Sum and Difference Beamspace Domain Refined Maximum Likelihood Algorithm for Elevation Estimation in Multipath Environment

Sheng Chen

Xidian University <https://orcid.org/0000-0003-3519-1137>

Yongbo Zhao (✉ ybzhao@xidian.edu.cn)

Xidian University <https://orcid.org/0000-0002-6453-0786>

Yili Hu

Xidian University

Xiaojiao Pang

Xidian University

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Sum and difference beamspace domain refined maximum likelihood algorithm for elevation estimation in multipath environment

Sheng Chen, Yong-bo Zhao*, Yi-li Hu, Xiao-jiao Pang

Email: 842561535@qq.com(SC), ybzhao@xidian.edu.cn(YZ* Correspondence), huyili66@126.com(YH), 1964230291@qq.com(XP)

National Lab of Radar Signal Processing, Xidian University, Xi'an 710071, China

Abstract: Beamspace domain super-resolution methods for elevation estimation in multipath environment has attracted significant attention. However, the difference beam is rarely used in super-resolution methods, especially in low elevation estimation. In order to improve the accuracy of beamspace domain maximum likelihood algorithm for elevation estimation, we apply the difference beam to the beamformer. Therefore, we develop a sum and difference beamspace domain maximum likelihood (SDBDML) algorithm for elevation estimation, which has good estimation accuracy and is not sensitive to antenna height and reflection coefficient errors. In order to further improve the estimation performance, we take the refined signal model into account in the SDBDML algorithm, and develop a sum and difference beamspace domain refined maximum likelihood (SDBDRML) algorithm. Compared with the SDBDML algorithm, the SDBDRML algorithm has higher estimation accuracy and lower computational burden, although it is sensitive to antenna height and reflection coefficient errors. The theoretical target elevation angle root mean square error (RMSE) and the computational complexity of proposed algorithms are analyzed. Finally, our computer simulations and real data processing results demonstrate the effectiveness of the proposed algorithms.

Key words: Elevation estimation; Beamspace domain; Sum and difference beam; Multipath environment; Maximum likelihood

1 Introduction

In recent decades, elevation estimation in multipath environment has attracted significant attention [1-3]. The direct signal and reflected signals passing along the Earth's surface enter the radar's main beam together. This paper assumes that the reflected signals only contain the specular reflection signal. The direct and reflected signals are indistinguishable in time, Doppler and space domain, resulting in a degradation of target elevation estimation performance. In addition, radar does not handle data from massive arrays very well in element-space domain (ED) because of the large data transmission, storage, and computation. Therefore, we often can only use the data in beamspace domain (BD) to estimate target angle.

The angle estimation algorithms based on sum and difference beams are classical methods in monopulse radar [4-6]. For monopulse methods, the reflected signal can only be regarded as interference and is elimi-

nated in multipath environment [7] in general, but the information about the space domain in the reflected signal is lost. The monopulse methods based on sum and difference beams are also used in array radar [8, 9], but the estimation performance is still limited.

Super-resolution methods, such as subspace algorithms and maximum likelihood (ML) schemes, are currently the primary methods for elevation estimation in multipath environment. The former ones, such as MUSIC [10] and ESPRIT [11], usually require more snapshots and cannot directly deal with coherent signals. On the other hand, spatial smoothing [12] can help solve coherent signals by losing a certain aperture before angle estimation. ML algorithm [13] can directly process coherent signals and can operate utilizing a single snapshot. The refined maximum likelihood (RML) algorithm [3, 14], makes full use of the prior knowledge of geometric information and surface reflection coefficient and uses a composite guide vector instead of a conventional guide vector. Thus, the RML algorithm has high estimation accuracy and low computational burden, but is sensitive to parameter errors due to the refined signal model mismatch [15]. The reflected signal can also be regarded as interference and is eliminated in super-resolution methods [16, 17].

Compared with the processing in ED, the processing in BD can obtain better performance by losing certain degrees of freedom, such as lower signal-to-noise ratio (SNR) resolution threshold, reduced amount of data transmission, storage, and computation [18, 19]. The three-dimensional (3D) beamspace domain maximum likelihood (BDML) algorithm [2] offers a closed-form solution for elevation estimation by the feature that the eigenvector of the beamspace correlation matrix associated with the smallest eigenvalue is orthogonal to the subspace spanned by the signals, so it has very low computational complexity. Besides, [2] also develops the two-dimensional (2D) BDML algorithm, which offers a closed-form solution without considering multipath. The 2D-BDML algorithm is suitable for the initial estimation of tracking or the scenes without multipath. On the basis of the 3D-BDML algorithm, the multipath scenario with interference or an extra target is discussed [20]. In [21], the covariance matrix in BD is reconstructed into the covariance matrix in ED, and then the angles of coherent signals are estimated using spatial smoothing MUSIC algorithm in ED. In [22], the BDML algorithm for elevation estimation in MIMO radar is proposed to reduce the computational burden. In [23], the BD U-ESPRIT algorithm performs 2D estimation of azimuth angle and elevation

angle for coherently sources in MIMO radar. The BD U-ESPRIT algorithm choose proper and few beams in 2D-BD to reduce computational complexity. In [24], the angle estimation for transmit beamspace-based in bistatic MIMO radar is developed. The authors design the transmit beamspace through convex optimization to focus the transmitted energy. However, this method is not suitable for conventional phased array radar.

