An Improved Cutting Force Model in Micro-milling Considering the Comprehensive Effect of Tool Runout, Size Effect and Tool Wear

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An improved cutting force model in micro-milling considering the comprehensive effect of tool runout, size effect and tool wear

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Abstract: Tool runout, cutting edge radius-size effect and tool wear have significant impacts on the cutting force of micro-milling. In order to predict the micro-milling force and the machining performance related to the cutting force, it is necessary to establish a cutting force model including tool runout, cutting edge radius and tool wear. In this study, an instantaneous uncut thickness (IUCT) model considering tool runout, a nonlinear shear/ploughing coefficient model including cutting-edge radius and a friction force coefficient model embedded with flank wear width, are constructed respectively. By integrating the IUCT, the nonlinear shear/ploughing coefficient and the friction force coefficient, a comprehensive micro-milling force model including the tool runout, size effect and tool wear is derived. Experiment results show that the proposed comprehensive model is efficient to predict the micro milling force.

Key words: Micro milling; Cutting force; Tool runout; Size effect; Tool wear

1 Introduction

Micro-milling, with the high machining efficiency and the ability of cutting diverse materials, is viewed as one of the most promising technology for fabricating the micro-electromechanical devices[1]. The cutting force, which is related to many machining performances, such as the vibration[2], deflection[3], stability[4] and energy consumption[5,6], plays an extremely important role in micro-milling. Constructing an accurate cutting force model is of great importance for understanding the micro milling process.

With the development of micro-milling technology, the mechanism of micro-milling force has been intensively investigated. Over the past decade, different factors such as tool runout [7], elastic recovery[8], tool wear[9], dead metal zone[10], edge radius size effect[11] and chip thickness accumulation[12], have been considered into the micro-milling force model. Among these factors, tool runout, edge radius size effect and tool wear have a particularly significant impact on micro milling force, and attract more attentions in the field of micro-milling research.

Due to the manufacturing and clamping errors, the tool runout is inevitable in the micro-milling process. As the feed speed per tooth in micro-milling is usually within a few microns, slight tool runout will significantly affect the uncut thickness (UCT) and the cutting force. Bao et al. [7] consider the UCT formed by the adjacent cutting teeth and built an analytic UCT model for micro-milling under tool runout. Li and Zhu[13] built a generic UCT model for flat micro end milling, in which the UCTs formed by all of the teeth are considered. Zhu et al. [14] extended the generic UCT model to ball end milling, and improved the force prediction accuracy in micro end ball milling.

Limited by the tool sharpening technology, the cutting edge is not absolutely sharp. The cutting-edge radius of the micro-milling tool is usually comparable to the uncut thickness. The comparability of the cutting-edge radius and the uncut thickness, viz. the edge radius size effect, leads to an obvious minimum uncut thickness (MUCT) phenomenon and ploughing effect[15], which greatly changes the micro-milling force. Liu et al. [16] deduced that the ratio of the MUCT to the edge radius is a function of the effective flow stress and the shear strength. By assuming that the total ploughing force and the shear

force at the MUCT point are equivalent, Son et al. [17] expressed the stagnant angle corresponding to the MUCT as a function of the friction angle. Based on the minimum cutting energy principle and the infinite stress principle[18], Malekian et al. [19] concluded that the stagnant angle equals to the friction angle.

Under the high rotation speed, the tiny-size micro-milling tool wears rapidly, resulting in a sharp increase in micro milling force. Bao et al.[20] built an experienced relationship between the tool wear and the amplitude of micro-milling force. Based on the friction stress distribution formular[21], Lu et al.[9] built a flank wear-included force model for Niki micro-milling. Compared to the flank wear, the effect of cutting-edge wear on the cutting force is much less studied. By assuming that both of the edge radius and the flank wear width increase with the deteriorating tool wear condition, Zhou et al. [22] attempted to build a cutting force model considering the edge wear and flank wear.

