Memristive Electromagnetic Induction Effects on Hopfield Neural Network

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Memristive electromagnetic induction effects on Hopfield neural network

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Abstract Due to the existence of membrane potential differences, the electromagnetic induction flows can be induced in the interconnected neurons of Hopfield neural network (HNN). To express the induction flows, this paper presents a unified memristive HNN model using hyperbolic-type memristors to link neurons. By employing theoretical analysis along with multiple numerical methods, we explore the electromagnetic induction effects on the memristive HNN with three neurons. Three cases are classified and discussed. When using one memristor to link two neurons bidirectionally, the coexisting bifurcation behaviors are disclosed with respect to the memristor coupling strength. When using two memristors to link three neurons, the antimonotonicity phenomena of periodic and chaotic bubbles are yielded, and initial-related multistable patterns are emerged. When using three memristors to link three neurons end to end, the extreme event owning riddled basin of attraction is demonstrated. In addition, we develop the printed circuit board (PCB)-based hardware experiments by synthesizing the memristive HNN and the experimental results well confirm the memristive electromagnetic induction effects.

Keywords: Dynamical effect · electromagnetic induction · Hopfield neural network (HNN) · hardware experiment · memristor · neuron

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1. Introduction

Memristor, a known nonlinear circuit element, is defined by Leon O. Chua for describing the relationship between flux and charge [1]. In virtue of the quasi-static expansion of Maxwell's equations, an electromagnetic field interpretation of this unique relationship has been presented. Till now, the memristor has been applied in wide scientific domains due to its distinct natures, such as nanoscale dimensions [2], nonlinearity [3, 4], synaptic plasticity effect [5], and so on. In neuroscience, from the point of view of electricity, we know that one neuron can be seen as a multichannel input- and output-signal processor or a non-autonomous nonlinear system that can modulate the external stimulus and thereby responses to it [6, 7]. Also, synapse can be seen as a two-port memristive device to connect two systems so as to realize the complex memory transmission characteristic [8]. Thus, memristor-based neurons or neural networks are now playing a vital effect in neuromorphic computation and brain-like applications [9–11].

In the past few years, availing of the memristor to express the electromagnetic induction that induced by membrane potential or electromagnetic radiation is a hot topic. In [11], Ma et al. thought that due to the transformation of intercellular and extracellular ion concentration or the differences of spatial distribution of ions, membrane potential of a neuron would be waved and thereby the time-changing electromagnetic flows were induced, the effects of which could be imitated by a flux-controlled memristor coupling with a neuron. On account of these, some memristive neuron models were proposed, from which mode transition or selection [6, 12], synchronous behaviors [13, 14], spatiotemporal patterns [15, 16], and coexisting modes [17] are uncovered profoundly. For some examples, in [6], to describe the membrane potential of Hindmarsh-Rose (HR) neuron model under the electromagnetic induction, Lv et al. constructed a memristive HR neuron model, where different electric modes of bifurcation, spiking, and chaotic bursting state were observed. In [16], based on a FitzHugh-Nagumo (FHN) neuron model, Takembo et al. constructed an n -neuron FHN chain network model under electromagnetic radiation, and the dynamical simulations proved that the function of the brain may impaired when it was driven to the external electromagnetic environments with strong radiation intensities.

Many researchers not only concentrated on the dynamical effects of a single neuron but also explored interactions of neurons in a network. In the neural network, electromagnetic induction flows can be induced when membrane potential differences are existed between each two interconnected neurons, whose effects are equivalent to the bidirectional induced currents emerged by a flux-controlled memristor linking each two neurons [8, 18, 19]. Accordingly, to pay attention to the electromagnetic induction effects on a unified network is a burning question.