To the best of our knowledge, beamspace super-resolution methods for elevation estimation in multipath environment, which considers difference beam, has not been investigated yet. In order to improve the accuracy of BDML algorithm for elevation estimation, we apply the difference beam to the beamformer and discuss two settings of sum and difference beams that can meet the needs of most elevation estimation scenarios. Then, a sum and difference beamspace domain maximum likelihood (SDBDML) algorithm is proposed. The SDBDML algorithm does not need the surface reflection parameters, i.e., it is not affected by the corresponding errors. Taking the refined signal model [3, 14] into account in the SDBDML algorithm, a sum and difference beamspace domain refined maximum likelihood (SDBDRML) algorithm is proposed. Our simulation results show that compared with the SDBDML algorithm, the SDBDRML algorithm has higher estimation accuracy and lower computational burden, although it is sensitive to antenna height and reflection coefficient errors.

This paper is structured as follows. First, Section 2 provides a multipath signal model. Then, Section 3 reviews the BDML algorithm briefly, proposes the SDBDML and SDBDRML algorithms for elevation estimation, and Section 4 gives the theoretical target elevation angle root mean square error (RMSE) and analyses the computational complexity of the proposed algorithms. Finally, Section 5 evaluates the proposed algorithms in various simulations and real data processing results, while Section 6 concludes this work.

2 Multipath signal model

Consider a digital array radar equipped with a uniform linear array. The geometry for a smooth earth model with multipath propagation is presented in Fig.1. The linear array with the M elements receives the two paths of target echoes from the different directions of arrival. One path returns directly from the target to the radar antenna, whereas another path returns from the reflecting surface, where the incident angle of the

direct and reflected signals are θ_1 and θ_2 , respectively. The height of the array radar centre is h_r . The distance between two adjacent array elements is d , which is half the wavelength. The height of the target is h_t . The distance from the target to the array radar centre is R_d .

Fig.1 Multipath geometry model of low-angle target

For the array radar system, the received signal in ED can be written as

$$\mathbf{x}(l) = [\mathbf{a}(\theta_1)\mathbf{M}(\theta_2)] \begin{bmatrix} \alpha(l) \\ \alpha(l)\rho e^{-j\varphi(\theta_1)} \end{bmatrix} + \mathbf{n}(l) = \mathbf{A}\mathbf{c}(l) + \mathbf{n}(l) \quad l = 1, \dots, L \quad (1)$$

where, L is the number of snapshots, $\mathbf{a}(\theta) \in \mathbb{C}^{M \times 1}$, called the array steering vector, is defined as

$$\mathbf{a}(\theta) = [e^{-j\pi d(M-1)\sin\theta/\lambda}, e^{-j\pi d(M-3)\sin\theta/\lambda}, \dots, e^{j\pi d(M-1)\sin\theta/\lambda}]^T \quad (2)$$

where λ is the working wavelength, $d = \lambda/2$, $\alpha(l)$ the complex amplitude related to target characteristics at the l th snapshot, ρ the surface reflection coefficient, and $[\mathbf{g}]^T$ denotes transpose operation. The surface reflection coefficient ρ is generally expressed as $\rho = \Gamma D \rho_s$, where Γ denotes the smooth surface reflection coefficient, D the divergence factor and ρ_s the surface roughness factor [14], but it can be regarded as a constant for the scenario in this paper [25]. $\varphi(\theta_1) \approx 4\pi h_r \sin(\theta_1)/\lambda$ is the phase-difference between the direct and reflected signals due to time delay. $\mathbf{n}(l) \in \mathbb{C}^{M \times 1}$ is the Gaussian white noise vector with zero mean at the l th snapshot, which is not correlated with the target signals. The variance matrix of $\mathbf{n}(l)$ is $\sigma_n^2 \mathbf{I}_M$. $\mathbf{I}_M$ represents an identity matrix of size $M \times M$.

Consider a unitary matrix beamformer, as

$$\mathbf{T} = \frac{1}{\sqrt{M}} [\mathbf{a}(\theta^1), \mathbf{a}(\theta^2), \dots, \mathbf{a}(\theta^K)] \quad (3)$$

where $\mathbf{T} \in \mathbb{C}^{M \times Q}$, $\mathbf{T}^H \mathbf{T} = \mathbf{I}_Q$, and $[\mathbf{g}]^H$ denotes the conjugate transpose operation. Then, the received data in BD is formed as

$$\mathbf{x}_B(l) = \mathbf{T}^H \mathbf{x}(l) = \mathbf{T}^H \mathbf{A} \mathbf{c}(l) + \mathbf{T}^H \mathbf{n}(l) = \mathbf{B} \mathbf{c}(l) + \mathbf{n}_B(l) \quad (4)$$

where $\mathbf{B} = \mathbf{T}^H \mathbf{A}$ and $\mathbf{n}_B(l) = \mathbf{T}^H \mathbf{n}(l)$ is still Gaussian white noise vector.