The works mentioned above have deeply investigated the individual effect of the tool runout, edge radius effect and tool wear on micro milling force. Besides, some other studies have explored the influence of the pairwise combination of the three factors on the cutting force of micro-milling. The combined effect of the tool wear and cutting-edge radius on the force was reported in study [22]. Jing et al. [8] considered the edge radius and runout, and built an analytic micro milling force model. Li et al.[23] adopted the spatial analytic geometry method to analysis the combined effect of the tool runout and wear the micro-milling force. Liu et al.[24] built a micro-milling force model including the tool runout and tool wear, and proposed a cutting force model-based tool wear monitoring method under varying tool runout. However, so far, to our best knowledge, no studies have considered the comprehensive effect of tool runout, cutting-edge radius and tool wear on the micro milling force. To bridge this gap, this study builds a mechanics micro milling force model considering the tool runout, cutting-edge radius and tool wear, and investigate the comprehensive effect of the three factors on the micro milling force.

The cutting force modeling process in this study is as follows. Firstly, the IUCT model including the tool runout is constructed. Then, a nonlinear shear/ploughing cutting coefficient model including the IUCT and the cutting-edge radius is derived, and the friction force coefficient in the flank wear region is represented as a function of the flank wear width. Finally, by integrating the IUCT, the shear/ploughing cutting coefficient and the friction force coefficient, a comprehensive micro-milling force model including tool runout, edge radius effect and the tool wear is constructed.

This paper evolves as follows. In section 2, the comprehensive micro milling force model is constructed. Section 3 proposes a genetic optimization-based model parameters calibration approach. In section 4, micro milling experiment is conducted to examine the efficiency of the proposed model. The paper is concluded in section 5.

2 The Improved Cutting Force Model for Micro-Milling

The proposed model is shown in Fig. 1. The input layer of the model consists of the tool runout, cutting edge radius and the flank wear. The second layer is the middle variables including the IUCT, the shear-ploughing coefficient and the friction coefficient. The output layer is the theoretical micro-milling force.

Fig.1 The flowchart of constructing the comprehensive micro-milling force model

The comprehensive model in Fig.1 is:

$$dF_c = \left[\int_0^h K_{c,sp} (h' | \lambda_s, r_e, \alpha) \cdot dh' + K_{c,vb} (VB | \lambda_v) \right] dz \quad (1)$$

$$dF_r = \left[\int_0^h K_{r,sp} (h' | \lambda_s, r_e, \alpha) \cdot dh' + K_{r,vb} (VB | \lambda_v) \right] dz \quad (2)$$

Notation dF_c is the tangential force, dF_r is radial force, dz is the unit axial cutting depth. Notation h is the instantaneous uncut thickness, which depends on the tool runout. Notation VB is the flank wear width, r_e is the cutting-edge radius related to the flank wear width, α is the rake angle. Notation $K_{c,sp}$ is the shear-ploughing coefficient in tangential direction, $K_{r,sp}$ is the shear-ploughing coefficient in radial direction. The shear-ploughing coefficient varies with the IUCT. If the partial IUCT h' is greater than the minimum uncut thickness h_m , the shear-ploughing coefficient represents the cutting force resulting from the shear effect (Fig.2a). Otherwise, the shear-ploughing coefficient corresponds to the ploughing force (Fig.2b). Notation $K_{c,vb}$ is the tangential friction force coefficient in flank wear region, $K_{r,vb}$ is the radial friction force coefficient. Mechanical parameter set $\lambda_s = \{\tau_s, \beta_s, \tau_m, \sigma_m\}$ includes the shear stress τ_s , friction angle β_s , normal cutting stress σ_m and the tangential cutting stress τ_m in ploughing region. Mechanical parameter set $\lambda_v = \{VB^*, \tau_v, \sigma_v\}$ consists of the normal friction stress σ_v , tangential friction stress τ_v and the width of elastic contact region VB^* .

Fig.2 The cutting force model: (a) shear effect with $h' > h_m$; (b) ploughing effect with $h' < h_m$

The construction of the IUCT model considering tool runout, the shear/ploughing coefficient model considering cutting edge radius and the friction force coefficient model with flank wear width are elaborated in the following three subsections.

