Echocardiography segmentation by Fractional Differential and Improved Canny, analysis by Fourier Descriptor

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Abstract. The omnidirectional M-mode echocardiogram provides a new method for human heart functional analyses. In this article, to sharpen object edges, we designed image processing kernel based on Fractional differential for image enhancement. After that, the contour of the left ventricle in a short axis is first extracted using both an improved Canny edge detection algorithm and the gray level searching algorithm in the radial direction as auxiliary. The modified Canny edge detection algorithm with the matching method between adjacent frames then is adopted for the subsequent frames to extract the left ventricular contours. The non-functional movements in the B-ultrasonic plane are determined by using the movement extracting method based on Fourier descriptors and the mass center with the inertia axis method, and the movements are removed from a compound motion. The Fourier descriptors are applied to get a series of image contour curves with the principal translation and rotation. Hence the curve of the cardiac motion can accurately show functional movements in any location of the heart. Using our technique, we can reduce multi-lines and excursion, as well as correct the omnidirectional M-mode echocardiography.

Keywords: B-ultrasonic image; heart; Cardiac motion; Fractional differential; Canny; Fourier descriptor.

1. Introduction

The heart is a vital organ in a human body and its pumping motion closely relates to its function. It is necessary to analyze the heart motion in the diagnosis of the heart disease ^[1-2]. The traditional M-mode echocardiography is used to analyze the myocardium function by displaying the movement-curve of the heart structure on the sampling line and by measuring various parameters of the wall motion (such as wall thickness, ventricular diameter, wall short-axis shortening fraction, ejection fraction, and cardiovascular exercise time, etc.) ^[3-4]. The main weakness of the detection is that the incidence angle of the ultrasound is not consistent with the direction of the movement, so obtained velocity error is great. The Anatomic M-mode (AMM) echocardiography is for post-processing an original image to obtain a motion curve of the interested local myocardium in wall or with the slice captured through the location of a sample line in the image ^[5-6]. The AMM echocardiography can only be set with three sampling lines in an image, but it

cannot get the curves of the myocardial motion of the walls during the same cardiac cycle. Our omnidirectional M-mode echocardiogram system improved the AMM echocardiography system and imaged the myocardial motion of different parts with a number of motion curves synchronously during the same cardiac cycle, and the main work is carried out based on the improved Canny edge detector and the Fourier descriptors. However, with the effect of the non-functional movements in the heart caused by breathing, blood surging and mutual effects among organs, a heart wall does not always move along a sample line. Obviously, the obtained gray level versus time waveform cannot reflect the functional movements of the some parts of the heart accurately^[7].

In this paper, to make keep edges while smooth away noises, we proposed a new Fractional differential template with a circle data structure, the coefficients in a 5x5 template are different in different places. The studied object extraction method, based on the improved Canny edge detection algorithm and the gray scale search algorithm, extracts the left ventricular contours of the short axis, and corrects the non-functional movements of the heart by obtaining the cardiac translation and ventricular deflection in the B-plane by using the movement extracting algorithm based on the Fourier descriptors. The aim is to improve the accuracy of the acquired cardiac waveform graph and then to increase the precision of the obtaining cardiac motion parameters. While for the B-mode ultrasound images of the cardiac short-axis, the motion tracking based on the ventricular contour is used to correct the non-functional movements of the heart in this study.

There has been extensive literature in the field of this research topic. Mihaela constructed a 3D MR image-based computer model of pathologic hearts, augmented with histology and optical fluorescence imaging to characterize the action potential propagation^[8]; and as the main information sources of magnetic resonance images, the detailed information can be discovered in the recent two review articles^[9-10].

Recently, some researchers were interested in using CT technique to make 3D information analysis^[11-14]. In addition to the magnetic resonance and CT images, Ultrasound images can also be used to do diagnosis in real time, and they are intuitive, no damage, timely accessed and with other characteristics. The ultrasound images can provide the comprehensive information for physicians to understand the anatomical structure and dynamic process of the heart. Besides, the method based on ultrasound images is easy and flexible to operate, and with the low cost in the diagnosis of the heart diseases. So it is still one of the most important ways to check the heart disease^[15]. The different model based algorithms or methods have been developed for the heart movement tracing or mapping in real time^[16-17]. In addition, it can also record the M-mode ultrasound images of the multiple cardiac cycles in any structure in the heart^[18].

In the B-plane, the non-functional movements include the translation of the cardiac structure and deflection in the direction of the ventricular long axis. The motion tracking, image registration and other functions were commonly used to correct the non-functional movements of the heart^[16-18]. The image registration was usually used for the medical images from different patients

or from the same patient but in different times, which could detect the similarity of images but the result was not satisfactory for the cardiac image sequences with the ventricular systolic and diastolic movements^[19]. The image tracking based on features was to track the whole or part movements in images to achieve the registration results. Although a genetic algorithm was adopted to track feature points for correcting the non-functional movements of the heart in the B-mode ultrasound images, it was difficult to find out all the points^[20]. Instead, using other image region tracing and detection algorithms could also be valuable, such as the Wavelet based algorithm^[21] and the recursive thresholding algorithm^[22].

2. Image enhancement by a new algorithm on Fractional differential

As usual, before image segmentation or object extraction, the image preprocessing is needed^[23]. To increase the detailed discontinuity information in our images, we made a new algorithm based on Fractional differential. Normally, the medical images are enhanced by using 1st or 2nd order differential algorithms such as Sobel, Robert, Laplacian and Canny etc. Although they can sharpen the edges and textures in images more or less, but meanwhile they will also increase noise very much, which make further image segmentation hard. As instance, the high pass operators often fail to enhance the vague or weak edges in images^[24-25]. The fractional difference (order 0.3-0.6) can detect the vague edges and textures, and keep noise lower; hence, it is suitable for the complicated medical images. The key task is how to design a simple and applicable fractional differential kernel.

