Seismological Effects on Spectral and Pseudospectral Acceleration Proximities Based on Random Vibration Theory

Haizhong Zhang

Kanagawa University

Yan-gang Zhao (✉ zhao@kanagawa-u.ac.jp)

Kanagawa University

Research Article

Keywords: Spectral absolute acceleration, pseudospectral acceleration, random vibration theory, seismological effects, inertial force.

Posted Date: July 20th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-713123/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Seismological effects on spectral and pseudospectral acceleration proximities based on random vibration theory

Haizhong Zhang¹⁾ and Yan-Gang Zhao^{1)*}

Abstract

Although spectral absolute acceleration, Sa , is a useful tool for estimating the inertial forces of structures in seismic designs, seismic codes typically specify only the pseudospectral acceleration, Spa . Many studies have been performed to clarify the relationship between these two spectra to relate them. A recent study indicated that this relationship could be affected by not only the structural damping ratio and period but also seismological parameters, such as magnitude and distance. However, how seismological parameters affect their relationship is not clearly understood. To clarify this issue, an approach that relates the two spectra, including seismological parameters, is proposed herein based on random vibration theory. The proposed approach is verified by comparison with the results of time-series analysis. Furthermore, the seismological effects are then explored and explained based on the proposed approach. It is found that Spa approaches Sa with increasing moment magnitude and source-to-site distance, particularly at long structural periods, with the main reason being attributable to the increase in the long-period components of earthquake ground motions. Finally, a practical formulation for estimating Sa from Spa considering the seismological effects is constructed and verified using real seismic records.

Keywords

Spectral absolute acceleration, pseudospectral acceleration, random vibration theory,

1) Department of Architecture, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama, 221-8686, Japan

* Corresponding author: zhao@kanagawa-u.ac.jp

seismological effects, inertial force.

1. Introduction

Response spectrum is presently the most widely used tool to characterize seismic loads for structural seismic designs. In most seismic codes, particularly those involving force-based design, the response spectrum for the design is typically specified as the 5%-damped pseudospectral acceleration, Spa , along with a damping modification factor for adjustment to other damping levels (Pu et al., 2016; Zhang and Zhao, 2020). Because Spa is defined according to the spectral relative displacement, Sd , as $Spa = \bar{\omega}^2 Sd$, with $\bar{\omega}$ being the structural circular frequency, the force estimated using Spa is proportional to the relative displacement, thus corresponding to the restoring force of the structure. Therefore, Spa is suitable in cases where the restoring force is of interest in the seismic design, e.g., in the design of structures where the damping is derived from the added energy dissipation devices (Lin and Chang, 2003). Nevertheless, when the inertial force is considered in the seismic design, e.g., to calculate the base shear in the designs of base-isolated structures or in foundation designs, the spectral absolute acceleration, Sa , is more suitable (Sadek et al., 2000; Mentrasti, 2008).

Early studies have shown that for small structural damping ratios, Sa can be approximated by Spa ; however, when the structural damping ratio is large and the structural period is long, Sa can significantly differ from Spa (Jenschke et al., 1964, 1965; Veletsos and Newmark, 1964; Newmark and Rosenblueth, 1971; Chopra, 2007) and is always greater than Spa (Newmark and Rosenblueth, 1971). Therefore, in such cases, directly using Spa to approximate Sa may underestimate the inertial forces and negatively affect the structural design.

Many studies have been conducted to clarify the relationship between Sa and Spa to relate them. Sadek et al. (2000) compared these two spectra based on statistical analyses of 72 accelerograms from 36 stations in western United States. Song et al. (2007) derived an

analytical equation to relate the two spectra by assuming earthquake excitation as a Gaussian stationary process. Mentrasi (2008) theoretically analyzed and explained the relationship between these two spectra based on exact integral analysis. Zhang et al. (2016) discussed these spectra and compared them based on the El Centro earthquake record.

The above studies have yielded useful results on the relationship between the two spectra as well as the effects of the structural parameters, namely structural period and damping ratio. Papagiannopoulos et al. (2013) further noted that the relationship between them is affected by not only the structural but also seismological parameters, e.g., magnitude and distance, based on the statistical analyses of 866 accelerograms recorded worldwide. However, the principle underlying how the seismological parameters affect the relationship between the two spectra has not been elucidated.

The aim of this paper is to clarify such seismological effects. The remainder of this paper is organized as follows. First, an approach to relate S_a and S_{pa} including seismological parameters is developed based on random vibration theory (RVT). This approach is then verified by comparison with the results of time-series analysis. Subsequently, based on the developed approach, the seismological effects on the relationship between the two spectra are systematically investigated and theoretically explained. Further, a practical formulation for estimating S_a from S_{pa} considering the seismological effects is constructed and verified using real seismic records. Finally, the conclusions of this study are shown.

2. Approach to relate S_a and S_{pa}

To investigate the seismological effects on the relationship between S_a and S_{pa} , there are two options: using real accelerograms or ground-motion prediction models. To identify consistent patterns within an appreciable margin of variability that is always displayed by strong-motion data, large numbers of records must be employed because one is unlikely to find

records that are essentially similar in all but one characteristic. Therefore, the ground-motion prediction models for S_{pa} and S_a are more suitable for use in this study. Although there are many available ground-motion prediction models for the 5%-damped S_{pa} , very few are available for 5%-damped S_a , and even fewer for S_a with various damping ratios. In this study, we adopt a Fourier amplitude spectrum (FAS) ground-motion model and RVT to estimate and relate the two spectra considering the ability of the RVT to relate the FAS to the response spectrum at any desired damping level. The proposed analytical approach allows the investigation of the seismological effects as well as a theoretical explanation for the observed phenomena.

