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Longitude and Latitude Based Received Signal Strength Difference

Localization Models

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Abstract: In order to use the latitude and longitude coordinates for received signal strength difference (RSSD) localization, the errors of several spherical distance calculation methods and the error of arc length relative to string length were compared. The distance-calculation RSSD localization equations were established, including spherical accurate calculation RSSD, spherical approximate calculation RSSD, and normal cylindrical projection RSSD. And then, the optimization RSSD localization models based on geodetic coordinates and corresponding to the above equations were established, and the models were verified using the point by point search method with good convergence. The numerical results show there are a lot of weak localization areas for the RSSD localization networks lack of central stations with 4,5,6 stations. Among networks with central stations, there are only a small number of weak-localization areas for the concave 4 stations network, while there are no weak-localization areas for the networks composed of more stations. When the measurement errors and the additional losses of radio wave propagation are not considered, the localization errors of the spherical accurate model, the spherical approximate model and the equiangular projection model are very small, among which the second model has the shortest localization time. The localization errors of equidistance projection model and equal-area projection model are large, neither of which is suitable for the middle latitude and high latitude areas.

Key words: Passive localization; RSSD; Longitude and latitude; Weak-localization area; Optimal modeling

Declarations

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1 Introduction

In the passive radio localization methods based on the signal transmit from the localization target, the distance-relation localization methods take the arrival distance of the signal as the localization parameter. According to different measurement parameters, the distance relation localization methods are divided into the time

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of arrival(TOA) method and the time difference of arrival(TDOA) method for measuring time, the received signal strength(RSS) method[1-4] and the received signal strength difference (RSSD) method[5-8] for measuring signal strength method. The RSSD localization method, also known as the strength difference of arrival (SDOA) localization method, which does not need to know the equivalent transmitting power of the signal source in advance, nor does it need to measure the absolute signal strength received by each station, only needs to measure the difference of received signal strength between stations, is a universal approach. The so-called received signal strength difference refers to the difference of the radio signal strength level expressed in decibels, essentially the ratio of the true values of the signal strengths.

The RSSD localization methods establish the distance-relationship equations by measuring the arrival strength level differences of radio waves, and then calculates the signal source position. There should be a certain relationship between the distance calculation method and the localization accuracy. However, there were no literatures studying the relationships. In addition, nowadays satellite navigation systems have become popular, geodetic coordinates are the easiest geographic parameters to obtain. While in the existing literatures, it is difficult to directly use the Geodetic coordinates to establish the RSSD localization model under Cartesian coordinates with distance as the axes, which need to be calculated or inquired separately to obtain. Furthermore, as the earth's surface is a curved surface, any method of calculating the distance between two points on the earth's surface in Cartesian coordinates in a plane will lead to errors. In Cartesian coordinates, the nonlinear least squares model of RSSD localization was built, and its solution method was studied[5]. A new geometrical closed-form solution called Direction of Exponent Uncertainty is proposed for RSSD localization was studied[6]. A robust two stage estimator of the optimization problem on account of RSSD is proposed in the presence of Gaussian mixture measurement noise based on the Huber's minimax model[7]. The radio source location and the height of the launch antenna are achieved by the genetic algorithm[8]. The accuracy of TDOA localization in geodetic coordinates based on five distance calculation methods was studied, and compared it with the traditional planar rectangular coordinate TDOA passive localization model in [9]. There were no literatures on the effect of various distance calculation methods in longitude and latitude coordinates on the localization performance of the RSSD localization and the RSS localization.

Consequently, how to use geodetic coordinates for RSSD localization is researched in this paper. A general RSSD localization equation on account of the latitude and longitude coordinates and the ideal-flat-ground wave propagation model are firstly established on the basis of reference [5]. Thereafter, the errors of several spherical distance calculation methods will be compared. At last, five RSSD localization models based on geodetic longitude and latitude are proposed to explore the optimal solution methods, and the numerical simulations are carried out to verify the model.

