Event-triggered Synchronization of Uncertain Delayed Generalized RDNNs

Weiyuan Zhang (✉ ahzwy@163.com)

Xianyang Normal University

Junmin Li

Xidian University

Keyi Xing

Xi'an Jiaotong University

Rui Zhang

Xi'an Jiaotong University

Xinyu Zhang

Xidian University

Research Article

Keywords: Event-triggered communication scheme, Robust generalized reaction-diffusion neural networks, Synchronization

Posted Date: July 21st, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-711289/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Event-triggered synchronization of uncertain delayed Generalized RDNNs

Weiyan Zhang ^{a,*}, Junmin Li ^b, Keyi Xing ^c, Rui Zhang ^{c,d}, Xinyu Zhang ^b

a) *Institute of Nonlinear Science, Xianyang Normal University, Xianyang 712000, P.R. China*

b) *School of Mathematics and Statistics, Xidian University, Xi'an 710071, P.R. China*

c) *State Key Laboratory for Manufacturing Systems Engineering, and Systems Engineering
Institute, Xi'an Jiaotong University, Xi'an 710049, P.R. China*

d) *State Key Laboratory for Mechanical Behavior of Materials, Xi'an Jiaotong University, Xi'an,
710049, P.R. China*

Abstract: This paper investigates the exponential synchronization analysis of master–slave chaotic uncertain delayed generalized reaction-diffusion neural networks (GRDNNs) with event-triggered control scheme. A delay GRDNNs system model for the analysis is constructed by investigating the effect of the network transmission delay. By constructing a novel Lyapunov–Krasovskii functional and using a delay system approach for designing event-triggered controllers and some inequality techniques like Jensen's inequality, Wirtinger's inequality and Halanay's inequality, the criteria are obtained for the event-triggered synchronization analysis and control synthesis of delayed GRDNNs. The synchronization criteria are formulated in terms of linear matrix inequalities. Finally, we conclude that the slave systems synchronize with the master systems. Two examples show the proposed theoretical results are feasible and effective.

Keywords: Event-triggered communication scheme, Robust generalized reaction-diffusion neural networks, Synchronization

1. Introduction

Neural networks (NNs) have been rather hot topic and been extensively studied in brain science and engineering fields due to their potential applications such as mammalian brains, solving certain optimization problems, fault diagnosis, etc. [1-4]. In general, NNs are categorized as either discrete-type [5] or continuous type [3,4]. The prominence of continuous-time NNs with time varying delays has been emphasized in many references. It is necessary to investigate the models of delayed NNs. As pointed in [6-9], in order to study the dynamical evolution law of NNs, two basic mathematical models are commonly adopted such as local field NN models[10] or static NN models[6] by the application of local field states or neural states of neurons. As said in [11], the above network models possibly relocated equally from one to another with some suppositions, but these suppositions do not always hold for a large number of applications. Hence, static network models and local field network models are never identical. Recently, some researchers have studied new unified NNs called generalized NNs (GNNs) to combine these two NN models and provided some criteria with a unified frame suitable for both local field NNs and static NNs [7-9,12].

Time delays are frequently encountered in hardware installations of NNs and many biological systems, which are ubiquitous in nature because of the finite signal communication time, switching

speed of amplifiers and memory effects, and they may induce instability, poor performance and oscillation of systems [1-4]. Thus, time delays always appear in domains ranging from population dynamics and network control to biological networks. NNs with time-varying delays are usually found and examined when ones analyze the dynamic behavior of systems [1-3,13-15]. In the previous studies on NNs, the time delay introduced is mainly time-varying delay signals $\tau(t)$ (given as $\tau(t) \leq \tau$), which belong to an upper bound interval $0 \leq \tau(t) \leq \tau$ [16-18]. In general, the lower bound value of $\tau(t)$ is not needed to be zero, thus, $\tau(t)$ has been stretched to the interval with lower and upper bounds, $0 \leq \tau_1 \leq \tau(t) \leq \tau_2$ [8,9,14,15]. In [14], An improved Lyapunov–Krasovskii functional is proposed by considering more important information about delay states, integral terms and the activation function. new less conservative delay-dependent stability conditions for generalized neural networks with interval time-varying delays are established by constructing a suitable Lyapunov–Krasovskii functional and applying the linear matrix inequality formulation. The authors in [12] investigated the synchronization for a class of GNNs based on Lyapunov stability theory and obtained both delay-derivative-dependent and delay-range-dependent conditions in terms of linear matrix inequalities. In addition, parameter uncertainties are unavoidable when modeling and implementing real NNs owing to measure errors, the parameter fluctuation and external disturbance, etc. We all know that the stability or synchronization of a well-designed system may often be destroyed due to its unavoidable uncertainty. Parametric uncertainties will also be introduced into the models of NNs [13,19-20]. If the uncertainty of a system is only due to the deviations and perturbations of its parameters, and if these deviations and perturbations are all bounded, then the system is called an interval system[20].

In 1665, Huygens found a fundamental natural phenomenon in the universe, that is, two weakly coupled pendulum clocks with hanging at the same beam synchronous in phase [21]. Chaos synchronization of two coupled NNs is an important phenomenon, which have drawn considerable attention because of wide practical applications in some fields with parallel recognition, secure communication, and so on. For example, a lot of NNs rely on a synchronous behavior for a proper functioning including pattern recognition and information transmission [1, 2]. NNs are easy to physically implement and hence have promising applications especially in secure communications based on synchronization [3, 4]. In the past years, NNs synchronization problems are hot topic and have been successfully used to many areas [4,22-27]. In fact, some natural systems can synchronize by themselves, but others cannot attain synchronization by themselves. In this case, some controllers need to be designed and are used to compel the systems to synchronize. In general, master NNs and slave NNs start to adjust from different initial state via the proper control strategy. Some control methods have been applied to obtain synchronization of NNs, such as pinning control [2], decentralized event-triggered control [4], intermittent control [23], sampled-data control [25], adaptive learning control [26], impulsive control [27, 28], and so on. In [25], the exponential synchronization for a class of hybrid coupled delayed RDNNs was considered under intermittent control with spacial sampled-data. In [28], the authors studied the synchronization analysis of coupled delayed RDNNs via a novel pinning impulsive controller.

In addition, networked control system has many remarkable advantages besides low cost, easy

diagnosis, long distance control, and so on. In [29], the authors have proposed a novel event-triggering scheme, then many papers focused on event-triggered control approach of many systems [4, 30-37]. In [34], event-triggered synchronization control was studied for switched delayed NNs with communication delays. In [37], event-triggered synchronization control was investigated for delayed NNs with quantization and actuator saturation. As we all known, maximizing the time intervals is an effective strategy to decrease the communication. In order to reduce the workload of communication network, the event triggered control approach was applied to the network-based synchronization system to decrease the amount of data packets sent by the sensor, and thereby saved the network resources [29-35,38]. However, there are few results on event-triggered control synchronization and analysis of uncertain GRDNNs. This paper will develop this control strategy to uncertain GRDNNs, and think about communication time delays between sensor and controller nodes for a class of distributed parameter networks system. At the same time, to decrease the cost, the GRDNNs may allocate a tiny embedded chip, which is often including few energy resources and limited computing capabilities. These limitations encourage the development of event-triggered control approach in the digital form [33-35].

Although research on synchronization has attracted so much attention, few of that had been devoted to the synchronization properties of the spatially and temporally GRDNNs. To the best of our knowledge, the synchronization of uncertain GRDNNs has not yet been considered. Therefore, it is important and interesting to study the synchronization of uncertain GRDNNs. In most existing works, it is assumed that the node state is only dependent on the time. Therefore, it is essential to investigate the relationship synchronization for the uncertain GRDNNs. The purpose of this study is to establish synchronization conditions of uncertain RGDNNs by applying the Lyapunov functional theory and event-triggered communication scheme. In event-triggered control, the measured error acts a key role during the event-triggered controller design. An event will be triggered to update the event-triggered controller when its magnitude reaches the prescribed value. This paper will develop this control strategy to uncertain delayed GRDNNs, and consider the communication delays between sensor and controller nodes. We extend uncertain delay GRDNNs model with communication delays and establish criteria for the synchronization by utilizing some inequality techniques. Also, we propose an even-triggered control method to synchronize two coupled uncertain GRDNNs system.

The paper is organized as follows. Section 2 provides some mathematical preliminaries and formulates the model of uncertainty delayed GRDNNs, some necessary definitions, lemmas and hypotheses are presented. In Section 3, some criteria for the synchronization sufficient of such coupled uncertain delayed GRDNNs are derived. In Section 4, numerical example is given to show the validity of the obtained results. The conclusions of this paper are presented in Section 5.

Notation: The notation A^T means the transpose of A . For symmetric matrices A and B , the notation $A > B$ means that the $A - B$ is positive definite. For a matrix C and symmetric

matrices A and B , $\begin{pmatrix} A & C \\ * & B \end{pmatrix}$ means symmetric matrix, in which the notation $*$ denotes the

entries implied by symmetry. Let I_N be an N -dimensional identity matrix and R^n denotes the

n -dimensional Euclidean space, and $R^{n \times m}$ is the set of $n \times m$ real matrices. $\Omega = [0, l]$ is a

compact set with smooth boundary $\partial\Omega$ and $mes\Omega > 0$, where $l > 0$ is a constant; $L^2(\Omega)$ is the space of real functions on Ω which are L^2 for the Lebesgue measure. It is a Banach space for the norm

$$\|u(t, x)\|_2 = \sqrt{\sum_{i=1}^n \|u_i(t, x)\|_2^2},$$

where $u(t, x) = (u_1(t, x), \dots, u_n(t, x))^T$ and $\|u_i(t, x)\|_2 = \left(\int_{\Omega} |u_i(t, x)|^2 dx\right)^{1/2}$. In addition, we define

$$\|u(t, x)\|_{\tau} = \sup_{-\tau \leq s \leq 0} \|u(t + s, x)\|_2.$$

2. Model description and preliminaries

Consider a class of robust delayed GRDNNs the master system as follows:

$$\begin{aligned} \frac{\partial u(t, x)}{\partial t} &= \frac{\partial}{\partial x} \left(D \frac{\partial u(t, x)}{\partial x} \right) - (A + \Delta A)u(t, x) + (B + \Delta B)g(Wu(t, x)) \\ &+ (C + \Delta C)g(Wu(t - \tau(t), x)) + J, t \geq 0, x \in \Omega, \\ u(t, x) &= 0, (t, x) \in (-\tau, +\infty) \times \partial\Omega, u(s, x) = \varphi(s, x), (s, x) \in (-\tau, 0] \times \Omega, \\ y(t, x) &= Mu(t, x) \end{aligned} \quad (1)$$

where $x = (x_1, x_2, \dots, x_m)^T \in \Omega$, $u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T \in R^n$ denotes the state vector; $D = \text{diag}(D_1, \dots, D_n)$, $D_i > 0$ represents the transmission diffusion coefficient,

$A = \text{diag}(a_1, \dots, a_n)$ is a known positive diagonal matrix with $a_i > 0$, $M = \text{diag}(m_1, \dots, m_n)$

is a known diagonal matrix, B, C and $W = (w_{ij})_{n \times n}$ represents the appropriately dimensioned known connection weight matrices between neurons;

$g(Wu(\cdot, x)) = (g_1(W_1 u(\cdot, x)), \dots, g_n(W_n u(\cdot, x)))^T$ denotes the activation vector function, W_j

represents the j th row vector of the matrix W and $J = (J_1, J_2, \dots, J_n)^T$ denotes a constant external input vector. $\tau(t)$ denotes the transmission time-varying delay, which satisfy

$0 \leq \tau(t) \leq \tau$, $\tau(t) \leq \mu$, where μ and τ are constants, $i=1, 2, \dots, n$. $\Delta A, \Delta B, \Delta C$ represent time-varying parameter uncertainties, which are supposed to be of the form

$$(\Delta A, \Delta B, \Delta C) = XF(t)(v_1, v_2, v_3), \quad (2a)$$

where X, v_1, v_2 , and v_3 are known real constant matrices, $F(t)$ is an unknown time-varying

matrix that satisfies

$$F^T(t)F(t) \leq I, \quad (2b)$$

where I is the identity matrix with appropriate dimension.

