. This was a popular multiscale scheme for simulating hydrodynamics with the adsorbed layers in the past time. Its obvious shortcoming is that it should normally require unaffordable computational time and storage for an engineering flow.

Instead of by assuming the continuum flowing fluids or solid layers for the adsorbed layers as done by Chauveteau et al.⁷ and Chan and Horn⁸, the author's earlier study¹⁹ took the adsorbed layer flow as essentially the molecular scale flow different from the intervening continuum fluid flow, well corresponding to the previous MDS-continuum hybrid schemes¹¹⁻¹⁶. The density and viscosity increases and the discontinuity and inhomogeneity effects across the layer thickness can be simultaneously considered in that model. Three closed-form explicit flow equations have been derived respectively for the two adsorbed boundary layer flows and for the intermediate continuum fluid flow¹⁹. By that multiscale approach, the computational time and storage can be greatly saved.

By using that multiscale approach, the author has studied the mode of multiscale hydrodynamics in the line contact where the intermediate continuum fluid exists in the whole contact and the thickness of the adsorbed boundary layer is comparable with the thickness of the intermediate continuum fluid film in the Hertzian contact zone²⁰.

The present paper attempts to study a new mode of multiscale hydrodynamics in the line contact where the continuum fluid vanishes and there is only the adsorbed boundary layer in the Hertzian contact zone. This is actually the mode of multiscale mixed hydrodynamics, where the pure boundary layer flow in the Hertzian contact zone should be described from the nanoscale flow equation while the flow in the inlet zone should be described by the multiscale approach. The present study is thus further progressive and should be of significant interest to the understanding of the hydrodynamic behavior in the line contact with very small surface separations.

Mixed hydrodynamic line contact

Figure 1(a) shows the studied mixed hydrodynamics in the line contact where for the surface separation smaller than $2h_{bf}$ is only present the adsorbed boundary layer; Here h_{bf} is the

thickness of the adsorbed layer on either of the solid surfaces. The pure boundary layer area includes the Hertzian contact zone (from $-1 \leq X \leq 1$) and the inlet area (from $X_0 \leq X \leq -1$) close to the Hertzian contact zone. In most of the inlet zone (for $X < X_0$) occurs multiscale hydrodynamics because of both the adsorbed layer effect and the intermediate continuum fluid hydrodynamic effect. In this line contact, h_{tot} is the surface separation, $h_{c,tot}$ is the surface separation at the Hertzian contact center (where the pressure gradient vanishes), h is the intermediate continuum fluid film thickness, and u_a and u_b are respectively the circumferential speeds of the upper and lower contact surfaces.

When $h_{c,tot} > 2h_{bf}$, the line contact will become as Fig.1(b) shows, and its hydrodynamics is the first mode of multiscale hydrodynamics in the line contact, which has been studied in Ref.20. For a full understanding and the purpose of comparison, in the present study the surface separation $h_{c,tot}$ covers a wide range which not only includes the hydrodynamics in Fig.1(a) but also includes those in Fig.1(b).

Analysis

The analysis here is based on the following assumptions:

- (a) The adsorbed layers on the two contact surfaces are identical;
- (b) The flow is isothermal and no side leakage occurs;
- (c) The compressibility of both the continuum fluid and the adsorbed layer is negligible;
- (d) No interfacial slippage occurs on any interface;
- (e) The contact surfaces are perfectly smooth.

Determination of the coordinate X_0

The dimensionless surface separation in the inlet zone (i.e. for $X \leq -1$) in the contact in Fig.1(a) is⁴:

$$H_{tot}(X) = H_{c,tot} - \frac{4}{\pi} \{ X(X^2 - 1)^{0.5} + \ln[-X - (X^2 - 1)^{0.5}] \} W \quad (1)$$

where $H_{tot} = h_{tot}/R$, $H_{c,tot} = h_{c,tot}/R$, $X = x/b$, and $W = w/(E_v R)$. Here,

$R = R_a R_b / (R_a + R_b)$, R_a and R_b are respectively the curvature radii of the upper and lower contact surfaces, b is the half Hertzian contact width, x is the dimensional coordinate, w is the load per unit contact length carried by the contact, $2/E_v = (1 - \nu_a^2)/E_a + (1 - \nu_b^2)/E_b$, E_a and E_b are respectively the elastic Young's moduli of the upper and lower contact surfaces, and ν_a and ν_b are respectively the Poisson's ratios of the two contact surfaces.