Manifold dynamics of biological neurons and neural networks are concerned for further understanding the complex nonlinear structures and functional behaviors of the brain [11, 20, 21]. Different from biological neurons, conductance-independent artificial neural network has received more and more attention for its high degree of flexibility and practicability. Hopfield neural network (HNN) is a classical neural network possessing simple algebraic expression but can display complex dynamical states, which is widely applying in numerous domains [22–24]. Because the dynamical behaviors are closely related to its applications, over the past years, a mass of modified HNN models were proposed, including fractional-order HNN model [23], time delayed HNN models [25], and hidden HNN model [26], and multiple dynamical characterizations are revealed accordingly.

By contrast, memristive HNN model brings some new views for cognizing the brain, and it has got long-term attention by scholars. Because some properties of memristor bear striking resemblance to synaptic

plasticity of neurons, by replacing the resistive weight with the memristive synaptic weight, some memristive HNN models were established to achieve the variable connection weight for neurons [26, 27]. Followed by, in recent years, considering the complex electromagnetic environment, a neural network under electromagnetic radiation was reported [28, 29]. Furthermore, when considering the membrane potential difference between two interconnected neurons in HNN, a memristive HNN model with the electromagnetic induction was raised. The authors of the manuscript [30, 31] have discussed a memristive HNN model with two neurons under the action of electromagnetic induction, where coexisting behaviors triggered by different initial conditions were revealed and were validated by hardware experiments. Nevertheless, we are ignorant of the electromagnetic induction on HNN induced by membrane potential differences of multiple neurons. Accordingly, it is necessary to establish a unified memristive HNN model to express the electromagnetic induction effects roundly, which has not been reported until now.

In this paper, based on the hyperbolic-type memristor, a unified memristive HNN model is presented. For simplicity, a classical tri-neuron HNN model is taken as an example, based on which memristor-coupled HNN model with three cases is considered in succession. Interesting dynamical effects and intricate dynamical evolutions are uncovered. The main contributions for this paper are threefold. i) A unified memristive HNN model is presented and its boundedness is proved theoretically. ii) Multiple dynamical methods are employed to numerically reveal the bifurcations and coexisting attractors' behaviors, which is helpful for us to mimic the real dynamical behaviors of collective neurons and to cognize the brain. iii) PCB-based memristive HNN circuit experiments are developed, and the results well confirm the dynamical effects.

The remaining contents are listed as follows. In Sec. 2, a unified memristive HNN model is established and its boundedness is proved. In Sec. 3, memristive electromagnetic induction effects on HNN are numerically revealed. In Sec. 4, an electronic neuron circuit platform is built and the dynamical effects are validated. And lastly, we summarize our work in Sec. 5.

2. Memristor-coupled network model

In this section, availing of an example of the HNN model and a threshold hyperbolic-type memristor model, we construct a unified memristive HNN model to express the electromagnetic induction effects. Besides, the model uniform boundedness is proved theoretically.

2.1. An example of the HNN model

The mathematical model of a Hopfield neural network (HNN) with n neurons is generally described as

$$\dot{\mathbf{X}} = -\mathbf{X} + \mathbf{W} \tanh(\mathbf{X}) + \mathbf{I} \quad (1)$$

where $\mathbf{X} = [x_1, x_2, \dots, x_n]^T$ represents the n -neuron membrane potentials, $\mathbf{W}$ is an $n \times n$ synaptic weight matrix, and $\mathbf{I} = [i_1, i_2, \dots, i_n]^T$ is an external current matrix.

The tri-neuron HNN has been widely studied. Thus, an example of HNN, with a 3×3 asymmetric synaptic weight matrix, can be referred to [32]. Herein, to facilitate the following analysis and calculation, two minute values, including inter-connection weight w_{22} and self-connection weight w_{33} , are neglected, and the weight w_{21} is adjusted as 2.8. A simplified synaptic weight matrix is thereby denoted as

$$\mathbf{W} = \begin{pmatrix} 3.8 & -1.9 & 0.7 \\ 2.8 & 0 & 1 \\ -6.6 & 1.3 & 0 \end{pmatrix} \quad (2)$$