2.1 Earthquake ground-motion model

Various methods are available in the literature to describe the FAS of the earthquake ground motions. The simplest method involves using seismology theory to compute the radiated FAS from a point source in terms of the various source, path, and site parameters. This study utilizes the seismological point-source theory to derive the FAS based on the description by Boore (2003). The FAS of the ground-motion acceleration, $Y(f)$ (cm/s), can be expressed as

$$Y(f) = [0.78 \frac{\pi}{\rho\beta^3} M_0 \frac{f^2}{1 + (f/f_c)^2}] [Z(R) \times \exp(\frac{-\pi f R}{Q(f)\beta})] [\exp(-\pi\kappa_0 f) A(f)] \quad (1)$$

where f = frequency (Hz); ρ = mass density of the crust (g/cm^3); β = shear-wave velocity of the crust (km/s); R = distance from the source (km); $Z(R)$ = geometric attenuation; κ_0 = site diminution (s); $Q(f)$ = anelastic attenuation; $A(f)$ = crust amplification; M_0 = seismic moment (dyne cm), which is related to the moment magnitude, M , as $M_0 = 10^{1.5M+10.7}$; f_c = corner frequency given as $f_c = 4.9 \times 10^6 \beta (\Delta\sigma/M_0)^{1/3}$, which represents the frequency below which the FAS decays; $\Delta\sigma$ = stress drop. The seismological parameters required in Eq. (1) for central and eastern North America (CENA) are used in this study because of recent work that have updated

the seismological parameters for these regions (Boore and Thompson, 2015), as detailed in Table 1 of Wang and Rathje (2016).

2.2 Expression for Sa/Spa

Based on the RVT, both Sa and Spa can be obtained from the FAS at any desired damping level. The RVT states that the peak value of a time-series signal is equal to the product of the peak factor and root-mean-square (rms) value, which can be expressed as

$$a_{\max} = pf \sqrt{\frac{1}{D\pi} \int_0^\infty |y(\omega)|^2 d\omega} \quad (2)$$

where $a_{\max}$ is the peak value of the signal, pf denotes the peak factor, and the square-root part in Eq. (2) represents the rms value of the signal, which is obtained from the signal duration D and FAS of the signal $y(\omega)$, and ω is the circular frequency ($\omega = 2\pi f$). Since the response spectrum is the peak response value of a single-degree-of-freedom (SDOF) oscillator, according to RVT, the response spectrum should be equal to the product of the peak factor and rms of the oscillator response. Boore (2003) derived an expression for Spa , which is expressed as

$$Spa(\bar{\omega}, \xi) = pf_{\xi p} \sqrt{\frac{1}{D_{\text{rms}}\pi} \int_0^\infty |YR(\omega, \bar{\omega}, \xi)|^2 d\omega} \quad (3)$$

where, $\bar{\omega}$ and ξ are the SDOF-oscillator circular frequency and damping ratio, respectively; $pf_{\xi p}$ is the peak factor of the oscillator response, and the square-root part in Eq. (3) represents the rms value of the oscillator response, which is obtained using the oscillator-response duration, D_{rms} , and oscillator-response FAS, $YR(\omega, \bar{\omega}, \xi)$. Here, $YR(\omega, \bar{\omega}, \xi)$ is equal to the product of the ground-motion FAS $Y(\omega)$ and modulus of the oscillator transfer function for Spa , $Hpa(\omega, \bar{\omega}, \xi)$, i.e., $YR(\omega, \bar{\omega}, \xi) = Y(\omega) |Hpa(\omega, \bar{\omega}, \xi)|$. $Hpa(\omega, \bar{\omega}, \xi)$ is expressed as

$$|Hpa(\omega, \bar{\omega}, \xi)| = \frac{1}{\sqrt{(2\xi \omega / \bar{\omega})^2 + ((\omega / \bar{\omega})^2 - 1)^2}} \quad (4)$$

The oscillator transfer function for Spa , $Hpa(\omega, \bar{\omega}, \xi)$, is obtained from the one for Sd , $Hd(\omega, \bar{\omega}, \xi)$, by multiplication with $\bar{\omega}^2$, i.e., $|Hpa(\omega, \bar{\omega}, \xi)| = \bar{\omega}^2 |Hd(\omega, \bar{\omega}, \xi)|$. $Hd(\omega, \bar{\omega}, \xi)$ is expressed as

$$|Hd(\omega, \bar{\omega}, \xi)| = \frac{1/\bar{\omega}^2}{\sqrt{(2\xi\omega/\bar{\omega})^2 + ((\omega/\bar{\omega})^2 - 1)^2}} \quad (5)$$

It should be noted that in RVT analysis, multiplying the spectral relative displacement Sd by $\bar{\omega}^2$ to obtain Spa is equivalent to multiplying the oscillator transfer function $Hd(\omega, \bar{\omega}, \xi)$ for Sd by $\bar{\omega}^2$ to obtain Spa . This is attributed to the property of Eq. (3), in which the integral of $YR(\omega, \bar{\omega}, \xi)$ is related to ω ; thus, one can insert $\bar{\omega}^2$ into the integral, i.e., transform $Hd(\omega, \bar{\omega}, \xi)$, or outside the radical sign, i.e., transform Sd , to obtain the same results.