2 RSSD localization equation based on longitude and latitude

Suppose there is a RSSD localization network with n radio monitoring stations, and the longitude and latitude coordinates of the monitoring stations are $M_i (\lambda_i, \varphi_i)$ and $M_j (\lambda_j, \varphi_j)$, $i, j=1, 2, \dots, n$, $i \neq j$. it is easy to prove that the formulas of radio signal strength level differences received by the monitoring stations M_i and M_j are (1) and (2).

$$\Delta ST_{ij} = ET_i - ET_j = PT_i - PT_j \quad (1)$$

$$\Delta SM_{ij} = EM_i - EM_j = PM_i - PM_j \quad (2)$$

Where, ΔST_{ij} 、 ΔSM_{ij} are the theoretical predicted value and measured value of the signal strength level difference at M_j and M_i respectively, and their units are all dB. ET_i , EM_i , ET_j and EM_j are the theoretical predicted values and measured values of the signal field strength of the two monitoring stations respectively, and the units are all dB μ V/m. PT_i , PM_i , PT_j and PM_j are respectively the theoretical predicted values and measured values of the signal received power of the two monitoring stations, and their units are all dBm.

The theoretical prediction formulas of signal received power are (3) and (4).

$$PT_i = EIRP - Lp(d_i) \quad (3)$$

$$PT_j = EIRP - Lp(d_j) \quad (4)$$

Where, Pr_i and Pr_j are respectively the power of the receiving antenna at the points where the radio waves reach M_i and M_j ; $EIRP$ is the omnidirectional transmitting power of the transmitting source; $Lp(d_i)$ and $Lp(d_j)$ are respectively the propagation loss of the signal from the transmitting source O to the points where M_i and M_j are located.

In [5], the localization equation was deduced by using the general formula of radio wave propagation. Considering that fixed radio monitoring was paid more attention to fixed radio source localization, the ideal flat ground propagation model of fixed radio communication[10] is adopted in Formula (5).

$$Lp=120+40\lg d-20\lg ht-20\lg hr \quad (5)$$

Where, h_t and h_r are the heights of the transmitting antenna and the receiving antenna relative to the ground respectively, in meters (m).

From (3), (4) and (5), Formula (6) and (7) can be obtained.

$$PT_i = EIRP - 20\lg d_i + 20\lg ht + 20\lg hr_i \quad (6)$$

$$PT_j = EIRP - 20\lg d_j + 20\lg ht + 20\lg hr_j \quad (7)$$

It is assumed that δ_{ij} is the error between the measured value and the theoretical predicted value, i.e., $\delta_{ij} = \Delta SM_{ij} - \Delta ST_{ij}$. The localization Equation (8) can be obtained from (1), (6) and (7).

$$20\lg \frac{d_j(\lambda, \varphi)}{d_i(\lambda, \varphi)} + 20\lg \frac{hr_i}{hr_j} = \Delta SM_{ij} - \delta_{ij} \quad (8)$$

Where, the measured value of signal strength level difference ΔSM_{ij} is the observed quantity, and the rectangular coordinate (λ, φ) of the transmitting source O is the unknown quantity.

Let $\Omega_{ij} = 10^{\frac{\Delta SM_{ij} - \delta_{ij}}{20} - \lg \frac{hr_i}{hr_j}}$, then simplify (8) and obtain (9).

$$\frac{d_j(\lambda, \varphi)}{d_i(\lambda, \varphi)} = \Omega_{ij} \quad (9)$$

3 Ground distance calculation method and corresponding localization equation

Distance calculation is one of the bases of RSSD localization. Here, several calculation methods are compared first, the RSSD localization equations based on different distance calculation methods are derived.