3 Methods

This section reviews the 3D-BDML algorithm briefly, and presents the proposed SDBDML and SDBDRML algorithms for elevation estimation.

3.1 Previous 3D-BDML algorithm

The 3D-BDML algorithm [2] considers a $M \times 3$ unitary matrix beamformer

$$\mathbf{T}_{3D} = \frac{1}{\sqrt{M}} \left[\mathbf{a}\left(-\frac{2}{M}\right) \mathbf{M}(0) \mathbf{M}\left(\frac{2}{M}\right) \right] \quad (5)$$

including a reference beam pointing 0° , and two auxiliary beams pointing to $-2/M$ and $2/M$. Then, the 3D-BDML estimations of θ_1 and θ_2 are the solution to the following optimization problem

$$\underset{\theta_1, \theta_2, \mathbf{c}(1:L), \mathbf{c}(L)}{\text{Minimize}} \sum_{l=1}^L \|\mathbf{x}_B(l) - \mathbf{B} \mathbf{c}(l)\|_2^2 \quad (6)$$

where $\|\mathbf{g}\|_2$ denotes the 2-norm and $\mathbf{B} = \mathbf{T}_{3D}^H \mathbf{A}$. Utilize the separability to obtain the respective least square error solution $\mathbf{c}_{LS}(l) = [\mathbf{B}^H \mathbf{B}]^{-1} \mathbf{B}^H \mathbf{x}_B(l)$, $l = 1, 2, \dots, L$, where $[\mathbf{g}]^{-1}$ denotes a matrix inverse operation.

Substitute $\mathbf{c}_{LS}(l)$ for $\mathbf{c}(l)$ in Eq. (6), we get

$$\underset{\theta_1, \theta_2}{\text{Minimize}} \sum_{l=1}^L \mathbf{x}_B^H(l) \mathbf{P}_B^\perp(\theta_1, \theta_2) \mathbf{x}_B(l) \quad (7)$$

where $\mathbf{P}_B^\perp(\theta_1, \theta_2) = \mathbf{I}_3 - \mathbf{B} [\mathbf{B}^T \mathbf{B}]^{-1} \mathbf{B}^T$. The solution of Eq. (7) is given in [2].

3.2 The SDBDML algorithm

Without loss of generality, suppose the number of array elements is even, and $M = 2K$, where K is an integer. We define the matrix as follows

$$\mathbf{T}_1 = \frac{1}{\sqrt{M}} [\mathbf{a}_1(\theta^1), \mathbf{a}_1(\theta^2), \dots, \mathbf{a}_1(\theta^Q)] \quad (8)$$

$$\mathbf{T}_2 = \frac{1}{\sqrt{M}} [\mathbf{a}_2(\theta^1), \mathbf{a}_2(\theta^2), \dots, \mathbf{a}_2(\theta^Q)] \quad (9)$$

where

$$\begin{aligned} \mathbf{a}_1(\theta^q) &= [e^{-j\pi d(M-1)\sin\theta^q/\lambda}, e^{-j\pi d(M-3)\sin\theta^q/\lambda}, \dots, e^{-j\pi d\sin\theta^q/\lambda}]^T \\ \mathbf{a}_2(\theta^q) &= [e^{j\pi d\sin\theta^q/\lambda}, e^{j3\pi d\sin\theta^q/\lambda}, \dots, e^{j\pi d(M-1)\sin\theta^q/\lambda}]^T \end{aligned} \quad (10)$$

It can be seen that $\mathbf{T} = \begin{bmatrix} \mathbf{T}_1 \\ \mathbf{T}_2 \end{bmatrix}$, $\mathbf{T}_1, \mathbf{T}_2 \in \mathbf{C}^{K \times Q}$, and $\mathbf{a}_2(\theta^q) = \mathbf{a}_1(\theta^q) e^{j\pi d M \sin\theta^q/\lambda}$. Then, the received data in BD can be rewritten as

$$\mathbf{x}_B(l) = \mathbf{T}^H \mathbf{x}(l) = \mathbf{T}_1^H \mathbf{x}_1(l) + \mathbf{T}_2^H \mathbf{x}_2(l) = \mathbf{x}_{Bs}(l) \quad (11)$$

where $\mathbf{x}(l) = \begin{bmatrix} \mathbf{x}_1(l) \\ \mathbf{x}_2(l) \end{bmatrix}$ and $\mathbf{x}_1(l), \mathbf{x}_2(l) \in \mathbf{C}^{K \times 1}$. It can be seen from Eq. (11) that the array is divided into two equal subarrays, and $\mathbf{x}_{Bs}$ is the sum of the subarrays in BD. We can easily get the difference of the subarrays in BD, as

$$\mathbf{x}_{Bd}(l) = \mathbf{T}_1^H \mathbf{x}_1(l) - \mathbf{T}_2^H \mathbf{x}_2(l) \quad (12)$$