In a digital image, there is a large relevance among the pixel grey scales in the neighborhood of the detecting pixel. The pixels are auto-correlated, their fractal geometric information show up the properties of complex textural features, and the fractional differential is one of Fractal Geometry Mathematical Foundations. It is natural for one to think about what effect is gained when applying the fractional differential to detect textural features in an image. In this paper, a new fractional differential algorithm, which can improve detailed texture information remarkably, is studied.

For Fractional differential, we use Grümwald-Letnikov definition^[24-25].

For $\forall v \in R$, if signal $s(t) \in [d, t]$ ($d < t, d \in R, t \in R$), the integral part $[v]$ meets the condition $m+1 < m \in Z$, Z is for the continuous derivative of the integer set order; if $v > 0$ and m is equal to $[v]$, then v order derivative can be:

$${}_d D_t^v s(t) = \lim_{h \rightarrow 0} s_h^v(t) = \lim_{\substack{h \rightarrow 0 \\ nh \rightarrow t-d}} h^{-v} \sum_{r=0}^n C_r^{-v} s(t-rh) \quad (1)$$

Where, $C_r^{-v} = (-v)(-v+1)\cdots(-v+r-1)/r!$

If the duration $s(t)$ is $t \in [d, t]$, the signal duration $[d, t]$ can be divided equally in the unit equal interval $h=1$:

$$n = \left\lceil \frac{t-d}{h} \right\rceil^{h=1} = [t-d] \quad (2)$$

In this way, the ν order fractional order of the differential expression in one dimensional signal $s(t)$ can be deducted as:

$$\begin{aligned} \frac{d^\nu s(t)}{dt^\nu} &\approx s(t) + (-\nu)s(t-1) + \frac{(-\nu)(-\nu+1)}{2}s(t-2) + \\ &+ \frac{(-\nu)(-\nu+1)(-\nu+2)}{6}s(t-3) + \dots + \frac{\Gamma(-\nu+1)}{n!\Gamma(-\nu+n+1)}s(t-n) \\ &= d_0s(t) + d_1s(t-1) + d_2s(t-2) + d_3s(t-3) + \dots + d_ns(t-n) \end{aligned} \quad (3)$$

These $n+1$ non-zero coefficient values are in order as:

$$\begin{cases} a_0 = 1 \\ a_1 = -\nu \\ a_2 = (-\nu)(-\nu+1)/2 = (\nu^2 - \nu)/2 \\ a_3 = (-\nu)(-\nu+1)(-\nu+2)/6 = (-\nu^3 + 3\nu^2 - 2\nu)/6 \\ a_4 = (-\nu)(-\nu+1)(-\nu+2)(-\nu+3)/24 = (\nu^4 - 6\nu^3 + 11\nu^2 - 6\nu)/24 \\ \dots \\ a_n = \Gamma(-\nu+1)/n!\Gamma(-\nu+n+1) \end{cases} \quad (4)$$

In a traditional Tians 5x5 template of Fractional calculus, as Fig.1 shown, except for the central point, the other points have only two different coefficients “ $1 \rightarrow a_1$ ” and “ $2 \rightarrow a_2$ ”, no matter it has a square data structure or a circle data structure. The values of a_1 and a_2 are related to Fractional calculus parameters in Eq.(4). For each detecting pixel, the detection result can be the convolution result of square data structure divided by the coefficient sum S_1 ; and can be the convolution result of circle data structure divided by the coefficient sum S_2

Fig.1. Traditional Tians 5x5 templates with square and circle data structures.

The coefficient sum in Fig.1(b):

$$S_1 = 16a_2 + 8a_1 = 16\left(\nu - \nu^2\right)/2 + 8\nu = 16\nu - 8\nu^2 \quad (5)$$

If we set: $\nu = 1$, then, $S_1 = 16\nu - 8\nu^2 = 16 - 8 = 8$, when $\nu = 0.5$, then,

$$S_1 = 16\nu - 8\nu^2 = 8 - 2 = 6$$

The coefficient sum in Fig.1(c):

$$S_2 = 12a_2 + 8a_1 = 12(v - v^2) / 2 + 8v = 14v - 6v^2 \quad (6)$$

If we set: $v = 1$, then, $S_2 = 14v - 6v^2 = 14 - 6 = 8$, when $v = 0.5$, then,

$$S_2 = 14v - 6v^2 = 7 - 1.5 = 5.5$$

It is very different to the Tians template, and the new algorithm has the 5x5 templates as shown in Fig.2. In the new template, with the square data structure, there are 5 different coefficients, and with the circle data structure, there 4 different coefficients. The different coefficient values are expressed in Eq.(7), and the coefficient sums in the square data structure and the circle data structure are S_3 (Eq.(8)) and S_4 (Eq.(9)) respectively.