The dimensionless intermediate continuum fluid film thickness in Fig.1(a) is:

$$H(X) = H_0 - \frac{4}{\pi} \{X(X^2 - 1)^{0.5} + \ln[-X - (X^2 - 1)^{0.5}]\}W, \quad \text{for } X \leq X_0 \quad (2)$$

where $H = h/R$, $H_0 = H_{c,tot} - 2H_{bf}$, and $H_{bf} = h_{bf}/R$.

The dimensionless coordinate X_0 of the boundary between the pure boundary layer area and the inlet area with the fluid film and the boundary layer mixing is thus solved from the following equation:

$$2H_{bf} = H_{c,tot} - \frac{4}{\pi} \{X_0(X_0^2 - 1)^{0.5} + \ln[-X_0 - (X_0^2 - 1)^{0.5}]\}W \quad (3)$$

Flow equations

For $X < X_0$

In this area, the total flow consists of the boundary layer flows and the intermediate continuum fluid flow; The flow continuity equation is²⁰:

$$q_{v,bf,A} + q_{v,bf,B} + q_{v,hf} = \frac{u_a + u_b}{2} h_{c,tot} \quad (4)$$

where $q_{v,bf,A}$ and $q_{v,bf,B}$ are respectively the volume flow rates per unit contact length of the upper and lower adsorbed layers, and $q_{v,hf}$ is the volume flow rate per unit contact length of the intermediate continuum fluid.

The flow rates $q_{v,bf,A}$ and $q_{v,bf,B}$ are respectively expressed as¹⁹:

$$\begin{aligned} q_{v,bf,A} = & u_a h_{bf} + \frac{u_b - u_a}{2} h_{bf} \frac{\varepsilon \lambda_{bf}}{2\lambda_{bf} + C_y(1 + \frac{\Delta x}{D})} + \frac{F_1 h_{bf}^3}{12\eta_{bf,1}^{eff}} \frac{\partial p}{\partial x} \\ & - \frac{h_{bf}^3}{2\eta_{bf,1}^{eff}} \frac{\partial p}{\partial x} \left(1 + \frac{1}{2\lambda_{bf}} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h_{bf}}\right) \frac{\varepsilon}{1 + \frac{\Delta x}{D}} \end{aligned} \quad (5)$$

and

$$\begin{aligned} q_{v,bf,B} = & u_b h_{bf} + \frac{u_a - u_b}{2} h_{bf} \frac{\varepsilon \lambda_{bf}}{2\lambda_{bf} + C_y(1 + \frac{\Delta x}{D})} + \frac{F_1 h_{bf}^3}{12\eta_{bf,1}^{eff}} \frac{\partial p}{\partial x} \\ & - \frac{h_{bf}^3}{2\eta_{bf,1}^{eff}} \frac{\partial p}{\partial x} \left(1 + \frac{1}{2\lambda_{bf}} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h_{bf}}\right) \frac{\varepsilon}{1 + \frac{\Delta x}{D}} \end{aligned} \quad (6)$$

where p is the hydrodynamic pressure, D is the fluid molecule diameter, $\lambda_{bf} = h_{bf}/h$, $\eta_{bf,1}^{eff}$ is

the effective viscosity of the physical adsorbed layer and $\eta_{bf,1}^{eff} = Dh_{bf} / [(n-1)(D + \Delta_x)(\Delta_l / \eta_{line,l})_{avr,n-1}]$, Δ_x is the separation between the neighboring fluid molecules in the flow direction in the adsorbed layer, $q_0 = \Delta_{j+1} / \Delta_j$ (Δ_j is the separation between the $(j+1)^{th}$ and j^{th} fluid molecules across the layer thickness) and q_0 is constant, $\eta_{line,j} / \eta_{line,j+1} = q_0^m$, $\varepsilon = (2DI + II) / [h_{bf}(n-1)(\Delta_l / \eta_{line,l})_{avr,n-1}]$, $F_1 = \eta_{bf,1}^{eff} (12D^2\psi + 6D\phi) / h_{bf}^3$, n is the equivalent number of the fluid molecules across the layer thickness, and Δ_{n-2} is the separation between the neighboring fluid molecules across the