Similarly, based on the RVT, Sa can be obtained by replacing the oscillator transfer function for Spa in Eq. (3) by that for Sa . The oscillator transfer function for Sa , $Ha(\omega, \bar{\omega}, \xi)$, is given by

$$|Ha(\omega, \bar{\omega}, \xi)| = \frac{\sqrt{(2\xi\omega/\bar{\omega})^2 + 1}}{\sqrt{(2\xi\omega/\bar{\omega})^2 + ((\omega/\bar{\omega})^2 - 1)^2}} \quad (6)$$

Then, the expression for the ratio of Sa to Spa can be obtained using the respective spectra:

$$\frac{Sa(\bar{\omega}, \xi)}{Spa(\bar{\omega}, \xi)} = \sqrt{\frac{\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega}{\int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega}} \times \frac{pf_\xi}{pf_{\xi p}} \quad (7)$$

where pf_ξ is the oscillator-response peak factor corresponding to Sa . Thus, Eq. (7) theoretically relates to the two spectra. Here, it should be noted that the oscillator-response duration for Sa is assumed to be the same as that for Spa in the derivation of Eq. (7). The expression for Sa/Spa

in Eq. (7) can be decomposed into two terms: the first term (i.e., $\sqrt{\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega} /$

$\sqrt{\int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega}$) is the ratio of the oscillator-response rms values for Sa and Spa ,

hereafter denoted as R_{rms} ; and the second term (i.e., $pf_{\xi} / pf_{\xi p}$) is the ratio of the oscillator-response peak factors for Sa and Spa , hereafter denoted as R_{pf} .

When applying Eq. (7) to estimate the ratio of the two spectra, the value of the peak factor must be determined. Many models have been developed for the estimation of the peak factor (Cartwright and Longuet-Higgins, 1956; Davenport, 1964; Vanmarcke, 1975). Among these, the model by Vanmarcke (1975) has been found to provide the most reasonable estimations of the response spectra in RVT analysis (Wang and Rathje, 2016). The cumulative distribution function, P , of the peak factor, pf , as provided by Vanmarcke (1975) is expressed as

$$P(pf < r) = [1 - e^{(-r^2/2)}] \times \exp[-2f_z \exp(-r^2/2) D_{gm} \frac{(1 - e^{-\delta^{1.2} r \sqrt{\pi/2}})}{(1 - e^{r^2/2})}] \quad (8)$$

Here, δ is a bandwidth factor, which is defined as a function of the spectral moments:

$$\delta = \sqrt{1 - \frac{m_1^2}{m_0 m_2}} \quad (9)$$

where m_0 , m_1 , and m_2 denote the zeroth-, first-, and second-order moments of the square of the FAS, and the n th-order spectral moment, m_n , is defined as

$$m_n = \frac{1}{\pi} \int_0^\infty \omega^n |Y(\omega)|^2 d\omega \quad (10)$$

In addition, f_z denotes the rate of zero crossings, which is also a function of the spectral moments, and is given by

$$f_z = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \quad (11)$$

The ground-motion duration D_{gm} in Eq. (8) is estimated based on the model of Boore and Thompson (2014, 2015). In RVT analysis, the expect value of the peak factor is always used, which can be obtained using Eq. (8) by the expression $\int_0^\infty [1 - P(pf < r)] dr$.

3. Verification

To investigate the accuracy of the proposed approach, the ratio of the two spectra, Sa/Spa , was calculated using Eq. (7) and compared with that obtained using traditional time-series analysis. The time series for the analysis was generated from ground-motion FAS using the stochastic method SIMulation (Boore, 2005) program via stochastic simulations (Boore, 1983). For each FAS, a suite of 100 time-series signals was generated, and the simulated time series matched the FAS on average. Then, the values of Sa , Spa , and the spectral ratios Sa/Spa for all the generated time series were calculated using the direct integration method by Nigam and Jennings (1969). For each FAS, the 100 corresponding results of the spectral ratios Sa/Spa for a given damping level were averaged and compared with those obtained using the proposed approach. A wide range of the structural parameters, i.e., oscillator period T_0 (0.01–10 s) and damping ratio ζ (10–50%), as well as main seismological parameters, including the moment magnitude M (4–8) and site-to-source distance R (20–200.01 km), were considered in the calculations. Some of these representative comparisons are shown in Figs. 1–3. The favorable agreement in these figures validate the proposed approach.

From the figures, the relationship between the two spectra can also be clearly observed. It is observed that the values of Sa/Spa are always greater than unity for all cases, which implies that Sa is always greater than Spa . Figure 1 also indicates that Sa/Spa is nearly equal to unity at small oscillator periods and increases with increasing oscillator periods and damping ratios; these values may be considerably greater than unity for very long oscillator periods and very large damping ratios. These observed properties for the relationship between the two spectra are consistent with those observed by statistical analyses of real seismic records and common knowledge (Jenschke et al., 1964, 1965; Veletsos and Newmark, 1964; Newmark and Rosenblueth, 1971; Sadek et al., 2000; Chopra, 2007; Song et al., 2007; Mentrasti, 2008; Papagiannopoulos et al., 2013; Zhang et al., 2016). The consistency of these observations provides additional confirmation of the proposed approach.