In this paper, we assume the following conditions hold.

(A1) For all $\varpi_1 \neq \varpi_2 \in R$, the neuron activation function $g_j(\cdot)$ satisfies

$$L_j^- \leq \frac{g_j(\varpi_1) - g_j(\varpi_2)}{\varpi_1 - \varpi_2} \leq L_j^+, j=1, 2, \dots, n,$$

with L_j^- and L_j^+ being constant scalars, $\bar{L}_1 = \text{diag}(L_1^-, \dots, L_n^-)$ and $\bar{L}_2 = \text{diag}(L_1^+, \dots, L_n^+)$.

Considering the slave system` as

$$\begin{aligned} \frac{\partial \mathcal{W}(t, x)}{\partial t} &= \frac{\partial}{\partial x} \left(D \frac{\partial \mathcal{W}(t, x)}{\partial x} \right) - (A + \Delta A) \mathcal{W}(t, x) + (B + \Delta B) g(W \mathcal{W}(t, x)) \\ &+ (C + \Delta C) g(W \mathcal{W}(t - \tau(t), x)) + J + U(t, x), t \geq 0, x \in \Omega, \\ \mathcal{W}(t, x) &= 0, (t, x) \in (-\tau, +\infty) \times \partial\Omega, \mathcal{W}(s, x) = \phi(s, x), (s, x) \in (-\tau, 0] \times \Omega, \\ \mathcal{W}(t, x) &= M \mathcal{W}(t, x) \end{aligned} \quad (3)$$

in which $U(t, x)$ is the controllers.

For our synchronization scheme, let us define the synchronization error and output error by, respectively:

$$e_i(t, x) = \mathcal{W}_i(t, x) - u_i(t, x), \bar{z}_i(t, x) = \mathcal{Y}_i(t, x) - y_i(t, x), i = 1, 2, \dots, n.$$

Then, we have the synchronization error as follows:

$$\begin{aligned} \frac{\partial e(t, x)}{\partial t} &= \frac{\partial}{\partial x} \left(D \frac{\partial e(t, x)}{\partial x} \right) - \bar{A} e(t, x) + \bar{B} g(W e(t, x)) \\ &+ \bar{C} g(W e(t - \tau(t), x)) + U(t, x), \\ e(t, x) &= 0, (t, x) \in (-\tau, +\infty) \times \partial\Omega, e(s, x) = \bar{\phi}(s, x), (s, x) \in (-\tau, 0] \times \Omega, \\ \bar{z}(t, x) &= M e(t, x) \\ D &= 0, \bar{B} = 0, \bar{C} = 0, W = I \end{aligned} \quad (4)$$

where $e(\cdot, x) = (e_1(\cdot, x), \dots, e_n(\cdot, x))^T$, $\bar{A} = A + \Delta A$, $\bar{B} = B + \Delta B$, $\bar{C} = C + \Delta C$,

$$\begin{aligned} f(W e(\cdot, x)) &= (f_1(W_1 e(\cdot, x)), \dots, f_n(W_n e(\cdot, x)))^T, \bar{\phi}(s, x) = \phi(s, x) - \phi(s, x), \\ f_j(W_j e(\cdot, x)) &= g_j(W_j \mathcal{W}(\cdot, x)) - g(W_j u(\cdot, x)). \end{aligned}$$

Thus, $f_j(W_j e(\cdot, x))$ satisfy the following condition:

$$L_j^- \leq \frac{f_j(W_j e(\cdot, x))}{W_j e(\cdot, x)} \leq L_j^+, f_j(0) = 0, e(\cdot, t) \neq 0, j=1, 2, \dots, n, \quad (5)$$

The following Lemmas are used to obtain the criteria of coupled master-slave system synchronization.

Lemma 1 (*Extended Wirtinger's Inequality, Hardy, Littlewood and Polya [39]*) Let $z \in H^1(\Omega), \Omega = (0, l)$ be a scalar function with $z(0) = 0$ or $z(l) = 0$ Then

$$\int_{\Omega} z^2(x) dx \leq \frac{4l^2}{\pi^2} \int_{\Omega} \left[\frac{dz}{dx} \right]^2 dx. \quad (6)$$

Moreover, if $z(0) = z(l) = 0$, then

$$\int_{\Omega} z^2(x) dx \leq \frac{l^2}{\pi^2} \int_{\Omega} \left[\frac{dz}{dx} \right]^2 dx. \quad (7)$$

Lemma 2 (*Gu, Kharitonov and Chen [40]*) Let scalars γ_1 and γ_2 satisfy $\gamma_2 > \gamma_1$, a vector function $x : [\gamma_1, \gamma_2] \rightarrow R^n$ such that the integrations concerned are defined, for any matrix $M > 0$, then

$$\left(\int_{\gamma_1}^{\gamma_2} x(s) ds \right)^T M \left(\int_{\gamma_1}^{\gamma_2} x(s) ds \right) \leq (\gamma_2 - \gamma_1) \int_{\gamma_1}^{\gamma_2} x(s)^T M x(s) ds. \quad (8)$$

Lemma 3 (*Halanay's Inequality, Halanay [41]*). Let $V : [t_0 - h, \infty) \rightarrow [0, \infty)$ be an absolutely continuous function and $0 < \delta_1 < 2\delta$ satisfying

$$\dot{V}(t) \leq -2\delta V(t) + \delta_1 \sup_{s \in [-h, 0]} V(t+s), t \geq t_0.$$

Then
$$V(t) \leq e^{-2\alpha(t-t_0)} \sup_{s \in [-h, 0]} V(t_0+s), t \geq t_0, \quad (9)$$

where δ_1 and δ are positive constants, $\alpha > 0$ is a unique positive solution of

$$\alpha = \delta - \frac{\delta_1 e^{2\alpha h}}{2}.$$

Lemma 4^[42] Let $X, J(t)$ and η be real matrices of appropriate dimensions, and $J(t)$ satisfying $J(t)^T J(t) \leq I$. Then, for any constant $\varepsilon > 0$, the following inequality holds:

$$XJ(t)\eta + \eta^T J(t)^T X^T \leq \varepsilon X^T X + \varepsilon^{-1} \eta^T \eta$$

Remark 1. It is worth pointing out that the structure of the parameter uncertainties including (2a) and (2b) has been widely utilized in many references [4,13,19,20] to deal with the stability or synchronization problems for uncertain NNs and other systems. The motivation for considering system (1) with uncertainties $\Delta A, \Delta B$ and ΔC , which stem from the fact that, in practice, it is

almost impossible to obtain exact mathematical models of dynamic systems because of the complexity for such systems. In (2a) and (2b), $\Delta A, \Delta B$ and ΔC represent time-varying parameter uncertainties with appropriate dimensions, $F(t)$ is the unknown and uncertain matrix with Lebesgue measurable elements bounded satisfies $F^T(t)F(t) \leq I$, which is an important condition in solving the robust stability or synchronization problems for uncertain systems. The Lemma 4 plays an important role in the reported criteria, (2b) is a key and necessary condition to ensure Lemma 4 hold. Furthermore, it is reasonable and unavoidable that the model of the controlled system with uncertainties owing to the existence of external disturbance, modelling error, and parameter fluctuation during the implementation. The parameter uncertainties in the system considered are said to be admissible if (2a) and (2b) hold. Therefore, it is of theoretical and practical importance to investigate robust stability, synchronization, etc. of the delayed NNs[4,13,19,20].

Consider system (1) with the Neumann boundary conditions. Set the points $0 = x_0 < x_1 < \dots < x_N = l$ divide $[0, l]$ into N sampling intervals. We suppose that N

sensors are placed in the middle $\bar{x}_j = \frac{x_j + x_{j+1}}{2} (j = 0, \dots, N-1)$ of these intervals.

$t_0, t_1, \dots, t_k, \dots$ are sampling instants with $t_0 < t_1 < \dots < t_k < \dots + \infty$, where $t_0 = 0$ is the initial time and $\lim_{k \rightarrow \infty} t_k = \infty$. The sampling intervals on time and space may be variable and satisfies

$$0 \leq t_{k+1} - t_k \leq h, x_{j+1} - x_j \leq \Delta. \quad (10)$$

Here, the measured output of system (4) is sampled and then transmitted to the controller of the slave system as follow:

$$\bar{z}(t, x) = y(t_k, \bar{x}_j) = Me(t_k, \bar{x}_j), t \in [t_k, t_{k+1}), \quad (11)$$

$$\bar{x}_j = \frac{x_j + x_{j+1}}{2}, x \in [x_j, x_{j+1}), j = 0, \dots, N-1,$$

in which $e(t_k, \bar{x}_j)$ is utilized to decide if the control system needs to send $\bar{z}(t_{k+j}, \bar{x}_j)$ to the controller by the following inequality:

$$\begin{aligned} & \left[\bar{z}(t_{k+\bar{j}}, \bar{x}_j) - \bar{z}(t_k, \bar{x}_j) \right]^T \Lambda_1 \left[\bar{z}(t_{k+\bar{j}}, \bar{x}_j) - \bar{z}(t_k, \bar{x}_j) \right] \\ & < \delta \bar{z}(t_{k+\bar{j}}, \bar{x}_j)^T \Lambda_2 \bar{z}(t_{k+\bar{j}}, \bar{x}_j) \end{aligned} \quad (12)$$

in which Λ_1, Λ_2 are positive weighting matrices, $\bar{j} = 1, 2, \dots, \delta \in [0, 1)$. $\bar{z}(t_{k+j}, \bar{x}_j)$ will not be sent until it can not satisfied condition (12).

Suppose the time-varying communication delays $\tau_k \in (0, \ell], \ell > 0, k = 1, 2, \dots$, from the sampling termination of the error system to the slave system in practical implementation can be detected by the difference between the instants at sampled states $e(t_k, \bar{x}_j)$ and the control states $e(t_k + \tau_k, \bar{x}_j)$. Thus, we can get

$$\bar{z}(t, x) = Me(t_k, \bar{x}_j), t \in [t_k + \tau_k, t_{k+j} + \tau_{k+j}), x \in [x_j, x_{j+1}), \quad (13)$$

Hence, the novel control law is designed as

$$U(t, x) = KMe(t_k, \bar{x}_j), t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), x \in [x_j, x_{j+1}), \quad (14)$$

with the gain matrix $K = \text{diag}(k_1, k_2, \dots, k_n)$ to be determined, $\bar{x}_j = \frac{x_j + x_{j+1}}{2}, j = 0, \dots, N-1$.

By (12)-(14), we obtain

$$\begin{aligned} & \left[e(t_{k+j}, \bar{x}_j) - e(t_k, \bar{x}_j) \right]^T M^T \Lambda_1 M \left[e(t_{k+j}, \bar{x}_j) - e(t_k, \bar{x}_j) \right] \\ & < \delta e(t_{k+j}, \bar{x}_j)^T M^T \Lambda_2 Me(t_{k+j}, \bar{x}_j) \end{aligned} \quad (15)$$

Next, we consider two cases:

Case i): if $t_{k+1} + \tau_{k+1} \leq t_k + h + \ell \leq \max\{\tau_k\}$, where h is a sampling period, defining

$$\bar{\tau}(t) = t - t_k, t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}) \quad (16)$$

Clearly, we have $\tau_k \leq \bar{\tau}(t) \leq (t_{k+1} - t_k) + \tau_{k+1} \leq h + \ell$ (17)

Case ii): if $t_k + h + \ell < t_{k+1} + h + \tau_{k+1}, \ell \leq \max\{\tau_k\}$, where h is a sampling period, considering the following intervals

$$[t_k + \tau_k, t_k + h + \ell], [t_k + ih + \ell, t_k + ih + h + \ell] \quad (18)$$

From $\tau_k \leq \ell$, we can find that there exists N_0 such that

$$t_k + N_0 h + \ell < t_{k+1} + \tau_{k+1} \leq t_k + N_0 h + h + \ell \quad (19)$$

moreover, $x(t_k, \bar{x}_j)$ and $x(t_k + ih, \bar{x}_j)$ with $i = 1, 2, \dots, N_0$ satisfy (15).

