Observer-Based Event-triggered Adaptive Containment Control for Multiagent Systems With Prescribed performance

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Abstract

This paper focuses on the problem of the observer-based event-triggered adaptive containment control for a class of nonlinear multiagent systems (MASs) with prescribed performance. First, the radial basis function neural networks (RBFNNs) are adopted to approximate the uncertain smooth nonlinear function, and the neural network-based state observer is designed to estimate the unmeasurable state. Besides, to reduce the control resource assumption and get a better balance between the system performance and network constraints, the switching threshold based event-triggered control strategy is introduced. Based on this, the novel distributed containment controller is designed by utilizing the adaptive backstepping technique and the dynamic surface control (DSC) technique to guarantee that the output of each follower converges to the convex hull formed by multileader. Moreover, the containment errors can be converged to the prescribed boundary and all signals in closed-loop system are semi-global uniformly ultimately bounded (SGUUB) as well. Finally, the simulation example is carried out to illustrate the efficiency of the proposed controller.

Keywords Adaptive Containment Control · Event-triggered · prescribed performance · State Observer

1 Introduction

Over the past decades, cooperative control of multiagent systems (MASs) has gained growing popularity on account of its extensive applications in various fields, such as sensor networks [1] [2] and marine vessels [3] [4] as well as spacecraft formation flying [5], to name just a few. According to the number of the leaders, cooperative control problem can be segmented into three categories including leaderless consensus control [6], leader-following consensus control [7] and the containment control [8]. The fundamental characteristic of the containment control is that the MASs are composed of multiple leaders and all followers converge to the convex hull limited by multileader. In view of this, quite a few researches on containment control are designed for linear multiagent systems [9–11].

As is known to all, nonlinear multiagent systems are more suitable for describing practical application scenarios than linear ones. However, the dynamics of nonlinear multiagent systems are often difficult to obtain, which means the traditional adaptive control method is not suitable any more. Recently, the approximate capacity of the neural network and the fuzzy logic system (FLS) have shed some insightful light on the investigations of the adaptive control problem for nonlinear multiagent systems. Mei et al. [12] applied the universal approximation property of radial basis function neural

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networks (RBFNNs) to tackle the obstacle of uncertain nonlinearities and proposed the NN-based distributed adaptive containment control scheme for second-order MASs with unknown nonlinear dynamics. Yoo [13] extended this approach to cope with consensus problems in high-order nonlinear strict-feedback multiagent systems under a directed graph topology, which can better be applied to the engineering systems. Besides, Wang and Tong [14] utilized FLSs to identify the unknown nonlinear functions and cope with the containment control problem for full-state-constrained nonlinear strict-feedback systems. However, the results described in [12–14] are based upon a quite ideal situation that the states of all followers are measurable, while it is inevitable that some states can not be obtained directly because of economic constraints or technological limitations. In this case, Tong and Li [15] introduced the fuzzy adaptive observer to estimate unmeasured states for single-input and single-output (SISO) nonlinear systems. Subsequently, an adaptive fuzzy backstepping dynamic surface control (DSC) method was introduced for multiple-input and multiple-output (MIMO) nonlinear systems in [16]. Note that, the DSC technique shares the merit in overcoming the "explosion of complexity" issue inherent in each adaptive backstepping design step. The general idea of this technique is to introduce the first-order filter in each process of the backstepping design procedure.

Recently, as system performance has become significantly important for practical systems, a number of control schemes with prescribed performance, especially coordinated transformation-based prescribed performance, become major concerns as open for nonlinear multiagent systems. [17–20]. The primary objective of prescribed performance is to guarantee that the control errors gather to the predefined performance bounds along with the convergence rate. In [20], the state feedback control approach is established to address the trouble of unknown pure-feedback nonlinear systems. Tong and Li [17] proposed the output feedback prescribed performance control method for SISO switched nonstrict-feedback nonlinear systems. The authors in [19] focused on the nonlinear large-scale systems consisting of immeasurable states and unknown time-varying delays. Ni et al. [21] investigated the fixed-time recurrent neural network control for uncertain strict-feedback nonlinear systems in the presence of unknown dead-zone output. More recently, by using FLSs, a quantized cooperative control strategy with the prespecified performance approach for MASs with unknown gains and input quantization was developed in [22]. Wang et al. [23] dealt with the issue of adaptive fuzzy containment control for nonlinear MASs with prescribed performance, unknown disturbance and unknown dead zone. However, the aforementioned results are time-triggered control, which will result in unnecessary resource consumption due to the continuously information transmission through the communication topology.

It should be mentioned that the research of event-triggered control schemes for nonlinear systems is of extraordinary significance as it can further improve the control efficiency as well as decrease heavy communication burden and cost than classical time-triggered strategy [24]. Xing et al. [25] put forward the original event-triggered control method named the switching threshold strategy based on the framework of the fixed threshold strategy [26] and the relative threshold strategy [27]. The proposed adaptive controller is able to compensate for the measurement errors so that the input-to-state stability (ISS) assumption is no longer needed. In [28], the event-triggered-based output feedback control mechanism was investigated for uncertain nonlinear systems. To decrease the communication channel bandwidth, the authors proposed the new 1-bit signal transmission rule which has been proved to be effective. Ma et al. [29] designed the observer-based adaptive event-triggered control method for the stochastic nonlinear systems along with full state constraints and actuator faults. In order to obtain accurate tracking performance, it is essential to extend the event-triggered control from nonlinear systems to MASs. Consequently, Zhang et al. [30] developed the event-triggered tracking control scheme for nonlinear MASs with unknown disturbances that can be estimated by a disturbance observer. The problem of distributed adaptive output control for uncertain nonlinear MASs with unmeasured states and uncertain control gains was addressed, and the Nussbaum function is used to compensate for the uncertain control gains in [31]. Besides, Qiu et al. [32] applied the fixed threshold event-triggered control mechanism to pure-feedback nonlinear systems with prescribed performance and unavailable system states. It is worth noting that the problems of event-triggered adaptive containment control for nonlinear MASs with immeasurable states and predefined performance are still to be studied.

Motivated by the analysis mentioned above, this paper is concerned with the observer-based event-triggered adaptive containment control problem for a class of high-order uncertain nonlinear multiagent systems in strict-feedback form with prescribed performance. RBFNNs are utilized to approximate the

unknown nonlinear function of the system dynamic, as well as the NN-based observer is applied to estimate the unavailable states. Combined with the Lyapunov stability theorem and backstepping techniques, the novel distributed adaptive containment control approach is proposed. The major contributions of this paper can be stated as follows: 1) By means of extending the switching threshold event-triggered strategy to the containment control of MASSs, not only the communication burden has been alleviated but also the system performance can be ensured. In addition, the proposed control scheme can achieve the containment control objective. 2) Under the framework of prescribed performance control, the containment errors can converge to a predefined arbitrarily small compact set with rapid convergence speed and high accuracy. 3) The proposed control method can overcome the differentiation explosion by using the DSC technique, and thus, the designed controllers become more effective than conventional adaptive backstepping controllers, the containment control also gains superior performance. All variables of the MASs are semi-global uniformly ultimately bounded (SGUUB) as well.

The rest of this paper is described as follows. In Section II, the system formulation and preliminary knowledge are given. The controller design and stability analysis are shown in Section III. The effectiveness of this proposed strategy is verified by the simulation results in Section IV. Section V finally draws the conclusion.