4. Seismological effects on Sa - Spa relationship

4.1 Exploration of the seismological effects

This section investigates the seismological effects on the relationship between Sa and Spa based on the proposed approach. From Figs. 2 and 3 in the previous section, the Sa/Spa trends for variation of the moment magnitude and site-to-source distance can be easily clarified. Figure 2 indicates that Sa/Spa is considerably affected by the moment magnitude. The value of Sa/Spa decreases with increase in the moment magnitude, particularly for long oscillator periods. Figure 3 indicates that the behavior of Sa/Spa with variation of the site-to-source distance R is typically consistent with the variation of the moment magnitude but to a much smaller degree of variation.

The effects of the site conditions on the relationship between the two spectra were also investigated. The site term in the FAS model of Eq. (1) corresponds to crustal amplification for hard rock. To investigate the effects of the surface site conditions, three soft soil sites composed of single layers of soil on hard rock were considered. Further, two levels of the soil damping ratio, h (1% and 10%), were considered. Values of Sa/Spa for the three soil sites were calculated using the proposed approach by considering all structural and seismological parameters in the previous section. The representative results corresponding to an oscillator damping ratio of 30% are shown in Fig. 4; additionally, the undamped site fundamental period, T_s , impedance ratio of the soil layer with respect to hard rock, I_p , and soil damping ratio, h , are presented. It is evident from the figure that Sa/Spa varies with the soil parameters, i.e., impedance ratio, soil damping ratio, and site fundamental period, but to a much smaller degree than with the moment magnitude (Fig. 2). In addition, the effects of the site conditions on Sa/Spa are complex, and no simple and regular pattern is readily observed. The values of Sa/Spa for the soil sites may be larger or smaller than those for the rock sites, depending on the

parameters of the soil sites. It is noted that Sa/Spa for the soil site in Fig. 4(b) is nearly the same as that for the rock site. When I_p is larger (Fig. 4(a)) or the site fundamental period T_s is longer (Fig. 4(c)), Sa/Spa for the soil site decreases compared with that for the rock site. When the soil damping ratio h is smaller (Fig. 4(d)), Sa/Spa for the soil site increases compared with that for the rock site. In addition, from comparison of the results in Fig. 4, it is observed that the effect of the site fundamental period on Sa/Spa is the most significant compared with other site parameters.

4.2 Explanation of the observed phenomenon

The observed phenomena can be theoretically explained based on the proposed approach. The representative results of the two terms in Eq. (7), i.e., R_{rms} and R_{pf} , for the cases in Fig. 2 are shown in Figs. 5 and 6 to investigate their contributions toward Sa/Spa . It is evident from Figs. 2 and 5 that the value of Sa/Spa is similar to that of R_{rms} , and all the characteristics, including the trends with variations in the structural and seismological parameters, are captured adequately by R_{rms} . In addition, Fig. 6 indicates that the value of R_{pf} is close to unity compared to R_{rms} . These observations indicate that Sa/Spa is dominated by R_{rms} , which facilitates explanation of the phenomenon based on R_{rms} .

To facilitate further analysis, the integral terms in the numerator and denominator of R_{rms} , i.e., $\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega$ and $\int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega$, are regarded as areas of the square of the oscillator-response FAS enclosed by the circular frequency axis. The integral term in the numerator, i.e., $\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega$, corresponds to Sa and that in the denominator, i.e., $\int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega$, corresponds to Spa . Thus, the proximity between the two spectra or the value of Sa/Spa is determined by the proximity between these two areas. The closer the two areas are, the more similar are the two spectra, and the closer is the value of Sa/Spa to unity. It

can be further observed from the first term R_{rms} in Eq. (7) that the difference between the two areas is caused by the difference between the oscillator transfer functions of the two spectra, i.e., $Ha(\omega, \bar{\omega}, \xi)$ and $Hpa(\omega, \bar{\omega}, \xi)$, because the ground-motion FAS $Y(\omega)$, which affects the oscillator-response FAS, is the same for the two spectra.

To investigate the differences between the oscillator transfer functions for Sa and Spa , their values for two oscillator damping ratios, namely 5% and 30%, are compared in Fig. 7. The oscillator period was chosen as 0.5 s for comparison. Here, it should be noted that the oscillator period T_0 is different from the period T ($T = 2\pi/\omega$) of the ground-motion FAS. It can be found that the two oscillator transfer functions are very similar for the smaller oscillator damping ratio (5%). However, with the increase in the oscillator damping ratio, the difference between the two oscillator transfer functions increases over short periods. Nevertheless, at periods around and greater than the oscillator period T_0 , the values of the two oscillator transfer functions are very similar. In addition, it should be noted that values of $Ha(\omega, \bar{\omega}, \xi)$ are always greater than those of $Hpa(\omega, \bar{\omega}, \xi)$, and the degree increases with increasing oscillator damping ratios, particularly for short periods. Based on these properties of the oscillator transfer functions, the Sa/Spa trends with variations in the oscillator damping ratio can be clarified. Because the difference between the two oscillator transfer functions increases with increasing oscillator damping ratio, the difference between the two integral areas, i.e., $\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega$ and $\int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega$, will increase. This causes an increase in the difference between the two spectra with increases in the oscillator damping ratios (Fig. 1). Further, because the value of the oscillator transfer function for Sa is always greater than that for Spa , $\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega > \int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega$, and the value of Sa is always larger than that of Spa (Fig. 1–3).