Fig.5. New Fractional differential 5x5 templates with square and circle data structures.

$$a_1 = -v; a_2 = a_1 / \sqrt{2} = v / \sqrt{2}; a_3 = a_2 = (v - v^2); a_4 = a_1 / 2 = v / 2; a_5 = a_2 / 2\sqrt{2} = (v - v^2) / 2\sqrt{2}. \quad (7)$$

The coefficient sum in Fig.2(b):

$$\begin{aligned} S_3 &= c + 4b_1 + 4b_2 + 4b_3 + 8b_4 + 4b_5 = c + 4v + 4v / \sqrt{2} + 4(v - v^2) + 4v + 2(v - v^2) / \sqrt{2} \\ &= c + 12v + 6v / \sqrt{2} - 4v^2 - 2v^2 / \sqrt{2} = c + (12 + 6 / \sqrt{2})v - (4 + 2 / \sqrt{2})v^2 \approx c + 16.26v - 5.42v^2 \end{aligned} \quad (8)$$

If $v = 1$, then, $S_3 = 10.84$, $c = -S_3$; if $v = 0.5$, then, $S_3 \approx 9.49$, we take $c = -9.49$

The coefficient sum in Fig.2(c):

$$\begin{aligned} S_4 &= 4b_1 + 4b_2 + 4b_3 + 8b_4 = 4v + 4v / \sqrt{2} + 4(v - v^2) + 4v \\ &= 12v + 4v / \sqrt{2} - 4v^2 = (12 + 4 / \sqrt{2})v - 4v^2 \approx 14.84v - 4v^2 \end{aligned} \quad (9)$$

If $v = 1$, then, $c = 1$, $S_4 = 10.84$ $c = -S_4$; if $v = 0.5$, then, $S_4 = 8.42$, we take $c = -8.42$.

3. Cardiac ventricular contour extraction and description

For the object tracking, there is a huge number of algorithms ^[26-27], generally, they have been studied completely or partly either based on similarity or based on discontinuity, which is depending on image resolution ^[28] and the applications ^[29]. In our application, the algorithm is proposed based on discontinuity, i.e, the improved Canny edge detector + Fractional differential image enhancement template. The details for the improved Canny edge detector is described as the follows.

3.1. Improved Canny edge detection algorithm

In the B-mode ultrasound image in the cardiac short-axis, the radial variation of the heart chamber's contour reflects the function properties of the systolic and diastolic of the heart, and the overall translation and rotation in the B-plane of the contour reflect the non-functional movements of the heart. At present it is possible to get the cardiac ventricular contours with an improved boundary extraction algorithm. The time interval between two standard DICOM images with 75 frames per second is approximately 13ms, which is much shorter than the cardiac cycle 0.75s~1s of a normal person. So there is a strong spatial correlation between frames. For this reason, one can divide the extraction procedure of the ventricular contours into two stages: (1) the extraction of the ventricular contour in the first frame; and (2) the extraction of the ventricular contours in the subsequent frames. After the comparison among experiments with a variety of boundary extraction algorithms, it proves that a special edge detection based object tracing algorithm^[20] with a number of post processing functions are definitely needed^[24]. In this paper, an improved Canny edge detection algorithm^[30-31] is studied by a modified gray scale search algorithm^[30] based on radial, and it is adopted for the first frame to extract the ventricular contour. An improved Canny edge detection algorithm with the matching method between adjacent frames is adopted for the subsequent frames to get the left ventricular contours, which is described as follows.

In the Canny algorithm, the one-dimensional Gaussian filter is firstly discussed because the two-dimensional Gaussian filter can be superimposed by two one-dimensional Gaussian filters. And the mathematical expression is:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-x^2 / 2\sigma^2), \quad (10)$$

Where, σ^2 denotes the extent of the smoothed image. If it is too small, the noise cannot be eliminated completely. On the contrary, the false edges are generated. There are two ways to solve the contradiction. One is to use different filters and different smoothing parameters to preprocess images, then to do edge detection, and finally to analyze the results for synthesizing the image edges. The alternative way is to use different smoothing methods for different parts of an image based on different noise and edge types as described in the following.

The Gaussian filter of a signal $F(x)$ can be represented as:

$$F_g(x) = F(x) + F(x)\sigma^2 / 2 + \dots + F^{2m}(x)\sigma^{2m} / \prod_{p=1}^m 2p + \dots, \quad (11)$$

where $F_g(x)$ is the filtering result, and $F^{2m}(x)$ is the $(2m)^{th}$ derivative of $F(x)$,

$\prod_{p=1}^m 2p = 2 \times 4 \times \dots \times 2m$. The expression omits the higher order derivatives as:

$$F_g(x) \cong F(x) + F(x)\sigma^2 / 2. \quad (12)$$

The error of the pretreatment is defined as:

$$|F_g(x) - F(x)| \leq \varepsilon, \quad (13)$$

Therefore,

$$\sigma^2(x) = 2\varepsilon / |F(x)|. \quad (14)$$

The latter method is used to improve: assuming that the available signal $S(x)$ is equal to the original signal $S(x)$ plus noise $N(x)$:

$$S(x) = F(x) + N(x). \quad (15)$$

Generally, the human visual system is more sensitive to edges and noise of flat portions. When the second derivative of edges is fairly large, σ^2 must be as small as possible. When the second derivative of the flat portions is fairly small, σ^2 must be as large as possible. The equation (16) is used instead of equation (10),

$$\sigma_s^2(x) = \sigma_f^2(x) + \sigma_n^2. \quad (16)$$

Here, $\sigma_s(x)$ is the smoothing parameter of the obtained signal, $\sigma_f(x)$ is the smoothing parameter of the original signal, and σ_n is the smoothing parameter of noise.

Fig.3 Contour detection by improved Canny operator and closed by gray searching and post processing algorithms.

According to the above discussion, we can have:

$$\sigma^2(x) = k\sigma_n^2 / \sigma_s^2(x), \quad (17)$$

Where, k is a scale factor. When x is in the image edges, $F(x)$ changes very quickly, therefore, $\sigma_f^2(x) \gg \sigma_n^2$, that is: $\sigma^2(x) \cong k\sigma_n^2 / \sigma_f^2(x)$. When x is in the flat section of the image, $\sigma_f^2(x) \ll \sigma_n^2$, $\sigma^2(x) \cong k$, we can see that σ^2 is changed with x change. So in an image, σ^2 is changed with the change of (x, y) .