2 Problem Formulation and Preliminaries

2.1 Graph Theory

The exchange of information among agents can be expressed as the directed graph $\mathcal{G} = (\mathcal{V}, \mathfrak{N}, \mathcal{A})$, with the set of vertexes $\mathcal{V} = \{1, 2, \dots, N, N+1, \dots, N+M\}$ and the set of edges $\mathfrak{N} = \{(i, j) | i, j \in \mathcal{V}, j \neq i\}$. The followers are labeled as $1, \dots, N$, while “ $N+1, \dots, N+M$ ” represent the leader. $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{(N+M) \times (N+M)}$ stands for the adjacency matrix, where $(i, j) \in \mathfrak{N}$ means that there exists an information flow from agent j to agent i , and $a_{i,j} = 1$ when $(i, j) \in \mathfrak{N}$, otherwise $a_{i,j} = 0$. The neighbors set of nodes i is defined as $\mathcal{N}_i = \{j | (i, j) \in \mathfrak{N}\}$. Let $\mathcal{L} = [l_{i,j}] \in \mathbb{R}^{(N+M) \times (N+M)}$ be the Laplacian matrix, and $l_{i,j} = -a_{i,j}$ for all $j \neq i$, otherwise, $l_{i,j} = \sum_{j \in \mathcal{N}_i} a_{i,j}$. The Laplacian matrix equals to $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_1, \dots, d_{N+M}\}$ with $d_i = \sum_{j \in \mathcal{N}_i} a_{i,j}$ denotes the degree matrix of the digraph $\mathcal{G}$. It is assumed that each follower owns at least one neighbor, while leaders own none. Then, the Laplacian $\mathcal{L}$ of the digraph $\mathcal{G}$ can be partitioned into

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ 0_{M \times N} & 0_{M \times M} \end{bmatrix}$$

Assumption 1: For each follower, there is at least one leader that possesses a directed path to it.

Lemma 1 [33]: Based on Assumption 1, each eigenvalues of matrix $\mathcal{L}_1$ has a positive real part. Each row of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ has a sum that equals to 1, and each entry of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is nonnegative.

Assumption 2: The outputs of the leaders $y_{l,r} (l = N+1, \dots, N+M)$ are adequately smooth function of t , and $y_{l,r}, \dot{y}_{l,r}, \ddot{y}_{l,r}$ are bounded, i.e., there exists a positive constant ϖ , such that

$$\|y_{l,r}\|^2 + \|\dot{y}_{l,r}\|^2 + \|\ddot{y}_{l,r}\|^2 \leq \varpi$$

Definition 1 [13]: A set $\mathfrak{C}$ is convex, if for any $X_1, X_2 \in \mathfrak{C}$ and any $\theta \in [0, 1]$, $\theta X_1 + (1 - \theta)X_2 \in \mathfrak{C}$. The convex hull of a set $\mathfrak{C}$, denoted $\mathfrak{Co}(X) (X = X_1, \dots, X_n)$, is the smallest convex set that contains all points in X . Concretely, $\mathfrak{Co}(X) = \{\sum_{k=1}^n \theta_k X_k | X_k \in X, \theta_k > 0, \sum_{k=1}^n \theta_k = 1\}$.

2.2 Model Formulation

Consider a strict-feedback nonlinear multiagent system composed of N agents. The dynamics of the i th ($i=1, \dots, N$) agents are described as follows:

$$\begin{aligned} \dot{x}_{i,j} &= x_{i,j+1} + f_{i,j}(\bar{x}_{i,j}), \quad j = 1, 2, \dots, n-1 \\ \dot{x}_{i,n} &= u_i + f_{i,n}(\bar{x}_{i,n}) \\ y_i &= x_{i,1}, \quad i = 1, 2, \dots, N \end{aligned} \tag{1}$$

where $\bar{x}_{i,m} = [x_{i,1}, x_{i,2}, \dots, x_{i,m}]^T \in \mathbb{R}^m$ are state vectors. $y_i \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are the system output and the control input, respectively. $f_{i,j}(\cdot)$ denotes the unknown smooth nonlinear function. In this paper, it is assumed that only the output variables y_i are available, while other states are unmeasurable during the controller design procedure.

Rewrite system (1) as:

$$\begin{aligned}\dot{\bar{x}}_{i,n} &= K_i \bar{x}_{i,n} + A_i y_i + \sum_{j=1}^n B_{i,j} f_{i,j}(\bar{x}_{i,j}) + C_i u_i \\ y_i &= D_i^T \bar{x}_{i,n}, \quad i = 1, 2, \dots, N\end{aligned}\tag{2}$$

where

$$\begin{aligned}K_i &= \begin{bmatrix} -k_{i,1} & & & \\ -k_{i,2} & I_{(N-1) \times (N-1)} & & \\ \vdots & & \ddots & \\ -k_{i,n} & 0 & \dots & 0 \end{bmatrix}, A_i = [k_{i,1}, k_{i,2}, \dots, k_{i,n}]^T, B_{i,j} = \begin{bmatrix} 0, \dots, 0, 1, 0, \dots, 0 \end{bmatrix}^T, \\ C_i &= [0, \dots, 0, 1]^T, \quad D_i = [1, 0, \dots, 0]^T\end{aligned}$$

The vector A_i is selected so that K_i is a strict Hurwitz matrix, thus, given any positive definite matrix Q_i with $Q_i = Q_i^T > 0$, there is a positive symmetric matrix P_i satisfying

$$K_i^T P_i + P_i^T K_i = -Q_i\tag{3}$$

2.3 RBFNN and State-Observer Design

The universal approximation property of RBFNNs are used to approximate an unknown smooth function $\Psi(X) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ defined on a compact set Ω_x , there exists an RBFNN as:

$$\Psi(X) = W^{*T} S(X)$$

where $X \in \Omega_x \subset \mathbb{R}^n$ is the input vector, $W^* \in \mathbb{R}^{l \times m}$ is the ideal weight matrix, $l > 1$ represents the number of neurons, $S(X) = [S_1(X), S_2(X), \dots, S_l(X)]^T$ is the Gaussian basis function vectors, and $S_i(X) = \exp[-(X - \varrho_i)^T(X - \varrho_i)/\varsigma_i^2]$ for $i = 1, 2, \dots, l$, where $\varrho_i = [\varrho_{i,1}, \varrho_{i,2}, \dots, \varrho_{i,n}]^T$, $i = 1, 2, \dots, l$ denote the center of the receptive field, ς_i represents the width of the Gaussian function.

As is known to all, RBFNN can approximate any continues nonlinear function to a desired accuracy on a compact space. On account of the excellent ability of RBFNNs, the unknown smooth function $F_{i,j}(\bar{x}_{i,j}) \in \mathbb{R}^m$ can be approximated as:

$$\hat{f}_{i,j}(\bar{x}_{i,j} | \hat{W}_{i,j}) = \hat{W}_{i,j}^T S_{i,j}(\bar{x}_{i,j})\tag{4}$$

where $\hat{W}_{i,j}$ represents the estimation of the optimal weight matrix $W_{i,j}^*$.

Let $\hat{\bar{x}}_{i,j} = [\hat{x}_{i,1}, \dots, \hat{x}_{i,j}]^T$ denote the estimation of the state vector $\bar{x}_{i,j}$, One has

$$\hat{f}_{i,j}(\hat{\bar{x}}_{i,j} | \hat{W}_{i,j}) = \hat{W}_{i,j}^T S_{i,j}(\hat{\bar{x}}_{i,j})\tag{5}$$

The optimal weight matrix $W_{i,j}^*$ is expressed as:

$$W_{i,j}^* := \arg \min_{\hat{W}_{i,j} \in \hat{\Xi}_{i,j} \in \mathbb{R}^{l \times m}} \left\{ \sup_{\substack{\hat{\bar{x}}_{i,j} \in \hat{\Xi}_{i,j} \\ \bar{x}_{i,j} \in \Xi_{i,j}}} \|\hat{f}_{i,j}(\hat{\bar{x}}_{i,j} | \hat{W}_{i,j}) - f_{i,j}(\bar{x}_{i,j})\| \right\}\tag{6}$$

where $\Xi_{i,j}$, $\hat{\Xi}_{i,j}$ and $\tilde{\Xi}_{i,j}$ are compact sets for $\bar{x}_{i,j}$, $\hat{\bar{x}}_{i,j}$ and $\hat{W}_{i,j}$, respectively.