layer thickness just on the adsorbed layer-fluid interface. Here, $I = \sum_{i=1}^{n-1} i(\Delta_l / \eta_{line,l})_{avr,i}$,

$$\psi = \sum_{i=1}^{n-1} i(l\Delta_{l-1} / \eta_{line,l-1})_{avr,i}, \quad II = \sum_{i=0}^{n-2} [i(\Delta_l / \eta_{line,l})_{avr,i} + (i+1)(\Delta_l / \eta_{line,l})_{avr,i+1}] \Delta_i,$$

$$\phi = \sum_{i=0}^{n-2} [i(l\Delta_{l-1} / \eta_{line,l-1})_{avr,i} + (i+1)(l\Delta_{l-1} / \eta_{line,l-1})_{avr,i+1}] \Delta_i,$$

$$i(\Delta_l / \eta_{line,l})_{avr,i} = \sum_{j=1}^i \Delta_{j-1} / \eta_{line,j-1}, \quad \text{and} \quad i(l\Delta_{l-1} / \eta_{line,l-1})_{avr,i} = \sum_{j=1}^i j\Delta_{j-1} / \eta_{line,j-1};$$

$\eta_{line,j-1}$ is the local viscosity between the j^{th} and $(j-1)^{th}$ fluid molecules across the layer thickness.

The flow rate $q_{v,hf}$ is expressed as¹⁹:

$$q_{v,hf} = \frac{u_a + u_b}{2} h - \frac{h^3}{12\eta} \frac{\partial p}{\partial x} + \frac{h^3}{\eta_{bf,1}^{eff}} \frac{\partial p}{\partial x} \left[\frac{F_2 \lambda_{bf}^2}{6} - \frac{\lambda_{bf}}{1 + \frac{\Delta_x}{D}} \left(\frac{1}{2} + \lambda_{bf} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h} \right) \right] \quad (7)$$

where η is the fluid bulk viscosity, and $F_2 = 6\eta_{bf,1}^{eff} D(n-1)(l\Delta_{l-1} / \eta_{line,l-1})_{avr,n-1} / h_{bf}^2$.

Based on the relation $\eta = \eta_a e^{\alpha p}$, where η_a is the fluid bulk viscosity in the ambient condition and α is the fluid viscosity-pressure index, substituting Eqs.(5)-(7) into Eq.(4) and re-arranging gives that:

$$\left\{ \frac{h^3}{12} - \frac{h^3}{C_{y,1}} \left[\frac{F_2 \lambda_{bf}^2}{6} - \frac{\lambda_{bf}}{1 + \frac{\Delta x}{D}} \left(\frac{1}{2} + \lambda_{bf} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h} \right) \right] - \frac{F_1 h_{bf}^3}{6C_{y,1}} \right. \\ \left. + \frac{\varepsilon h_{bf}^3}{C_{y,1} \left(1 + \frac{\Delta x}{D} \right)} \left(1 + \frac{1}{2\lambda_{bf}} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h_{bf}} \right) \right\} e^{-\alpha p} \frac{dp}{dx} = \frac{(u_a + u_b)\eta_a}{2} (h_{tot} - h_{c,tot})$$

for $X < X_0$ (8)

where $C_{y,1} = \eta_{bf,1}^{eff} / \eta$.

For $X_0 \leq X \leq -1$

In this area, there is only the pure boundary layer flow, which can be described by the nanoscale flow equation²¹. The flow continuity equation for this area is²¹:

$$\frac{Sh_{tot}^3}{12C_{y,2}} e^{-\alpha p} \frac{dp}{dx} = \frac{(u_a + u_b)\eta_a}{2} (h_{c,tot} - h_{tot}) \quad (9)$$

where $C_{y,2} = \eta_{bf,2}^{eff} / \eta$, $\eta_{bf,2}^{eff}$ is the effective viscosity of the boundary layer confined in the whole surface separation, and S is the parameter describing the non-continuum effect of the boundary layer.

Solving the differential equations for the contact in Fig.1(a)

Define the following dimensionless parameters:

$$U = (u_a + u_b)\eta_a / (2E_v R), \quad P = p / p_h, \quad G = \alpha E_v, \quad \lambda = b / (4R)$$

Here, p_h is the maximum Hertzian contact pressure.