According to the features, a different σ^2 in a different position is applied to realize the purpose. Figure 3 shows the example of the algorithm performance: the original image is detected by the improved Canny edge detector, meanwhile the ventricular contour is traced by a boundary tracing algorithm, then the two results are fused, finally a number of post processing functions are applied to obtain the final result. The combination of the two methods for extracting the ventricular contour can have the better result.

The images in Figs.4-6 are from the left ventricular short axis in the B-mode image sequences of a man. The B-mode images can show the heart movement of the patient in three cardiac cycles. The cycle length is in 2.5 minutes and the number of the total frames is 200. All the analysis data in this study are from a group of B-mode images.

The next important thing is to determine two thresholds for improving the Canny edge detector, it can be described as: the cross-entropy is a measure of the information difference of the two probability distributions ($P = \{p_1, p_2, \dots, p_N\}$, $Q = \{q_1, q_2, \dots, q_N\}$), its definition is:

$$D(P, Q) = \sum_{i=1}^N p_i \ln \frac{p_i}{q_i}. \quad (18)$$

Its symmetric form is called the symmetric cross-entropy and the definition can be defined as

$$D(P : Q) = \sum_{i=1}^N p_i \ln \frac{p_i}{q_i} + \sum_{i=1}^N q_i \ln \frac{q_i}{p_i}. \quad (19)$$

The threshold determination is often made according to the principle of the suitable optimization in a thresholding algorithm. An image is divided into two classes: objects (o) and background (b), and each of the classes are supposed to have a normal distribution, in which the parameters can be obtained from the histogram of an original image:

$$p(g/i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left\{-\frac{g - \mu_i(t)^2}{2\sigma_i^2(t)}\right\}, i = o, b, \quad (20)$$

Where, t is a threshold, g is a gray scale value (0-255). The variances in the two categories are estimated respectively as follows:

$$\sigma_o^2(t) = \frac{1}{P_o} \sum_{g=0}^t h(g)(g - \mu_o(t))^2, \quad (21)$$

$$\sigma_b^2(t) = \frac{1}{P_b} \sum_{g=t+1}^L h(g)(g - \mu_b(t))^2, \quad (22)$$

Where, the priori probability of the object class is $P_o = \sum_{g=0}^t h(g)$, the priori probability of the

background class is $P_b = \sum_{g=t+1}^L h(g)$, their means within clusters are $u_o(t) = \frac{1}{P_o} \sum_{g=0}^t gh(g)$ and

$u_b(t) = \frac{1}{P_b} \sum_{g=t+1}^L gh(g)$ respectively. Here, t is a threshold, g is a grey value, and L is a grey

upper bound. Then the posterior probability is obtained by using the Bayesian probability:

$$p(i / g) = P_i p(g / i) / \sum_{i=o,b} P_i p(g / i). \quad (23)$$

The difference between the objects and the background in an image is measured by using the cross-entropy, combined with the Bayesian judgment ^[32], and the difference might be represented by the average value of a cross-entropy for every pixel in the original image P . For the object and background regions with posterior probability $p(o|s)$ and $p(b|s)$, the optimal threshold via the maximum posterior probability of pixels in the different regions is obtained. The Inter-class cross-entropy based on posterior probability of the single pixel is

$$D(o : b; s) = \frac{1}{3} [1 + p(o|s)] \ln \frac{1 + p(o|s)}{1 + p(b|s)} + \frac{1}{3} [1 + p(b|s)] \ln \frac{1 + p(b|s)}{1 + p(o|s)}. \quad (24)$$

The difference between classes is:

$$D(o : b) = \sum_{s \in o} \frac{p(s)}{P_o} D(o : b; s) + \sum_{s \in b} \frac{p(s)}{P_b} D(o : b; s). \quad (25)$$

Replacing pixel gray scale s with the gray value g for simplifying the calculation is for replacing a probability distribution with a gray scale histogram. The equation (25) can be re-written as:

$$D(o : b; g) = \sum_{g=0}^T \frac{h(g)}{P_o} D(o : b; g) + \sum_{g=T+1}^L \frac{h(g)}{P_b} D(o : b; g), \quad (26)$$

Where, L is a grey value for upper bound, and T is a grey value threshold.

To obtain the optimal threshold T^* based on the maximum cross-entropy between classes, it can be achieved through a searching operation:

$$D(o : b; T^*) = \max_T D(o : b; T) \quad (27)$$

The edge detection and post processing results are displayed in Fig.1, and the three sequences of processed images are illustrated in Figs.4-6.

3.2. Non-functional movement parameters of ventricular contours

The heart rate of a normal adult is about 75 times per minute and the average cardiac cycle is about 0.8 second. In a cardiac cycle, the systole is 0.1 second and the diastole is 0.7 second for the atrial. And for the ventricle, the systole is 0.3 second and the diastole is 0.5 second. In each period of the cardiac systole and diastole, both the direction and the force for the ventricle are various, so that the overall displacement of each frame and the deflection in the B-mode irradiation plane are different. As it can be seen from the contour map, the obtained contour is of irregular curved, which is called elliptic. It contains the higher harmonic components, which is caused by the movement of the ventricle and the existence of the papillary muscle with the noise in the B-mode ultrasound images. With the systole and diastole of the heart, the size of the elliptic varies constantly. The changes in the direction of the shaft diameter reflect the functional movements of the heart – the systole and diastole. And the overall displacement and deflection reflect the non-functional movements of the heart. As the Fourier descriptors and the centroid principal axes algorithm^[33-34] can both describe the overall movement of the object and ignore the details (high frequency components), the Fourier descriptors and the classical centroid principal axes algorithm^[35-37] are adopted in analyzing and calculating the contour of the left ventricle to obtain the overall movement data.