Utilizing the weight matrix (2) and taking no account of the external currents, the numerical simulations of the HNN model are shown in Fig. 1. One can see from Fig. 1(a), the phase portraits of two symmetric period-1 limit cycles initiated from the initials $(-0.01, 0, 0)$ (red) and $(0.01, 0, 0)$ (blue) are coexisting in the $x_1 - x_2 - x_3$ phase space. Besides, as shown in Fig. 1(b), two attracting domains depicted by local attraction basins are located in $x_1(0) - x_2(0)$ initial plane, where ‘LP1’ and ‘UP1’ represent the lower period-1 and upper period-1 behaviors, respectively. As a result, this HNN model takes on bistable period-1 behaviors.

Fig. 1 Based on (2), coexisting symmetric period-1 patterns in the HNN **a** phase portraits on the $x_1 - x_2 - x_3$ phase space **b** local attraction basin in the $x_1(0) - x_2(0)$ initial plane when $x_3(0) = 0$

2.2. Unified memristive HNN model

Referring to [18], a monotone, differentiable, and threshold memristor is used to express the electromagnetic induction, and the mathematical form is written as

$$\begin{cases} I_M = kG(\varphi)V_M = k \tanh(\varphi)V_M \\ \dot{\varphi} = f(V_M, \varphi) = V_M - \varphi \end{cases} \quad (3)$$

where k , $G(\varphi) = \tanh(\varphi)$, V_M , and I_M stand for the memristor coupling coefficient, memductance function, output voltage of memristor, and input current of memristor, respectively. In this expression, V_M and I_M stand for the membrane potential difference between two interconnected neurons and the induced current flowing through the memristor.

Using one flux-controlled memristor model to link each two neurons bidirectionally and taking no account of the external currents, a unified memristive HNN mathematical model with n neurons and n memristor arrays can be established as

$$\begin{cases} \dot{X} = -X + W \tanh(X) + KV_M \tanh(\Phi) \\ \dot{\Phi} = V_M - \Phi \end{cases} \quad (4)$$

where K and V_M denote the memristor coupling strength matrix and the membrane potential difference matrix. For $n = 3$, the two matrixes can be denoted as

$$K = \begin{pmatrix} k_1 & 0 & -k_3 \\ -k_1 & k_2 & 0 \\ 0 & -k_2 & k_3 \end{pmatrix}, \quad V_M = \begin{pmatrix} V_{M1} & 0 & 0 \\ 0 & V_{M2} & 0 \\ 0 & 0 & V_{M3} \end{pmatrix} \quad (5)$$

where $V_{M1} = x_1 - x_2$, $V_{M2} = x_2 - x_3$, and $V_{M3} = x_3 - x_1$.

Note that, the hyperbolic-type memristor is used to express the electromagnetic induction induced by the

membrane potential difference between two interconnected neurons. Thus, in (4), the $KV_M \tanh(\Phi)$ term can be regarded as the induction current of the memristor. Besides, $\Phi = (\phi_1, \phi_2, \phi_3)^T$ represents a magnetic flux matrix in the memristor array.

To intuitively express the electromagnetic induction flows induced by potential differences between the interconnected neurons, the abridged general view of the memristive HNN model with three neurons is depicted in Fig. 2, where the two-way induction currents are flowing through one memristor to mimic the electromagnetic induction flows. Therefore, each memristor is used to link two neurons bidirectionally and six induction currents $\pm I_{Mi}$ ($i = 1, 2, 3$) are yielded thereby.

Fig. 2 Abridged general view of a memristor-coupled HNN model

In this paper, memristor is used to express the electromagnetic induction induced by membrane potential difference between two neurons. In neuroscience, as a matter of fact, a memristor can also be employed to express the synaptic plasticity of neurons, i.e., to replace the resistive weight with the memristive synaptic weight, and to express the electromagnetic induction induced by the external electromagnetic radiation or the inner membrane potential of neurons. To sum up, several examples of three expressions of the memristive HNN models are listed in Table 1.