In addition, $[t_k + \tau_k, t_{k+1} + \tau_{k+1}) = \Omega_1 \cup \Omega_2 \cup \Omega_3$, in which $\Omega_1 = [t_k + \tau_k, t_k + h + \ell]$,

$$\Omega_2 = \bigcup_{i=1}^{N_0-1} \{\Omega_2^i\} = \bigcup_{i=1}^{N_0-1} [t_k + ih + \ell, t_k + ih + h + \ell], \Omega_3 = [t_k + N_0 h + \ell, t_{k+1} + \tau_{k+1}). \quad (20)$$

$$\text{Let } \theta(t) = \begin{cases} t - t_k, & t \in \Omega_1 \\ t - t_k - ih, & t \in \Omega_2 \\ t - t_k - N_0 h, & t \in \Omega_3 \end{cases}, \text{ thus we can get } \begin{cases} \tau_k \leq \theta(t) \leq h + \ell, & t \in \Omega_1 \\ \tau_k \leq \theta(t) \leq h + \ell, & t \in \Omega_2 \\ \tau_k \leq \theta(t) \leq h + \ell, & t \in \Omega_3 \end{cases} \quad (21)$$

$$\text{and } 0 \leq \tau_k \leq \theta_M, \text{ for all } t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), \theta_M = h + \ell, \text{ i.e. } 0 \leq \theta(t) \leq \theta_M. \quad (22)$$

Therefore, for case i), defining $\bar{e}(t, \bar{x}_j) = 0, t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$. For case ii), defining

$$\bar{e}(t, \bar{x}_j) = \begin{cases} 0, & \text{for } t \in \Omega_1 \\ e(t_k, \bar{x}_j) - e(t_k + ih, \bar{x}_j), & t \in \Omega_2 \\ e(t_k, \bar{x}_j) - e(t_k + N_0 h, \bar{x}_j), & t \in \Omega_3 \end{cases} \quad (23)$$

From definition of $\bar{e}(t, \bar{x}_j)$ and function $\theta(t)$, we can have control condition (12) as

$$\bar{e}(t, \bar{x}_j)^T M^T \Lambda_1 M \bar{e}(t, \bar{x}_j) < \delta e(t - \theta(t), \bar{x}_j)^T M^T \Lambda_2 M e(t - \theta(t), \bar{x}_j) \quad (24)$$

According to the sampled error output $\bar{z}(t, x)$, ones can obtain

$$\begin{aligned} \frac{\partial e(t, x)}{\partial t} &= \frac{\partial}{\partial x} \left(D \frac{\partial e(t, x)}{\partial x} \right) - \bar{A} e(t, x) + \bar{B} f(We(t, x)) \\ &+ \bar{C} f(We(t - \tau(t), x)) + K M e(t - \theta(t), \bar{x}_j) - K M \bar{e}(t, \bar{x}_j) \end{aligned} \quad (25)$$

We note that $e(t - \theta(t), \bar{x}_j) = e(t - \theta(t), x) - \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds$,

$$\bar{e}(t, \bar{x}_j) = \bar{e}(t, x) - \int_{\bar{x}_j}^x \bar{e}_s(t, s) ds.$$

3. Main results

In this section, our event triggering scheme can ensure the stability of the closed-loop system (25). The following novel criteria can guarantee synchronous of the master system (1) and slave system (3) with $\Delta A = \Delta B = \Delta C = 0$ via the event trigger scheme (12).

Theorem 1. Assume (A1) hold. Given scalars $\tau > 0, 0 < \mu < 1, \delta \geq 0, \theta_M \geq 0, 0 \leq \alpha < 1$, if there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \alpha_1$, matrices $R_7 > 0, R_8 > 0, R_9 > 0, G_0, R_j, j = 1, \dots, 6$, and diagonal matrix K , positive diagonal matrices $\Lambda_1 > 0, \Lambda_2 > 0, \Gamma > 0, \bar{\Gamma} > 0$,

$P = \text{diag}(p_1, p_2, \dots, p_n) > 0$ such that the following matrix inequalities hold:

$$\begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} > 0, \quad (26)$$

$$\begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} > 0, \quad (27)$$

$$-\alpha_1 PD + \left(\frac{\Delta}{\varepsilon_1 \pi} + \frac{\Delta}{\varepsilon_2 \pi} \right) PKM + \delta \left(\frac{\Delta}{\varepsilon_3 \pi} + \frac{\Delta^2}{\pi^2} \right) M^T \Lambda_2 M < 0 \quad (28)$$

$$\Phi = (\Phi_{11 \times 11}) < 0, \quad (29)$$

where $\Phi_{11} = 2\alpha P - 2PA + R_1 + R_8 + R_4 - e^{-2\alpha\tau} R_7 - e^{-2\alpha\theta_M} R_9 - L_1 W W^T \Gamma + \varepsilon_1 \frac{\Delta}{\pi} PKM$,
 $\Phi_{12} = e^{-2\alpha\theta_M} R_9 + PKM$, $\Phi_{14} = e^{-2\alpha\tau} R_7 - e^{-2\alpha\tau} G_0$, $\Phi_{15} = e^{-2\alpha\tau} G_0$, $\Phi_{16} = PB + R_2 + R_5 + L_2 W \Gamma$,
 $\Phi_{17} = PC$, $\Phi_{19} = -P - A^T P^T$, $\Phi_{1,10} = -2PKM$, $\Phi_{22} = -e^{-2\alpha\theta_M} R_9 + \delta \left(1 + \varepsilon_3 \frac{\Delta}{\pi} \right) M^T \Lambda_2 M$,
 $\Phi_{23} = -e^{-2\alpha\theta_M} R_9$, $\Phi_{29} = M^T K^T P^T$, $\Phi_{33} = -e^{-2\alpha\theta_M} R_8 - e^{-2\alpha\theta_M} R_9$, $\Phi_{44} = -(1 - \mu) e^{-2\alpha\tau} R_1$
 $-2e^{-2\alpha\tau} R_7 + e^{-2\alpha\tau} G_0 - L_1 W W^T \bar{\Gamma}$, $\Phi_{45} = e^{-2\alpha\tau} R_7 - e^{-2\alpha\tau} G_0$, $\Phi_{47} = -e^{-2\alpha\tau} (1 - \mu) R_2 + L_2 W \bar{\Gamma}$,
 $\Phi_{55} = -e^{-2\alpha\tau} R_4 - e^{-2\alpha\tau} R_7$, $\Phi_{5,11} = -e^{-2\alpha\tau} R_5$, $\Phi_{66} = R_3 + R_6 - \Gamma$, $\Phi_{69} = B^T P^T$,
 $\Phi_{77} = -(1 - \mu) e^{-2\alpha\tau} R_3 - \bar{\Gamma}$, $\Phi_{79} = C^T P^T$, $\Phi_{88} = (-2 + 2\alpha) PD$, $\Phi_{99} = \tau^2 R_7 + \varepsilon_2 \frac{\Delta}{\pi} PKM$
 $+ \theta_M^2 R_9 - 2P$, $\Phi_{9,10} = -PKM$, $\Phi_{10,10} = -M^T \Lambda_1 M$, $\Phi_{11,11} = -e^{-2\alpha\tau} R_6$,

the other terms are zero, then the error system (4) with $\Delta A = \Delta B = \Delta C = 0$ under the event trigger (14) is exponentially stable, which can ensure the exponential synchronization of the master system (1) and the slave system (3). Moreover, the considered controller gain matrix in (14) can be given by $K = P^{-1}Y$.

Proof Define the Lyapunov–Krasovskii functional as

$$V(t) = \sum_{i=1}^4 V_i(t), \quad (30)$$

where $V_1(t) = \int_{\Omega} e(t, x)^T P e(t, x) dx$,

$$V_2(t) = \int_{\Omega} \int_{t-\tau}^t \exp[2\alpha(s-t)] \begin{pmatrix} e(s, x) \\ f(We(s, x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(s, x) \\ f(We(s, x)) \end{pmatrix} ds dx$$

$$+ \int_{\Omega} \int_{t-\theta_M}^t \exp[2\alpha(s-t)] e(s, x)^T R_8 e(s, x) ds dx$$

$$+ \int_{\Omega} \int_{t-\tau}^t \exp[2\alpha(s-t)] \begin{pmatrix} e(s, x) \\ f(We(s, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(s, x) \\ f(We(s, x)) \end{pmatrix} ds dx,$$

$$V_3(t) = \int_{\Omega} \frac{\partial e(t, x)^T}{\partial x} \bar{P} \frac{\partial e(t, x)}{\partial x} dx, \quad \text{where } \bar{P} = PD,$$

$$V_4(t) = \tau \int_{\Omega} \int_{-\tau}^0 \int_{t+\varsigma}^t \exp[2\alpha(s-t)] \frac{\partial e(s, x)^T}{\partial s} R_7 \frac{\partial e(s, x)}{\partial s} ds d\varsigma dx$$

$$+ \theta_M \int_{\Omega} \int_{-\theta_M}^0 \int_{t+\varsigma}^t \exp[2\alpha(s-t)] \frac{\partial e(s, x)^T}{\partial s} R_9 \frac{\partial e(s, x)}{\partial s} ds d\varsigma dx,$$

We can get the following equality by calculating the derivative of $V(t)$,

$$\dot{V}(t) = \sum_{i=1}^4 \dot{V}_i(t)$$

The derivative of $V_1(t)$, $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, along the trajectories of (22), we can find

$$\dot{V}_1(t) = 2 \int_{\Omega} e(t, x)^T P \frac{\partial e(t, x)}{\partial t} dx, \quad (31)$$

The derivatives of $V_2(t)$, $V_3(t)$ and $V_4(t)$, $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, are as follows:

$$\begin{aligned} \dot{V}_2(t) + 2\alpha V_2(t) &\leq \int_{\Omega} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix} dx \\ &- (1 - \mu) \exp(-2\alpha\tau) \int_{\Omega} \begin{pmatrix} e(t - \tau(t), x) \\ f(We(t - \tau(t), x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(t - \tau(t), x) \\ f(We(t - \tau(t), x)) \end{pmatrix} dx \\ &+ \int_{\Omega} e(t, x)^T R_8 e(t, x) dx - \exp(-2\alpha\theta_M) \int_{\Omega} e(t - \theta_M, x)^T R_8 e(t - \theta_M, x) dx \end{aligned} \quad (32)$$

$$\begin{aligned} &+ \int_{\Omega} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix} dx \\ &- \exp(-2\alpha\tau) \int_{\Omega} \begin{pmatrix} e(t - \tau, x) \\ f(We(t - \tau, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(t - \tau, x) \\ f(We(t - \tau, x)) \end{pmatrix} dx, \\ \dot{V}_3(t) &= 2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial x} \bar{P} \frac{\partial^2 e(t, x)}{\partial x \partial t} dx \end{aligned} \quad (33)$$

$$\begin{aligned} &\dot{V}_4(t) + 2\alpha V_4(t) \\ &\leq \tau^2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} R_7 \frac{\partial e(t, x)}{\partial t} dx - \tau \exp(-2\alpha\tau) \int_{\Omega} \int_{t-\tau}^t \frac{\partial e(s, x)^T}{\partial s} R_7 \frac{\partial e(s, x)}{\partial s} ds dx \\ &+ \theta_M^2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} R_9 \frac{\partial e(t, x)}{\partial t} dx - \theta_M \exp(-2\alpha\theta_M) \int_{\Omega} \int_{t-\theta_M}^t \frac{\partial e(s, x)^T}{\partial s} R_9 \frac{\partial e(s, x)}{\partial s} ds dx \end{aligned} \quad (34)$$