Suppose the abscissa of an image is in the real axis, the ordinates of an image are the imaginary axes and the ventricular contour is composed of N points. Any point in the ventricular contour is around a circle curve, which can be used to get a one-dimensional complex sequence:

$$s(k) = x(k) + jy(k) \quad k = 0, 1, \dots, N-1. \quad (28)$$

The Discrete Fourier Transform (DFT) of $s(k)$ is:

$$S(u) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) \exp[-j2\pi uk / N], \quad u = 0, 1, \dots, N-1 \quad (29)$$

Specifically, u is the frequency, $S(u)$ is the spectral coefficient of $s(k)$. From the nature of the Fourier transform, the high-frequency components of the Fourier transform are for the some detailed information: the low-frequency components of that are for the general shape of the contour and the DC components correspond to the centroid position surrounded by the contour can be presented as:

$$S(0) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) = \frac{1}{N} \sum_{k=0}^{N-1} [x(k) + jy(k)] \quad (30)$$

With the inverse Fourier transform, the contour curve $s(k)$ can be reconstructed by the Fourier coefficients $S(u)$:

$$S(k) = \frac{1}{N} \sum_{u=0}^{N-1} S(u) \exp\left[\frac{j2\pi uk}{N}\right] \quad u = 0, 1, \dots, N-1. \quad (31)$$

As the high-frequency components of the Fourier transform correspond to the details of the contour and the low-frequency components of the Fourier transform related to the shape of the contour, we can use the first M Fourier coefficients based on the low-frequency components to rebuild the boundary to get an approximate contour with a closed curve:

$$\hat{s}(k) = \frac{1}{N} \sum_{u=0}^{M-1} S(u) \exp\left[\frac{j2\pi uk}{N}\right], \quad u = 0, 1, \dots, M-1. \quad (32)$$

In light of the reconstruction experiment of the contour, $\hat{s}(k)$ can make a smooth contour without gaps, and ignore the edge details with tiny fluctuations. It can also reduce the effect from the papillary muscle near the ventricle, and decrease the impact of the noise and the edge error during extracting the contour when $M=40$.

In order to eliminate the influence of the noise, the centroid of the ventricular contour is calculated by re-constructing boundary. The ventricle contours of two continuous frames are set as $s_i(k)$ and $s_{i+1}(k)$, and then the centroid position of the i^{th} frame is as follows:

$$x_{i0} = \frac{1}{N} \sum_{k=0}^{N-1} \hat{x}(k), \quad y_{i0} = \frac{1}{N} \sum_{k=0}^{N-1} \hat{y}(k). \quad (33)$$

If the translation of the contour in the plane is $(\Delta x, \Delta y)$, then

$$\Delta x = x_{(i+1)0} - x_{i0}, \quad \Delta y = y_{(i+1)0} - y_{i0}. \quad (34)$$

When the size of the ventricular contour varies with the cardiac systole and diastole, the contour curve can be expressed as $s_c(k) = C \cdot s(k)$, where, C is a constant. The Fourier transform $S_c(u) = C \cdot S(u)$ is that the Fourier coefficients are magnified C times. Thus, the impact of the ventricular size change caused by the heart systole and diastole can be eliminated by the normalized Fourier coefficients.

The parameter k_0 is set as shift to the sequence origin for different starts. When the start point of the contour varies, we can get $s_p(k) = s(k - k_0)$. And the Fourier transform of $s_p(k)$ is:

$$S_p(u) = S(u) \exp(-j2\pi uk_0 / N). \quad (35)$$

The coordinate origin is positioned at the centroid of the ventricular area. The contour curve turned around by angle θ in a space domain can be expressed as $s_r(k) = s(k) \exp(j\theta)$. Its Fourier transform is:

$$S_r(u) = S(u) \exp(j\theta). \quad (36)$$

The Fourier coefficients after rotation are equal to that of the origin Fourier coefficients times $\exp(j\theta)$. To be precise, the phases of different frequency components in the Fourier coefficients are increased by an angle θ . So the rotation of the boundary can be detected by the phase changes of the Fourier coefficients.

Eliminating the DC component by the equation (26), the ratio of the first M Fourier coefficients from contours of two continuous images is:

$$\frac{S_{j+1}(u)}{S_j(u)} = \frac{|S_{j+1}(u)| \exp[i\varphi_{j+1}(u)]}{|S_j(u)| \exp[i\varphi_j(u)]} = C \cdot \exp[i\Delta\varphi(u)] \quad u = 1, 2, \dots, M-1. \quad (37)$$

In Equation (37), C is the amplitude ratio of the Fourier coefficients, which respects the scale of the boundary. And $\Delta\varphi(u)$ represent the phase changes of the Fourier coefficients with different frequencies.

The phase changes of the Fourier coefficients for the boundaries of two continuous images can be got by the Equations (25), (26) and (27).

$$\Delta\varphi(u) = \theta + (-j2\pi k_0 / N), \quad (38)$$

Suppose: $z = -j2\pi k_0 / N$, it has:

$$\Delta\varphi(u) = \theta + zu \quad u = 1, 2, \dots, M-1. \quad (39)$$

The phase changes of the Fourier coefficients in different frequencies are known. The intercept θ and slope z of the straight line can be determined by Hough transform from the linear equation (24). And the point in $u, \Delta\varphi(u)$ space corresponds to a straight line in the space with parameters θ, z [30].

$$\theta = -zu + \Delta\varphi(u). \quad (40)$$

The lines in the parameter θ, z space corresponding to the collinear points in $u, \Delta\varphi(u)$ space intersect each other in one point. The values of the intersection θ, z are the intercept, and the slope of the linear Equation (30) is determined. If the intercept in equation (29) can be obtained, the rotation angle of the boundary curve can be determined in a plane [36-37].