2.1 Uncut Thickness Model considering Tool Runout

By changing the equivalent tool radius and the angle between the radius, axial tool runout makes the trochoidal trajectory and the IUCT of each tooth change. Because the axial cutting depth is much smaller than the length of tool, the radial tool runout could be regarded as the translation of the cutting part at the bottom of the micro-milling tool, and thus the tool runout could be represented by the length r_o and the angle γ_o of the translation vector. According to studies [13,24], the IUCT of the k -th equivalent radius at cutting depth z at reference position angle ϕ could be written as:

$$h_{k,z}(\phi) = \max \left\{ \min_m \left[R_{k,z} - R_{m,z} + f_z \sin(\phi_k^z) \frac{M \cdot \Delta \theta_{m,k}^z}{2\pi} \right], 0 \right\} \quad (3)$$

Notation $R_{k,z}$ is the k -th equivalent radius at cutting depth z , $\Delta \theta_{m,k}^z$ is the angle that the k -th equivalent radius clockwise leads the m -th equivalent radius at depth z . ϕ_k^z is the rotation angle of the k -th equivalent radius at depth z , f_z is the feed per tooth. The detailed calculation process of determining the equivalent radius $R_{k,z}$ and the equivalent angles $\Delta \theta_{m,k}^z$ by the tool runout parameters (r_o, γ_o) could refer to study [24].

2.2 Shear/Ploughing Coefficient Model considering Cutting Edge Radius

The comparability of the IUCT and cutting-edge radius, viz. the tool edge radius size effect, lead to the minimum uncut thickness and nonlinear IUCT-varying shear/ploughing coefficient. In this study, an analytic MUCT model is derived by assuming the normal ploughing force and the normal shear force are equivalent at the MUCT point (Fig.3). The assumption could be written as:

$$r_e \sigma_m d\theta dz = \frac{\tau_s \cos(\beta_s)}{\sin^2\left(\frac{\theta_s - \beta_s}{2}\right)} dh dz \quad (4)$$

The left of Eq.(4) is the norm ploughing force $dF_{p,n}$ in Fig.3, the right is the normal shear force $dF_{s,n}$ under the minimum energy principle[18], θ_s is the stagnant angle corresponding to MUCT. By solving Eq. (4), the analytic stagnant angle is derived as:

$$\theta_s = \pi - \arcsin \left\{ \frac{\sigma_m}{\sqrt{4\tau_s^2 \cos^4 \beta_s + (\tau_s \sin 2\beta_s + \sigma_m)^2}} \right\} - \arctan \left\{ \tan \beta_s + \frac{\sigma_m}{2\tau_s \cos^2 \beta_s} \right\} + \beta_s \quad (5)$$

The MUCT value is:

$$h_m = r_e (1 - \cos \theta_s) \quad (6)$$

Fig.3 The MUCT model based on the assumption that the normal ploughing force and the normal shear force are equivalent at the MUCT point

The angle of the intersection point of rake face and round nose is θ_{lim} , and the corresponding UCT is $h_{lim} = r_e - r_e \cos \theta_{lim}$. According to the UCT value, the whole cutting region could be divided into three parts: the ploughing region with $h < h_m$, the shear region with UCT-varying shear force coefficient ($h_m \leq h < h_{lim}$) and the part with constant shear force coefficient ($h \geq h_{lim}$). The shear/ploughing coefficient in the three different part is:

$$K_{c,sp}(h | \lambda_s, r_e, \alpha) = \begin{cases} \frac{\tau_s \sin(\theta_{lim} - \beta_s)}{\sin^2\left(\frac{\theta_{lim} - \beta_s}{2}\right)} & h \geq h_{lim} \\ \frac{\tau_s \sin(\theta - \beta_s)}{\sin^2\left(\frac{\theta - \beta_s}{2}\right)} & h_m \leq h < h_{lim} \\ \sigma_m + \tau_m \cot \theta & 0 \leq h < h_m \end{cases} \quad (7)$$

$$K_{r,sp}(h | \lambda_s, r_e, \alpha) = \begin{cases} \frac{\tau_s \cos(\theta_{lim} - \beta_s)}{\sin^2\left(\frac{\theta_{lim} - \beta_s}{2}\right)} & h \geq h_{lim} \\ \frac{\tau_s \cos(\theta - \beta_s)}{\sin^2\left(\frac{\theta - \beta_s}{2}\right)} & h_m \leq h < h_{lim} \\ \sigma_m \cot \theta - \tau_m & 0 \leq h < h_m \end{cases} \quad (8)$$