By using Lemma 2 combined with reciprocally convex approach [43], we obtain

$$\begin{aligned}
& -\tau \exp(-2\alpha\tau) \int_{t-\tau}^t \frac{\partial e(s,x)^T}{\partial s} R_7 \frac{\partial e(s,x)}{\partial s} ds \\
& = -\tau \exp(-2\alpha\tau) \left(\int_{t-\tau(t)}^t \frac{\partial e(s,x)^T}{\partial s} R_7 \frac{\partial e(s,x)}{\partial s} ds + \int_{t-\tau}^{t-\tau(t)} \frac{\partial e(s,x)^T}{\partial s} R_7 \frac{\partial e(s,x)}{\partial s} ds \right) \quad (35)
\end{aligned}$$

$$\begin{aligned}
& \leq -\exp(-2\alpha\tau) \begin{pmatrix} e(t,x) - e(t-\tau(t),x) \\ e(t-\tau(t),x) - e(t-\tau,x) \end{pmatrix}^T \begin{pmatrix} R_7 & G_0 \\ * & R_7 \end{pmatrix} \begin{pmatrix} e(t,x) - e(t-\tau(t),x) \\ e(t-\tau(t),x) - e(t-\tau,x) \end{pmatrix} \\
& -\theta_M \exp(-2\alpha\theta_M) \int_{\Omega} \int_{t-\theta_M}^t \frac{\partial e(s,x)^T}{\partial s} R_9 \frac{\partial e(s,x)}{\partial s} ds dx \\
& \leq \exp(-2\alpha\theta_M) \begin{pmatrix} e(t,x) \\ e(t-\theta(t),x) \\ e(t-\theta_M,x) \end{pmatrix}^T \begin{bmatrix} -R_9 & R_9 & 0 \\ * & -R_9 & R_9 \\ * & * & -R_9 \end{bmatrix} \begin{pmatrix} e(t,x) \\ e(t-\theta(t),x) \\ e(t-\theta_M,x) \end{pmatrix} \quad (36)
\end{aligned}$$

$$\begin{aligned}
& \mathbb{V}_4^{\mathbb{E}}(t) + 2\alpha V_4(t) \leq \tau^2 \int_{\Omega} \frac{\partial e(t,x)^T}{\partial t} R_7 \frac{\partial e(t,x)}{\partial t} dx \\
& -\exp(-2\alpha\tau) \begin{pmatrix} e(t,x) - e(t-\tau(t),x) \\ e(t-\tau(t),x) - e(t-\tau,x) \end{pmatrix}^T \begin{pmatrix} R_7 & G_0 \\ * & R_7 \end{pmatrix} \begin{pmatrix} e(t,x) - e(t-\tau(t),x) \\ e(t-\tau(t),x) - e(t-\tau,x) \end{pmatrix} \\
& +\theta_M^2 \int_{\Omega} \frac{\partial e(t,x)^T}{\partial t} R_9 \frac{\partial e(t,x)}{\partial t} dx \quad (37) \\
& +\exp(-2\alpha\theta_M) \begin{pmatrix} e(t,x) \\ e(t-\theta(t),x) \\ e(t-\theta_M,x) \end{pmatrix}^T \begin{bmatrix} -R_9 & R_9 & 0 \\ * & -R_9 & R_9 \\ * & * & -R_9 \end{bmatrix} \begin{pmatrix} e(t,x) \\ e(t-\theta(t),x) \\ e(t-\theta_M,x) \end{pmatrix}
\end{aligned}$$

Using the free weighting matrix method, we can get that

$$\begin{aligned}
& \int_{\Omega} \left[2e(t,x)^T P + 2 \frac{\partial e(t,x)^T}{\partial t} P \right] \left\{ -\frac{\partial e(t,x)}{\partial t} \right. \\
& + \left[\frac{\partial}{\partial x} \left(D \frac{\partial e(t,x)}{\partial x} \right) - Ae(t,x) + Bf(We(t,x)) + Cf(We(t-\tau(t),x)) \right. \\
& \left. \left. + KMe(t-\theta(t), \bar{x}_j) - KM\bar{e}(t, \bar{x}_j) \right] \right\} dx = 0 \quad (38)
\end{aligned}$$

with $e(t-\theta(t), \bar{x}_j) = e(t-\theta(t), x) - \int_{\bar{x}_j}^x e_s(t-\theta(t), s) ds$,

$\bar{e}(t, \bar{x}_j) = \bar{e}(t, x) - \int_{\bar{x}_j}^x \bar{e}_s(t, s) ds$.

Integrating by parts and substitution of the Dirichlete boundary conditions, one can lead to

$$\int_{\Omega} e(t,x)^T P \frac{\partial}{\partial x} \left(D \frac{\partial e(t,x)}{\partial x} \right) dx = - \int_{\Omega} \frac{\partial e(t,x)^T}{\partial x} P D \frac{\partial e(t,x)}{\partial x} dx \quad (39)$$

From Young's inequality for any scalar $\bar{\varepsilon}_1 > 0$, the following inequality (40) is true:

$$\begin{aligned}
& 2 \sum_{j=0}^{N-1} \int_{x_j}^{x_{j+1}} e(t, x)^T PKM \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right] dx \\
&= 2 \sum_{i=1}^n \sum_{j=0}^{N-1} \int_{x_j}^{x_{j+1}} p_i k_i m_i \left[e_i(t - \theta(t), x) - e_i(t - \theta(t), \bar{x}_j) \right] dx \\
&\leq \bar{\varepsilon}_1 \int_{\Omega} e(t, x)^T PKM e(t, x) dx \\
&+ \bar{\varepsilon}_1^{-1} \sum_{j=0}^{N-1} \int_{x_j}^{x_{j+1}} \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right]^T PKM \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right] dx
\end{aligned} \tag{40}$$

According to Lemma 1, we have the following inequality

$$\begin{aligned}
& \int_{x_j}^{x_{j+1}} \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right]^T PK \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right] dx \\
&= \int_{x_j}^{\bar{x}_j} \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right]^T PK \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right] dx \\
&+ \int_{\bar{x}_j}^{x_{j+1}} \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right]^T PK \left[e(t - \theta(t), x) - e(t - \theta(t), \bar{x}_j) \right] dx \\
&\leq \frac{\Delta^2}{\pi^2} \int_{x_j}^{x_{j+1}} \frac{\partial e(t - \theta(t), x)^T}{\partial x} PK \frac{\partial e(t - \theta(t), x)}{\partial x} dx
\end{aligned} \tag{41}$$

Choosing next $\bar{\varepsilon}_1 = \frac{\Delta}{\pi} \varepsilon_1$, we get from (40) and (41) that

$$\begin{aligned}
& \int_{\Omega} 2e(t, x)^T PKM \int_{\bar{x}_j}^x e_s(t - \theta(t), s)^T ds dx \\
&\leq \varepsilon_1 \frac{\Delta}{\pi} \int_{\Omega} e(t, x)^T PKM e(t, x) dx \\
&+ \frac{\Delta}{\varepsilon_1 \pi} \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} PKM \frac{\partial e(t - \theta(t), x)}{\partial x} dx
\end{aligned} \tag{42}$$

Thus, from (40)-(42), we have

$$\begin{aligned}
& \int_{\Omega} 2e(t, x)^T PKM e(t - \theta(t), \bar{x}_j) dx \\
&= \int_{\Omega} 2e(t, x)^T PKM \left(e(t - \theta(t), x) - \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds \right) dx \\
&= \int_{\Omega} \left[2e(t, x)^T PKM e(t - \theta(t), x) - 2e(t, x)^T PKM \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds \right] dx \\
&\leq \int_{\Omega} 2e(t, x)^T PKM e(t - \theta(t), x) dx + \varepsilon_1 \frac{\Delta}{\pi} \int_{\Omega} e(t, x)^T PKM e(t, x) dx \\
&+ \frac{\Delta}{\varepsilon_1 \pi} \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} PKM \frac{\partial e(t - \theta(t), x)}{\partial x} dx
\end{aligned} \tag{43}$$

Similarly to (42) and (43), by Young's inequality for any scalar $\varepsilon_2, \varepsilon_3 > 0$, we find inequality (44)

and equality (45) as follows:

$$\begin{aligned} \int_{\Omega} -2 \frac{\partial e(t, x)^T}{\partial t} PKM \int_{\bar{x}_j}^x e_s(t - \theta(t), s)^T ds dx &\leq \varepsilon_2 \frac{\Delta}{\pi} \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} PKM \frac{\partial e(t, x)}{\partial t} dx \\ &+ \frac{\Delta}{\varepsilon_2 \pi} \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} PKM \frac{\partial e(t - \theta(t), x)}{\partial x} dx \end{aligned} \quad (44)$$

$$\begin{aligned} &\int_{\Omega} 2 \frac{\partial e(t, x)^T}{\partial t} PKM e(t - \theta(t), \bar{x}_j) dx \\ &= \int_{\Omega} 2 \frac{\partial e(t, x)^T}{\partial t} PKM \left(e(t - \theta(t), x) - \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds \right) dx \\ &= \int_{\Omega} 2 \frac{\partial e(t, x)^T}{\partial t} PKM e(t - \theta(t), x) dx - \int_{\Omega} 2 \frac{\partial e(t, x)^T}{\partial t} PKM \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds dx \\ &= \int_{\Omega} 2 \frac{\partial e(t, x)^T}{\partial t} PKM e(t - \theta(t), x) dx + \varepsilon_2 \frac{\Delta}{\pi} \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} PKM \frac{\partial e(t, x)}{\partial t} dx \\ &+ \frac{\Delta}{\varepsilon_2 \pi} \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} PKM \frac{\partial e(t - \theta(t), x)}{\partial x} dx \end{aligned} \quad (45)$$

From (24), we can conclude

$$\begin{aligned} &\int_{\Omega} \bar{e}(t, \bar{x}_j)^T M^T \Lambda_1 M \bar{e}(t, \bar{x}_j) dx < \int_{\Omega} \delta e(t - \theta(t), \bar{x}_j)^T M^T \Lambda_2 M e(t - \theta(t), \bar{x}_j) dx \\ &= \int_{\Omega} \delta \left(e(t - \theta(t), x) - \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds \right)^T M^T \Lambda_2 M \left(e(t - \theta(t), x) - \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds \right) dx \\ &= \int_{\Omega} \delta e(t - \theta(t), x)^T M^T \Lambda_2 M e(t - \theta(t), x) - 2 \delta e(t - \theta(t), x)^T M^T \Lambda_2 M \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds dx \\ &+ \int_{\Omega} \delta \int_{\bar{x}_j}^x e_s(t - \theta(t), s)^T ds M^T \Lambda_2 M \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds dx \\ &\leq \int_{\Omega} e(t - \theta(t), x)^T \delta \left(1 + \varepsilon_3 \frac{\Delta}{\pi} \right) M^T \Lambda_2 M e(t - \theta(t), x) dx \\ &+ \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} \delta \left(\frac{\Delta}{\varepsilon_3 \pi} + \frac{\Delta^2}{\pi^2} \right) M^T \Lambda_2 M \frac{\partial e(t - \theta(t), x)}{\partial x} dx, \end{aligned} \quad (46)$$

Similar to inequality (39), we know

$$2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} P \frac{\partial}{\partial x} \left(D \frac{\partial u(t, x)}{\partial x} \right) dx = -2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial x} \bar{P} \frac{\partial^2 e(t, x)}{\partial x \partial t} dx \quad (47)$$

From assumption (5), we have

$$\left(f_j(W_j e(t, x)) - L_j^- W_j e(t, x) \right) \left(f_j(W_j e(t, x)) - L_j^+ W_j e(t, x) \right) \leq 0 \quad (48)$$

which is equivalent to

$$\chi(t, x)^T \begin{pmatrix} L_j L_j^+ W_j W_j^T I_i I_i^T & -\frac{L_j^- + L_j^+}{2} W_j I_i I_i^T \\ -\frac{L_j^- + L_j^+}{2} W_j I_i I_i^T & I_i I_i^T \end{pmatrix} \chi(t, x) \leq 0 \quad (49)$$

where $\chi(t, x) = \begin{pmatrix} e(t, x)^T & f(We(t, x))^T \end{pmatrix}^T$, I_i is the unit column vector having 1 element on its i -th row and zeros elsewhere. Thus for any appropriately dimensioned diagonal matrix $\Gamma > 0$, we have the following inequality holds [44]:

$$0 \leq \chi(t, x)^T \begin{pmatrix} -L_1 W W^T \Gamma & L_2 W \Gamma \\ * & -\Gamma \end{pmatrix} \chi(t, x) \quad (50)$$

where $L_1 = \text{diag}(L_1^- L_1^+, \dots, L_n^- L_n^+)$, $L_2 = \text{diag}\left(\frac{L_1^- + L_1^+}{2}, \dots, \frac{L_n^- + L_n^+}{2}\right)$.