To explain the Sa/Spa trend with variation in oscillator period, the values of the two

oscillator transfer functions for different oscillator periods (0.1 and 1 s) are compared in Fig. 8. The oscillator damping ratio is set as 30% for comparison. It is observed that owing to the increase in the oscillator period, the region of the oscillator transfer functions with periods less than the oscillator period T_0 increases. Because the values of these two transfer functions are very different in this region ($Ha(\omega, \bar{\omega}, \xi) > Hpa(\omega, \bar{\omega}, \xi)$), as shown in Fig. 8, the differences between the two areas given by the numerator $\int_0^\infty |Y(\omega)Ha(\omega, \bar{\omega}, \xi)|^2 d\omega$ and denominator $\int_0^\infty |Y(\omega)Hpa(\omega, \bar{\omega}, \xi)|^2 d\omega$ of R_{rms} increase with increasing oscillator periods. This explains the increase in the differences between the two spectra or the values of Sa/Spa with increasing oscillator periods (Figs. 1–3).

To explain the Sa/Spa trend with variations in the seismological parameters, namely the moment magnitude, site-to-source distance, and site conditions, the key factor governing the seismological effects is investigated in accordance with R_{rms} in Eq. (7). It is observed that the seismological parameters affect R_{rms} by changing the ground-motion FAS. The other components affecting the oscillator-response FAS in R_{rms} , i.e., the oscillator transfer functions, are independent of the seismological parameters. Moreover, it is known that the distribution of the ground-motion FAS with frequency instead of absolute values of the FAS affects the first term. This conclusion can be easily verified by multiplying the FAS by a constant value; since a constant value exists in both the numerator and denominator of R_{rms} , they will cancel each other and will not affect R_{rms} . Therefore, the key factor affecting the seismological effects is the distribution of ground-motion FAS with frequency, i.e., the frequency content of the ground motions.

To investigate the variation of the frequency content of ground motions with the moment magnitude and site-to-source distance, the values of the ground-motion FAS for two moment magnitudes and two site-to-source distances are compared in Fig. 9. It is noted that although

the FAS values at all periods increase with increasing moment magnitudes, the increases in the long periods are more significant. This indicates that the long-period components of the earthquake ground motions increase relatively compared to the short-period components with increasing moment magnitudes. Similarly, although the FAS values at all periods decrease with increasing site-to-source distances, the decreases at short periods are more significant. This means that the long-period components of the earthquake ground motions increase relative to the short-period components with increasing site-to-source distances. In addition, it is observed that the degree of variation of the frequency content of the ground motions with the site-to-source distances is smaller than that of the moment magnitude.

Based on the above properties of variation of the ground-motion frequency content with moment magnitude and site-to-source distance, the Sa/Spa variation trends can be explained. It is worth emphasizing again that the spectral ratio Sa/Spa can be understood by investigating the differences between the two areas of the square of the oscillator-response FAS in Eq. (7). The difference between these two areas is attributed to the difference between their oscillator transfer functions over short periods. In addition, the two transfer functions are very similar at periods around and longer than the oscillator period T_0 . It can be seen from Fig. 9 that when the components are concentrated more over a long period, in which the two oscillator transfer functions are similar, the two spectra will be more similar. Therefore, when the long-period components of the ground motions increase with increasing moment magnitudes and site-to-source distances, similar portions of the two areas will increase relatively, as shown in Fig. 9. This explains why Spa approaches Sa with increasing moment magnitudes and site-to-source distances (Fig. 2).

To explain the effects of site conditions on the relationship between the two spectra, the FAS values for the soil sites in Fig. 4 are compared with those for the rock sites, which is shown in Fig. 10. It is noted that the effects of the soil conditions on the ground-motion frequency

content are not as simple as those of the moment magnitude and site-to-source distance. The site effect on the frequency content is strongly dependent on the period T . The components around the first few site resonance periods are amplified; however, those between these site resonance periods or at very short periods are de-amplified, as reported in previous studies (Zhang and Zhao, 2017, 2018). The components longer than the site fundamental period remain nearly unchanged. In addition, the amplification/de-amplification degree is dependent on the impedance ratio (Figs. 10 (a) and (b)) and soil damping ratio (Figs. 10 (c) and (d)). Moreover, the regions where the FAS values are de-amplified by the soil site in the shorter periods are strongly affected by the fundamental period of the site; as the site fundamental period increases, the de-amplification region becomes wider (Figs. 10 (b) and (c)).

Based on the properties of the effects of the site conditions on the frequency content, the Sa/Spa trends with variation of site conditions can be explained. When the impedance ratio and soil damping are large (Fig. 10 (a)) or the site fundamental period is long (Fig. 10 (c)), the decreases in the components in the short period are more significant than the increases in the components around the site resonance periods. This means that the short-period components of the ground motion decrease. Because the two oscillator transfer functions are different at short periods and are very similar at periods around and greater than the oscillator period T_0 , when the short-period components of the ground motion decrease, the different parts of these two areas will decrease. Therefore, the difference between the two areas in R_{rms} decreases, and Sa/Spa approaches unity, as shown in Figs. 4 (a) and (c). When the soil damping is very small (Fig. 10 (d)), the degree of increase in the components around the site resonance periods is more significant than the degree of decrease of the components over short periods. This implies that the short-period components of the ground motions increase. Therefore, the difference between the two areas in the first term increases, and Sa/Spa increases compared to that for the hard rock site, as shown in Fig. 4 (d).