2.3 Friction Force Coefficient in Flank Wear Region

The relationship between the flank tool wear and the friction force coefficient has been clearly revealed in the existed studies. According to the friction stress distribution formular, the relationship between the friction force coefficient and the flank wear could be written as:

$$K_{c,vb}(VB | \lambda_v) = \begin{cases} \frac{\tau_v}{3} \cdot VB & VB < VB^* \\ \tau_v \left(VB - \frac{2}{3} VB^* \right) & VB \geq VB^* \end{cases} \quad (9)$$

$$K_{c,vb}(VB | \lambda_v) = \begin{cases} \frac{\sigma_v}{3} \cdot VB & VB < VB^* \\ \sigma_v \left(VB - \frac{2}{3} VB^* \right) & VB \geq VB^* \end{cases} \quad (10)$$

Many studies showed that the cutting-edge radius varies with the flank wear width. However, due to the uncertain build up edge and micro chipping that affect the effective cutting-edge radius, it is difficult to build a deterministic analytic model to represent the relationship between the edge radius and the flank tool wear width. To reveal the dependency of the cutting-edge radius on the flank wear width, in the experimental validation part, empirical relationship between the calibrated edge radius and the measured flank wear width is analyzed by statistical correlation analysis method.

3 Calibration of the Model Parameters

Genetic optimization algorithm is adopted to calibrate the model parameters. Besides the two unknown mechanical parameter sets λ_s and λ_v , the tool runout and the efficient edge radius are also calibrated via the genetic optimization algorithm. Practically, the static tool runout could be directly measured by optic measurements. However, the optic measurement-based method is incompetent to measure the dynamic rotation-speed-dependent runout in cutting process. Due to the build up edge and microchipping, it is also difficult to directly measure the effective cutting-edge radius by optical microscope or atomic force microscope. Therefore, in this study, the dynamic tool runout and the effective edge radius are also calibrated via the genetic optimization algorithm. The flank wear band is regular and the flank wear width

could be directly measured by the optical microscope. In this study, the flank wear width is measure by Olympus Toolmakers microscope during the recess of tool holder.

Including the mechanical parameter sets λ_s and λ_v , tool runout parameters, effective cutting-edge radius, there are 12 parameters need to be calibrated. The purpose of the optimization-based calibration is to find out the 12 optimum parameters, such that the gap between the theoretical force and the measured force is smallest. The measured forces in feed and normal directions are utilized to calibrate the parameters of the proposed model. In order to reduce the computation cost of the calibration process, instead of jointly optimizing the 12 parameters, the 12 parameters to be calibrated are optimized by three steps. Firstly, the cutting force signal, generated by the fresh tool with known edge radius and flank wear width, is adopted to calibrate the machinal parameters λ_s , and the parameter set λ_s is set as a shared input of the subsequent calibration processes with worn tool. Then, with the sharing mechanical parameters λ_s , the friction force coefficient is calibrated under the worn tool. Finally, the parameters set λ_v that reflects the relationship between the flank wear and the friction force coefficient, is calibrated with the calibrated friction force coefficients and the measured flank wear widths. The parameters need to be calibrated are listed in [Table 1](#). The stepwise calibration process is shown in [Fig.4](#).

Table 1

Parameters to be calibrated

	Parameter to be calibrated	Notation	Unit
Calibrated with fresh tool	Friction angle	β_s	rad
	normal cutting stress	σ_m	$N/\mu m^2$
	tangential cutting stress	τ_m	$N/\mu m^2$
	Shear stress	τ_s	$N/\mu m^2$
Calibrated with worn tool	Cutting edge radius	r_e	μm
	Length of tool runout	r_o	μm
	Angle of tool runout	γ_o	rad
	tangential friction force coefficient	$K_{c,vb}$	$N/\mu m$
	radial friction force coefficient	$K_{r,vb}$	$N/\mu m$
	tangential friction stress	τ_v	$N/\mu m^2$
	radial friction stress	σ_v	$N/\mu m^2$
	width of elastic contact region	VB^*	μm

Fig.4 The parameters calibration process

4 Experimental Validation

4.1 Experimental setup

Micro slot milling is conducted to validate the proposed comprehensive micro-milling force model. The machine used in the experiments is MIKRON HSM600U vertical milling machine. The tool is CS2008-0200. Steel T4 is used as work-piece material. The cutting force is measured with a Kistler9109AA2 3-channel dynamometer. The sampling rate is 24kHz. The flank wear width is measured by Olympus Toolmakers. There are 10 cutting passes. In each cutting pass, a 3-cm-long slot is machined. The cutting parameters used in this study are listed in Table 2. The geometric parameters of the fresh tool are listed in Table 3.