Similarly, for any appropriately dimensioned diagonal matrix $\bar{\Gamma} > 0$, we have the following inequality:

$$0 \leq \chi(t - \tau(t), x)^T \begin{pmatrix} -L_1 W W^T \bar{\Gamma} & L_2 W \bar{\Gamma} \\ * & -\bar{\Gamma} \end{pmatrix} \chi(t - \tau(t), x) \quad (51)$$

Combining (31)-(38), we get

$$\begin{aligned} \mathbb{V}(t) + 2\alpha V(t) &\leq \int_{\Omega} 2e(t, x)^T P \frac{\partial e(t, x)}{\partial t} dx + 2\alpha \int_{\Omega} e(t, x)^T P e(t, x) dx \\ &+ \int_{\Omega} \left[2e(t, x)^T P + 2 \frac{\partial e(t, x)^T}{\partial t} P \right] \left\{ -\frac{\partial e(t, x)}{\partial t} \right. \\ &+ \left[\frac{\partial}{\partial x} \left(D \frac{\partial e(t, x)}{\partial x} \right) - A e(t, x) + B f(We(t, x)) + C f(We(t - \tau(t), x)) \right. \\ &+ K M e(t - \theta(t), \bar{x}_j) - K M \bar{e}(t, \bar{x}_j) \left. \right\} dx \\ &+ \int_{\Omega} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix} dx \\ &- \exp(-2\alpha\tau)(1 - \mu) \int_{\Omega} \begin{pmatrix} e(t - \tau(t), x) \\ f(We(t - \tau(t), x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(t - \tau(t), x) \\ f(We(t - \tau(t), x)) \end{pmatrix} dx \\ &+ \int_{\Omega} e(t, x)^T R_8 e(t, x) dx - \exp(-2\alpha\theta_M) \int_{\Omega} e(t - \theta_M, x)^T R_8 e(t - \theta_M, x) dx \end{aligned}$$

$$\begin{aligned}
& + \int_{\Omega} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix} dx \\
& - \exp(-2\alpha\tau) \int_{\Omega} \begin{pmatrix} e(t-\tau, x) \\ f(We(t-\tau, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(t-\tau, x) \\ f(We(t-\tau, x)) \end{pmatrix} dx \\
& + \tau^2 \int_{\Omega} \frac{\partial e(t, x)}{\partial t} R_7 \frac{\partial e(t, x)}{\partial t} dx \\
& - \exp(-2\alpha\tau) \int_{\Omega} \begin{pmatrix} e(t, x) - e(t-\tau(t), x) \\ e(t-\tau(t), x) - e(t-\tau, x) \end{pmatrix}^T \begin{pmatrix} R_7 & G_0 \\ * & R_7 \end{pmatrix} \begin{pmatrix} e(t, x) - e(t-\tau(t), x) \\ e(t-\tau(t), x) - e(t-\tau, x) \end{pmatrix} dx \\
& + \theta_M^2 \int_{\Omega} \frac{\partial e(t, x)}{\partial t} R_9 \frac{\partial e(t, x)}{\partial t} dx + 2\alpha \int_{\Omega} \frac{\partial e(t, x)}{\partial x} \bar{P} \frac{\partial e(t, x)}{\partial x} dx \\
& + 2 \int_{\Omega} \frac{\partial e(t, x)}{\partial x} \bar{P} \frac{\partial^2 e(t, x)}{\partial x \partial t} dx \\
& + \exp(-2\alpha\theta_M) \int_{\Omega} \begin{pmatrix} e(t, x) \\ e(t-\theta(t), x) \\ e(t-\theta_M, x) \end{pmatrix}^T \begin{bmatrix} -R_9 & R_9 & 0 \\ * & -R_9 & R_9 \\ * & * & -R_9 \end{bmatrix} \begin{pmatrix} e(t, x) \\ e(t-\theta(t), x) \\ e(t-\theta_M, x) \end{pmatrix} dx
\end{aligned} \tag{52}$$

One can use the derived (39)– (47), (24) and (51), substitute (39) and (47) into (38) and add the right-hand sides of (52) into, for $\mathcal{V}^{\mathcal{K}}(t) + 2\alpha\bar{V}(t) - \alpha_1 \sup_{s \in [-\frac{\tau}{\alpha}, 0]} V(s)$, we can have the following result:

$$\begin{aligned}
\mathcal{V}^{\mathcal{K}}(t) + 2\alpha V(t) & \leq 2 \int_{\Omega} e(t, x)^T P \frac{\partial e(t, x)}{\partial t} dx + 2\alpha \int_{\Omega} e(t, x)^T P e(t, x) dx \\
& + \int_{\Omega} \left[-2e(t, x)^T P \frac{\partial e(t, x)}{\partial t} - 2 \frac{\partial e(t, x)}{\partial x} \bar{P} \frac{\partial e(t, x)}{\partial x} \right. \\
& \left. - 2 \frac{\partial e(t, x)}{\partial t} P \frac{\partial e(t, x)}{\partial t} - 2 \frac{\partial e(t, x)}{\partial x} \bar{P} \frac{\partial^2 e(t, x)}{\partial x \partial t} \right] dx \\
& + \int_{\Omega} \left[2e(t, x)^T P + 2 \frac{\partial e(t, x)}{\partial t} P \right] \left[-Ae(t, x) + Bf(We(t, x)) \right. \\
& \left. + Cf(We(t-\tau(t), x)) - KM\bar{e}(t, \bar{x}_j) \right] dx \\
& + \int_{\Omega} 2e(t, x)^T PKMe(t-\theta(t), x) dx + \varepsilon_1 \frac{\Delta}{\pi} \int_{\Omega} e(t, x)^T PKMe(t, x) dx \\
& + \frac{\Delta}{\varepsilon_1 \pi} \int_{\Omega} \frac{\partial e(t-\theta(t), x)}{\partial x} PKM \frac{\partial e(t-\theta(t), x)}{\partial x} dx
\end{aligned}$$

$$\begin{aligned}
& + \int_{\Omega} 2 \frac{\partial e(t, x)^T}{\partial t} PKM e(t - \theta(t), x) dx + \varepsilon_2 \frac{\Delta}{\pi} \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} PKM \frac{\partial e(t, x)}{\partial t} dx \\
& + \frac{\Delta}{\varepsilon_2 \pi} \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} PKM \frac{\partial e(t - \theta(t), x)}{\partial x} dx \\
& + \int_{\Omega} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix} dx \\
& - \exp(-2\alpha\tau)(1-\mu) \int_{\Omega} \begin{pmatrix} e(t - \tau(t), x) \\ f(We(t - \tau(t), x)) \end{pmatrix}^T \begin{pmatrix} R_1 & R_2 \\ * & R_3 \end{pmatrix} \begin{pmatrix} e(t - \tau(t), x) \\ f(We(t - \tau(t), x)) \end{pmatrix} dx \\
& + \int_{\Omega} e(t, x)^T R_8 e(t, x) dx - \exp(-2\alpha\theta_M) \int_{\Omega} e(t - \theta_M, x)^T R_8 e(t - \theta_M, x) dx \\
& + \int_{\Omega} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(t, x) \\ f(We(t, x)) \end{pmatrix} dx \\
& - \exp(-2\alpha\tau) \int_{\Omega} \begin{pmatrix} e(t - \tau, x) \\ f(We(t - \tau, x)) \end{pmatrix}^T \begin{pmatrix} R_4 & R_5 \\ * & R_6 \end{pmatrix} \begin{pmatrix} e(t - \tau, x) \\ f(We(t - \tau, x)) \end{pmatrix} dx \\
& - 2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial x} \bar{P} \frac{\partial e(t, x)}{\partial x \partial t} dx + \tau^2 \int_{\Omega} \frac{\partial e(t, x)^T}{\partial t} R_7 \frac{\partial e(t, x)}{\partial t} dx \\
& + 2\alpha \int_{\Omega} \frac{\partial e(t, x)^T}{\partial x} PD \frac{\partial e(t, x)}{\partial x} dx \\
& - \exp(-2\alpha\tau) \int_{\Omega} \begin{pmatrix} e(t, x) - e(t - \tau(t), x) \\ e(t - \tau(t), x) - e(t - \tau, x) \end{pmatrix}^T \begin{pmatrix} R_7 & G_0 \\ * & R_7 \end{pmatrix} \begin{pmatrix} e(t, x) - e(t - \tau(t), x) \\ e(t - \tau(t), x) - e(t - \tau, x) \end{pmatrix} dx \\
& + \int_{\Omega} e(t - \theta(t), x)^T \delta \left(1 + \varepsilon_3 \frac{\Delta}{\pi} \right) M^T \Lambda_2 M e(t - \theta(t), x) dx \\
& + \int_{\Omega} \frac{\partial e(t - \theta(t), x)^T}{\partial x} \delta \left(\frac{\Delta}{\varepsilon_3 \pi} + \frac{\Delta^2}{\pi^2} \right) M^T \Lambda_2 M \frac{\partial e(t - \theta(t), x)}{\partial x} dx \\
& - \int_{\Omega} \bar{e}(t, \bar{x}_j)^T M^T \Lambda_1 M \bar{e}(t, \bar{x}_j) dx \\
& + \int_{\Omega} \chi(t - \tau(t), x)^T \begin{pmatrix} -L_1 W W^T \bar{\Gamma} & L_2 W \bar{\Gamma} \\ * & -\bar{\Gamma} \end{pmatrix} \chi(t - \tau(t), x) dx \\
& + \int_{\Omega} \chi(t, x)^T \begin{pmatrix} -L_1 W W^T \Gamma & L_2 W \Gamma \\ * & -\Gamma \end{pmatrix} \chi(t, x) dx
\end{aligned}$$

$$\begin{aligned}
& +\theta_M^2 \int_{\Omega} \frac{\partial e(t,x)^T}{\partial t} R_9 \frac{\partial e(t,x)}{\partial t} dx \\
& +\exp(-2\alpha\theta_M) \begin{pmatrix} e(t,x) \\ e(t-\theta(t),x) \\ e(t-\theta_M,x) \end{pmatrix}^T \begin{bmatrix} -R_9 & R_9 & 0 \\ * & -R_9 & R_9 \\ * & * & -R_9 \end{bmatrix} \begin{pmatrix} e(t,x) \\ e(t-\theta(t),x) \\ e(t-\theta_M,x) \end{pmatrix} \\
& = \xi^T \Phi \xi + \frac{\Delta}{\varepsilon_1 \pi} \int_{\Omega} \frac{\partial e(t-\theta(t),x)^T}{\partial x} PKM \frac{\partial e(t-\theta(t),x)}{\partial x} dx \\
& + \frac{\Delta}{\varepsilon_2 \pi} \int_{\Omega} \frac{\partial e(t-\theta(t),x)^T}{\partial x} PKM \frac{\partial e(t-\theta(t),x)}{\partial x} dx \\
& + \int_{\Omega} \frac{\partial e(t-\theta(t),x)^T}{\partial x} \delta \left(\frac{\Delta}{\varepsilon_3 \pi} + \frac{\Delta^2}{\pi^2} \right) M^T \Lambda_2 M \frac{\partial e(t-\theta(t),x)}{\partial x} dx
\end{aligned} \tag{52}$$

$$\text{where } \xi = \begin{bmatrix} e(t,x)^T & e(t-\theta(t),x)^T & e(t-\theta_M,x)^T & e(t-\tau(t),x)^T \\
\end{bmatrix}$$

$$\begin{aligned}
& e(t-\tau,x)^T \quad f(We(t,x))^T \quad f(We(t-\tau(t),x))^T \quad \frac{\partial e(t,x)^T}{\partial x} \\
& \frac{\partial e(t,x)^T}{\partial t} \quad \bar{e}(t,\bar{x}_j)^T \quad f(We(t-\tau,x))^T \quad \left. \right]^T.
\end{aligned}$$