The dimensionless form of Eq.(8) is:

$$\frac{1}{4} F(\lambda_{bf}) H^3 e^{-PG\lambda} \frac{dP}{dX} = U(H - H_0) \quad (10)$$

where

$$F(\lambda_{bf}) = \frac{1}{12} - \frac{1}{C_{y,1}} \left[\frac{F_2 \lambda_{bf}^2}{6} - \frac{\lambda_{bf}}{1 + \frac{\Delta x}{D}} \left(\frac{1}{2} + \lambda_{bf} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2} \lambda_{bf}}{h_{bf}} \right) \right] - \frac{F_1 \lambda_{bf}^3}{6C_{y,1}} \\ + \frac{\varepsilon \lambda_{bf}^3}{C_{y,1} \left(1 + \frac{\Delta x}{D} \right)} \left(1 + \frac{1}{2\lambda_{bf}} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h_{bf}} \right) \quad (11)$$

Integrating Eq.(10) gives that:

$$J_2 = \int_{-\infty}^{X_0} \frac{H - H_0}{H^3 F(\lambda_{bf})} dX = \int_0^{P(X_0)} \frac{e^{-PG\lambda}}{4U} dP = \frac{1 - e^{-P(X_0)G\lambda}}{4UG\lambda} \quad (12)$$

Solving Eq.(12) gives that:

$$P(X_0) = -\frac{\ln(1 - 4UG\lambda J_2)}{G\lambda} \quad (13)$$

The dimensionless form of Eq.(9) is:

$$\frac{SH_{tot}^3}{48C_{y,2}} e^{-PG\lambda} \frac{dP}{dX} = U(H_{c,tot} - H_{tot}) \quad (14)$$

Integrating Eq.(14) gives that:

$$P(X_0) = -\frac{\ln(-48UG\lambda J_3)}{G\lambda} \quad (15)$$

where $J_3 = \int_{X_0}^{-1} C_{y,2}(H_{tot} - H_{c,tot})dX / (SH_{tot}^3)$.

Solving the coupled equations (13) and (15) gives that:

$$J_2 - 12J_3 = \frac{1}{4UG\lambda} = \frac{1}{UG} \left(\frac{\pi}{8W}\right)^{1/2} \quad (16)$$

For a given fluid-contact surface interaction and given values of U , G and W , the surface separation $H_{c,tot}$ can be solved from Eq.(16).

Solution for the contact in Fig.1(b)

The solution for the contact in Fig.1(b) has been shown in Ref.20. For comparison, the equation for solving the dimensionless intermediate continuum fluid film thickness H_c at the center of the Hertzian contact zone in Fig.1(b) is repeated as follows²⁰:

$$-0.2041 \left(\lg \frac{W}{H_c}\right)^2 + 0.8876 \lg \frac{W}{H_c} + \lg \left[\frac{4GU}{\pi F(\lambda_{bf,e})} \left(\frac{8}{\pi}\right)^{1/2} \right] - \frac{3}{2} \lg W - 1.2725 = 0,$$

$$\text{for } 0.01 \leq \frac{W}{H_c} \leq 200 \quad (17)$$

where $H_c = h_c / R$, the function $F(\lambda_{bf,e})$ is shown by Eq.(11), and $\lambda_{bf,e} = h_{bf} / (kh_c)$. The value of k has been found to be around unity²⁰. For a given fluid-contact surface interaction and given values of U , G and W , the film thickness H_c can be solved from Eq.(17). The dimensionless surface separation at the center of the Hertzian contact zone in Fig.1(b) is then $H_{c,tot} = H_c + 2H_{bf}$.