Defining the corresponding minimal approximation errors $\lambda_{i,j}$ and the approximation errors $\mu_{i,j}$ respectively, as

$$\lambda_{i,j} = f_{i,j}(\bar{x}_{i,j}) - \hat{f}_{i,j}(\hat{x}_{i,j}|W_{i,j}^*) \quad (7)$$

$$\mu_{i,j} = f_{i,j}(\bar{x}_{i,j}) - \hat{f}_{i,j}(\hat{x}_{i,j}|\hat{W}_{i,j}) \quad (8)$$

Assumption 3: The corresponding minimal approximation errors $\lambda_{i,j}$ and the approximation errors $\mu_{i,j}$ are bounded, i.e., existing the constant $\lambda_{i,j}^0 > 0, \mu_{i,j}^0 > 0$ such that $\|\lambda_{i,j}\| < \lambda_{i,j}^0, \|\mu_{i,j}\| < \mu_{i,j}^0$. Defining $\eta_{i,j} = \lambda_{i,j} - \mu_{i,j}, \eta_{i,j}^0 = \lambda_{i,j}^0 + \mu_{i,j}^0$, then we have $\|\eta_{i,j}\| \leq \lambda_{i,j}^0 + \mu_{i,j}^0 = \eta_{i,j}^0, i = 1, \dots, N, j = 1, \dots, n$.

Since the states of each follower in system (1) except outputs are unavailable in the control design, the NN-based state observers is constructed to estimate the unavailable states:

$$\dot{\hat{x}}_{i,n} = K_i \hat{x}_{i,n} + A_i y_i + \sum_{j=1}^n B_{i,j} \hat{f}_{i,j}(\hat{x}_{i,j}|\hat{W}_{i,j}) + C_i u_i \quad (9)$$

$$y_i = D_i^T \hat{x}_{i,n}, \quad i = 1, 2, \dots, N$$

Define the observer error as $e_i = [e_{i,1}, e_{i,2}, \dots, e_{i,n}]^T = \bar{x}_{i,n} - \hat{x}_{i,n} = [(x_{i,1} - \hat{x}_{i,1})^T, \dots, (x_{i,n} - \hat{x}_{i,n})^T]^T$, then from (2) and (9), the observer error dynamic can be written as

$$\begin{aligned} e_i &= K_i e_i + \sum_{j=1}^n B_{i,j} f_{i,j}(\bar{x}_{i,j}) - \sum_{j=1}^n B_{i,j} \hat{f}_{i,j}(\hat{x}_{i,j}|\hat{W}_{i,j}) \\ &= K_i e_i + \mu_i, \quad i = 1, \dots, N \end{aligned} \quad (10)$$

where $\mu_i = [\mu_{i,1}, \dots, \mu_{i,n}]^T$

By combining (1), (9), and (10), one obtains:

$$\begin{aligned} \dot{\hat{x}}_{i,j} &= \hat{x}_{i,j+1} + k_{i,j}(x_{i,1} - \hat{x}_{i,1}) + \hat{f}_{i,j}(\hat{x}_{i,j}|\hat{W}_{i,j}), \quad j = 1, 2, \dots, n-1 \\ \dot{\hat{x}}_{i,n} &= u_i + k_{i,n}(x_{i,1} - \hat{x}_{i,1}) + \hat{f}_{i,n}(\hat{x}_{i,n}|\hat{W}_{i,n}) \end{aligned} \quad (11)$$

2.4 Event-triggered Design

A switching threshold event-triggered control strategy is given below. The intermediate control function is defined as follows:

$$r_i = \begin{cases} -(1 + \varepsilon_i) \left[\alpha_{i,n} \tanh\left(\frac{s_{i,n} \alpha_{i,n}}{\epsilon}\right) + \bar{a} \tanh\left(\frac{s_{i,n} \bar{a}}{\epsilon}\right) \right] & , \quad \varepsilon_i |u_i(t)| + a \geq m_i \\ \alpha_{i,n} - \bar{m}_i \tanh\left(\frac{s_{i,n} \bar{m}_i}{\epsilon}\right) & , \quad \varepsilon_i |u_i(t)| + a < m_i \end{cases} \quad (12)$$

where $\alpha_{i,n}$ is virtual control function to be designed later.

Define the triggering event as

$$u_i(t) = r_i(t_i^k) \quad \forall t \in [t_i^k, t_i^{k+1}) \quad (13)$$

$$t_i^{k+1} = \inf \left\{ t \in \mathbb{R} \mid \tau_i(t) \geq \max \{ \varepsilon_i |u_i(t)| + a, m_i \} \right\} \quad (14)$$

where $\tau_i(t) = r_i(t) - u_i(t)$ denotes the measurement error. $t_i^k, k \in \mathbb{Z}^+$ is the update time, i.e., when (14) is triggered, the time will be recorded by t_i^{k+1} , and the control value will be marked as $u(t_i^{k+1})$. $\varepsilon_i, m_i, \bar{m}_i > m_i, a$ and $\bar{a} > a/(1 - \varepsilon_i)$ are positive design parameters, $0 < \varepsilon_i < 1$. During the period $t \in [t_i^k, t_i^{k+1})$, the control signal holds as $r_i(t_i^k)$.

From (13) and (14), we have

$$\tau_i(t) = \begin{cases} \delta_{i,1}(t) \varepsilon_i u_i(t) + \delta_{i,2}(t) a & , \quad \varepsilon_i |u_i(t)| + a \geq m_i \\ \delta_{i,3}(t) m_i & , \quad \varepsilon_i |u_i(t)| + a < m_i \end{cases} \quad (15)$$

where $\delta_{i,1}(t), \delta_{i,2}(t)$ and $\delta_{i,3}(t)$ are continuous time-varying parameters with $\delta_{i,1}(t_i^k) = \delta_{i,2}(t_i^k) = \delta_{i,3}(t_i^k) = 0, \delta_{i,1}(t_i^{k+1}) = \pm 1, \delta_{i,2}(t_i^{k+1}) = \pm 1, \delta_{i,3}(t_i^{k+1}) = \pm 1, |\delta_{i,1}(t)| \leq 1, |\delta_{i,2}(t)| \leq 1, |\delta_{i,3}(t)| \leq 1, \forall t \in [t_i^k, t_i^{k+1})$. Then, one has

$$u_i(t) = \begin{cases} \frac{r_i}{1 + \delta_{i,1}(t)\varepsilon_i} - \frac{\delta_{i,2}(t)a}{1 + \delta_{i,1}(t)\varepsilon_i} & , \quad \varepsilon_i|u_i(t)| + a \geq m_i \\ r_i - \delta_{i,3}(t)m_i & , \quad \varepsilon_i|u_i(t)| + a < m_i \end{cases} \quad (16)$$

Remark 1: This switching threshold-based controller has the advantages of the fixed threshold strategy and the relative threshold strategy. As there is a high magnitude of control signal $u_i(t)$, the relative threshold strategy is chosen to obtain longer update intervals; while when $u_i(t)$ is approaching to zero, the fixed threshold strategy is applied to obtain better system performance. The Zeno behavior can be avoided as well.

Lemma 2 [25]: The hyperbolic tangent function $\tanh(\cdot)$ satisfies the following property:

$$0 \leq |\iota| - \iota \tanh \frac{\iota}{\epsilon} \leq 0.2785\epsilon$$

where ϵ is an arbitrary positive constant, and $\iota \in \mathbb{R}$.

2.5 Prescribed Performance

According to [20], the prescribed performance is accomplished by ensuring that the containment error surfaces $v_{i,1} = \sum_{j=1}^N a_{i,j}(y_i - y_j) + \sum_{l=N+1}^{N+M} a_{i,l}(y_i - y_{l,r})$ evolve strictly within predefined decaying bounds as follows:

$$-\eta_{i \min} \omega_i(t) < v_{i,1}(t) < \eta_{i \max} \omega_i(t), \quad \forall t \geq 0 \quad (17)$$

where $\eta_{i \min}$ and $\eta_{i \max}$ are positive parameters. The prescribed performance function $\omega_i(t) = (\omega_{i,0} - \omega_{i,\infty})e^{-\zeta_i t} + \omega_{i,\infty}$ is a strictly decreasing smooth function satisfying $\lim_{t \rightarrow \infty} \omega_i(t) = \omega_{i,\infty} > 0, \omega_{i,0} = \omega_i(0) > \omega_{i,\infty}, \zeta_i > 0$ such that $-\eta_{i \min} \omega_i(0) < v_{i,1}(0) < \eta_{i \max} \omega_i(0)$, and we can obtain $|v_{i,1}(t)| < \max\{\eta_{i \min} \omega_i(0), \eta_{i \max} \omega_i(0)\}$.