3.1 Accurate spherical calculation method based on geodetic coordinates [11,12]

According to the spherical accurate method, the great circle distance between the signal source O and the monitoring station M_i is expressed as (10).

$$d_i(\lambda, \varphi) = \frac{\pi R}{180} \cdot \arccos \left[\sin \varphi_i \sin \varphi + \cos \lambda_i \cos \lambda \cos (\lambda - \lambda_i) \right] \quad (10)$$

Where, d is distance in kilometers. π is circular constant, and R is average radius of the earth 6371.1km. (λ, φ) and (λ_i, φ_i) are the longitude and latitude coordinates of O and M_i respectively, in degrees ($^\circ$).

3.2 Spherical approximate calculation method based on geodetic coordinate

In the coordinate system of the earth's surface expressed by latitude and longitude, the length of the meridians are the same and do not change with the longitude. But the length of latitude lines decrease with the increase of the latitude, the longest of which is at the equator. Let r denote the scaling factor of the latitude relative to the equator [9], and r_i denote the average scaling factor of the of equal-latitude lines relative to the equator that the two points I and O pass through, then

$$r_i = \cos \frac{\varphi_i + \varphi}{2} \quad (11)$$

$$d_i(\lambda, \varphi) = \frac{\pi R}{180} \sqrt{r_i^2 (\lambda - \lambda_i)^2 + (\varphi - \varphi_i)^2} \quad (12)$$

The error between the spherical approximation method and the spherical accurate method is shown in Fig.1.

Fig. 1 Error of approximate method versus accurate method

Fig. 2 Error of equidistant projection method versus accurate method

3.3 Projection distance calculation method of ortho-axis cylinder based on geodetic coordinates

The existing two-dimensional RSSD passive localization model is based on the plane Cartesian coordinate system. The distance between the signal source O and the monitoring station M_i is expressed as (13).

$$d_i(x, y) = \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad (13)$$

Where, (x, y) and (x_i, y_i) are the rectangular coordinates of O and M_i respectively.

The Cartesian coordinates correspond to the plane maps, which are projections of the earth's surface.

Here, the projection distance calculation methods based on spherical ortho-axis cylindrical projection^[17,18] are proposed, and the distance between two points is calculated by using the projection methods of spherical ortho-axis cylindrical projection and Pythagorean theorem comprehensively. The details are as follows.

If the horizontal axis of the projection plane is the x axis and the vertical axis is the y axis, then the three horizontal axis projection formulas of the equidistance projection, the equal-angle projection and the equal-area projection are all (14).

$$x = \frac{\pi R}{180} \quad (14)$$

And the vertical axis projection formulas are different. Wherein, the projection formula of spherical equidistant regular cylinder is (15). Substituting (14) and (15) into (13), the projection distance formula of spherical equidistant regular cylinder can be obtained as (16). The error of the spherical equidistant cylindrical projection method compared with the spherical accurate method is shown in Fig.2.

$$y = \frac{\pi R}{180} \quad (15)$$

$$d_i(\lambda, \varphi) = \frac{\pi R}{180} \sqrt{(\lambda_i - \lambda)^2 + (\varphi_i - \varphi)^2} \quad (16)$$

The projection formula of spherical equiangular regular cylinder is (17). Substituting it into (13), the projection distance formula of spherical equiangular regular cylinder can be obtained as (18). The error of the spherical equal-angle orthocylinder projection method compared with the spherical accurate method is shown in Fig.3.

$$y = R \ln \left[\tan \left(45^\circ + \frac{\varphi}{2} \right) \right] \quad (17)$$

$$d_i(\lambda, \varphi) = R \sqrt{\left[\frac{\pi}{180} (\lambda_i - \lambda) \right]^2 + \left[\frac{\tan(45^\circ + \frac{\varphi_i}{2})}{\tan(45^\circ + \frac{\varphi}{2})} \right]^2} \quad (18)$$

The projection formula of spherical regular cylinders with equal area is (19). Substituting into (13), the projection distance formula of regular cylinders with equal spherical area can be obtained as (20). The error between the projection method of spherical equal-area regular cylinders and the spherical accurate method is shown in Fig.4

$$y = R \sin \quad (19)$$

$$d_i(\lambda, \varphi) = R \sqrt{\left[\frac{\pi}{180} (\lambda_i - \lambda) \right]^2 + (\sin \varphi_i - \sin \varphi)^2} \quad (20)$$

Fig. 3 Error of equal Angle projection method compared with exact method

Fig. 4 Error of equal area projection method compared with exact method

3.4 Comparison of spherical string length and great circle distance

The radio waves in the ultrashort wave band mainly propagate in the form of straight line near the ground, and the straight line is parallel to the spherical string. If the effects of altitude, antenna height and atmospheric refraction are ignored, the linear propagation distance can be approximately considered to be equal to the length of the spherical string.