The translation of a contour in a plane can be detected by the DC component changes in the Fourier transform of the ventricular contour. The contour rotation can be detected by the phase

changes of the Fourier coefficients and the Hough transform.

4. Experiments and discussion

In the experimental, all the images are preprocessed by a new kernel of Fractional differential, then they are segmented by the improved Canny edge detector, finally they are analyzed by using Fourier descriptor. The details are as follows.

4.1. Video processing and analysis

The comparative data are corresponding to the short axis B-images for the left ventricular, the numbers of which are in 11-20, 22, 24, 26, 28, 30, 32, 38, 40, 42, 44, 46, or 48-59. The frame rate of the B-images is 80 frames per second. The average heart rate of patients is 74 beats per minute. Note that a cardiac cycle corresponds to about 65 images. The images from the 11th frame to the 38th frame are in the stage of the ventricular systole. And the images from the 40th frame to the 58th frame are in the stage of the ventricular diastole. The centroid position changes of the left ventricle in the cardiac B-images obtained by the method described above can be seen clearly in Figs.4-6. Figure 5 and Fig.6 show the two different sequences (112-242, 12-42) of images.

Fig.4 Sequence #1: Contours of left ventricular wall.

Fig.5 Sequence #2: Contours of left ventricular wall.

Fig.6 Sequence #3: Contours of left ventricular wall.

Fig.7 Central coordinates of left ventricular contours from a sequence of images: (a) the movement description by Fourier descriptors; and (b) the movement description by principle of classical mechanics.

In Fig.7(b), when the ventricle contracts to its smallest size, the y-axis coordinate in the contour center is decreased significantly, which is because the lower parts of the ventricular contour are contracted very narrowly (see Figs.4-6) so that the y-axis coordinate used to calculate the contour centroid is decreased (the centroid position is shift up). Using the Fourier descriptor to reconstruct the boundary curve, the center coordinates of the contour are determined as shown in Fig.7(a), which reduces the effects of the heart structure like the papillary muscle and brings the ventricular contour close to the actual case.

Table 1 Angle list of main axis in the chamber contour.

Frame number	$y_a /(^{\circ})$	$y_b /(^{\circ})$	$\Delta y /(^{\circ})$	Frame number	$y_a /(^{\circ})$	$y_b /(^{\circ})$	$\Delta y /(^{\circ})$
11	76.56	76.842	-0.282	42	91.6	91.980	-0.380
12	76.75	76.481	0.269	44	87.83	87.557	0.273
13	76.06	76.343	-0.283	46	81.26	81.173	0.087
14	75.52	75.734	-0.214	47	77.74	77.634	0.106
15	74.92	74.856	0.064	48	76.96	76.982	-0.023
16	73.63	73.598	0.0316	49	77.92	77.834	0.086
17	74.62	74.878	-0.258	50	78.58	77.509	1.071
18	74.76	74.763	-0.003	51	75.58	75.610	-0.030
20	75.77	75.771	-0.001	52	73.88	74.999	-1.019
22	77.67	77.756	-0.086	53	75.83	75.832	-0.002
26	82.18	82.997	-0.817	54	74.24	74.270	-0.030
28	82.46	82.398	0.062	55	74.09	74.083	0.007
30	83.64	83.533	0.107	56	73.6	73.660	-0.060
32	82.94	82.737	0.203	57	76.62	76.412	0.208
38	87.73	87.707	0.023	58	73.31	73.354	-0.044
40	90.49	91.012	-0.522	59	74.55	74.477	0.073

From Fig.7, the centroid of the ventricular contour shifts slightly in the process of the

ventricular systole and diastole, that is, the ventricular wall does the same shift. Though the translation of each frame is not more than two pixels in distance, where one pixel is equal to 0.44 mm, the cumulative displacement of multiple frames still has a significant impact on the sampling waveform. So it is necessary to correct the translation motion of the heart.

In order to compare the deflection angles of the principal axes, the principal direction changing in the ventricular contours are calculated in Table 1. Where, y_a is the principal angle calculated by the Fourier descriptors, y_b is the principal angle calculated by the centroid principal axes algorithm. As it can be seen from Table 1, the principal direction of the contour is changed a little during the ventricular systole and diastole. When the principal angle increases in the systole, the angle in the diastole will decrease. When the ventricle contracts to its smallest size, the principal axis is almost vertical. The variation rule and the numerical value of the principal axis calculated by two ways are almost the same.

From the above results, both methods can be adopted to isolate the functional movements and the non-functional movements of the ventricle. The non-functional movement parameters of the ventricular contours calculated by the Fourier descriptors are close to that of the actual movement of the heart.

4.2. Correction of omnidirectional M-mode echocardiography

The displacement of the contour curve in each frame and the deflection in the B-irradiated plane obtained by the Fourier descriptors are used to establish a reference model of the non-functional movements and to guide the position and the direction of the sampling line for doing the equivalent movement, which makes the points of the wall structures analyzed always move up and down in the direction line.

Assuming that the motion vector of any point in the wall is $\bar{S}(x, y)$, and the calculated non-functional movement vector is $\bar{M}(x, y)$. Then the ventricular functional movements are

$$\bar{D}(x, y) = \bar{S}(x, y) - \bar{M}(x, y). \quad (41)$$

Fig.8 Comparison of contour revising before and after additional operations: (a) Contour comparison between 47th and 48th frame before revising; and (b) Contour comparison between 47th and 48th frame after revising.