Table 2

Cutting parameters used in this study

Spindle speed	Axial cutting depth	Cutting speed	Feed speed	Feed per tooth
18000rpm	80μm	45.24m/min	144mm/min	4μm

Table 3

Geometric parameters of the micro-milling tool

Tooth number	Tool diameter	Rake angle	Clearance angle	Initial cutting edge radius	Initial flank wear width
2	0.5mm	0°	6°	2μm	0μm

4.2 Parameter calibration results

The cutting force signal acquired during the first cutting pass is adopted to calibrate the four mechanical parameters: the shear stress, friction angle in shear region, normal and tangential cutting stresses in

ploughing region. It is assumed that the tool at the first cutting pass is fresh. The tool edge radius at the first cutting pass is the initial cutting-edge radius ($2\mu m$), and the flank wear width at the first cutting pass is $0\mu m$. By taking the initial cutting-edge radius and the flank wear width into the proposed cutting force model and comparing the theoretical force and the measured cutting force, the four mechanical parameters along with the runout parameters are calibrated. The calibration results are listed in [Table 4](#). Taking the calibrated shear stress, friction angle in shear region and the normal cutting stress in ploughing region into the MUCT model proposed in section 2.2, the ratio of the MUCT value to the cutting-edge radius is calculated as 0.3537. This result is consistent to the findings in most of the studies [\[25-27\]](#) which indicate the ratio is in the range of $[0.2, 0.4]$.

Table 4

Calibration results of cutting pass 1

β_s	σ_m	τ_m	τ_s	r_o	γ_o
0.557	0.027	0.026	0.001	1.022	2.046

The mechanical parameters $\lambda_s = \{\tau_s, \beta_s, \tau_m, \sigma_m\}$ calibrated at the first cutting pass are shared with the subsequent cutting passes. At the following cutting passes, the cutting-edge radius and the flank wear width vary with the tool wear condition. The flank wear width is directly measured. The effective cutting-edge radius, the flank wear width-dependent friction force coefficients and the runout parameters are jointly calibrated via the calibration algorithm proposed in section 3. The calibrated results are shown in [Table 5](#). As the tool is re-clamped at the beginning of each cutting pass, the runout parameter cannot keep constant for different cutting passes. This could be noticed from [Table 5](#). The varying process of the flank wear width and the effective cutting-edge radius is shown in [Fig.5](#). It clearly shows that the cutting-edge radius increases as the flank wear width increases. The correlation coefficient of the cutting-edge radius and the flank wear width is calculated as 0.8842, implying that the effective cutting-edge radius highly depends on the flank wear width.

Table 5

Calibration results of all cutting passes

Pass	r_e	r_o	γ_o	$K_{c,vb}$	$K_{r,vb}$	VB
1	0	1.022	2.046	0	0	0
2	3.653	2.022	1.668	0.005	0.003	14
3	4.184	2.140	1.715	0.012	0.005	15
4	3.991	1.197	1.344	0.011	0.01	20
5	4.218	2.823	1.694	0.012	0.01	22.5
6	3.717	2.717	1.664	0.02	0.009	24.5
7	4.482	0.372	0.907	0.015	0.01	27
8	4.275	0.595	0.864	0.016	0.019	29
9	4.734	0.929	1.306	0.021	0.011	33.5
10	5.618	0.703	1.221	0.035	0.015	35

Fig.5 The flank wear and the cutting-edge wear: (a) flank wear width and cutting-edge radius VS cutting pass; (b) cutting edge radius VS flank wear width.