Then, we get the sum and difference data in BD

$$\mathbf{x}_{Bsd}(l) = \begin{bmatrix} \mathbf{x}_{Bs}(l) \\ \mathbf{x}_{Bd}(l) \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1^H \mathbf{x}_1(l) + \mathbf{T}_2^H \mathbf{x}_2(l) \\ \mathbf{T}_1^H \mathbf{x}_1(l) - \mathbf{T}_2^H \mathbf{x}_2(l) \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1^H & \mathbf{T}_2^H \\ \mathbf{T}_1^H & -\mathbf{T}_2^H \end{bmatrix} \begin{bmatrix} \mathbf{x}_1(l) \\ \mathbf{x}_2(l) \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_2 \\ \mathbf{T}_1 & -\mathbf{T}_2 \end{bmatrix}^H \begin{bmatrix} \mathbf{x}_1(l) \\ \mathbf{x}_2(l) \end{bmatrix} \quad (13)$$

Define the sum and difference matrix beamformer

$$\mathbf{T}_{sd} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_1 \\ \mathbf{T}_2 & -\mathbf{T}_2 \end{bmatrix} \quad (14)$$

The left half of $\mathbf{T}_{sd}$ represents sum beamformer, and the right half represents difference beamformer.

In what follows, we demonstrate the advantage of $\mathbf{T}_{sd}$ over the $\mathbf{T}_{3D}$ by array patterns. Consider the linear array with the 30 elements, $d = \lambda / 2$, the beams in $\mathbf{T}_{3D}$ pointing to $-2/M$, 0° , and $2/M$, and the beams in $\mathbf{T}_{sd}$ pointing to $-2/M$ and $2/M$. The array patterns of $\mathbf{T}_{3D}$ and $\mathbf{T}_{sd}$ are depicted in Fig. 2. This figure shows that $\mathbf{T}_{sd}$ with only two directions has wider spatial coverage and higher effective gain than $\mathbf{T}_{3D}$ with three directions. Besides, the beam settings can be adjusted according to actual needs. For example, for the case where the spatial range of interest is large, select the beams pointing to $-4/M$, 0° and $4/M$ to construct the sum and difference matrix beamformer. The corresponding array patterns are depicted in Fig. 3. This figure shows that the spatial coverage is wider than the coverage in Fig. 2. The two beam pointing conditions of $\mathbf{T}_{sd}$ in Figs. 2 and 3 meet the needs of most elevation estimation scenarios.

(a) Array patterns for $\mathbf{T}_{3D}$ with the beams pointing to $-2/M$, 0° , and $2/M$

(b) Array patterns for T_{sd} with the beams pointing to $-2/M$ and $2/M$

Fig. 2 Array patterns for a uniform linear array with 30 elements and $d = \lambda / 2$

Fig. 3 Array patterns for a uniform linear array with 30 elements, $d = \lambda / 2$, and T_{sd} with the beams pointing to $-4/M$, 0° and $4/M$.

In addition, any method of generating difference beams can be used to obtain the sum and difference matrix beamformer. Further, the sum and difference data in BD can be rewritten as

$$\mathbf{x}_{Bsd}(l) = \mathbf{T}_{sd}^H \mathbf{x}(l) = \mathbf{T}_{sd}^H \mathbf{A} \mathbf{c}(l) + \mathbf{T}_{sd}^H \mathbf{n}(l) = \mathbf{B}_{sd} \mathbf{c}(l) + \mathbf{n}_{Bsd}(l) \quad (15)$$

where $\mathbf{B}_{sd} = \mathbf{T}_{sd}^H \mathbf{A}$ and $\mathbf{n}_{Bsd}(l) = \mathbf{T}_{sd}^H \mathbf{n}(l)$. Next, in order to make better use of ML algorithm, we analyze the properties of $\mathbf{T}_{sd}$. Because $\mathbf{T}$ is a unitary matrix, we have $\mathbf{T}^H \mathbf{T} = \mathbf{T}_1^H \mathbf{T}_1 + \mathbf{T}_2^H \mathbf{T}_2 = \mathbf{I}_Q$. From Eq. (14), we

165 have

$$166 \quad \mathbf{T}_{sd}^H \mathbf{T}_{sd} = \begin{bmatrix} \mathbf{T}_1^H \mathbf{T}_1 + \mathbf{T}_2^H \mathbf{T}_2 & \mathbf{T}_1^H \mathbf{T}_1 - \mathbf{T}_2^H \mathbf{T}_2 \\ \mathbf{T}_1^H \mathbf{T}_1 - \mathbf{T}_2^H \mathbf{T}_2 & \mathbf{T}_1^H \mathbf{T}_1 + \mathbf{T}_2^H \mathbf{T}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_Q & \mathbf{T}_1^H \mathbf{T}_1 - \mathbf{T}_2^H \mathbf{T}_2 \\ \mathbf{T}_1^H \mathbf{T}_1 - \mathbf{T}_2^H \mathbf{T}_2 & \mathbf{I}_Q \end{bmatrix} \quad (16)$$