According to Lemma 3 and the conditions of Theorem 1, we have

$$\begin{aligned}
& \mathfrak{V}(t) + 2\alpha V(t) - \alpha_1 \sup_{s \in [-\tau_M, 0]} V(s) \leq \mathfrak{V}(t) + 2\alpha V(t) - \alpha_1 V(t-\tau(t)) \\
& \leq \mathfrak{V}(t) + 2\alpha V(t) - \alpha_1 \int_{\Omega} 2e(t-\tau(t),x)^T PDe(t-\tau(t),x) dx \\
& + \int_{\Omega} \frac{\partial e(t-\tau(t),x)^T}{\partial x} \left(-\alpha_1 PD + \left(\frac{\Delta}{\varepsilon_1 \pi} + \frac{\Delta}{\varepsilon_2 \pi} \right) PKM \right. \\
& \left. + \delta \left(\frac{\Delta}{\varepsilon_3 \pi} + \frac{\Delta^2}{\pi^2} \right) M^T \Lambda_2 M \right) \frac{\partial e(t-\tau(t),x)}{\partial x} dx \leq 0,
\end{aligned} \tag{53}$$

then the error system (4) with $\Delta A = \Delta B = \Delta C = 0$ under the event-triggered control (14) is stable.

This implies that the two systems (1) and (3) are synchronized. This completes the proof.

Remark 2. In (30), the Lyapunov–Krasovskii functional $V(t)$ is continuous, derivative and positive definite. As we know, the choice of the Lyapunov–Krasovskii functional is crucial for obtaining less conservative criteria. Various kinds of the Lyapunov–Krasovskii functional have been constructed to discuss the NNs with time varying delay. For the derivative of the Lyapunov–Krasovskii functional, it is necessary to estimate the derivative for deriving the criteria.

In this paper, there are some techniques including the free-weighting matrix, Jensen's inequality, Wirtinger's inequality, Halanay's inequality, combined with reciprocally convex and so on have been used by constructing a novel Lyapunov–Krasovskii functional and applying a delay system approach for designing event-triggered controllers of delayed GRDNNs. The synchronization criteria are formulated in terms of linear matrix inequalities.

Remark3. In recent years, a great number of research investigations have analyzed interval-delayed NNs [6-9,12-14,18-20]. Development of delay-dependent stability or synchronization conditions has received increasing attention from research communities, which have become important topics of research. Recently, many effective methods and techniques including the delay-partitioning method [45], reciprocally convex combination method [9], free-weighting matrix approach [12,18] and augmented Lyapunov–Krasovskii functionals approach [14] have been utilized to derive much less conservative results. For interval-delayed NNs, the authors in [9,45,46] have found the admissible maximum upper bound τ of time varying delay $\tau(t)$ for guaranteeing stability of system

considered by giving different values of scalar μ . The corresponding results are shown and listed in Tables. In this paper, we investigate the exponential synchronization analysis of master–slave chaotic uncertain delayed GRDNNs with event-triggered control scheme. The event-triggered synchronization criteria obtained of delayed GRDNNs are delay-dependent. In fact, Theorem 1 is stabilization criterion of system considered, too. Interval influence stability of such system can be discussed by using approach similar to references[9,45,46].

Remark 4. To reduce the network workload event-triggered control scheme of finite-dimensional systems has been investigated extensively over the past decades. There are a few references on event-triggered control of distributed parameter systems (DPSs), which are potentially of great interest in a long distance control of chemical reactors[47] or air polluted areas [48]. In [49], Event-triggered control of DPSs was used via model reduction approach leading to local results concerning practical stability. Furthermore, this method seems to be inapplicable to the systems with spatially-dependent diffusion coefficients. In [36], the authors propose distributed event-triggered control of DPSs under the point measurements and under the averaged measurements. However, there are few theoretical results for reaction diffusion neural networks using the event-triggered control approach. In the present work, we develop event-triggered controllers for DPSs governed by uncertain semilinear diffusion equations to solve distributed parameter neural networks synchronization control problems.

Remark 5. In Theorem 1, the Lyapunov-based analysis of (25) with the “compensation” of the terms $KM \int_{\bar{x}_j}^x e_s(t - \theta(t), s) ds$ with $\theta(t) = 1$ for $t \neq t_k$ and $KM \int_{\bar{x}_j}^x \bar{e}_s(t, s) ds$ is difficult.

In this paper, the approach that we have developed is based on the combination of the Lyapunov–Krasovskii functional for (25) with Halanay's inequality, which was applied to solve robust sampled-data control problem for parabolic systems governed by uncertain semilinear diffusion equations with distributed control on a finite interval in [50].

Remark6. Theorem 1 provides a sufficient condition on synchronization under the Event-triggered control. We assume a sampled-data controller design for a one dimensional delayed Generalized RDNNs, where the sampled data in time measurements of the state are taken in a finite number of fixed sampling spatial points. It is suggested that the sampling intervals in time and in space may be variable, but bounded. The sampling instants (in time) may be uncertain. The approach in this paper

is based on the novel combination of Lyapunov–Krasovskii functionals with Wirtinger's, Young's and Halanay's inequalities etc..

In Theorem 1, we assume that parameter uncertainties are zero, that is $\Delta A = \Delta B = \Delta C = 0$. When $\Delta A \neq 0, \Delta B \neq 0, \Delta C \neq 0$ in system (4), one can get Theorem 2 as follows:

Theorem 2. Let (A1) hold. Given scalars $\tau > 0, 0 < \mu < 1, \delta \geq 0, \theta_M \geq 0, 0 \leq \alpha < 1$, if there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \alpha_1, \lambda$ matrices $R_7 > 0, R_8 > 0, R_9 > 0, G_0, R_j, j = 1, \dots, 6$, and diagonal matrix K , positive diagonal matrices $\Lambda_1 > 0, \Lambda_2 > 0, \Gamma > 0, \bar{\Gamma} > 0$, $P = \text{diag}(p_1, p_2, \dots, p_n) > 0$ such that the following matrix inequality with (26)–(28) hold:

$$\begin{pmatrix} \Pi & \bar{X} \\ * & -\lambda I \end{pmatrix} < 0, \quad (54)$$

where $\Pi = (\Pi_{11 \times 11})$, $\Pi_{11} = \Phi_{11} + \lambda \nu_1^T \nu_1$, $\Pi_{12} = \Phi_{12}, \Pi_{14} = \Phi_{14}, \Pi_{15} = \Phi_{15}$,

$$\begin{aligned} \Pi_{16} &= \Phi_{16} - \lambda \nu_1^T \nu_2, \Pi_{17} = \Phi_{17} - \lambda \nu_1^T \nu_3, \Pi_{18} = \Phi_{18}, \Pi_{19} = \Phi_{19}, \Pi_{1,10} = \Phi_{1,10}, \\ \Pi_{22} &= \Phi_{22}, \Pi_{23} = \Phi_{23}, \Pi_{29} = \Phi_{29}, \Phi_{33} = \Pi_{33}, \Pi_{44} = \Phi_{44}, \Phi_{45} = \Pi_{45}, \Pi_{47} = \Phi_{47}, \\ \Pi_{55} &= \Phi_{55}, \Pi_{5,11} = \Phi_{5,11}, \Pi_{66} = \Phi_{66} + \lambda \nu_2^T \nu_2, \Pi_{67} = \lambda \nu_2^T \nu_3, \Pi_{69} = \Phi_{69}, \\ \Pi_{77} &= \Phi_{77} + \lambda \varepsilon_3^T \varepsilon_3, \Pi_{79} = \Phi_{79}, \Pi_{88} = \Phi_{88}, \Pi_{89} = \Phi_{89}, \Pi_{99} = \Phi_{99}, \Pi_{9,10} = \Phi_{9,10}, \end{aligned}$$

$\Pi_{10,10} = \Phi_{10,10}, \Pi_{11,11} = \Phi_{11,11}$, the same terms Φ_{ij} as those represented in Theorem 1 are satisfied,

then the error system (22) under the event trigger (14) is exponentially stable, which can ensure the exponential synchronization of the master system (1) and the slave system (3). Moreover, the considered controller gain matrix in (14) can be given by $K = P^{-1}Y$.

Proof. Replace A, B, C in LMI (23) by $\bar{A}, \bar{B}, \bar{C}$, respectively.

Similar to the proof of Theorem 1, we can know that

$$\Phi + \bar{X}F(t)\eta + (\bar{X}F(t)\eta)^T < 0, \quad (51)$$

where $\eta = [-\nu_1 \ 0 \ 0 \ 0 \ 0 \ \nu_2 \ \nu_3 \ 0 \ 0 \ 0 \ 0]^T$,

$$\bar{X} = \begin{bmatrix} X^T P^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X^T P^T & 0 & 0 \end{bmatrix}^T.$$

By Lemma 4, we can find that (51) is equivalent to

$$\Phi + \lambda^{-1} \bar{X} \bar{X}^T + \lambda \eta^T \eta < 0, \quad (52)$$

Applying Schur complement verify that (52) is equivalent to (50).

4. An illustrative example

An illustrative example is show to examine the event-triggered synchronization characteristic of delayed GRDNNs.

Example 1. In this example, we consider the following chaotic delayed GRDNNs with system parameters:

$$D=I, A=\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}, B=\begin{pmatrix} 0.8 & -0.5 \\ -0.2 & 1.1 \end{pmatrix}, C=\begin{pmatrix} -0.5 & -0.6 \\ 0.2 & 0.3 \end{pmatrix}, W=0.8I, M=I,$$

$$\tau(t) = 0.4 + 0.2\cos(t),$$

$$J=0.2I, x \in \Omega = \{x \mid 0 \leq x \leq 1\}, t \geq 0, g_j(t) = 0.5(|t+1| - |t-1|), j=1, 2.$$

Clearly, $g_j(\cdot), j=1, 2$ satisfy Assumption (A1). We can be verify that $\bar{L}_1=0, \bar{L}_2=I$.

A straightforward calculation gives $\alpha(t) \leq 0.2 < 1, \tau = 0.6, \mu = 0.3$.

Choosing $\Delta = \frac{\pi}{4}, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1, \alpha = 0.5, \theta_M = 0.01, \delta = 0.2, \alpha_1 = 2$.