As in Fig.5, for the profile of the Earth, β is the great circular arc between two points; B is the length of the spherical string corresponding to the great circle. According to the Pythagorean Theorem, (21) and (22) can be obtained.

$$\sin \frac{\beta}{2} = \frac{b/2}{R} \quad (21)$$

$$b = 2R \sin \frac{\beta}{2} \quad (22)$$

Fig.5 Relation between spherical string length and radians

Fig.6 Comparison between great circle distance and spherical string length.

In the range of 333km, the maximum communication distance between ground and air that can be reached by ground ultrashort wave[13], the comparison between the distance of the great circle and the length of the spherical string is shown in Fig.6. As can be seen from the figure, the error of the great circle distance replacing the straight line distance is very small, no more than 0.016% within 333 km, and no more than 0.00039% within 50km which is the common communication distance of land radio communication. Therefore, it is reasonable that the spherical string is replaced by the great circle distance and the line of equal latitude between two points by the spherical accurate method and the spherical approximation method, and the error is very small.

3.5 RSSD localization equations based on different distance calculation methods

By substituting Equations (10), (12), (16), (18) and (20) into Equation (9), the spherical RSSD localization equation (23) based on accurate calculation can be obtained .

$$\frac{\arccos\left[\sin\varphi_j \sin\varphi + \cos\lambda_j \cos\lambda \cos(\lambda - \lambda_j)\right]}{\arccos\left[\sin\varphi_i \sin\varphi + \cos\lambda_i \cos\lambda \cos(\lambda - \lambda_i)\right]} = \Omega_{ij} \quad (23)$$

The spherical the RSSD localization equation (24) based on approximate calculation can be obtained.

$$\frac{r_{ij}^2 (\lambda - \lambda_j)^2 + (\varphi - \varphi_j)^2}{r_i^2 (\lambda - \lambda_i)^2 + (\varphi - \varphi_i)^2} = \Omega_{ij}^2 \quad (24)$$

The spherical RSSD localization equation (25) based on equidistant projection can be obtained.

$$\frac{(\lambda_j - \lambda)^2 + (\varphi_j - \varphi)^2}{(\lambda_i - \lambda)^2 + (\varphi_i - \varphi)^2} = \Omega_{ij}^2 \quad (25)$$

The spherical RSSD localization equation (26) based on of equiangular projection can be obtained.

$$\frac{\left[\frac{\pi}{180}(\lambda_j - \lambda)\right]^2 + \left[\ln \frac{\tan(45^\circ + \frac{\varphi_j}{2})}{\tan(45^\circ + \frac{\varphi}{2})}\right]^2}{\left[\frac{\pi}{180}(\lambda_i - \lambda)\right]^2 + \left[\ln \frac{\tan(45^\circ + \frac{\varphi_i}{2})}{\tan(45^\circ + \frac{\varphi}{2})}\right]^2} = \Omega_{ij}^2 \quad (26)$$

The spherical RSSD localization equation (27) based on equal area projection can be obtained

$$\frac{\left[\frac{\pi}{180}(\lambda_j - \lambda)\right]^2 + (\sin \varphi_j - \sin \varphi)^2}{\left[\frac{\pi}{180}(\lambda_i - \lambda)\right]^2 + (\sin \varphi_i - \sin \varphi)^2} = \Omega_{ij}^2 \quad (27)$$

If the number of stations which can receive a same signal is n , and the signal strength level differences between the waves reaching these stations can be measured, then a set of localization equations composed of $\frac{n(n-1)}{2}$ equations can be obtained. Due to the difference between the actual situation of radio wave propagation and the theoretical model, as well as the existence of measurement errors, there is often a certain localization deviation, and the localization circles are difficult to intersect at one point, but intersect at multiple points near the signal source, which leads to no solution of the set of localization equations, because there is no common intersection point of all the localization curves. Therefore, the algebraic analytic method is not practical.