Solution for the Newtonian fluid

Classical hydrodynamics assumed that there is no adsorbed layer effect and the fluid is Newtonian in the whole contact in Fig.1(a). For this case, the dimensionless surface separation $H_{c,N}$ at the center of the Hertzian contact zone in Fig.1(a) is solved from the following equation

²⁰;

$$-0.2041(\lg \frac{W}{H_{c,N}})^2 + 0.8876 \lg \frac{W}{H_{c,N}} + \lg(24.3825GU) - \frac{3}{2} \lg W - 1.2725 = 0, \quad \text{for } 0.01 \leq \frac{W}{H_{c,N}} \leq 200 \quad (18)$$

Calculation

The calculations were made for the case $G=4500$ and $W=1.0E-3$, for a relatively strong fluid-contact surface interaction. For a steel-steel contact, the load gives $p_h=2.6\text{GPa}$. For the given fluid-contact surface interaction, the parameter $C_{y,1}$ is expressed as²⁰:

$$C_{y,1}(\bar{H}_{bf}) = 1.8335 - \frac{1.4252}{\bar{H}_{bf}} + \frac{0.5917}{\bar{H}_{bf}^2} \quad (19)$$

where $\bar{H}_{bf} = h_{bf} / h_{cr,bf,1}$, and $h_{cr,bf,1}$ is a critical thickness and here taken as 20nm .

The parameter $C_{y,2}$ is formulated as²²:

$$C_{y,2}(\bar{H}_{tot}) = 1.8335 - \frac{1.4252}{\bar{H}_{tot}} + \frac{0.5917}{\bar{H}_{tot}^2} \quad (20)$$

where $\bar{H}_{tot} = h_{tot} / h_{cr,bf,2}$, and $h_{cr,bf,2}$ is a critical thickness. On the location $X = X_0$, it should be satisfied that $C_{y,1}(\bar{H}_{bf}) = C_{y,2}(2h_{bf} / h_{cr,bf,2})$. Thus, $h_{cr,bf,2} = 2h_{cr,bf,1} = 40\text{nm}$.

The parameter S is formulated as²²:

$$S(\bar{H}_{tot}) = \begin{cases} -1 & , \text{ for } \bar{H}_{tot} \geq 1 \\ [0.4 - 1.374(\bar{H}_{tot} - 0.035)^{-0.534}]^{-1} & , \text{ for } 0.035 < \bar{H}_{tot} < 1 \end{cases} \quad (21)$$

In the present calculation, the other input parameter values were taken as follows²⁰:

$$D=0.5\text{nm}, \quad \Delta_{n-2} / D = \Delta x / D = 0.15, \quad m=1.5, \quad n=8, \quad q_0=1.2$$

These values give that $h_{bf} = 4.32\text{nm}$.

Results

Figure 2(a) shows the variations of the calculated $h_{c,tot}$ value with the dimensionless rolling speed U respectively for $R = 0.1\text{mm}$ and $R = 0.01\text{mm}$. They are respectively compared with those calculated from the conventional Newtonian fluid model for the same cases. For a high rolling speed, the present calculation is nearly equal to the conventional calculation, and this indicates the negligible effect of the adsorbed layer. However, for a critically low rolling speed, the present calculation is much greater than the conventional one, showing the effect of the adsorbed layer very significantly increasing the surface separation. For a very low rolling speed,

although the conventional Newtonian fluid model gives the almost vanishing surface separation, the present model still gives the small surface separation, which is important for avoiding the direct surface bumping and efficiently reducing the surface wear. The present results are fairly agreeable with the experimental results⁸. It is interestingly observed that for a given low dimensionless rolling speed, the present calculated dimensional surface separation $h_{c,tot}$ is nearly constant, independent on the equivalent curvature radius R of the contact surfaces. According to the normalized parameter $U = (u_a + u_b)\eta_a / (2E_v R)$, this actually means that for the same low dimensional rolling speed $(u_a + u_b)/2$, the dimensional surface separation $h_{c,tot}$ is greater for a smaller R , which gives a higher dimensionless rolling speed U for the same operating condition according to Fig.2(a). This may be the significant microscale effect of hydrodynamics, which should be of potential application value.