167 where

$$168 \quad \mathbf{T}_1^H \mathbf{T}_1 = \frac{1}{M} \begin{bmatrix} \mathbf{a}_1^H(\theta^1) \mathbf{a}_1(\theta^1) & \mathbf{L} & \mathbf{a}_1^H(\theta^1) \mathbf{a}_1(\theta^2) \\ & \mathbf{M} & \mathbf{O} & \mathbf{M} \\ \mathbf{a}_1^H(\theta^2) \mathbf{a}_1(\theta^1) & \mathbf{L} & \mathbf{a}_1^H(\theta^2) \mathbf{a}_1(\theta^2) \end{bmatrix} \quad (17)$$

169 The element at the u row and v column of $\mathbf{T}_2^H \mathbf{T}_2$ is

$$170 \quad (\mathbf{T}_2^H \mathbf{T}_2)_{uv} = \frac{1}{M} \mathbf{a}_2^H(\theta^u) \mathbf{a}_2(\theta^v) = \frac{1}{M} \mathbf{a}_1^H(\theta^u) e^{-j\pi d M \sin \theta^u / \lambda} \mathbf{a}_1(\theta^v) e^{j\pi d M \sin \theta^v / \lambda} = (\mathbf{T}_1^H \mathbf{T}_1)_{uv} (\mathbf{F})_{uv} \quad (18)$$

$$171 \quad (\mathbf{F})_{uv} = e^{j\pi d M (\sin \theta^v - \sin \theta^u) / \lambda} \quad (19)$$

172 For the case with the beams pointing to $-2/M$ and $2/M$, we have

$$173 \quad \mathbf{F} = \begin{bmatrix} 1 & e^{j\pi d M (\sin \theta^2 - \sin \theta^1) / \lambda} \\ e^{j\pi d M (\sin \theta^1 - \sin \theta^2) / \lambda} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (20)$$

174 For the case with the beams pointing to $-4/M$, 0° and $4/M$, we have

$$175 \quad \mathbf{F} = \begin{bmatrix} 1 & e^{j\pi d M (\sin \theta^2 - \sin \theta^1) / \lambda} & e^{j\pi d M (\sin \theta^3 - \sin \theta^1) / \lambda} \\ e^{j\pi d M (\sin \theta^1 - \sin \theta^2) / \lambda} & 1 & e^{j\pi d M (\sin \theta^3 - \sin \theta^2) / \lambda} \\ e^{j\pi d M (\sin \theta^1 - \sin \theta^3) / \lambda} & e^{j\pi d M (\sin \theta^2 - \sin \theta^3) / \lambda} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (21)$$

176 For Eq. (20) and Eq. (21), we use $d = \lambda / 2$ and the approximation with very small error, i.e., $\sin \theta \approx \theta$. Thus,

177 we have $\mathbf{T}_2^H \mathbf{T}_2 = \mathbf{T}_1^H \mathbf{T}_1$, so $\mathbf{T}_{sd}$ is a unitary matrix and $\mathbf{n}_{Bsd}$ is still Gaussian white noise vector, which is

178 consistent with the basic assumption of ML estimation [26]. Next, substituting $\mathbf{T}_{sd}$ for $\mathbf{T}$, and $\mathbf{B}_{sd}$ for $\mathbf{B}$ in

179 Eq. (7), we get

$$180 \quad \underset{\theta_1, \theta_2}{\text{Maximize}} \sum_{l=1}^L \mathbf{x}_{Bsd}^H(l) \mathbf{P}_{Bsd}(\theta_1, \theta_2) \mathbf{x}_{Bsd}(l) \quad (22)$$

where $\mathbf{P}_{Bsd}(\theta_1, \theta_2) = \mathbf{B}_{sd} [\mathbf{B}_{sd}^H \mathbf{B}_{sd}]^{-1} \mathbf{B}_{sd}^H$.

In addition, from the multipath geometry in Fig. 1, θ_2 can be easily obtained by the approximation

$$\theta_2 = f(\theta_1) \approx -\arcsin(\sin \theta_1 + 2h_r / R_d) \quad (23)$$

and we have $\mathbf{B}_{sd} = \mathbf{T}_{sd}^H \mathbf{A}_I = \mathbf{T}_{sd}^H [\mathbf{a}(\theta_1) \mathbf{M}(f(\theta_1))]$. Thus, we only need to estimate the one-dimensional parameter of θ_1 , and θ_2 can be obtained from Eq. (23). The ML function can be rewritten as

$$\text{Maximize}_{\theta_1} \sum_{l=1}^L \mathbf{x}_{Bsd}^H(l) \mathbf{P}_{Bsd}(\theta_1, f(\theta_1)) \mathbf{x}_{Bsd}(l). \quad (24)$$

Refer to [3, 27], when the number of elements or SNR is large enough, the theoretical target elevation angle RMSE of the SDBDML algorithm can be deduced to

$$E_{\text{SDBDML}}(\hat{\theta}_1) \approx \sqrt{\frac{M\sigma_n^2}{2} \text{Re}^{-1} \{ \mathbf{S}_P^H \mathbf{D}^H \mathbf{P}_{Bsd}^\perp \mathbf{D} \mathbf{S}_P \}} \quad (25)$$

where $\text{Re}\{\cdot\}$ denotes the real part, $\mathbf{S}_P = \sum_{l=1}^L \mathbf{c}(l)$, $\mathbf{D} = \mathbf{T}_{sd}^H \frac{\partial}{\partial \theta_1} \mathbf{A}_I$, $\mathbf{P}_{Bsd}^\perp = \mathbf{I} - \mathbf{P}_{Bsd}(\theta_1, f(\theta_1))$.