To achieve the desired performance, the constrained containment error is converted into an equivalent unconstrained one. Letting

$$v_{i,1}(t) = \omega_i(t) \Theta_i(\beta_i(t)), \quad \forall t \geq 0 \quad (18)$$

where β_i is the converted error, and the smooth function $\Theta_i(\beta_i)$ is strictly monotonic increasing, and

$$\Theta_i(\beta_i) = \frac{\eta_{i \max} e^{\beta_i} - \eta_{i \min} e^{-\beta_i}}{e^{\beta_i} + e^{-\beta_i}}$$

The inverse function can be represented as:

$$\beta_i(t) = \Theta_i^{-1} \left(\frac{v_{i,1}(t)}{\omega_i(t)} \right) = \frac{1}{2} \ln \frac{\Theta_i + \eta_{i \min}}{\eta_{i \max} - \Theta_i} \quad (19)$$

and

$$\dot{\beta}_i(t) = \pi_i \left(\dot{v}_{i,1} - \frac{\dot{\omega}_i v_{i,1}}{\omega_i} \right) \quad (20)$$

where $\pi_i = \frac{1}{2\omega_i} \left[\frac{1}{\Theta_i + \eta_{i \min}} - \frac{1}{\Theta_i - \eta_{i \max}} \right]$

Define the following state transformation:

$$s_{i,1} = \beta_i(t) - \frac{1}{2} \ln \frac{\eta_{i \min}}{\eta_{i \max}} \quad (21)$$

the transformation state dynamics are

$$\dot{s}_{i,1} = \pi_i \left(\dot{v}_{i,1} - \frac{\omega_i v_{i,1}}{\omega_i} \right) \quad (22)$$

Similar to [20], it is obvious that $v_{i,1}$ satisfies the prescribed performance expressed by (17) when $s_{i,1}$ is bounded.

Remark 2: Different from the control strategy in [19] and [22] which can not be applied to the MASs with multileader, this paper attempts to deal with the containment control problems for MASs. Moreover, the unmeasured states and the switching threshold-based event-triggered strategy are incorporated in this work.

3 Controller Design and Stability Analysis

In this section, an adaptive neural network-based controller and parameter adaptive laws are presented by the backstepping technique and DSC technique. All signals of the closed-loop system (1) are SGUUB, and the containment errors $v_{i,1}$ are as small as possible.

To design n-step adaptive backstepping consensus controller, the coordinate transformations are defined as:

$$\begin{aligned} s_{i,j} &= \hat{x}_{i,j} - \nu_{i,j} \\ z_{i,j} &= \nu_{i,j} - \alpha_{i,j-1}, \quad i = 1, \dots, N, \quad j = 2, \dots, n \end{aligned} \quad (23)$$

where $s_{i,j}$ is an error surface, $\nu_{i,j}$ is a filtered virtual control, which is obtained through a first-order filter on virtual control function $\alpha_{i,j-1}$, and $z_{i,j}$ is the output error of the first-order filter.

Lemma 3 [23]: Letting $y_i = [y_1, \dots, y_N]^T$, and $s_{*,1} = [s_{1,1}, \dots, s_{N,1}]^T$. Then, the following inequality holds:

$$\|y_i + \mathcal{L}_1^{-1} \mathcal{L}_2 y_{l,r}\| \leq \|s_{*,1}\| / \|\bar{\sigma}(\mathcal{L}_1)\|$$

where $\bar{\sigma}$ is the minimum singular value of $\mathcal{L}_1$.

Step 1: Combining (1), (7), (10) with (23), the time derivative of $v_{i,1} = \sum_{j=1}^N a_{i,j}(y_i - y_j) + \sum_{l=N+1}^{N+M} a_{i,l}(y_i - y_{l,r})$ is

$$\begin{aligned} \dot{v}_{i,1} &= \sum_{j=1}^N a_{i,j}(\dot{y}_i - \dot{y}_j) + \sum_{l=N+1}^{N+M} a_{i,l}(\dot{y}_i - \dot{y}_{l,r}) \\ &= d_i \dot{x}_{i,1} - \sum_{j=1}^N a_{i,j} \dot{x}_{j,1} - \sum_{l=N+1}^{N+M} a_{i,l} \dot{y}_{l,r} \\ &= d_i (\hat{x}_{i,2} + f_{i,1}(x_{i,1}) + e_{i,2}) - \sum_{j=1}^N a_{i,j} (\hat{x}_{j,2} + f_{j,1}(x_{j,1}) + e_{j,2}) - \sum_{l=N+1}^{N+M} a_{i,l} \dot{y}_{l,r} \\ &= d_i \left(\hat{x}_{i,2} + \hat{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1}) + \tilde{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1}) + \lambda_{i,1} + e_{i,2} \right) \\ &\quad - \sum_{j=1}^N a_{i,j} \left(\hat{x}_{j,2} + \hat{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1}) + \tilde{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1}) + \lambda_{j,1} + e_{j,2} \right) - \sum_{l=N+1}^{N+M} a_{i,l} \dot{y}_{l,r} \\ &= d_i \left(\alpha_{i,1} + z_{i,2} + s_{i,2} + \hat{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1}) + \tilde{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1}) + \lambda_{i,1} + e_{i,2} \right) \\ &\quad - \sum_{j=1}^N a_{i,j} \left(\hat{x}_{j,2} + \hat{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1}) + \tilde{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1}) + \lambda_{j,1} + e_{j,2} \right) - \sum_{l=N+1}^{N+M} a_{i,l} \dot{y}_{l,r} \end{aligned} \quad (24)$$

where $\tilde{W}_{i,j} = W_{i,j}^* - \hat{W}_{i,j}$, $j = 1, \dots, n$.

Consider the following Lyapunov function candidate

$$V_{i,1} = e_i^T P_i e_i + \frac{1}{2} s_{i,1}^2 + \frac{1}{2\gamma_{i,1}} \tilde{W}_{i,1}^T \tilde{W}_{i,1} + \sum_{j=1}^N \frac{a_{i,j}}{2\chi_{j,1}} \tilde{W}_{j,1}^T \tilde{W}_{j,1} \quad (25)$$

where $\gamma_{i,1}$, $\chi_{j,1}$ are both positive design constants.

The time derivative of $V_{i,1}$ along (3), (10) and (22) is

$$\dot{V}_{i,1} = -e_i^T Q_i e_i + 2e_i^T P_i \mu_i + s_{i,1}^T \pi_i \left(\dot{v}_{i,1} - \frac{\dot{\omega}_i v_{i,1}}{\omega_i} \right) - \frac{1}{\gamma_{i,1}} \tilde{W}_{i,1}^T \dot{\tilde{W}}_{i,1} - \sum_{j=1}^N \frac{a_{i,j}}{\chi_{j,1}} \tilde{W}_{j,1}^T \dot{\tilde{W}}_{j,1} \quad (26)$$

Using the Young's inequality $2a^T b \leq \|a\|^2 + \|b\|^2$, one has

$$2e_i^T P_i \mu_i \leq \|e_i\|^2 + \|P_i\|^2 \|\mu_i^0\|^2 \quad (27)$$

$$\begin{aligned} s_{i,1}^T \pi_i \left[d_i(\lambda_{i,1} + e_{i,2}) - \sum_{j=1}^N a_{i,j}(\lambda_{j,1} + e_{j,2}) \right] &\leq \pi_i^2 \|s_{i,1}\|^2 + \frac{1}{2} \|\mathcal{L}_1\|_F^2 \|\lambda_{i,1}^0\|^2 + \frac{1}{2} \|\mathcal{L}_1\|_F^2 \|e_{i,2}\|^2 \\ &\leq \pi_i^2 \|s_{i,1}\|^2 + \frac{1}{2} \|\mathcal{L}_1\|_F^2 \|\lambda_{i,1}^0\|^2 + \frac{1}{2} \|\mathcal{L}_1\|_F^2 \|e_{i,2}\|^2 \end{aligned} \quad (28)$$