Table 1 Comparison of pivotal indexes for the memristive HNN models.

References	Memristor functions	Physical directions	Memristor models	Memductance functions
[26]	Synaptic plasticity effect	Scalar	$I_M = kG(\phi)V_M, \dot{\phi} = V_M$	$G(\phi) = a\phi + b\phi^2$
[27]	Synaptic plasticity effect	Scalar	$I_M = kG(\phi)V_M, \dot{\phi} = \sin(\phi) + V_M$	$G(\phi) = \phi$
[28]	Electromagnetic induction effect	Single-directional vector	$I_M = kG(\phi)V_M, \dot{\phi} = V_M$	$G(\phi) = a + 3b\phi^2$
[29]	Electromagnetic induction effect	Single-directional vector	$I_M = kG(\phi)V_M, \dot{\phi} = \alpha V_M - \beta\phi$	$G(\phi) = a - b \phi $
[30]	Electromagnetic induction effect	Bi-directional vector	$I_M = kG(\phi)V_M, \dot{\phi} = V_M$	$G(\phi) = \phi$
[31]	Electromagnetic induction effect	Bi-directional vector	$I_M = kG(\phi)V_M, \dot{\phi} = V_M - \phi$	$G(\phi) = a - 3b\phi^2$
This paper	Electromagnetic induction effect	Bi-directional vector	$I_M = kG(\phi)V_M, \dot{\phi} = V_M - \phi$	$G(\phi) = \tanh(\phi)$

One can see from Table 1 that due to the different nonlinear properties of memductance function, various memristor models can be adopted to construct memristive HNN model. Besides, when using a memristive

model to express the synaptic plasticity effects, the memristive synaptic weight is a scalar, and when using a memristive model to express the electromagnetic induction effects, the physical direction of induction currents flowing through the memristor is a vector. In this paper, we use non-ideal memristors to connect neurons end to end, thus the induction currents are the bi-directional vectors.

2.3. Model uniform boundedness

Boundedness is a vital property of a nonlinear dynamical system. In this paper, for $n = 3$, uniform boundedness of the model (4) is deduced in theory, proving that all the motions, including chaotic motions, are trapped into a bounded region.

1) *Basic Definition of Uniform Boundedness*: Consider a general nonlinear dynamical system as

$$\dot{x} = h(t, x) \quad (6)$$

where $h: R_+ \times B \rightarrow R^n$ is continuous, and $B \subset R^n$ is a domain that contains the origin.

Definition 1 [33]: The solutions of system (6) are uniformly bounded if there exists a positive constant c_1 , independent of $t_0 \geq 0$, and for every $c_2 \in (0, c_1)$, there is $h_1 = h_1(c_2) > 0$, independent of t_0 , such that

$$\|x(t_0)\| \leq c_2 \Rightarrow \|x(t)\| \leq h_1, \forall t \geq t_0 \quad (7)$$

2) *Uniform Boundedness Analysis*: Denote $Y = [X, \Phi]$, and take

$$A = \begin{pmatrix} -I & \mathbf{0} \\ B & -I \end{pmatrix} \quad (8a)$$

$$g(Y) = \begin{bmatrix} W \tanh(X) + KV_M \tanh(\Phi) \\ \mathbf{0} \end{bmatrix} \quad (8b)$$

where I is a unit matrix, A is the linearized matrix of (4), and B can be regarded as the linearized matrix of the state equation Φ against the state variable X , which is a 3×3 matrix denoted as

$$B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \quad (9)$$

Then the memristive HNN model (4) is rewritten by

$$\dot{Y} = AY + g(Y) \quad (10)$$

For the initial condition $Y(t_0)$, by the variation of parameters formula, any solution $Y(t)$ of system (10) can be written as

$$Y(t) = Y(t_0)e^{A(t-t_0)} + \int_{t_0}^t e^{A(t-s)} g(Y(s)) ds \quad (11)$$

It is easy to know that all the characteristic roots of constant matrix A have negative real parts, so there exist positive constants L and α , if there is a constant D with $|g(Y)| \leq D$, then for $t \geq t_0$, we have

$$|Y(t)| = L|Y(t_0)|e^{-\alpha(t-t_0)} + LD/\alpha \quad (12)$$

Therefore, it is concluding that, for the tri-neuron memristor-coupled HNN model, the model (4) is uniform boundedness.