It can be solved the LMIs (26)-(29) in Theorem 1 by applying the Matlab LMI control Toolbox. We can have the feasible solution as

$$\begin{aligned} P &= \begin{pmatrix} 1.5756 & 0 \\ 0 & 1.5756 \end{pmatrix}, G_0 = \begin{pmatrix} -7.4358 & -1.1314 \\ -1.1314 & -5.5223 \end{pmatrix}, R_1 = \begin{pmatrix} 1.2987 & -1.5775 \\ -1.5775 & 1.9142 \end{pmatrix}, \\ R_2 &= \begin{pmatrix} 0.5883 & 0.2663 \\ 0.2663 & 0.6794 \end{pmatrix}, R_3 = \begin{pmatrix} 1.4489 & 0.7169 \\ 0.7169 & 1.2203 \end{pmatrix}, R_4 = \begin{pmatrix} 1.2245 & 0.5761 \\ 0.5761 & 1.7706 \end{pmatrix}, \\ R_5 &= \begin{pmatrix} 4.1471 & 3.7162 \\ 3.7162 & 3.4681 \end{pmatrix}, R_6 = \begin{pmatrix} 1.7729 & 0.2729 \\ 0.2729 & 1.4860 \end{pmatrix}, R_7 = \begin{pmatrix} 1.8896 & -0.3976 \\ -0.3976 & 1.3951 \end{pmatrix}, \\ R_8 &= \begin{pmatrix} 1.3861 & 0.7341 \\ 0.7341 & 1.9371 \end{pmatrix}, R_9 = \begin{pmatrix} 3.0571 & 0.9613 \\ 0.9613 & 3.0571 \end{pmatrix}, \Lambda_1 = \begin{pmatrix} 2.4158 & 0 \\ 0 & 2.4158 \end{pmatrix}, \\ \Lambda_2 &= \begin{pmatrix} 2.4709 & 0 \\ 0 & 2.4709 \end{pmatrix}, \Gamma = \begin{pmatrix} 5.5355 & 0 \\ 0 & 5.5355 \end{pmatrix}, \bar{\Gamma} = \begin{pmatrix} 3.5954 & 0 \\ 0 & 3.5954 \end{pmatrix}, \\ K &= \begin{pmatrix} -4.0590 & 0 \\ 0 & -4.0590 \end{pmatrix} \end{aligned}$$

We assume the initial conditions of $u_i(t, x)$ and $\mathcal{U}_i(t, x), i=1, 2$ are

$u_1(t, x) = -0.1, u_2(t, x) = 0.1, \mathcal{U}_1(t, x) = -0.2, \mathcal{U}_2(t, x) = 0.1$, in addition, let the boundary

conditions of $u_i(t, x)$ and $u_i'(t, x), i = 1, 2$ be Neumann boundary conditions.

Fig.1 Change of the Master system $u_1(t, x)$ described in Example 1.

Fig.2 Change of the Master system $u_2(t, x)$ described in Example 1.

Fig.3 Evolution of the error system $e_1(t, x)$ described in Example 1.

Fig.4 Evolution of the error system $e_2(t, x)$ described in Example 1.

Fig.5 Change of the error system $e_1(t, x)$ when $x = 0.6$ in Example 1.

Fig.6 Change of the error system $e_2(t, x)$ when $x = 0.6$ in Example 1.

The change processes of the state $u_i(t, x)$ and error $e_i(t, x), i = 1, 2$, are shown in Figure 1 to Figure 4. Therefore, one can know from Theorem 1 that the master-slave systems are exponential synchronization in this example.

Example 2. In this example, we consider the following master system (1) and slave system (3) with system parameters:

$$D=I, A=\begin{pmatrix} 6 & 0 \\ 0 & 8 \end{pmatrix}, B=\begin{pmatrix} 0.6 & -0.8 \\ 0.2 & 1.5 \end{pmatrix}, C=\begin{pmatrix} -1.5 & 0.5 \\ -0.2 & 0.6 \end{pmatrix}, W=0.5I, M=I,$$

$$\Delta A = \sin(t) \begin{pmatrix} 0.01 & 0.03 \\ 0.04 & 0.12 \end{pmatrix}, \Delta B = \sin(t) \begin{pmatrix} 0.03 & -0.01 \\ 0.12 & -0.04 \end{pmatrix}, \Delta C = \sin(t) \begin{pmatrix} 0.02 & 0.01 \\ 0.08 & 0.04 \end{pmatrix},$$

$$X = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix}, v_1 = \begin{pmatrix} 0.1 \\ 0.3 \end{pmatrix}^T, v_2 = \begin{pmatrix} 0.3 \\ -0.1 \end{pmatrix}^T, v_3 = \begin{pmatrix} 0.2 \\ 0.1 \end{pmatrix}^T, J = \begin{pmatrix} 1.8 \\ 1.5 \end{pmatrix} F(t) = \sin(t),$$

$$\tau(t) = 0.3 + 0.2 \sin(t), x \in \Omega = \{x \mid 0 \leq x \leq 1\}, t \geq 0, g_j(t) = \tanh(t), j = 1, 2.$$

We can see that $g_j(\cdot), j = 1, 2$ satisfy Assumption (A1). We can be verify that

$$\bar{L}_1 = 0, \bar{L}_2 = 0.5I. \text{ A straightforward calculation gives } \bar{\alpha}(t) \leq 0.5 < 1, \tau = 0.5, \mu = 0.4.$$

$$\text{Choosing } \Delta = \frac{\pi}{4}, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1, \alpha = 0.2, \theta_M = 0.01, \delta = 0.2, \alpha_1 = 2.$$

It can be solved the LMIs (26)-(29) and (54) in Theorem 2 by applying the Matlab LMI control Toolbox. We can have the feasible solutions as follows:

$$\begin{aligned} P &= \begin{pmatrix} 4.3725 & 0 \\ 0 & 4.3725 \end{pmatrix}, G_0 = \begin{pmatrix} -3.2368 & -1.7524 \\ -1.7524 & -3.1673 \end{pmatrix}, R_1 = \begin{pmatrix} 1.9054 & -1.0351 \\ -1.0351 & 1.7471 \end{pmatrix}, \\ R_2 &= \begin{pmatrix} 0.6353 & 0.3438 \\ 0.3438 & 0.7796 \end{pmatrix}, R_3 = \begin{pmatrix} 1.0341 & 0.6174 \\ 0.6174 & 1.0217 \end{pmatrix}, R_4 = \begin{pmatrix} 1.1146 & -0.5151 \\ -0.5151 & 1.3617 \end{pmatrix}, \\ R_5 &= \begin{pmatrix} 3.1493 & 1.0568 \\ 1.0568 & 2.7359 \end{pmatrix}, R_6 = \begin{pmatrix} 1.1352 & 0.4467 \\ 0.4467 & 1.4860 \end{pmatrix}, R_7 = \begin{pmatrix} 1.7785 & 0.3137 \\ 0.3137 & 1.6491 \end{pmatrix}, \\ R_8 &= \begin{pmatrix} 1.8043 & 0.9041 \\ 0.9041 & 1.8043 \end{pmatrix}, R_9 = \begin{pmatrix} 3.3476 & 0.8067 \\ 0.8067 & 3.0571 \end{pmatrix}, \Lambda_1 = \begin{pmatrix} 2.9884 & 0 \\ 0 & 2.9884 \end{pmatrix}, \\ \Lambda_2 &= \begin{pmatrix} 2.0086 & 0 \\ 0 & 2.0086 \end{pmatrix}, \Gamma = \begin{pmatrix} 5.0784 & 0 \\ 0 & 5.0784 \end{pmatrix}, \bar{\Gamma} = \begin{pmatrix} 3.0371 & 0 \\ 0 & 3.0371 \end{pmatrix}, \\ K &= \begin{pmatrix} -5.9271 & 0 \\ 0 & -5.9271 \end{pmatrix}, \lambda = 0.8055, \end{aligned}$$

when $\ell = 0.06$ and $\mu = 0.5$, some calculation results of $h_{\max}$ are shown in Table 1 for various values of δ , we find the corresponding upper bounds for sampling interval h is essentially smaller.

Table 1 Maximum sampling interval h for different δ					
δ	0.02	0.04	0.05	0.2	0.5
$h_{\max}$	0.1357	0.1024	0.0725	0.0671	0.0471

We assume the initial conditions of $u_i(t, x)$ and $\mathcal{W}_i(t, x), i = 1, 2$ are

$u_1(t, x) = 0.8, u_2(t, x) = 1.1, \mathcal{U}_1(t, x) = 0.5, \mathcal{U}_2(t, x) = 1.2$, in addition, let the boundary conditions of $u_i(t, x)$ and $\mathcal{U}_i(t, x), i = 1, 2$ be Dirichlet boundary conditions.

Fig.7 Change of the Master system $u_1(t, x)$ described in Example 2.

Fig.8 Change of the Master system $u_2(t, x)$ described in Example 2.

Fig.9 Evolution of the error system $e_1(t, x)$ described in Example 2.

Fig.10 Evolution of the error system $e_2(t, x)$ described in Example 2.

Fig.11 Change of the error system $e_1(t, x)$ when $x = 0.6$ in Example 2.

Fig.12 Change of the error system $e_2(t, x)$ when $x = 0.6$ in Example 2.

Fig.13 Release instants and release interval under the event-trigger (24)

Fig. 14 The diagram of networked master–slave synchronization for the delayed GRDNNs

The simulation results are shown in Figs. 7–13, where Figs. 7 and 8 show the change processes of $u_1(t, x)$ and $u_2(t, x)$ in system (1), respectively. Figs. 9 and 10 exhibit change processes of the synchronization errors $e_1(t, x)$ and $e_2(t, x)$, respectively. Figs. 11 and 12 depict change processes of the synchronization errors $e_1(t, x)$ and $e_2(t, x)$ when $x = 0.6$, respectively. In this example, we can take event-triggering schemes to show the release instants and release intervals with known control gain and triggered matrix, which are illustrated in Fig. 13. The diagram of networked master–slave synchronization for the uncertain delayed GRDNNs is shown in Fig. 14.

According to Theorem 2, the slave system (3) and the master system (1) are exponential synchronized as shown in Figs. 9 and 10. The numerical simulations obviously demonstrate the effectiveness of the developed event-triggered control method to the exponential synchronization of chaotic uncertain delayed GRDNNs with Dirichlet boundary conditions.

Remark7:In [9], a generalized NN model without reaction-diffusion terms was firstly proposed, which includes both the local field NN model and static NN model as its special cases, model

stability criteria were Given. In fact, diffusion effects always exist in the NNs and electric circuits when electrons are moving in asymmetric electromagnetic fields. Hence, ones desire to study the activation of neurons varying in space as well as in time. At the same time, RDNNs have been investigated to show unpredictable behaviors such as bifurcation, chaotic attractors and periodic oscillations and so on, which induced to study on its chaos synchronization, which is a key step for both understanding brain science and designing RDNNs for practical use. Inspired by the analysis above, in this paper, synchronization problem for a class of generalized NN model with reaction-diffusion terms was studied. As is well known, when $D = 0, \bar{C} = 0$ and $W = I$, model (1)

becomes to the local field NNs[7-9]; when $D = 0, \bar{B} = 0, \bar{C} = 0$ and $W = I$, model (1) becomes to the static NNs[6,9,12]; when $D = 0$ and $W = I$, model (1) becomes to a classic delayed cellular NNs model[11,12,19]. Thus, in case of $D \neq 0$, model (1) becomes to corresponding NNs model with reaction-diffusion terms. Therefore, model (1) can be called a generalized RDNNs model, we make full use of information with the reaction-diffusion terms in the studies for synchronization problem.

Remark 8: The motivation for presenting the experimental results is to demonstrate that the proposed event-triggered communication scheme can be utilized to synchronization strategy for delayed uncertain GRDNNs. The improved synchronization novel results are obtained via given controllers. The simulation is performed applying Matlab software. In Theorem 2, the size of the matrix inequality deduces bigger due to more relationships have been taken consideration of, and this will make it difficult to find feasible solutions for these linear matrix inequalities. In order to solve this problem, some optimization algorithms will be used to taken consideration of both the upper bounds of certain parameters as well as the computation complexity. By taking advantage of the classical implicit format solving the partial differential equations and the method of steps for differential difference equations, we provide two illustrative numerical examples and their simulations to show the feasibility and effectiveness of the theoretical results.

5. Conclusions

This paper studied the event-triggered synchronization control for uncertain delayed generalized RDNNs with communication delays. An event-triggered algorithm was used to synchronize the uncertain delayed generalized RDNNs. A model is presented for the synchronization error system with state delay by utilizing the delay system approach. The criteria for the event-triggered synchronization problem and synthesis of uncertain delayed generalized RDNNs are established in terms of LMIs. Two illustrated examples are considered to show the effectiveness of the derived results.