5. Practical *Sa/Spa* formulation

5.1 Formulation construction

Based on the conclusions derived in Section 4, this section constructs a simple *Sa/Spa* formulation considering the seismological effects for practical use in seismic design. To this end, a large number of function forms are trialed to fit the *Sa/Spa* results in Section 3. Finally, considering a balance between accuracy and simplicity, a simple *Sa/Spa* formulation is proposed, which is expressed as

$$\frac{Sa}{Spa} = 1 + (0.14 \xi^{1.54} \zeta^{-0.57}) T_0^{\xi^{-0.2/(5\sqrt{\zeta}+1)}} \quad (12)$$

where ζ is a parameter reflecting the seismological effects on *Sa/Spa*, which is detailed below. This equation satisfies the boundary condition that, when the oscillator period T_0 and damping ratio ξ decrease to zero, *Sa/Spa* equals unity. Equation (12) also captures the *Sa/Spa* trends with variations in the oscillator period and damping ratio observed in Figs. 1–3 and previous studies using real seismic records (Sadek et al., 2000; Papagiannopoulos et al., 2013).

Because the seismological parameters (especially the moment magnitude M) can significantly influence *Sa/Spa*, as observed above, Eq. (12) incorporates the seismological effects. In principle, the seismological parameters, e.g., the moment magnitude M and site-to-source distance R , should be explicitly included in the *Sa/Spa* formulation. However, most of these parameters are not available in seismic codes, a formulation including such parameters is unrealistic for practical seismic design. Section 4 indicates that the seismological effects on *Sa/Spa* are governed by the frequency content of the ground motion, therefore, a parameter related to the frequency content can be adopted to reflect the seismological effects. Various frequency-content parameters have been defined based on the FAS and response spectra of ground motions (Craifaleanu, 2011). Because only *Spa* is

typically specified in seismic codes, four parameters available from Spa , introduced by Craifaleanu (2011) —specifically, the predominant period T_{gSV} , modified spectral characteristic period T_{mean}^* , modified central period T_{cen}^* , and modified shape factor q^* — are tested. It is found that (not shown here), although the four parameters are closely related to moment magnitude and site-to-source distance, when they are applied to derive the Sa/Spa formulation, the estimated results are not sufficiently stable. Therefore, a new frequency-content parameter ζ is proposed based on Spa to reflect the seismological effects. This parameter is expressed as

$$\zeta = \frac{Spa(6s)}{PGA} \quad (13)$$

where, $Spa(6s)$ and PGA are the values of Spa at 6 s and peak ground acceleration (i.e., Spa at 0 s) for a 5% damping ratio, respectively. It can be seen from Eq. (13) that ζ can be easily obtained from the Spa specified in seismic codes.

It can be easily known from Eq. (13) that ζ is closely related to the frequency content of the ground motion. When the short-period components are predominant and the bandwidth is narrow, $Spa(6)$ is small relative to PGA, thus ζ is small. Conversely, when the bandwidth increases with the long-period component, $Spa(6)$ increases relative to PGA, hence ζ will increase. Therefore, ζ typically increases with the long-period components and can simply reflect the frequency content of the ground motion. In fact, the use of the Spa values at periods 0 and 6 s was determined by trialing numerous values at different periods; it is found that these two values result in Eq. (12) generating the best precision.

The Sa/Spa results calculated by Eq. (12) were compared with those in Section 3; some representative comparisons are shown in Fig. 11. It is noted that Eq. (12) performs very well in the prediction of Sa/Spa , particularly in the period range, $T_0 < 6$ s, which is generally of interest in practical seismic design.

5.2 Comparison with real seismic records

The Sa/Spa results produced by Eq. (12) were further compared with those of real seismic records. As the seismological parameters in Eq. (1), e.g., stress drop and geometric attenuation, for CENA were used for the above analyses, for consistency, the seismic records for the verification were also selected from the regions of CENA. All seismic ground motions in the Pacific Earthquake Engineering Research Center (PEER) database from CENA were searched. A total of 367 seismic records (734 accelerograms) with moment magnitude larger than 4.0 and rupture distance between 10 to 300 km were selected. Because in the regions of CENA, there is a lack of large earthquakes, the largest moment magnitude of the selected records was limited to 5.85. The selected records were then classified into four groups considering a balance in the number of records, as shown in Fig. 12. It is evident that the Sa/Spa results by the proposed formulation are highly congruent with those of the real seismic records.

In addition, the Sa/Spa results obtained in previous studies (Sadek et al., 2000; Song et al. 2007; Mentrasi 2008) are plotted in Fig. 12. It is clear from the figure that the proposed formulation performs better than existing ones that do not incorporate the seismological effects. The Sa/Spa results produced by the existing formulation basically correspond to those with large moment magnitude.

6. Conclusions

To investigate the effects of seismological parameters on the relationship between spectral and pseudospectral accelerations, i.e., Sa and Spa , an approach relating the two spectra including seismological parameters is proposed based on random vibration theory. Using the proposed approach, the effects of the seismological parameters on the relationship between the two spectra are systematically explored and theoretically explained. Finally, a practical

formulation for estimating Sa from Spa considering the seismological effects is constructed.