4 Optimized RSSD localization model

Among the analytical methods for solving the localization equations, Chan method is applicable to the plane Cartesian coordinates, and can also be used in the spherical approximate calculation model which is similar to the plane Cartesian coordinates. The recursive methods, such as Taylor series iteration method (NLS), are with convergence problems. The above methods are not applicable to the accurate calculation model of the sphere, because its localization equation belongs to the transcendental equation with trigonometric function. Here, the optimization localization modeling method based on unconstrained nonlinear programming [9,14] is adopted. The deviation cumulant of the difference between the predicted value and the measured value of the signal strength level from the hypothetical localization to any two stations is taken as the objective function, and the longitude and latitude of the localization of the transmitting source is taken as the decision variable. The optimization objective function of RSSD localization is (28).

$$f(\lambda, \varphi) = \text{Min} \sum_{i=1 \dots K; j=1 \dots K}^{i>j} \left| \Delta SM_{ij} - 20 \lg \frac{d_j(\lambda, \varphi)}{d_i(\lambda, \varphi)} - 20 \lg \frac{hr_i}{hr_j} \right| \quad (28)$$

Where, K is the number of searchable points in the longitude and latitude dimensions, so the total number of searchable grids is K^2 .

By substituting (10), (12), (16), (18) and (20) into (28), the optimal RSSD localization model for accurate spherical distance calculation can be obtained as (29)

$$f(\lambda, \varphi) = \text{Min} \sum_{m=1 \dots K; n=1 \dots K}^{m>n} \left| 20 \lg \frac{\arccos[\sin \varphi_j \sin \varphi + \cos \lambda_j \cos \lambda \cos(\lambda - \lambda_j)]}{\arccos[\sin \varphi_i \sin \varphi + \cos \lambda_i \cos \lambda \cos(\lambda - \lambda_i)]} - 20 \lg \Omega_{ij} \right| \quad (29)$$

The optimal RSSD localization model for spherical approximate calculation is (30).

$$f(\lambda, \varphi) = \text{Min} \sum_{i=1 \dots K; j=1 \dots K}^{i>j} \left| 10 \lg \frac{r_{ij}^2(\lambda - \lambda_j)^2 + (\varphi - \varphi_j)^2}{r_i^2(\lambda - \lambda_i)^2 + (\varphi - \varphi_i)^2} - 20 \lg \Omega_{ij} \right| \quad (30)$$

The optimal RSSD localization model for spherical equidistance projection is (31).

$$f(\lambda, \varphi) = \text{Min} \sum_{i=1 \dots K; j=1 \dots K}^{i>j} \left| \Delta SM_{ij} - 10 \lg \frac{(\lambda_j - \lambda)^2 + (\varphi_j - \varphi)^2}{(\lambda_i - \lambda)^2 + (\varphi_i - \varphi)^2} - 20 \lg \frac{hr_i}{hr_j} \right| \quad (31)$$

The optimal RSSD localization model for spherical equiangular projection is (32).

$$f(\lambda, \varphi) = \text{Min}_{i=1 \dots K; j=1 \dots K} \sum_{i>j} \left| \Delta SM_{ij} - 10 \lg \frac{\left[\frac{\pi}{180} (\lambda_j - \lambda) \right]^2 + \left[\ln \frac{\tan(45^\circ + \frac{\varphi_j}{2})}{\tan(45^\circ + \frac{\varphi}{2})} \right]^2}{\left[\frac{\pi}{180} (\lambda_i - \lambda) \right]^2 + \left[\ln \frac{\tan(45^\circ + \frac{\varphi_i}{2})}{\tan(45^\circ + \frac{\varphi}{2})} \right]^2} - 20 \lg \frac{hr_i}{hr_j} \right| \quad (32)$$