According to the idea of information theory, the SDBDML algorithm has higher estimation accuracy than the 3D-BDML algorithm, because the beamformer of the SDBDML algorithm has wider spatial coverage and higher effective gain and uses geometry information. In addition, $h_r = R_d$, so neither antenna height error nor target distance error will affect the estimation accuracy of the SDBDML algorithm.

3.3 The SDBDRML algorithm

In this subsection, we take the refined signal model [3, 14] into account in the SDBDML algorithm. Utilizing the prior knowledge of surface reflection coefficient, and using a composite guide vector instead of a conventional guide vector, $\mathbf{B}_{sd}$ can be replaced by

$$\mathbf{b}_{sd} = \mathbf{B}_{sd} \begin{bmatrix} 1 \\ \rho e^{-j\varphi} \end{bmatrix} = \mathbf{T}_{sd}^H [\mathbf{a}(\theta_1) \quad \mathbf{a}(f(\theta_1))] \begin{bmatrix} 1 \\ \rho e^{-j\varphi(\theta_1)} \end{bmatrix} = \mathbf{T}_{sd}^H [\mathbf{a}(\theta_1) + \rho e^{-j\varphi(\theta_1)} \mathbf{a}(f(\theta_1))]. \quad (26)$$

Substituting $\mathbf{b}_{sd}$ for $\mathbf{B}_{sd}$ in Eq. (24), we get

$$\underset{\theta_1}{\text{Maximize}} \mathbf{F}(\theta_1) \quad (27)$$

where

$$\mathbf{F}(\theta_1) = \sum_{l=1}^L \mathbf{x}_{Bsd}^H(l) \mathbf{P}_{bsd}(\theta_1) \mathbf{x}_{Bsd}(l) = \sum_{l=1}^L \mathbf{x}_{Bsd}^H(l) \mathbf{b}_{sd} [\mathbf{b}_{sd}^H \mathbf{b}_{sd}]^{-1} \mathbf{b}_{sd}^H \mathbf{x}_{Bsd}(l) = \sum_{l=1}^L \frac{\mathbf{x}_{Bsd}^H(l) \mathbf{b}_{sd} \mathbf{b}_{sd}^H \mathbf{x}_{Bsd}(l)}{\mathbf{b}_{sd}^H \mathbf{b}_{sd}}. \quad (28)$$

The ML function can be simplified to

$$\underset{\theta_1}{\text{Maximize}} \sum_{l=1}^L \left| \frac{\mathbf{b}_{sd}^H \mathbf{x}_{Bsd}(l)}{\sqrt{\mathbf{b}_{sd}^H \mathbf{b}_{sd}}} \right|. \quad (29)$$

Refer to [3, 27], when the number of elements or SNR is large enough, the theoretical target elevation angle RMSE of the SDBDRML algorithm can be deduced to

$$E_{\text{SDBDRML}}(\hat{\theta}_1) = \sqrt{\frac{M \sigma_n^2}{2 |S_{p1}|^2} (\mathbf{D}_1^H \mathbf{P}_{bsd}^\perp \mathbf{D}_1)^{-1}} \quad (30)$$

where $S_{p1} = \sum_{l=1}^L \alpha(l)$, $\mathbf{P}_{bsd}^\perp = \mathbf{I} - \mathbf{P}_{bsd}(\theta_1)$, $\mathbf{D}_1 = \mathbf{T}_{sd}^H \frac{\partial \mathbf{b}_{sd}}{\partial \theta_1}$.

Because the SDBDRML algorithm uses the prior knowledge of surface reflection coefficient and reduces the signal dimension, the SDBDRML algorithm has higher estimation accuracy and lower computational burden. However, the SDBDRML algorithm relies heavily on surface reflection conditions, i.e., its estimation performance will severely reduce due to signal model mismatch [15] when the surface reflection parameters are unknown. On the contrary, the SDBDML algorithm does not use the surface reflection parameters, i.e., it is not affected by the corresponding errors.

4 Computational complexity analysis

In this section, the computational complexity of the proposed algorithms, the MUSIC based on covariance matrix reconstructed (MUSIC-CMR) algorithm in [21], the 2D-BDML and 3D-BDML algorithms in [2] are analyzed.

Let the number of spatial search grid be P . The computational complexity of the MUSIC-CMR algo-

rithm is $O(Q^2L + Q^3 + QM + (M - U)^3 + P(M - U)^2)$, where U is the number of subarrays. The computational complexity of the 3D-BDML and 2D-BDML algorithms is $O(Q^2L + Q^3 + QM)$. The SDBDML and SDBDRML algorithms mainly considers the transformation from ED to BD and the computation of objective function. The computational complexity of the transformation from ED to BD is $O(QM)$. The computational complexity of the objective function of the two algorithms are $O(PQ^2L + Q^3)$ and $O(PQL)$. The computational complexity of these algorithms are summarized in Table 1. The order of algorithms in Table 1 is from small to large according to the computational complexity.