Figure 2(b) shows the results similar as in Fig.2(a) respectively for $R = 1mm$ and $R = 10mm$. Figure 3(a) plots the $h_{c,tot}$ versus U curves respectively for different R . It is shown that for a given relatively high dimensionless rolling speed U , a higher value of R gives a significantly greater value of $h_{c,tot}$. This qualitatively agrees with the conventional recognition. However, for a given critically low dimensionless rolling speed, the dimensional surface separation $h_{c,tot}$ is independent on the equivalent curvature radius R of the two contact surfaces. This contradicts to the conventional results. It should just be due to the adsorbed layer effect. Figure 3(b) plots the $h_{c,tot}$ versus u curves respectively for different R when $\eta_a = 0.1Pa \cdot s$ and $E_v = 209GPa$. Here, u is the dimensional rolling speed and $u = (u_a + u_b)/2$. Only for sufficiently high dimensional rolling speeds, the increase of R results in a higher value of $h_{c,tot}$, agreeing with the conventional recognition. However for a critically low dimensional rolling speed such as smaller than 0.1m/s, by contrary, the reduction of R significantly increases the value of $h_{c,tot}$, which is on the scales of 1nm or 10nm. This strongly indicates the microscale effect of hydrodynamics due to the adsorbed layer. Conventional hydrodynamics fail to reveal this important effect.

Conclusions

A multiscale analysis is presented for the mixed hydrodynamics in the line contact where the surface separation in the Hertzian contact zone is so low that it is filled with the adsorbed boundary layer, while in most of the inlet zone the total flow consists of the adsorbed layer flows and the intermediate continuum fluid flow. The flow equations were respectively given for the Hertzian and inlet zones. Solving the coupled flow continuity equations in these two zones fast gives the surface separation for a given operating condition.

The calculations were made for a steel-steel contact under the load which gives the maximum Hertzian contact pressure 2.6GPa. A relatively strong fluid-contact surface interaction was taken. It gives the thickness of the adsorbed layer on either of the contact surfaces as 4.32nm. No interfacial slippage was assumed on any interface. The calculations were also extended to the cases where the surface separation in the Hertzian contact zone is so thick that it is filled with both the adsorbed boundary layers and the intermediate continuum fluid.

The calculated results show that for high rolling speeds, the hydrodynamics in a line contact follows conventional hydrodynamic theory (with the adsorbed boundary layer effect neglected); However for a critically low rolling speed, it largely deviates from the conventional hydrodynamic theory prediction with a much higher surface separation owing to the significant adsorbed layer effect. The results qualitatively agree with the experiments.

The significant microscale effect of hydrodynamics has been observed. That is: for a critically low dimensional rolling speed (u), the reduction of the equivalent curvature radius (R) of the two contact surfaces significantly increases the value of the surface separation $h_{c,tot}$ which is on the scales of 1nm or 10nm, just because of the adsorbed boundary layer effect.

Author contributions:

Y. B. Z. proposed the idea of this paper, derived the analysis, made the calculation, discussed the results, drew the conclusion, and wrote the paper.

Competing interests:

The author declares no competing interests.

References

1. Kalker, J. J. On elastic line contacts. *J. Appl. Mech.* **39**, 1125-1132(1972).
2. Pinkus, O. & Sternlicht, B. *Theory of hydrodynamic lubrication*, McGraw-Hill, New York (1961).
3. Wolveridge, P. E., Baglin, K. P. & Archard, J. F. The starved lubrication of cylinders in line contact. *Proc. Inst. Mech. Eng.* **185**, 1159-1169(1970).
4. Grubin, A. N. Fundamentals of the hydrodynamic theory of lubrication of heavily loaded cylindrical surfaces, in Kh. F. Ketova (Ed.), Central Scientific Research Institute for Technology and Mechanical Engineering, D.S.I.R. London Translations, Wellington **337**, 115(1949).
5. Asay, D. B. & Kim, S. H. Effects of adsorbed water layer structure on adhesion force of silicon oxide nanoasperity contact in humid ambient. *J. Chem. Phys.* **124**, 174712(2006).
6. Cheng, S., Luan, B. & Robbins, M. O. Contact and friction of nanoasperities: Effects of adsorbed monolayers. *Phys. Rev. E* **81**, 016102(2010).
7. Chauveteau, G., Tirrell, M. & Omari, A. Concentration dependence of the effective viscosity of polymer solutions in small pores with repulsive or attractive walls. *J. Colloid Interface Sci.* **100**, 41-54(1984).
8. Chan, D. Y. C. & Horn, R. G. The drainage of thin liquid films between solid surfaces. *J. Chem. Phys.* **83**, 5311-5324(1985).
9. Zhang, Y. B. Power loss in multiscale mass transfer. *Front. Heat Mass Transf.* **13**, 22(2019).
10. Zhang, Y. B. Modeling of molecularly thin film elastohydrodynamic lubrication. *J. Balkan*

Trib. Assoc. **10**, 394-421(2004).