It can be concluded that the optimization model for spherical equiarea projection RSSD localization is (33).

$$f(\lambda, \varphi) = \text{Min}_{i=1 \dots K; j=1 \dots K} \sum_{i>j} \left| \Delta SM_{ij} - 10 \lg \frac{\left[\frac{\pi}{180} (\lambda_j - \lambda) \right]^2 + (\sin \varphi_j - \sin \varphi)^2}{\left[\frac{\pi}{180} (\lambda_i - \lambda) \right]^2 + (\sin \varphi_i - \sin \varphi)^2} - 20 \lg \frac{hr_i}{hr_j} \right| \quad (33)$$

Considering that the propagation distance of radio waves in the ultrashort band is usually no more than 50km, it is assumed that the transmitting source is located within the RSSD localization network, and the geometric center of the RSSD localization network or the station receiving the strongest signal is taken as the center, the solution of the optimization objective function is calculated by point-by-point search method[19], and the search step calculation is carried out in the direction of the longitude line and the latitude line respectively. There is no case of non-convergence. If the localization accuracy needs to be further improved, the search step can be reduced. In the optimization algorithm, the search step size is proportional to the localization accuracy and inversely proportional to the square of the operation quantity.

5 Comparison of localization accuracy

Considering the level measurement error is $\pm 1.5\text{dB}$ to first-level and second-level monitoring receivers, $\pm 3\text{dB}$ to third-level monitoring receivers[15], as well as the localization errors of different RSSD localization networks without measurement error need to be researched, the signal strength measurement error of 0dB and the random uniform distribution between -3 and 3dB are set.

Three localization networks consisting of 4, 5 and 9 monitoring stations are evenly distributed within a range of 20km from east to west and from north to south. Among them, the localization network with 5 stations contain four vertices and one center point of quadrilateral, while the rest are square arrays. Within the localization networks, the location of signal source is randomly selected by Monte Carlo method with avoiding the same location as the monitoring station, and a total of 100 different points are selected. The grid search method of point-by-point calculation is used to calculate the localization within 50 km of the latitude and longitude to the station with the strongest received signal, and the root mean square value of the localization errors is obtained.

Considering that the error expressed by a specific distance unit is not universal, while the percentage representation is universal. The percentage of localization accuracy defined as the denominator of the average length and width of the network. When the diameter of the localization network is known, the specific distance data can be obtained by multiplying the percentage. So, the root mean square error is in %, not in m or km.

Simulation parameters are shown in Table 1

Tab 1 Setting of simulation parameters in RSSD localization

Parameter	Value
Composition of location network	The localization networks consisting of 4, 5, 9 and 16 monitoring stations are evenly distributed within a range of 20km from east to west and 20km from north to south.
Location of the	Within the localization network, the location of signal source is randomly selected by Monte

source of the signal	Carlo method with avoiding the same location as the monitoring station, and a total of 100 different points are selected.
Measurement error	(1) No error.
of signal strength	(2) The error is distributed randomly and evenly between -3 and 3dB.

The simulation results show that when the signal strength error is not considered, among the five RSSD localization optimization models, in terms of localization time, the approximate calculation model and the equidistance model are the same and the shortest, the equal-area projection model is 1.8 times, the equiangular projection model is 3 times, and the accurate calculation model is 2.9 times. The localization errors are shown in Fig.7. It can be seen from Fig.7 that when there is no measurement error and the additional loss of radio wave propagation is not considered, the errors of the spherical accurate calculation model, the spherical approximate calculation model and the equiangular projection model are very small, and they are the universal RSSD localization models in the latitudes not exceed 85° . The localization errors of the equidistant projection method and the equal-area projection method increase with the increase of latitude. Among them, the equidistant projection method is smaller in low latitudes, 4.5-21% in middle latitudes, and the error is larger in high latitudes. The localization error of the equal-area projection method is twice that of the equal-area projection method when the latitude is below 70° .