Table 1 Computational complexity

Algorithms	Computational complexity
2D-BDML	$O(Q^2L + Q^3 + QM)$
3D-BDML	$O(Q^2L + Q^3 + QM)$
SDBDRML	$O(PQL + QM)$
SDBDML	$O(PQ^2L + Q^3 + QM)$
MUSIC-CMR	$O(Q^2L + Q^3 + QM + (M - U)^3 + P(M - U)^2)$

5 Results and discussion

In this Section, the elevation estimation performances of the algorithms mentioned in Section 4.2 in multipath environment are discussed through several simulation scenarios and real data. The beams pointing with $-2/M$ and $2/M$ for proposed algorithms are denoted by the 2D-SDBDML and 2D-SDBDRML algorithms, respectively. In MUSIC-CMR algorithm, the number of beams and subarrays are set to 5 and 15.

5.1 Simulations

Consider a digital array radar equipped with a uniform linear array with 30 elements where the distance between two adjacent array elements is $d = 0.5$ m. We set $\lambda = 1$ m, $h_r = 8$ m, $R_d = 20$ km, and $\rho = 0.9 \exp(j\pi)$. For all simulations, the number of Monte Carlo trials is set to 500. These experiments are all performed using MATLAB R2016b on a PC with an Intel i7-7700 at 3.60 GHz and 8GB of RAM. Let the

SNR be defined as

$$\text{SNR} = |\alpha|^2 / \sigma_n^2 \quad (31)$$

Initially, we demonstrate the experimental and the theoretical target elevation angle RMSE of proposed algorithms against SNR. Assume that the radar receives only a single snapshot, and the target elevation angle is 4.5° . Fig. 4 illustrates the target elevation angle RMSE of proposed algorithms against SNR. This figure shows that the estimation error decreases with the increase of SNR. The experimental error is consistent with the theoretical error in high SNR, but the experimental error is higher than the theoretical error in low SNR because of approximation error. The 2D-SDBDRML algorithm performs better than the 2D-SDBDML algorithm because of using the prior knowledge of surface reflection coefficient.

Fig. 4 Theoretical and experimental RMSE of angle estimation against SNR

The subsequent trials involve target elevation angle RMSE comparison of the algorithms mentioned in Section 4.2. For $\text{SNR} = 10 \text{ dB}$, the target elevation angle RMSE of the five algorithms against angle are shown in Figs. 5 and 6, for 64 snapshots and a single snapshot, respectively. These figures show that the MUSIC-CMR algorithm is greatly affected by the number of snapshots, and the 2D-BDML algorithm is not suitable for elevation estimation in multipath environment. The 3D-BDML algorithm performs better than the previous two algorithms, but the performance is poor in high elevation area because of the insufficient spatial coverage. In addition, the RMSE of the 3D-BDML algorithm is larger when the angle is 3.5° , which is caused by the phase of $\rho e^{-j\varphi(\theta_1)}$ [2]. According to the simulation parameters, the phase of ρ is π , and

$\varphi(\theta_1) \approx 4\pi h_r \sin \theta_1 / \lambda \approx 2\pi$, so the phase of $\rho e^{-j\varphi(\theta_1)}$ is $-\pi$, which results in the failure of the 3D-BDML algorithm. The proposed algorithms performs better than other three algorithms, especially in high elevation area, because the beams setting of the proposed algorithms has better spatial coverage, which is consistent with Fig. 2. The RMSE of most algorithms are larger when the angle is small. The reason of this phenomenon is that the phase-difference between of direct and reflected signals gets closer to 180° , resulting in the mutual cancellation of the two signals. Fig. 7 illustrates the normalized power of ideal signal in ED in multipath environment against angle. This figure shows that the signal power in low elevation area is obviously lower. In Figs. 5 and 6, the RMSE of the 2D-BDML algorithm in low elevation area is not larger than the RMSE in high elevation area. That is because the beams direction of 2D-BDML algorithm are 0° and $2/M$, which has a much higher gain for the direct signal than for the reflected signal and avoids the cancellation of the two signals.

Fig. 5 RMSE of angle estimation against angle for 64 snapshots

Fig. 6 RMSE of angle estimation against angle for a single snapshot

Fig. 7 The normalized power of ideal signal in ED in multipath environment against angle

Thirdly, we show the target elevation angle RMSE of the SDBDML algorithm with different beams setting against angle. For $\text{SNR} = 10$ dB, Fig. 8 illustrates the RMSE of the SDBDML algorithm with different beams setting against angle. Specifically, the beams directions represented by the four lines in Fig. 8 are given in Table 2. Fig. 8 shows that the performance of the 1D-SDBDML algorithm is poor because of its insufficient beam degree of freedom and spatial coverage. The performance of the 2D-SDBDML algorithm and 3D-SDBDML algorithm is basically the same, which shows that simply increasing the number of beams is limited to improve the performance. The 3D2-SDBDML algorithm performs better than other three algorithms in high elevation area, because the beams setting has wider spatial coverage, which is consistent with

Fig. 3. In addition, the performance of 3D2-SDBDML algorithm in low elevation area is not the best, because the beams setting is relatively scattered, which leads to the decrease of the effective gain in this area.