Where, $\bar{M}(x, y)$ is used to guide the movement of the sampling line, that is, the sampling line moves as $\bar{M}(x, y)$. It makes the structure point to be analyzed always move up and down in the direction line, to obtain functional movements of the wall structural point in the heart.

Figure 8 is one example of the two continuous frames that are before and after the correction of the omnidirectional M-mode echocardiography.

Fig.9 Comparison of M-mode echocardiography before and after eliminating non-functional exercise: (a) Sampling on the right and upward side; (b) Sampling on the upward side; (c) Sampling on the left and downward side; and (d) Sampling on the right and downward side.

The method is applied to construct the omnidirectional M-mode echocardiography. Figure 7 shows the omnidirectional M-mode echocardiography sampled by the fixed direction line (the direction line of the non-functional movements is not removed) and the tracking sampling line (the direction line of the non-functional movements is removed). Figure 9 also shows the comparison results in four sampling positions. In each group, the left picture is the location of the sampling line, and the right one is the sampling result. From Fig.9, the sampling line basically is for tracking the specific structure after the correction of the non-functional movements. In Fig.9(a), the movement in the overall cardiac cycle can be obtained. In Fig.9(a, b, d), the obtained omnidirectional M-mode echocardiography cycle is more apparent, and the waveform has the strong repeat-ability. In Fig.9(b), the obtained contour is clear and there is no specific point lost or wandered basically. In Fig.9(b, c), the Echocardiography is complete, and no sampling structure is lost or other organizational structures appear. The crest of the Echocardiography is sharper, which reflects the movement when the heart is impulse and the movement is not smoothed by the overall movement.

5. Conclusions

From our experiments, we proved that it is feasible to extract the left ventricular contours of the short axis with the Fractional differential and improved Canny edge detection algorithms. This technique is supported by the gray scale search algorithm based on radial, and we demonstrate the accuracy of our final results. For correcting the non-functional movements, we proposed to use the studied motion tracking method based on the ventricular contours. By correcting the non-functional movements, the sampling direction lines can basically be used to track the movement of a certain part in the heart of the same structure at different times, and the functional movements in the specific parts of the cardiac structures can be reflected accurately. The non-functional movements in the B ultrasonic plane can be represented by the overall displacement and the principal axis deflection of the ventricular contours. The Fourier descriptors are adopted to obtain the principal translation and rotation of a series of object contour curves accurately. However, this technique tends to exhaust huge computational expenses.

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References

- [1] Brittain E L, Nwabuo C , Xu M , et al. Echocardiographic Pulmonary Artery Systolic Pressure in the Coronary Artery Risk Development in Young Adults (CARDIA) Study: Associations With Race and Metabolic Dysregulation [J]. *Journal of the American Heart Association*, 2017, 6(4):e005111.
- [2] Corredor C M. Dynamic Assessment of the Heart: Echocardiography in the Intensive Care Unit [M]. Springer International Publishing, 2016.
- [3] Patel K V, Metzinger M , Park B , et al. Longitudinal Associations of Fitness and Obesity in Young Adulthood With Right Ventricular Function and Pulmonary Artery Systolic Pressure in Middle Age: The CARDIA Study[J]. *Journal of the American Heart Association*, 2021.
- [4] JEero, Haapala, A., Henna, Haapala, L. , Heidi, & Syvaaja, et al. (2020). Longitudinal associations of physical activity and pubertal development with academic achievement in adolescents. *Sport Health Science*, 265-273.
- [5] Garcia-Hermoso, D Martinez-Gomez, J Rosario Fernández-Santos, et al. Longitudinal associations of physical fitness and body mass index with academic performance [J]. *Scandinavian Journal of Medicine & Science in Sports*, 2021.
- [6] Booth J N, Anstey D E, Bello N A , et al. Race and sex differences in asleep blood pressure: The Coronary Artery Risk Development in Young Adults (CARDIA) study[J]. *Journal of Clinical Hypertension*, 2019.
- [7] Brittain E L, Nwabuo C , Xu M , et al. Echocardiographic Pulmonary Artery Systolic Pressure in the Coronary Artery Risk Development in Young Adults (CARDIA) Study: Associations With Race and Metabolic Dysregulation[J]. *Journal of the American Heart Association*, 2017, 6(4):e005111.
- [8] P. Mihaela, et al., Construction of 3D MR image-based computer models of pathologic hearts, augmented with histology and optical fluorescence imaging to characterize action potential propagation, *J. Medical Image Analysis*, 16, 2, 505-523 (2012).
- [9] P. Caroline and J. N. Dacher, A review of segmentation methods in short axis cardiac MR images, *J. Medical Image Analysis*, 15, 2, 169-184 (2011).
- [10] G. Vikas, A. Hortense and Kirişli, et al., Cardiac MR perfusion image processing techniques: A survey, *J. Medical Image Analysis*, 16, 4, 767-785 (2012).
- [11] C. O. Schirra and C. Bontus, et al., Improvement of cardiac CT reconstruction using local motion vector fields, *J. Computerized Medical Imaging and Graphics*, 33, 2, 122-130 (2009).
- [12] M. Prummer, J. Hornegger, G. Lauritsch, L. Wigstrom, E. Girard-Hughes and R. Fahrig, Cardiac C-Arm CT: A Unified Framework for Motion Estimation and Dynamic CT, *J. IEEE Transactions on Medical Imaging*, 28, 11, 1836 – 1849 (2009).