The main conclusions of this study are summarized as follows:

1. The comparisons with the results obtained from traditional time-series analyses show that the proposed approach is valid.
2. The relationship between the spectral absolute acceleration Sa and pseudospectral acceleration Spa is significantly affected by variations in the seismological parameters. The spectral absolute acceleration Sa approaches the pseudospectral acceleration Spa with increasing moment magnitudes and source-to-site distances, particularly for longer oscillator periods.
3. The effects of site conditions on the relationships between the two spectra are dependent on specific site parameters, and the fundamental period of the site has a significant effect on the Sa/Spa ratio.
4. The key factor governing the seismological effects is the frequency content of the ground motions. The observed phenomena can be reasonably explained by the variation of the frequency content with the seismological parameters.
5. The proposed simple Sa/Spa formulation is demonstrated to perform very well for a wide range of periods utilized in practical seismic design by comparing with the results of real seismic records.

Acknowledgments

This study was partially supported by the National Natural Science Foundation of China (Grant No. 51738001); the support is gratefully acknowledged. The authors are also grateful to Baojian Hang for help with illustration of the figures in this paper.

Declarations

Funding The research leading to these results received funding from the National Natural Science Foundation of China (Grant No. 51738001).

Conflicts of interest/Competing interests The authors have no conflicts of interest to declare that are relevant to the content of this article.

Data availability All data generated or analyzed during this study are included in this published article.

Code availability Available upon request.

Authors' contributions Haizhong Zhang: conceptualization, methodology, writing-original draft preparation, investigation. Yan-Gang Zhao: data curation, visualization, supervision, writing-reviewing and editing.

References

Boore, D.M., 1983. Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra. *Bulletin of the Seismological Society of America* **73**, 1865–1894.

Boore, D.M., 2003. Simulation of ground motion using the stochastic method. *Pure and Applied Geophysics* **60**, 635–676.

Boore, D.M., 2005. SMSIM—Fortran programs for simulating ground motions from earthquakes: version 2.3, U.S. Geol. Surv. Open-File Rept. OFR 96-80-A, Menlo Park, California.

Boore, D.M. and Thompson, E.M., 2014. Path durations for use in the stochastic-method simulation of ground motions. *Bulletin of the Seismological Society of America* **104**, 2541–2552.

Boore, D.M. and Thompson, E.M., 2015. Revisions to some parameters used in stochastic-method simulations of ground motion. *Bulletin of the Seismological Society of*

447 *America* **105**, 1029–1041.

448 Cartwright, D.E. and Longuet-Hlggms, M.S., 1956. The statistical distribution of the maxima
 449 of a random function. *Proceedings of the Royal Society of London. Series A, Mathematical*
 450 *and Physical Sciences* **237**, 212–223.

451 Chopra, A.K., 2007. Elastic response spectrum: A historical note. *Earthquake Engineering and*
 452 *Structural Dynamics* **36**, 3–12.

453 Davenport, A.G., 1964. Note on the distribution of the largest value of a random function with
 454 application to gust loading. *Proceedings of the Institution of Civil Engineers* **28**, 187–196.

455 Jenschke, V.A., Clough, R.W., and Penzien, J., 1964. Analysis of earth motion accelerograms.
 456 Report No. SESM 64-1, University of California, Berkeley, CA.

457 Jenschke, V.A., Clough, R.W., and Penzien, J., 1965. Characteristics of strong ground motions.
 458 *Proceedings of the Third World Conference on Earthquake Engineering* **III**, New Zealand,
 459 125–142.

460 Lin, Y.Y. and Chang, K.C., 2003. A study on damping reduction factor for buildings under
 461 earthquake ground motion. *Journal of Structural Engineering* **129(2)**, 206–214.

462 Mentrasti, L., 2008. Estimate of spectral and pseudo-spectral acceleration proximity.
 463 *Engineering Structures* **30(9)**, 2338–2346.

464 Nigam, N. and Jennings, P., 1969. Calculation of response spectra from strong-motion
 465 earthquake records. *Bulletin of the Seismological Society of America* **59(2)**, 909–922.

466 Newmark, N.M. and Rosenblueth, E., 1972. Fundamentals of earthquake engineering.
 467 Englewood Cliffs: Prentice Hall.

468 Papagiannopoulos, G.A., Hatzigeorgiou, G.D., and Beskos, D.E, 2013. Recovery of spectral
 469 absolute acceleration and spectral relative velocity from their pseudo-spectral counterparts.
 470 *Earthquakes and Structures* **4 (5)**, 1–000.