Substituting (24), (27), (28) into (26) results in

$$\begin{aligned} \dot{V}_{i,1} &\leq -l_{min} \|e_i\|^2 + \|P_i\|^2 \|\mu_i^0\|^2 + \frac{1}{2} \|\mathcal{L}_1\|_F^2 \|\lambda_{i,1}^0\|^2 - \frac{1}{\gamma_{i,1}} \tilde{W}_{i,1}^T \dot{\tilde{W}}_{i,1} - \sum_{j=1}^N \frac{a_{i,j}}{\chi_{j,1}} \tilde{W}_{j,1}^T \dot{\tilde{W}}_{j,1} \\ &\quad + s_{i,1}^T \pi_i \left[d_i(\alpha_{i,1} + z_{i,2} + s_{i,2} + \hat{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1}) + \tilde{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1})) - \pi_i s_{i,1} \right. \\ &\quad \left. - \sum_{j=1}^N a_{i,j}(\hat{x}_{j,2} + \hat{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1}) + \tilde{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1})) - \sum_{l=N+1}^{N+M} a_{i,l} \dot{y}_{l,r} - \frac{\dot{\omega}_i v_{i,1}}{\omega_i} \right] \end{aligned} \quad (29)$$

where $l_{min} = \min(\lambda_{min}(Q_i) - 1 - \frac{1}{2} \|\mathcal{L}_1\|_F^2)$, and $\lambda_{min}(Q_i)$ is the smallest eigenvalue of matrix Q_i .

Design the virtual control function $\alpha_{i,1}$ and the parameter adaptive law $\tilde{W}_{i,1}$, $\tilde{W}_{j,1}$ as follows:

$$\begin{aligned} \alpha_{i,1} &= \frac{1}{d_i} \left[-\frac{c_{i,1}}{\pi_i} s_{i,1} + \pi_i s_{i,1} - \sum_{j=1}^N a_{i,j}(\hat{x}_{j,2} + \hat{W}_{j,1}^T S_{j,1}(\hat{x}_{j,1})) + \sum_{l=N+1}^{N+M} a_{i,l} \dot{y}_{l,r} + \frac{\dot{\omega}_i v_{i,1}}{\omega_i} \right] \\ &\quad - \hat{W}_{i,1}^T S_{i,1}(\hat{x}_{i,1}) \end{aligned} \quad (30)$$

$$\dot{\tilde{W}}_{i,1} = \gamma_{i,1} \left(\pi_i d_i s_{i,1} S_{i,1}(\hat{x}_{i,1}) - \sigma_{i,1} \hat{W}_{i,1} \right) \quad (31)$$

$$\dot{\tilde{W}}_{j,1} = \chi_{j,1} \left(\pi_i s_{i,1} S_{j,1}(\hat{x}_{j,1}) - \varrho_{j,1} \hat{W}_{j,1} \right) \quad (32)$$

where $c_{i,1} > 0$, $\sigma_{i,1} > 0$ and $\varrho_{j,1} > 0$ are design parameters.

By invoking (30) - (32), we can obtain

$$\begin{aligned} \dot{V}_{i,1} &\leq -l_{min} \|e_i\|^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i s_{i,1}^T (z_{i,2} + s_{i,2}) + \sigma_{i,1} \tilde{W}_{i,1}^T \hat{W}_{i,1} \\ &\quad + \sum_{j=1}^N a_{i,j} \varrho_{j,1} \tilde{W}_{j,1}^T \hat{W}_{j,1} + \rho_{i,1} \end{aligned} \quad (33)$$

where $\rho_{i,1} = \|P_i\|^2 \|\mu_i^0\|^2 + \frac{1}{2} \|\mathcal{L}_1\|_F^2 \|\lambda_{i,1}^0\|^2$.

Define first-order filter with time constant $\tau_{i,j}$ as:

$$\tau_{i,j} \dot{\nu}_{i,j} + \nu_{i,j} = \alpha_{i,j-1}, \quad \nu_{i,j}(0) = \alpha_{i,j-1}(0) \quad (34)$$

as $\dot{z}_{i,j} = \dot{\nu}_{i,j} - \dot{\alpha}_{i,j-1}$, we obtain $\dot{z}_{i,j} = -\frac{z_{i,j}}{\tau_{i,j}} + M_{i,j}(s_{i,1}, \dots, s_{i,j}, z_{i,2}, \dots, z_{i,j}, \hat{W}_{i,1}, \dots, \hat{W}_{i,j}, y_{l,r}, \dot{y}_{l,r}, \ddot{y}_{l,r})$

Step 2: from (5), (11), one obtains

$$\dot{\hat{x}}_{i,2} = \hat{x}_{i,3} - k_{i,2}\hat{x}_{i,1} + k_{i,2}x_{i,1} + \hat{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) \quad (35)$$

Consider the following Lyapunov function candidate

$$V_{i,2} = V_{i,1} + \frac{1}{2}s_{i,2}^2 + \frac{1}{2}z_{i,2}^2 + \frac{1}{2\gamma_{i,2}}\tilde{W}_{i,2}^T\tilde{W}_{i,2} \quad (36)$$

where $\gamma_{i,2}$ is a positive design constant.

The time derivative of $V_{i,2}$ along (7), (8), (23), (33) - (35) is

$$\begin{aligned} \dot{V}_{i,2} &= -l_{min}\|e_i\|^2 - c_{i,1}s_{i,1}^2 + s_{i,1}^T\pi_i d_i(z_{i,2} + s_{i,2}) + \sigma_{i,1}\tilde{W}_{i,1}^T\hat{W}_{i,1} + \rho_{i,1} \\ &\quad + s_{i,2}^T\left(s_{i,3} + z_{i,3} + \alpha_{i,2} - k_{i,2}\hat{x}_{i,1} + k_{i,2}x_{i,1} + \hat{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) + \tilde{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) - \tilde{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) - \dot{\nu}_{i,2}\right) \\ &\quad - \frac{1}{\tau_{i,2}}z_{i,2}^2 + z_{i,2}M_{i,2}(\cdot) - \frac{1}{\gamma_{i,2}}\tilde{W}_{i,2}^T\dot{\tilde{W}}_{i,2} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} \\ &= -l_{min}\|e_i\|^2 - c_{i,1}s_{i,1}^2 + s_{i,1}^T\pi_i d_i(z_{i,2} + s_{i,2}) + \sigma_{i,1}\tilde{W}_{i,1}^T\hat{W}_{i,1} + \rho_{i,1} \\ &\quad + s_{i,2}^T\left(s_{i,3} + z_{i,3} + \alpha_{i,2} - k_{i,2}\hat{x}_{i,1} + k_{i,2}x_{i,1} + \hat{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) + \tilde{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) + \lambda_{i,2} - \mu_{i,2} - \dot{\nu}_{i,2}\right) \\ &\quad - \frac{1}{\tau_{i,2}}z_{i,2}^2 + z_{i,2}M_{i,2}(\cdot) - \frac{1}{\gamma_{i,2}}\tilde{W}_{i,2}^T\dot{\tilde{W}}_{i,2} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} \end{aligned} \quad (37)$$

According to Young's inequality, we have

$$s_{i,j}(\lambda_{i,j} - \mu_{i,j}) \leq \frac{1}{2}s_{i,j}^2 + \frac{1}{2}\|\eta_{i,j}^0\|^2 \quad (38)$$

By invoking (38), we can obtain

$$\begin{aligned} \dot{V}_{i,2} &\leq -l_{min}\|e_i\|^2 - c_{i,1}s_{i,1}^2 + s_{i,1}^T\pi_i d_i(z_{i,2} + s_{i,2}) + \sigma_{i,1}\tilde{W}_{i,1}^T\hat{W}_{i,1} \\ &\quad + s_{i,2}^T\left(s_{i,3} + z_{i,3} + \alpha_{i,2} - k_{i,2}\hat{x}_{i,1} + k_{i,2}x_{i,1} + \hat{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) + \tilde{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) + \frac{1}{2}s_{i,2} - \dot{\nu}_{i,2}\right) \\ &\quad - \frac{1}{\tau_{i,2}}z_{i,2}^2 + z_{i,2}M_{i,2}(\cdot) - \frac{1}{\gamma_{i,2}}\tilde{W}_{i,2}^T\dot{\tilde{W}}_{i,2} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} + \rho_{i,1} + \frac{1}{2}\|\eta_{i,2}^0\|^2 \end{aligned} \quad (39)$$