- [13] A. A. Isola and A. Ziegler, et al., Motion compensated iterative reconstruction of a region of interest in cardiac cone-beam CT, *J. Computerized Medical Imaging and Graphics*, 34, 2, 149-159 (2010).
- [14] J. M. Peyrat, H. Delingette, M. Sermesant, C. Y. Xu and N. Ayache, Registration of 4D Cardiac CT Sequences Under Trajectory Constraints With Multichannel Diffeomorphic Demons, *J. IEEE Transactions on Medical Imaging*, 29, 7, 1351 – 1368 (2010).
- [15] J. Bai, K. Liu, Y. Jiang, K. Ying, P. Zhang and J. Shao, A two-dimensional CVIB imaging system with a speckle tracking algorithm, *J. Ultrasonics*, 48, 5, 394-402 (2008).
- [16] T. Arts and F. W. Prinzen, et al., Mapping Displacement and Deformation of the Heart With Local Sine-Wave Modeling, *J. IEEE Transactions on Medical Imaging*, 29, 5, 1114– 1123 (2010).
- [17] A. Prakosa and M. Sermesant, et al., Generation of Synthetic but Visually Realistic Time Series of Cardiac Images Combining a Biophysical Model and Clinical Images, *J. IEEE Transactions on Medical Imaging*, 32, 1, 99 – 109 (2013).
- [18] Q. Lin, W. J. Wu and L. Q. Huang, et al., An omnidirectional M-mode echocardiography system and its clinical application, *J. Computerized Medical Imaging and Graphics*, 30, 333–338 (2006).
- [19] Weixing Wang, Nan Yang, et al. A review of road extraction from remote sensing images [J]. *Journal of Traffic and Transportation Engineering (English Edition)*, 2016, 3(3):271-282.
- [20] L. Q. Huang and Q. Lin, Research on accurate detection of omnidirectional M-mode cardiography, *J. Chinese Journal of Scientific Instrument*, 30, 5, 1105-1109 (2009).
- [21] Weixing Wang, Runqing Li, Kevin Wang, Fangnian Lang, Weiwei Chen and Bin Zhao, Crack and Fracture central line delineation on Steger and Hydrodynamics with improved Fractional differential, *International Journal of Wavelets, Multiresolution and Information Processing*, 18(5) 2050037 (2020) (21 pages).
- [22] W. X. Wang, Image analysis of aggregates, *J. International Journal: Computers & Geosciences*, 25, 71-81 (1999).
- [23] Liu, S., Rahman, M.A., Lin, C.-F., Wong, C.Y., Jiang, G., Liu, S.C., Kwok, N., Shi, H. Image contrast enhancement based on intensity expansion-compression (2017) *Journal of Visual Communication and Image Representation*, 48, pp. 169-181.
- [24] Wang Weixing, Li Hongxia, et al., Pavement crack detection on geodesic shadow removal with local oriented filter on LOF and improved Level set, *Construction Building Materials*, 2020, 237(2020)117750.
- [25] Wang Weixing, Chen Weiwei, et al., Extraction of tunnel centerline and cross sections on Fractional calculus and 3D invariant moments and best-fit ellipse, *Optics & Laser technology*, 2020, 128(2020), 106220.
- [26] Keli Hu, Jun Ye, En Fan, Shigen Shen, Longjun Huang, Jiatian Pi. A novel object tracking algorithm by fusing color and depth information based on single valued neutrosophic cross-entropy[J]. *Journal of Intelligent & Fuzzy Systems*, 2017, 32(3): 1775–1786.
- [27] Pi J, Hu K*, Zhang X, Gu Y, Zhan Y. Robust Object Tracking with Compressive Sensing and Patches Matching[J]. *IEICE TRANSACTIONS on Information and Systems*. 2016, 99(6):1720-3.
- [28] Lixin Fan, En Fan, Changhong Yuan, Keli Hu. Weighted fuzzy track association method based on Dempster-Shafer theory in distributed sensor networks[J]. *International Journal of Distributed Sensor Networks*, 2016, 12(7), 10 Pages.
- [29] Z. He, Y. Cao, L. Du, B. Xu, J. Yang, Y. Cao, S. Tang and Y. Zhuang, "MRFN: Multi-Receptive-Field Network for Fast and Accurate Single Image Super-Resolution," *IEEE Transactions on Multimedia*, vol. 22, no. 4, pp. 1042–1054, 2020.
- [30] J. Canny and S. Castan, Advanced edge detection techniques, *J. Techniques in Computational Vision*, 23, 5, 1-6 (1994).
- [31] W. X. Wang, F. Bergholm and B. Yang, Froth delineation based on image classification, *J. Mineral Engineering*, 16, 11, 1183-1192 (2003).
- [32] W. X. Wang, Binary image segmentation of aggregates based on polygonal approximation and classification of concavities, *J. Pattern Recognition*, 31, 10, 1503-1524 (1998).
- [33] A. K. Singh, A. K. Singh, S. K. Vaishy, R. S. Yadav and A. K. Singh, Fourier Analysis of Prostate Tissue with Statistical Approach, *J. Med. Imaging Health Inf.* 2, 182-187 (2012).
- [34] W. X. Wang, Image analysis of particles by modified Ferret method – best-fit rectangle, *J. Powder Technology*, 165, 1, 1-10 (2006).
- [35] E. O. Ahmed, and D. D. A. Mohamed, Estimation of general 2D affine motion using Fourier descriptors, *J. Pattern Recognition*, 35, 1, 223-228 (2002).
- [36] C. Zheng, Z. Y. Yang and H. L. He, et al., Motion Detection of Closed Boundary Using Fourier Descriptors and Hough Transform, *J. Computer Engineering and Application*, 28, 68-69, 80 (2005).
- [37] Y. Q. Wu and X. L. Fu, The Method of Detecting and Correcting Oblique License Plate Based on the Corner Points Information and Inertial Principal Axis, *J. Journal of Engineering Graphics*, 6, 127-131 (2009).