Table 2 The meaning of the four lines in Fig. 8

Line type	Beams directions
Solid	0°
Plus	$-2/M, 2/M$
Dotted	$-2/M, 0^\circ, 2/M$
Square	$-4/M, 0^\circ, 4/M$

Fig. 8 RMSE of target elevation angle with different beams setting against angle

Finally, we analyze the influence of the errors of key parameters on the performance of the 2D-SDBDRML algorithm. Assuming that the SNR is 10dB and the target elevation angle is 4.5° , the target elevation angle RMSE of the 2D-SDBDRML algorithm against the errors of antenna height, the amplitude and phase of reflection coefficient are shown in Figs. 9-11, respectively. These figures show that the errors of antenna height and phase of reflection coefficient have a significant influence on the performance of the 2D-SDBDRML algorithm, while the amplitude error of reflection coefficient has a minor effect on the performance of the algorithm.

Fig. 9 RMSE of target elevation angle against antenna height error

Fig. 10 RMSE of target elevation angle against amplitude error of reflection coefficient

Fig. 11 RMSE of target elevation angle against phase error of reflection coefficient

5.2 Real data

The real data is applied to further verify the performance of the proposed algorithms. The array radar is equipped with a uniform linear array with 20 elements where the distance between two adjacent array elements is $d = 0.5$ m, and the height of the array radar centre is 10 m. The radar works in the meter wave band and receives only a single snapshot. The beam width is about 5° , the terrain scenario is hilly, and the target is an airplane.

Figs. 12 and 13 show the target distance and azimuth against frames, respectively. The distance between the airplane and radar first increased from about 60 km to 130 km and then decreased to 60 km. Figs. 14 and 15 show the processed results of the real data by using the algorithms mentioned in Section 4.2. The target elevation angle varies from 4° to 10° . It can be seen that the MUSIC-CMR algorithm is not suitable for this scene. The estimation errors of the 2D-BDML and 3D-BDML algorithms are basically less than 2° and 1.5° , respectively. The estimations of these two algorithms are less than the true elevation except for individual frames, because the spatial coverage of these two algorithms is insufficient in high elevation area. In addition, there are some singularities in the estimations of these two algorithms, which shows that these two algorithms are not very robust. The proposed algorithms performs better than other three algorithms. The estimation errors of the 2D-SDBDML and 2D-SDBDRML algorithms are basically less than 0.5° and 0.25° , respectively. There are also some singularities in the estimations of the 2D-SDBDML algorithm, but the overall situation is good. The 2D-SDBDRML algorithm obtains the optimal estimation without singularity and achieves the accuracy of one twentieth of the beam width.

Fig. 12 Target distance against frames

Fig. 13 Target azimuth against frames

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(a) The MUSIC-CMR, 2D-BDML, and 3D-BDML algorithms

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(b) The 2D-SDBDML and 2D-SDBDRML algorithms

333 Fig. 14 Estimation of target elevation angle against frames

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(a) The MUSIC-CMR, 2D-BDML, and 3D-BDML algorithms

(b) The 2D-SDBDML and 2D-SDBDRML algorithms

Fig. 15 Estimation error of target elevation angle against frames

6 Conclusions

In this paper, the problem of BD elevation estimation in multipath environment is discussed. Two ML algorithms based on sum and difference beamspace domain are proposed. The SDBDML algorithm has higher estimation accuracy than the 3D-BDML algorithm because of using the difference beams and geometry information, although the computational burden is higher. Compared with the SDBDML algorithm, the SDBDRML algorithm has higher estimation accuracy and lower computational burden because of using the prior knowledge of surface reflection coefficient and reducing the signal dimension. However, the SDBDRML algorithm relies heavily on surface reflection conditions, i.e., its estimation performance will severely reduce when the surface reflection parameters are unknown.

This paper analyzes two beam settings for proposed algorithms, which can be adjusted according to actual needs. Generally speaking, the beams pointing to $-2/M$ and $2/M$ is enough for elevation estimation in multipath environment.

Abbreviations

ED: Element-space domain; BD: Beamspace domain; ML: Maximum likelihood; RML: Refined maximum likelihood; SNR: Signal-to-noise ratio; 3D: Three-dimensional; BDML: Beamspace domain maximum likelihood; 2D: Two-dimensional; SDBDML: Sum and difference beamspace domain maximum likelihood; SDBDRML: Sum and difference beamspace domain refined maximum likelihood; RMSE: Root mean square

error; MUSIC-CMR: MUSIC based on covariance matrix reconstructed

Declarations

Availability of data and materials

The datasets generated and/or analysed during the current study are not publicly available due to confidentiality requirements.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

SC and YZ designed the research. SC, YH and XP processed the data. SC drafted the manuscript. All authors read and approved the final manuscript.

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