11. Zhang, Y. B. Lubrication generated between two sliding parallel plane surfaces by micro asperities: Part I-Performance analysis. *J. Comput. Theor. Nanosci.* **8**, 1111-1119(2011).
12. Zhang, Y. B. Lubrication analysis for a line contact covering from boundary lubrication to hydrodynamic lubrication: Part I- Micro contact results. *J. Comput. Theor. Nanosci.* **11**, 62-70(2014).
13. Atkas, O. & Aluru, N. R. A combined continuum/DSMC technique for multiscale analysis of microfluidic filters. *J. Comput. Phys.* **178**, 342-372(2002).
14. Liu, J., Chen, S., Nie, X. & Robbins, M. O. A continuum-atomistic simulation of heat transfer in micro- and nano- flows. *J. Comput. Phys.* **227**, 279-291(2007).
15. Sun, J., He, Y. & Tao, W. Q. Scale effect on flow and thermal boundaries in micro-/nano-channel flow using molecular dynamics-continuum hybrid simulation method. *Int. J. Numer. Meth. Eng.* **81**, 207-228(2010).
16. Nie, X. B., Chen, S. & Robbins, M. O. A continuum and molecular dynamics hybrid method for micro- and nano- fluid flow. *J. Fluid Mech.* **500**, 55-64(2004).
17. Yang, X. & Zheng, Z. C. Effects of channel scale on slip length of flow in micro/nano channels. *ASME J. Fluids Eng.* **132**, 061201(2010).
18. Yen, T. H., Soong, C. Y. & Tzeng, P. Y. Hybrid molecular dynamics-continuum simulation for nano/mesoscale channel flows. *Microfluid. Nanofluid.* **3**, 665-675(2007).
19. Zhang, Y. B. Modeling of flow in a very small surface separation. *Appl. Math. Mod.* **82**, 573-586(2020).
20. Zhang, Y. B. Multiscale hydrodynamics in line contacts. *Mech. Res. Commun.* **111**, 103658(2021).
21. Zhang, Y. B. The flow equation for a nanoscale fluid flow. *Int. J. Heat Mass Transf.* **92**, 1004-1008(2016).
22. Zhang, Y. B. Lubrication analysis for a line contact covering from boundary lubrication to hydrodynamic lubrication: Part I- Micro contact results. *J. Comput. Theor. Nanosci.* **11**, 62-70(2014).

Figure legends:

Fig.1(a) Hydrodynamics in the line contact with very small surface separations, present mode of multiscale hydrodynamics in the line contact.

Fig.1(b) Hydrodynamics in the line contact with very small surface separations, first mode of multiscale hydrodynamics in the line contact²⁰.

Fig.2(a),(b) Plots of the value of the dimensional surface separation $h_{c,tot}$ against the dimensionless rolling speed U for different R and its comparison with the conventional calculation result $h_{c,N}$ for the same operating condition.

Fig.3(a) Plots of the value of the dimensional surface separation $h_{c,tot}$ against the dimensionless or dimensional rolling speed respectively for different R .

Fig.3(b) Plots of the value of the dimensional surface separation $h_{c,tot}$ against the dimensionless or dimensional rolling speed respectively for different R , $\eta_a = 0.1Pa \cdot s$, and $E_v = 209GPa$.

Figures

Fig.1(a)

Fig.1(b)

Figure 1

(a) Hydrodynamics in the line contact with very small surface separations, present mode of multiscale hydrodynamics in the line contact. (b) Hydrodynamics in the line contact with very small surface separations, first mode of multiscale hydrodynamics in the line contact²⁰.

Fig.2(a)

Fig.2(b)

Figure 2

(a),(b) Plots of the value of the dimensional surface separation against the dimensionless rolling speed U for different R and its comparison with the conventional calculation result for the same operating condition.

Fig.3(a)

Fig.3(b)

Figure 3

(a) Plots of the value of the dimensional surface separation against the dimensionless or dimensional rolling speed respectively for different R . (b) Plots of the value of the dimensional surface separation against the dimensionless or dimensional rolling speed respectively for different R , λ , and σ .