Acknowledgements

This work is partially supported by the National Natural Science Foundation of China under Grants No.61573013, the Special research projects in Shaanxi Province Department of Education under Grant No. 17JK0824, Young and middle-aged top talents projects of Xianyang Normal University under Grant No.XSYBJ201801.

Compliance with ethical standards

Conflict of Interest

The authors declare that we have no conflicts of interests regarding the publication of this article.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent

I consent the journal to review the paper. I inform that the manuscript has not been submitted to other journal for simultaneous consideration. The manuscript has not been published previously. The study is not split up into several parts to increase the quantity of submissions and submitted to various journals or to one journal over time. No data have been fabricated or manipulated (including images) to support my conclusions. No data, text, or theories by others are presented as if they were of my own. Proper acknowledgements to other works are provided, and I use no material that is copyrighted. I consent to submit the paper, and I have contributed sufficiently to the scientific work and I am responsible and accountable for the results.

References

- [1] R. Rakkiyappan, S. Dharani, Q. Zhu. Synchronization of reaction–diffusion neural networks with time-varying delays via stochastic sampled-data controller, *Nonlinear Dyn* (2015) 79:485 – 500.
- [2] JL Wang , HN Wu , T Huang , SY Ren , J Wu, Pinning Control for Synchronization of Coupled Reaction-Diffusion Neural Networks With Directed Topologies, *IEEE Transactions on Systems Man & Cybernetics Systems*, 2017 , 46 (8) :1109-1120.
- [3] Oh-Min Kwon, Myeong-Jin Park, Sang-Moon Lee, Ju H. Park, and Eun-Jong Cha, Stability for Neural Networks With Time-Varying Delays via Some New Approaches, *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 2, Feb. 2013,181-193.
- [4] Sibel Senana, M. Syed Alib, R. Vadivelb, Sabri Arik, Decentralized event-triggered synchronization of uncertain Markovian jumping neutral-type neural networks with mixed delays, *Neural Networks* , 2017 , 86 :32-41.
- [5] Y. Liu, Z. Wang, J. Liang, and X. Liu, stability and synchronization of discrete-time Markovian jumping neural networks with mixed mode dependent time delays, *IEEE Trans. Neural Netw.*, vol. 20, no. 7, pp. 1102 – 1116, Jul. 2009.
- [6] H.-B. Zeng, J. H. Park, C.-F. Zhang, and W. Wang, Stability and dissipativity analysis of static neural networks with interval time varying delay, *J. Franklin Inst.*, vol. 352, no. 3, pp. 1284–1295, Mar. 2015.
- [7] Y. Liu, S. M. Lee, O. M. Kwon, and J. H. Park, New approach to stability criteria for generalized neural networks with interval time varying delays, *Neurocomputing*, vol. 149, pp. 1544–1551, Feb. 2015.
- [8] C.-K. Zhang, Y. He, L. Jiang, Q. H. Wu, and M. Wu, Delay dependent stability criteria for generalized neural networks with two delay components, *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25,no. 7, pp. 1263–1276, Jul. 2014.
- [9] X. Zhang, Q. Han, Global asymptotic stability for a class of generalized neural networks with interval time-varying delays, *IEEE Trans. Neural Netw.*, 22:1180–1192, 2011.

- [10] Z. Xu, H. Qiao, J. Peng, A comparative study on two modeling approaches in neural networks, *Neural Netw.*, 17:73–85, 2004.
- [11] J. Liang and J. Cao, A based-on LMI stability criterion for delayed recurrent neural networks, *Chaos, Solitons Fractals*, vol. 28, no. 1, pp. 154–160, 2006.
- [12] Q. Gan, T. Liu, C. Liu, T. Lv, Synchronization for a class of generalized neural networks with interval time-varying delays and reaction-diffusion terms, *Nonlinear Analysis: Modelling and Control*, Vol. 21, No. 3, 379–399, 2016.
- [13] R. Vadivel, M. Syed Ali, Young Hoon Joo, Drive-response synchronization of uncertain Markov jump generalized neural networks with interval time varying delays via decentralized event-triggered communication scheme, *Journal of the Franklin Institute* (2020), doi:<https://doi.org/10.1016/j.jfranklin.2020.05.004>
- [14] C. Wang, Y. He, W. Lin, Stability analysis of generalized neural networks with fast-varying delay via a relaxed negative-determination quadratic function method, *Applied Mathematics and Computation*, 391(2021)125631
- [15] O.M. Kwon, S.H. Lee, M.J. Park, S.M. Lee, Augmented zero equality approach to stability for linear systems with time-varying delay, *Appl. Math. Comput.* 381 (2020) 125329, doi: 10.1016/j.amc.2020.125329 .
- [16] T. Li, L. Guo, C. Sun, and C. Lin, Further results on delay-dependent stability criteria of neural networks with time-varying delays, *IEEE Trans. Neural Netw.*, vol. 19, no. 4, pp. 726–730, Apr. 2008.
- [17] Y. He, G. Liu, and D. Rees, New delay-dependent stability criteria for neural networks with time-varying delay, *IEEE Trans. Neural Netw.*, vol. 18, no. 1, pp. 310–314, Jan. 2007.
- [18] R. Saravanakumar, M. Syed Ali, Choon Ki Ahn, Hamid Reza Karimi, Peng Shi, Stability of Markovian Jump Generalized Neural Networks with Interval Time-Varying Delays, *IEEE Trans. Neural Netw. Learn. Syst.*, 28 (2017) 8:1840-1850.
- [19] X. Liao, K. Wong, Z. Wu, G. Chen, Novel robust stability criteria for interval-delayed Hopfield neural networks, *IEEE Trans. Circuits Syst. I* 48 (11) (2001) 1355 – 1359.
- [20] C. Li, X. Liao, R. Zhang, Global robust asymptotical stability of multi-delayed interval neural networks: an LMI approach, *Phys. Lett. A* 328(6) (2004) 452 – 462.
- [21] L.M. Pecora , T.-L. Carroll , Synchronization in chaotic systems, *Phys. Rev. Lett.* 64 (8) (1990) 821–825 .
- [22] W. Zhang, J. Li, K. Xing, C. Ding, Synchronization for distributed parameter NNs with mixed delays via sampled-data control, *Neurocomputing*, 2016 (175) 29:265-277.
- [23] S. Chen, G. Song, B. Zheng, T. Li, Finite-time synchronization of coupled reaction – diffusion neural systems via intermittent control, *Automatica* 109 (2019) 108564.
- [24] Y. Wu, J. Cao, Q. Li, Ahmed Alsaedi, Fuad E. Alsaadi, Finite-time synchronization of uncertain coupled switched neural networks under asynchronous switching, *Neural Networks* 85 (2017) 128–139.
- [25] B.-L. Lu , H.-J. Jiang , C. Hu , A. Abdurahman , Synchronization of hybrid coupled reaction-diffusion neural networks with time delays via generalized intermittent control with spacial sampled-data, *Neural Netw.* 105 (2018) 75–87.
- [26] J. Li, W. Zhang, M. Chen Synchronization of delayed reaction–diffusion neural networks via an adaptive learning control approach. *Computers and Mathematics with Applications*, 2013(65): 1775–1785.

- [27] Y. Sheng, Z. Zeng, Impulsive synchronization of stochastic reaction–diffusion neural networks with mixed time delays, *Neural Networks* 103 (2018) 83–93.
- [28] X. Xie, X.Liu , H. Xu, X. Luo, G.Liu, Synchronization of coupled reaction-diffusion neural networks: Delay-dependent pinning impulsive control, *Commun Nonlinear Sci Numer Simulat* 79 (2019) 104905.
- [29] D. Yue, E. Tian, and Q.-L. Han, A delay system method for designing event-triggered controllers of networked control systems, *IEEE Trans. Autom. Control*, 58 (2013) 2, 475–481.
- [30] Y. Huang, Y. Liu, Switching event-triggered control for a class of uncertain nonlinear systems, *Automatica* 108 (2019) 108471.
- [31] W. He, T. Luo, Y. Tang, W. Du, Y. Tian, F. Qian, Secure Communication Based on Quantized Synchronization of Chaotic Neural Networks Under an Event-Triggered Strategy, *IEEE Transactions on Neural Networks and Learning Systems*, Early Access Article, 2019.
- [32] A. Selivanov, E. Fridman Event-triggered H_{∞} control: a switching approach. *IEEE Trans. Automatic Control* , 61 (10), 3221–3226, 2016.
- [33] Z Fei , C Guan , H Gao, Exponential Synchronization of Networked Chaotic Delayed Neural Network by a Hybrid Event Trigger Scheme, *IEEE Trans. Neural Netw. Learn. Syst.*, 2017 , PP (99) :1-10.
- [34] S. Wen, Z. Zeng, Michael Z. Q. Chen, T. Huang, Synchronization of Switched Neural Networks With Communication Delays via the Event-Triggered Control, *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 10, oct. 2017, 2334–2343.
- [35] Y. Fan, X. Huang, H. Shen, J. Cao, Switching event-triggered control for global stabilization of delayed memristive neural networks: An exponential attenuation scheme, *Neural Networks* 117 (2019) 216 – 224.
- [36] L. Li, W. Zou, S. Fei, Event-triggered synchronization of delayed neural networks with actuator saturation using quantized measurements, *Journal of the Franklin Institute* 356 (2019) 6433–6459.
- [37] A. Selivanov, Emilia Fridman, Distributed event-triggered control of diffusion semilinear PDEs, *Automatica*, 2016 (68) 344–351.
- [38] J Chen, B Chen, Z Zeng , Synchronization in Multiple Neural Networks With Delay and Disconnected Switching Topology via Event-Triggered Impulsive Control Strategy , *IEEE Transactions on Industrial Electronics* , 2021(68)3: 2491 – 2500.
- [39] Hardy, G. H., Littlewood, J. E., Polya, G. *Inequalities*. Cambridge: Cambridge University Press, 1988.
- [40] Gu, K, Kharitonov, V, Chen, J., *Stability of time-delay systems*. Boston: Birkhauser, 2003.
- [41] Halanay, A.. *Differential equations: stability, oscillations, time lags*. New York: Academic Press, 1966.
- [42] Syed Ali, M., Saravanakumar, R., & Arik, S. Delay-dependent stability criteria of uncertain Markovian jump neural networks with discrete interval and distributed time-varying delays. *Neurocomputing*, 158 (2015)167-173.
- [43] P. G. Park, J. W. Ko, and C. Jeong, Reciprocally convex approach to stability of systems with

- time-varying delays, *Automatica*, 47 (1)(2011) 235–238.
- [44] Wang, Z., Liu, Y., Yu, L., Liu, X., Exponential stability of delayed recurrent neural networks with Markovian jumping parameters. *Phys. Lett. A* 356 (2006) 346–352.
- [45] H. Zhang, Z. Liu, G.-B. Huang, and Z. Wang, Novel weighting delay-based stability criteria for recurrent neural networks with time varying delay, *IEEE Trans. Neural Netw.*, vol. 21, no. 1, pp. 91–106, Jan. 2010.
- [46] T. Li, L. Guo, C. Sun, and C. Lin, "Further results on delay-dependent stability criteria of neural networks with time-varying delays," *IEEE Trans. Neural Netw.*, vol. 19, no. 4, pp. 726–730, Apr. 2008.
- [47] Smagina, Y., & Sheintuch, M.. Using Lyapunov's direct method for wave suppression in reactive systems. *Systems & Control Letters*, 2006 (7) 55: 566–572.
- [48] Court, J.R., Demetriou, M.A., & Gatsonis, N.A.. Spatial gradient measurement through length scale estimation for the tracking of a gaseous source. In *American control conference*, 2012: 2984–2989.
- [49] Yao, Z., & El-Farra, N.H.. Resource-aware model predictive control of spatially distributed processes using event-triggered communication. In *52nd IEEE conference on decision and control*, 2013: 3726–3731.
- [50] E. Fridman, Anatoly Blighovsky, Robust sampled-data control of a class of semilinear parabolic systems, *Automatica* 48 (2012) 826–836.