Construct the virtual control function $\alpha_{i,2}$ and the parameter adaptive law $W_{i,2}$ as:

$$\alpha_{i,2} = -c_{i,2}s_{i,2} - \pi_i d_i s_{i,1} - \frac{1}{2}s_{i,2} + k_{i,2}\hat{x}_{i,1} - k_{i,2}x_{i,1} - \hat{W}_{i,2}^T S_{i,2}(\hat{x}_{i,2}) + \dot{\nu}_{i,2} \quad (40)$$

$$\dot{\tilde{W}}_{i,2} = \gamma_{i,2}\left(s_{i,2}S_{i,2}(\hat{x}_{i,2}) - \sigma_{i,2}\hat{W}_{i,2}\right) \quad (41)$$

where $c_{i,2} > 0$ and $\sigma_{i,2} > 0$ are design parameters.

From (39), (40) and (41), one has

$$\begin{aligned} \dot{V}_{i,2} &\leq -l_{min}\|e_i\|^2 - c_{i,1}s_{i,1}^2 + s_{i,1}^T\pi_i d_i z_{i,2} - c_{i,2}s_{i,2}^2 + s_{i,2}^T(s_{i,3} + z_{i,3}) \\ &\quad + \sigma_{i,1}\tilde{W}_{i,1}^T\hat{W}_{i,1} + \sigma_{i,2}\tilde{W}_{i,2}^T\hat{W}_{i,2} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} + \rho_{i,2} - \frac{1}{\tau_{i,2}}z_{i,2}^2 + z_{i,2}M_{i,2}(\cdot) \end{aligned} \quad (42)$$

where $\rho_{i,2} = \rho_{i,1} + \frac{1}{2}\|\eta_{i,2}^0\|^2$.

Step j ($3 \leq j < n$): Similar to the derivation in Step 2, from (23), we can obtain

$$\dot{\hat{x}}_{i,j} = s_{i,j+1} + z_{i,j+1} + \alpha_{i,j} - k_{i,j}\hat{x}_{i,1} + k_{i,j}x_{i,1} + \hat{W}_{i,j}^T S_{i,j}(\hat{x}_{i,j}) - \dot{\nu}_{i,j} \quad (43)$$

Choose the Lyapunov function candidate as

$$V_{i,j} = V_{i,j-1} + \frac{1}{2}s_{i,j}^2 + \frac{1}{2}z_{i,j}^2 + \frac{1}{2\gamma_{i,j}}\tilde{W}_{i,j}^T\tilde{W}_{i,j} \quad (44)$$

where $\gamma_{i,j}$ is a positive design constant.

The time derivative of $V_{i,j}$ along (42) and (43) is

$$\begin{aligned} \dot{V}_{i,j} \leq & -l_{min}\|e_i\|^2 + s_{i,1}^T\pi_i d_i z_{i,2} - \sum_{k=1}^{j-1} c_{i,k}s_{i,k}^2 + \sum_{k=1}^{j-1} \sigma_{i,k}\tilde{W}_{i,k}^T\hat{W}_{i,k} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} \\ & + s_{i,j-1}s_{i,j} + \sum_{k=2}^{j-1} s_{i,k}z_{i,k+1} - \sum_{k=1}^{j-2} \left(\frac{z_{i,k+1}^2}{\tau_{i,k+1}} - z_{i,k+1}M_{i,k+1}(\cdot) \right) + \rho_{i,j-1} + \frac{1}{2}\|\eta_{i,j}^0\|^2 \\ & + s_{i,j} \left(s_{i,j+1} + z_{i,j+1} + \alpha_{i,j} - k_{i,j}\hat{x}_{i,1} + k_{i,j}x_{i,1} + \hat{W}_{i,j}^T S_{i,j}(\hat{x}_{i,j}) + \frac{1}{2}s_{i,j} + \tilde{W}_{i,j}^T S_{i,j}(\hat{x}_{i,j}) - \dot{\nu}_{i,j} \right) \\ & - \frac{1}{\tau_{i,j}}z_{i,j}^2 + z_{i,j}M_{i,j}(\cdot) - \frac{1}{\gamma_{i,j}}\tilde{W}_{i,j}^T\dot{\tilde{W}}_{i,j} \end{aligned} \quad (45)$$

Construct the virtual control function $\alpha_{i,j}$ and the parameter adaptive law $\dot{W}_{i,j}$ as:

$$\alpha_{i,j} = -c_{i,j}s_{i,j} - s_{i,j-1} - \frac{1}{2}s_{i,j} + k_{i,j}\hat{x}_{i,1} - k_{i,j}x_{i,1} - \hat{W}_{i,j}^T S_{i,j}(\hat{x}_{i,j}) + \dot{\nu}_{i,j} \quad (46)$$

$$\dot{\tilde{W}}_{i,j} = \gamma_{i,j} \left(s_{i,j}S_{i,j}(\hat{x}_{i,j}) - \sigma_{i,j}\hat{W}_{i,j} \right) \quad (47)$$

where $c_{i,j} > 0$ and $\sigma_{i,j} > 0$ are design parameters.

From (45), (46) and (47), one has

$$\begin{aligned} \dot{V}_{i,j} \leq & -l_{min}\|e_i\|^2 + s_{i,1}^T\pi_i d_i z_{i,2} - \sum_{k=1}^j c_{i,k}s_{i,k}^2 + \sum_{k=1}^j \sigma_{i,k}\tilde{W}_{i,k}^T\hat{W}_{i,k} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} \\ & + s_{i,j}s_{i,j+1} + \sum_{k=2}^j s_{i,k}z_{i,k+1} - \sum_{k=1}^{j-1} \left(\frac{z_{i,k+1}^2}{\tau_{i,k+1}} - z_{i,k+1}M_{i,k+1}(\cdot) \right) + \rho_{i,j} \end{aligned} \quad (48)$$

where $\rho_{i,j} = \rho_{i,j-1} + \frac{1}{2}\|\eta_{i,j}^0\|^2$.

Step n: In the final design step, from (12) and (25), one obtains

$$\dot{s}_{i,n} = u_i - k_{i,n}\hat{x}_{i,1} + k_{i,n}x_{i,1} + \hat{W}_{i,n}^T S_{i,n}(\hat{x}_{i,n}) - \dot{\nu}_{i,n} \quad (49)$$

Choose the Lyapunov function candidate as

$$V_{i,n} = V_{i,n-1} + \frac{1}{2}s_{i,n}^2 + \frac{1}{2}z_{i,n}^2 + \frac{1}{2\gamma_{i,n}}\tilde{W}_{i,n}^T\tilde{W}_{i,n} \quad (50)$$

where $\gamma_{i,n}$ is a positive design constant.