Fig.7 The localization accuracy of 5 methods without considering the measurement error of signal strength

The same result can be obtained by changing the distance between simulation stations. Therefore, the RSSD localization model for approximate calculation is the best.

6 localization weak area analysis

Definition: For a radio localization network in which the single-point localization can be achieve theoretically, the area where the localization deviation caused by a small observation error is greater than a certain limit is called the weak-localization area.

The weak-localization area is caused by the fact that the intersection angle of the localization straight line or the tangent line of localization curve is very small when multiple localization lines intersect near the signal source. For example, the area near the straight line where the two stations are located (blind area of AOA localization) when

performing a 2-station AOA localization; The areas far away from the localization network are obviously weak-localization areas. It is found that for some RSSD localization networks, there are also weak-localization areas in the network and out but near the network.

If the signal strength error is evenly distributed in the range of $-0.1 \sim 0.1$ dB, the weak-localization area of RSSD is defined as the area where the localization error is more than 2 times of the sum of the x-axis search step and y-axis search step. Based on 5000 Monte Carlo simulations, the figures of the weak-localization area for the RSSD networks with 4, 5, 6 and 7 stations are drawn in Fig.8, which shows some representative cases of RSSD localization network locating weak areas in inside area of the network and adjacent off-network area.

Fig.8 Localization weak area of RSSD localization network

Through the results, it is found that about inside area of the network and adjacent off-network area:

(1) There are a lot of weak areas for the localization networks with 4, 5 and 6 stations, which are lack of a central station. In the case of the convex 4 stations, There is a large loop weak-localization area near the boundary of the network. The weak-localization area of the network with 7 stations is small enough to be ignored.

(2) Among networks with central stations, there are only a small number of weak-localization areas for the concave 4 stations network, while there are no weak-localization area for the networks composed of more stations.

For the convenience of comparison, the AOA localization of the two stations was plotted through simulation, as shown in Fig.9. The weak-localization area of the TDOA network of the three stations is shown in Fig.10.

Fig.9 Weak-localization area of AOA network with 2 stations

Fig. 10 Weak-localization area of TDOA network with 3 stations

7 Conclusion

In order to use the geodetic longitude and latitude coordinates of the monitoring stations to carry out RSSD location of the radio source, several geodetic longitude and latitude based RSSD localization models are proposed. Through modeling and analysis, the following conclusions are obtained:

(1) The spherical accurate method and spherical approximation method are reasonable to replace the spherical string with the great circle distance between two points of the sphere, and the error is very small. The spherical projection distance calculation method is a comprehensive application of the spherical projection methods and Pythagorean theorem.

(2) There are a lot of weak areas for the localization networks with 4, 5 and 6 stations, which are lack of a central station. There are only a small number of weak-localization areas for the concave 4 stations network. There are no weak-localization area for the networks composed of more stations.

(3) When the measurement error and additional loss of radio wave propagation are not taken into account, the errors of the spherical accurate calculation model, the spherical approximate calculation model and the spherical angle projection model are very small, and they are universal in all latitudes of the world. The spherical

approximation model has the shortest localization time. The errors of spherical equidistance projection and spherical equal-area projection are large, the latter is greater than the former, and both increase with the increase of latitude, and can only be used in low latitude area. The localization error of the spherical equiangular projection model is very small, because the abscissa and ordinate after the projection are magnified in equal proportion, so the ratio of RSSD localization parameters to distance has no obvious change.

The RSSD localization model based on spherical approximate calculation is the best model because of the best localization time efficiency and the smallest localization error. The cost of RSSD localization network is lower than that of AOA localization network, and there is neither being sensitive to multipath effect like AOA localization, nor being difficult to locate narrowband signals like TDOA localization. Therefore, it is worth popularizing and applying.

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