The calibrated friction force coefficients and the measured flank wear width are utilized to calibrate the tangential friction stress, radius friction stress and the constant width of elastic contact region. The calibrated friction force coefficients are shown in Fig.6. It shows that, the growth curve of the friction force coefficient could be approximately represented by a polyline with the width of elastic contact region as the turning point. To obtain the mechanical parameters, the calibrated friction force coefficients are fitted by the polylines defined in Eq. (9)-(10). The constant width of elastic contact region is calibrated as $VB^*=18.678\mu\text{m}$. The friction force stresses associated with the slop of the fitting polyline are calibrated as $\sigma_v = 0.125 \cdot 10^{-2} \text{ N}/\mu\text{m}^2$, $\tau_v = 0.076 \cdot 10^{-2} \text{ N}/\mu\text{m}^2$. Comparing the calibrated shear stress, cutting stress in ploughing region and the friction stress in flank wear region, it could be found that the cutting stress in ploughing is much bigger than the other two stresses. This implies that the micro-milling force is most concentrated in the ploughing region.

Fig.6 The relationship between the friction force coefficient and the flank wear width.

4.3 Micro milling force prediction results

The cutting forces at the 10 passes are predicted via the proposed comprehensive model. The residual force is defined as the predicted force minus to the measured force. The estimated error is defined as the ratio of the second norm of the residual force to the second norm of the measured force. The predicted error is listed in [Table 6](#). It shows that the proposed comprehensive model is accurate to predict the micro-milling force.

Table 6

Prediction error

Pass	1	2	3	4	5	6	7	8	9	10
Error (%)	21.20	17.09	16.30	18.99	24.80	28.84	13.75	24.03	22.29	23.93

Three conventional cutting force models are also utilized to predict the micro milling force in this study. Different from the proposed comprehensive model, the three conventional models only consider two factors. The first conventional model includes tool runout and cutting-edge radius. The second model considers tool runout and tool wear. The third one considers the cutting-edge radius and tool wear. [Table 7](#) lists the prediction results of the three conventional models. It could be found that the proposed comprehensive model is more accurate than the three traditional models. This could also be concluded from [Fig.7](#). [Table 7](#) shows that, the model without considering the cutting-edge radius has a much higher prediction error than the other three models. This implies the cutting-edge radius has the most significant effect on the micro-milling force.

Fig.7 Cutting force prediction results at cutting pass 2 (a) Comprehensive model including tool runout, cutting-edge radius and tool wear; (b) Tool Runout & Cutting-edge radius; (c) Tool Runout & Tool wear; (d) Cutting-edge radius & Tool wear.

Table 7

Comparison between different models

Model	Prediction error (%)
Tool Runout & Cutting-edge radius	24.59
Tool Runout & Tool wear	31.03
Cutting-edge radius & Tool wear	23.75
Tool Runout & Cutting-edge radius & Tool wear	17.09

5 Conclusion

In order to accurately predict the micro-milling force, this study construct a micro-milling fore model considering the comprehensive effect of the tool runout, cutting edge radius and tool wear. The IUCT model inducing tool runout, nonlinear shear/ploughing coefficient model considering tool wear-dependent cutting-edge radius and the friction force coefficient function embedded with the flank wear are separately constructed. By integrating the IUCT, shear/ploughing coefficient and the friction force coefficient into the mechanism milling force model, a comprehensive micro-milling force model considering the tool runout, cutting edge radius and tool wear is derived. A genetic optimization-based stepwise calibration method is proposed to calibrate the parameters of the model. Experimental results show that the proposed model is efficient to predict the micro-milling force under varying tool runout, cutting edge radius and tool wear condition. Some conclusions are as follows.

- 1) Including tool runout, cutting edge radius and tool wear, the proposed comprehensive model is more accurate than the conventional models.
- 2) The cutting-edge radius increases as the flank wear width increases. The effective cutting-edge radius could be utilized to indicate the tool wear condition of micro milling.
- 3) The cutting stress in ploughing region is much higher than the shear stress and the friction stress in the flank wear region. The micro-milling force is most concentrated in the ploughing region.
- 4) Among the tool runout, cutting edge radius and tool wear, the cutting-edge radius has the most significant impact on micro-milling force.

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Declarations

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Consent to participate. All of the authors consent to participate.

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Authors' contributions. Tongshun Liu constructed the theoretical model and wrote the manuscript. Yayun Liu designed and directed the project. Kedong Zhang analyzed the data.

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