The time derivative of $V_{i,n}$ along (48) and (49) is

$$\begin{aligned} \dot{V}_{i,n} \leq & -l_{min}\|e_i\|^2 + s_{i,1}^T\pi_i d_i z_{i,2} - \sum_{k=1}^{n-1} c_{i,k}s_{i,k}^2 + \sum_{k=1}^{n-1} \sigma_{i,k}\tilde{W}_{i,k}^T\hat{W}_{i,k} + \sum_{j=1}^N a_{i,j}\varrho_{j,1}\tilde{W}_{j,1}^T\hat{W}_{j,1} \\ & + s_{i,n-1}s_{i,n} + \sum_{k=2}^{n-1} s_{i,k}z_{i,k+1} - \sum_{k=1}^{n-1} \left(\frac{z_{i,k+1}^2}{\tau_{i,k+1}} - z_{i,k+1}M_{i,k+1}(\cdot) \right) + \rho_{i,n-1} + \frac{1}{2}\|\eta_{i,n}^0\|^2 \\ & - \frac{1}{\gamma_{i,n}}\tilde{W}_{i,n}^T\dot{\tilde{W}}_{i,n} + s_{i,n} \left(u_i - k_{i,n}\hat{x}_{i,1} + k_{i,n}x_{i,1} + \hat{W}_{i,n}^T S_{i,n}(\hat{x}_{i,n}) + \tilde{W}_{i,n}^T S_{i,n}(\hat{x}_{i,n}) + \frac{1}{2}s_{i,n} - \dot{\nu}_{i,n} \right) \end{aligned} \quad (51)$$

Construct the virtual control function $\alpha_{i,n}$ and the parameter adaptive law $\dot{W}_{i,n}$ as:

$$\alpha_{i,n} = -c_{i,n}s_{i,n} - s_{i,n-1} - \frac{1}{2}s_{i,n} + k_{i,n}\hat{x}_{i,1} - k_{i,n}x_{i,1} - \hat{W}_{i,n}^T S_{i,n}(\hat{x}_{i,n}) + \dot{\nu}_{i,n} \quad (52)$$

$$\dot{\hat{W}}_{i,n} = \gamma_{i,n} \left(s_{i,n} S_{i,n}(\hat{x}_{i,n}) - \sigma_{i,n} \hat{W}_{i,n} \right) \quad (53)$$

where $c_{i,n} > 0$ and $\sigma_{i,n} > 0$ are design parameters.

From (12) and (16), one has

$$u_i(t) = \begin{cases} -\frac{1}{1+\delta_{i,1}(t)\varepsilon_i} (1+\varepsilon_i) \left[\alpha_{i,n} \tanh\left(\frac{s_{i,n}\alpha_{i,n}}{\epsilon}\right) + \bar{a} \tanh\left(\frac{s_{i,n}\bar{a}}{\epsilon}\right) \right] - \frac{\delta_{i,2}(t)a}{1+\delta_{i,1}(t)\varepsilon_i}, \\ \quad \varepsilon_i |u_i(t)| + a \geq m_i \\ \alpha_{i,n} - \bar{m}_i \tanh\left(\frac{s_{i,n}\bar{m}_i}{\epsilon}\right) - \delta_{i,3}(t)m_i, \\ \quad \varepsilon_i |u_i(t)| + a < m_i \end{cases} \quad (54)$$

Since $\forall a \in \mathbb{R}$ and $\epsilon > 0$, $-\iota \tanh(\iota/\epsilon) \leq 0$, in the interval $[t_k, t_{k+1})$, from (12) we have $s_{i,n}r_i \leq 0$. As $\delta_{i,1}(t) \in [-1, 1]$, $\delta_{i,2}(t) \in [-1, 1]$, we have $s_{i,n}r_i/(1+\delta_{i,1}(t)\varepsilon_i) \leq s_{i,n}r_i/(1+\varepsilon_i)$, and $\delta_{i,2}(t)a/(1+\delta_{i,1}(t)\varepsilon_i) \leq a/(1-\varepsilon_i)$.

From (51) - (54), one has

$$\begin{aligned} \dot{V}_{i,n} &\leq -l_{min} \|e_i\|^2 + s_{i,1}^T \pi_i d_i z_{i,2} - \sum_{k=1}^n c_{i,k} s_{i,k}^2 + \sum_{k=1}^n \sigma_{i,k} \tilde{W}_{i,k}^T \hat{W}_{i,k} + \sum_{j=1}^N a_{i,j} \varrho_{j,1} \tilde{W}_{j,1}^T \hat{W}_{j,1} \\ &\quad + \sum_{k=2}^{n-1} s_{i,k} z_{i,k+1} - \sum_{k=1}^{n-1} \left(\frac{z_{i,k+1}}{\tau_{i,k+1}} - z_{i,k+1} M_{i,k+1}(\cdot) \right) + \rho_{i,n} \end{aligned} \quad (55)$$

with

$$\rho_{i,n} = \begin{cases} \rho_{i,n-1} + \frac{1}{2} \|\eta_{i,n}^0\|^2 + 0.557\epsilon & , \quad \varepsilon_i |u_i(t)| + a \geq m_i \\ \rho_{i,n-1} + \frac{1}{2} \|\eta_{i,n}^0\|^2 + 0.2785\epsilon & , \quad \varepsilon_i |u_i(t)| + a < m_i \end{cases}$$

According to Young's inequality, we have

$$\tilde{W}_{i,k}^T \hat{W}_{i,k} \leq -\frac{1}{2} \tilde{W}_{i,k}^T \tilde{W}_{i,k} + \frac{1}{2} W_{i,k}^{*T} W_{i,k}^* \quad (56)$$

$$\tilde{W}_{j,1}^T \hat{W}_{j,1} \leq -\frac{1}{2} \tilde{W}_{j,1}^T \tilde{W}_{j,1} + \frac{1}{2} W_{j,1}^{*T} W_{j,1}^* \quad (57)$$

$$\pi_i d_i s_{i,1}^T z_{i,2} \leq \frac{\pi_i^2 d_i^2}{2} s_{i,1}^2 + \frac{1}{2} z_{i,2}^2 \quad (58)$$

$$s_{i,k} z_{i,k+1} \leq \frac{1}{2} s_{i,k}^2 + \frac{1}{2} z_{i,k+1}^2 \quad (59)$$

$$z_{i,k+1} M_{i,k+1}(\cdot) \leq \frac{z_{i,k+1}^2 M_{i,k+1}^2(\cdot)}{2\xi} + \frac{\xi}{2} \quad (60)$$

From (55) - (60), one has

$$\begin{aligned} \dot{V}_{i,n} &\leq -l_{min} \|e_i\|^2 - c_{i,1} s_{i,1}^2 + \frac{\pi_i^2 d_i^2}{2} s_{i,1}^2 - \sum_{k=2}^n (c_{i,k} - \frac{1}{2}) s_{i,k}^2 - \sum_{k=1}^n \frac{1}{2} \sigma_{i,k} \tilde{W}_{i,k}^T \tilde{W}_{i,k} \\ &\quad - \sum_{j=1}^N \frac{1}{2} a_{i,j} \varrho_{j,1} \tilde{W}_{j,1}^T \tilde{W}_{j,1} - \sum_{k=1}^{n-1} \left(\frac{1}{\tau_{i,k+1}} - \frac{M_{i,k+1}^2(\cdot)}{2\xi} - \frac{1}{2} \right) z_{i,k+1}^2 + \rho_i \end{aligned} \quad (61)$$

where $\rho_i = \rho_{i,n} + \frac{\xi}{2}(n-1) + \sum_{k=1}^n \frac{1}{2} \sigma_{i,k} W_{i,k}^{*T} W_{i,k}^* + \sum_{j=1}^N \frac{1}{2} a_{i,j} \varrho_{j,1} W_{j,1}^{*T} W_{j,1}^*$.

Based on the proposed event-triggered control, the main results can be concluded as follows.

Theorem 1: Consider the nonlinear MASs (1) in the fixed directed communication topology with prescribed performance and unmeasured states with the adaptive updating laws (31)(31)(41)(47)(53)

and proposed adaptive controller (30)(40)(46)(52)(54) subject to triggered signal (13)(14). Based on Assumption 1-3, all signals of the closed-loop system are SGUUB and the containment control objective is achieved with the containment errors converge to a small residual set nearing the origin with prescribed performance. Moreover, the Zeno behavior is thoroughly excluded.