471 Pu, W., Kasai, K., Karoki, E.K., and Huang, B., 2016. Evaluation of the damping modification

- factor for structures subjected to near-fault ground motions. *Bulletin of Earthquake Engineering* **14**, 1519–1544.
- Sadek, F., Mohraj, B., and Riley, M.A., 2000. Linear static and dynamic procedures for structures with velocity-dependent dampers. *Journal of Structural Engineering* **126**(8), 887–895.
- Song, J., Chu, Y.L., Liang, Z., and Lee, G.C., 2007. Estimation of peak relative velocity and peak absolute acceleration of linear SDOF systems. *Earthquake Engineering & Engineering Vibration* **6**(1), 1–10.
- Vanmarcke, E.H., 1975. On the distribution of the first-passage time for normal stationary random processes. *Journal of Applied Mechanics* **42**, 215–220.
- Veletsos, A.S. and Newmark, N.M., 1964. Response spectra for single-degree-of freedom elastic and inelastic systems. Report No. RTD-TDR-63-3096, vol. III, Air Force Weapons Laboratory, Albuquerque, NM.
- Wang, X. and Rathje, E.M., 2016. Influence of peak factors on site amplification from random vibration theory based site-response analysis. *Bulletin of the Seismological Society of America* **106**, 1–14.
- Zhang, D., Bai, Y., and Gao, J., 2016. Probe into several important concepts in Chinese current seismic response spectra. *Journal of Building Structures* **37**, 1000–6869. (In Chinese)
- Zhang, H.Z., Saito, T., and Zhao, Y.G., 2017. Simple calculation method of seismic motion amplification ratio corresponding to fundamental period. *Journal Structural and Construction Engineering AIJ* **82**, 597–604. (In Japanese)
- Zhang, H.Z. and Zhao, Y.G., 2018. A simple approach for estimating the first resonance peak of layered soil profiles. *Journal of Earthquake and Tsunami* **12**, 185005.
- Zhang, H.Z. and Zhao, Y.G., 2020. Damping modification factor based on random vibration theory using a source-based ground-motion model. *Soil Dynamics and Earthquake*

Figure 1. Variation of spectral ratio Sa/Spa with oscillator damping ratio ξ for cases with moment magnitude M and site-to-source distance R of (a) $M = 5, R = 20$ km, (b) $M = 7, R = 20$ km, (c) $M = 5, R = 126.20$ km, and (d) $M = 7, R = 126.20$ km.

Figure 2. Variation of spectral ratio Sa/Spa with moment magnitude M for cases with oscillator damping ratio ξ and site-to-source distance R of (a) $\xi = 0.3, R = 20$ km, (b) $\xi = 0.5, R = 20$ km, (c) $\xi = 0.3, R = 126.20$ km, and (d) $\xi = 0.5, R = 126.20$ km.

(a)

(b)

(c)

(d)

Figure 3. Variation of spectral ratio Sa/Spa with site-to-source distance R for cases with oscillator damping ratio ξ and moment magnitude M of (a) $\xi = 0.3, M = 5$, (b) $\xi = 0.5, M = 5$, (c) $\xi = 0.3, M = 7$, and (d) $\xi = 0.5, M = 7$.

(a)

(b)

(c)

(d)

Figure 4. Variation of spectral ratio S_a/S_{pa} with site parameters for soil sites with fundamental period T_s impedance ratio I_p , and soil damping ratio h of (a) $T_s = 0.5$ s, $I_p = 0.5$, $h = 0.1$, (b) $T_s = 0.5$ s, $I_p = 0.2$, $h = 0.1$, (c) $T_s = 1$ s, $I_p = 0.2$, $h = 0.1$, and (d) $T_s = 0.5$ s, $I_p = 0.2$, $h = 0.01$.

Figure 5. Variation of the first term with moment magnitude M for cases with oscillator damping ratio ζ and site-to-source distance R of (a) $\zeta = 0.3, R = 20$ km, (b) $\zeta = 0.5, R = 20$ km, (c) $\zeta = 0.3, R = 126.20$ km, and (d) $\zeta = 0.5, R = 126.20$ km.

Figure 6. Variation of the second term with moment magnitude M for cases with oscillator damping ratio ζ and site-to-source distance R of (a) $\zeta = 0.3$, $R = 20$ km, (b) $\zeta = 0.5$, $R = 20$ km, (c) $\zeta = 0.3$, $R = 126.20$ km, and (d) $\zeta = 0.5$, $R = 126.20$ km.

Figure 7. Comparison of oscillator transfer functions for spectral and pseudospectral accelerations for two oscillator damping ratios (5% and 30%).

Figure 8. Comparison of oscillator transfer functions for spectral and pseudospectral accelerations for different oscillator periods (0.1 s and 1 s).

Figure 9. Variation of the frequency content of ground-motion FAS with (a) moment magnitude and (b) site-to-source distance.

Figure 10. Variations of the frequency content of ground-motion FAS with site conditions for (a) $T_s = 0.5$ s, $I_p = 0.5$, $h = 0.1$, (b) $T_s = 0.5$ s, $I_p = 0.2$, $h = 0.1$, (c) $T_s = 1$ s, $I_p = 0.2$, $h = 0.1$, and (d) $T_s = 0.5$ s, $I_p = 0.2$, $h = 0.01$.

Figure 11. Comparisons of Sa/Spa results from the proposed approach, time-series analysis, and Eq. (12) for the cases with oscillator damping ratio ξ and site-to-distance R of (a) $\xi = 0.3$, $R = 20$ km, (b) $\xi = 0.5$, $R = 20$ km, (c) $\xi = 0.3$, $R = 126.20$ km, and (d) $\xi = 0.5$, $R = 126.20$ km.

Figure 12. Comparisons of Sa/Spa results from Eq. (12), real seismic records, and the formulations of Sadek et al. (2000), Song et al. (2007), and Mentrasti (2008) for the cases with oscillator damping ratio ξ of (a) $\xi = 0.1$, (b) $\xi = 0.2$, (c) $\xi = 0.3$, and (d) $\xi = 0.4$.