Proof: Take into account the Lyapunov function candidate for the whole system as

$$V = \sum_{i=1}^N V_{i,n} \quad (62)$$

The time derivative of V is

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^N \left\{ -l_{min} \|e_i\|^2 - c_{i,1} s_{i,1}^2 + \frac{\pi_i^2 d_i^2}{2} s_{i,1}^2 - \sum_{k=2}^n (c_{i,k} - \frac{1}{2}) s_{i,k}^2 - \sum_{k=1}^n \frac{1}{2} \sigma_{i,k} \tilde{W}_{i,k}^T \tilde{W}_{i,k} \right. \\ &\quad \left. - \sum_{j=1}^N \frac{1}{2} a_{i,j} \varrho_{j,1} \tilde{W}_{j,1}^T \tilde{W}_{j,1} - \sum_{k=1}^{n-1} \left(\frac{1}{\tau_{i,k+1}} - \frac{M_{i,k+1}^2(\cdot)}{2\xi} - \frac{1}{2} \right) z_{i,k+1}^2 + \rho_i \right\} \\ &\leq -\kappa V + R \end{aligned} \quad (63)$$

where

$$\begin{aligned} \kappa &= \min \left\{ l_{min}, 2 \left(c_{i,1} - \frac{\pi_i^2 d_i^2}{2} \right), 2(c_{i,j} - \frac{1}{2}), \frac{\gamma_{i,p}}{\sigma_{i,p}}, \frac{\chi_{j,1}}{\varrho_{j,1}}, 2 \left(\frac{1}{\tau_{i,k+1}} - \frac{M_{i,k+1}^2(\cdot)}{2\xi} - \frac{1}{2} \right) \right\}, \\ i &= 1, \dots, N, \quad j = 2, \dots, n, \quad p = 1, \dots, n, \quad k = 1, \dots, n-1. \\ R &= \sum_{i=1}^N \rho_i \end{aligned}$$

Integrating (63) over $[0, t]$, it is easy to obtain that

$$V(t) \leq e^{-\kappa t} V(0) + \frac{R}{\kappa} [1 - e^{-\kappa t}] \quad (64)$$

Using $\frac{\|s_{i,1}\|^2}{2} \leq V$, we have $\|s_{i,1}\|^2 \leq 2e^{-\kappa t} V(0) + \frac{2R}{\kappa} [1 - e^{-\kappa t}]$. Therefore, the error surfaces $\|s_{i,1}\|$ converge to the compact set $\Omega = \{s_{i,1} \mid \|s_{i,1}\| \leq \sqrt{\frac{2R}{\kappa}}\}$, which means $s_{i,1}$ is bounded, where $s_{i,1} = [s_{1,1}, \dots, s_{N,1}]^T$. In view of Lemma 3, we can concluded that the containment errors are bounded with prescribed performance. According to Assumption 2 and 3, it can be obtained that all signals of the closed-loop system are SGUUB.

Furthermore, from $\tau_i(t) = r_i(t) - u_i(t)$, we have $\frac{d|\tau_i(t)|}{dt} = \text{sign}(\tau_i(t)) \dot{\tau}_i(t) \leq |\dot{r}_i(t)|$. According to (12), it is known that $\dot{r}_i(t)$ is bounded which means there exists positive constant H_i with $\dot{r}_i(t) \leq H_i$. From $\tau(t_i^k) = 0$, and $\lim_{t_i \rightarrow t_i^{k+1}} \tau(t_i) = m_i$, $k \in \mathbb{Z}^+$, one can obtain that there exists t^* , and $t_i^{k+1} - t_i^k \geq t^* \geq m_i/H_i > 0$. Therefore, Zeno behavior is avoided.

The proof is completed.

4 Simulation Results

To validate the effectiveness of the proposed containment control method, an example of three followers and two leaders is utilized, and the fixed communication graph $\mathcal{G}$ is depicted in Fig.1. The dynamics of three followers are modeled as

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + \frac{1}{4} x_{i,1} \sin(x_{i,1}) \\ \dot{x}_{i,2} = u_i + x_{i,2} \sin(x_{i,1}) \\ y_i = x_{i,1} \end{cases} \quad i = 1, 2, 3$$

The dynamics of the leaders are described as

$$\begin{cases} y_{4d} = \sin(t + 1.86) - 0.6 \\ y_{5d} = \sin(t + 1.86) + 0.6 \end{cases}$$

The prescribed performance function is $\omega_i(t) = (4.5 - 1.2)e^{-1*t} + 1.2$. Choose correlative design parameters as $k_{1,1} = k_{2,1} = k_{3,1} = 15, k_{1,2} = k_{2,2} = 13, k_{3,2} = 14, \gamma_{1,1} = 1.6, \gamma_{1,2} = 8, \gamma_{2,1} = 3.3, \gamma_{2,2} = 8, \gamma_{3,1} = 1.6, \gamma_{3,2} = 8, \sigma_{1,1} = \sigma_{1,2} = \sigma_{2,1} = \sigma_{2,2} = \sigma_{3,1} = \sigma_{3,2} = 3, \chi_{1,1} = \chi_{2,1} = \chi_{3,1} = 8, \varrho_{1,1} = \varrho_{2,1} = \varrho_{3,1} = 3, \eta_{1\min} = \eta_{2\min} = \eta_{3\min} = 0.6, \eta_{1\max} = \eta_{2\max} = \eta_{3\max} = 0.9, \tau = 0.02, c_{1,1} = 16, c_{2,1} = 16, c_{3,1} = 15, c_{1,2} = 110, c_{2,2} = 100, c_{3,2} = 30, k_{1,2} = 288, k_{2,2} = 268, k_{3,2} = 127, m_1 = m_2 = m_3 = 0.8, \bar{m}_1 = \bar{m}_2 = \bar{m}_3 = 1, a = 0.1, \bar{a} = 0.5, \epsilon = 8, \varepsilon_1 = 0.5, \varepsilon_2 = 0.4, \varepsilon_3 = 0.5$.

Choose initial conditions as $x_{1,1}(0) = 0.5, x_{2,1}(0) = 0.5, x_{3,1}(0) = 0.4, x_{1,2}(0) = 0.5, x_{2,2}(0) = 0.5, x_{3,2}(0) = 0.4, \hat{x}_{1,1}(0) = 0.5, \hat{x}_{2,1}(0) = 0.5, \hat{x}_{3,1}(0) = 0.4$.

Fig. 1: Communication graph.

Fig. 2: Trajectories of $x_{i,1}(i = 1, 2, 3)$ and leaders y_{4d} and y_{5d} .

Fig. 2-7 describe the simulation results. Fig. 2 shows the containment result that the outputs of all followers asymptotically converge to the convex hull formed by the two leaders. Fig. 3 plots the second-order states of three followers. Fig. 4 presents the states $x_{i,1}$ and $x_{i,2}(i = 1, 2, 3)$ and their corresponding observers, and it is shown that the system outputs can be well estimated by NN-based observer. The containment errors and performance bounds are shown in Fig. 5, which indicate the control schemes can impose $v_{i,1}$ to remain within the prescribed bounds. Fig. 6 shows the trajectories of the event-triggered control signals u_i . The event-triggered times for three followers are expressed in Fig. 7. From the simulation results, the efficiency of the proposed control scheme can be verified.

Fig. 3: Trajectories of $x_{i,2}(i = 1, 2, 3)$.

Fig. 4: States and their observers.

5 Conclusion

In this paper, the adaptive event-triggered containment control scheme has been investigated for a class of strict-feedback nonlinear MASs with prescribed performance and unmeasured states which can be estimated by NN-based observers. Based on the backstepping control and DSC technique, the adaptive consensus controllers are developed. It is proved that the containment control purpose has been accomplished. Especially, the control signals have been updated by switching threshold event-triggered mechanism, which provides more elasticity in the balance of network constraints and system performance. On account of the prescribed performance function, the local neighborhood containment errors converge into a prescribed set. According to the Lyapunov stability theory, it is demonstrated that all signals of the closed-loop system are SGUUB. Finally, the effectiveness of the proposed containment control approach is confirmed by simulation example.

Fig. 5: Curves of containment errors $v_{i,1}(i = 1, 2, 3)$ and performance bounds.

Fig. 6: Control signals $u_i(i = 1, 2, 3)$.

Fig. 7: The triggered number under the switching threshold strategy.

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Compliance with ethical standards

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Conflict of interest

The authors declare that they have no conflict of interest.

Ethical approval

This article does not contain any experiments with human or animal participants performed by any of the authors.

Informed consent

Informed consent was obtained from all individual participants included in the study.

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