3. Memristive electromagnetic induction effects

In this section, three cases are classified in succession, including the memristive HNN model with M_1 , the memristive HNN model with M_1 and M_2 , and the memristive HNN model with three memristors, corresponding

to the values of matrix $\mathbf{K}$. On account of the example of the HNN model, memristive electromagnetic induction effects are thoroughly revealed by multiple numerical methods.

3.1. Case I: Memristive HNN model with M_1

Firstly, one memristor M_1 used to link neurons 1 and 2 is taken into account, the memristor coupling strength matrix in this case is denoted as

$$\mathbf{K} = \begin{pmatrix} k_1 & 0 & 0 \\ -k_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (13)$$

where k_1 stands for the memristor coupling strength between neurons 1 and 2.

Stability of the equilibrium point is also a vital property of the HNN dynamical system, which is closely related to the application of itself [34]. Setting the left side of (4) to zero and configuring the equilibrium points as $P(\eta_1, \eta_2, \eta_3, \eta_{\phi 1}, \eta_{\phi 2}, \eta_{\phi 3})$, then there are $\eta_3 = -6.6 \tanh(\eta_1) + 1.3 \tanh(\eta_2)$, $\eta_{\phi 1} = \eta_1 - \eta_2$, $\eta_{\phi 2} = \eta_2 - \eta_3$, and $\eta_{\phi 3} = \eta_3 - \eta_1$.

Accordingly, the equilibrium points P are able to be solved as long as two variables η_1 and η_2 are determined. Two transcendental equations with respect to two parameters are confirmed as

$$\begin{cases} H_1 = -\eta_1 - \eta_2 + 6.6 \tanh(\eta_1) - 1.9 \tanh(\eta_2) + 1.7 \\ H_2 = -\eta_2 + 2.8 \tanh(\eta_1) + a - k_1(\eta_1 - \eta_2) \tanh(\eta_1 - \eta_2) \end{cases} \quad (14)$$

where $a = \tanh[-6.6 \tanh(\eta_1) + 1.3 \tanh(\eta_2)]$.

Due to the difficulty of obtaining the arithmetic solutions, a graphic analysis method is employed to obtain the analytical solutions of (14) using MATLAB platform. When the memristor coupling intension k_1 is set as 0.12 and 0.18 respectively, the equilibrium points P can be determined by examining the intersections of functions, as shown in Fig. 3.

Fig. 3 For $k_1 = 0.12$ and $k_1 = 0.18$, two parameters η_1 and η_2 achieved by examining the intersections of functions in (14)

Observing Fig. 3, when η_1 and η_2 are in the regions $[-0.6, 0.6]$ and $[-0.9, 0.7]$, the black H_1 curve with determined function remains unchanged, but the red and blue H_2 curves involve two different values of k_1 . Therefore, three examined intersections including one zero equilibrium point P_0 as well as two nonzero equilibrium points P_1 and P_2 can be precisely calculated respectively.

Availing of the $\mathbf{K}$ matrix in (13), when the Jacobin matrix of the model (4) in case I is deduced, the equilibrium points, eigenvalues, and stabilities for two representatives $k_1 = 0.12$ and $k_1 = 0.18$ can be calculated. Two zero eigenvalues at P_0 are both calculated as $-1, -1, -1, 1.0124, -0.1062 \pm j2.0600$. Besides, nonzero eigenvalues at P_1 and P_2 for $k_1 = 0.12$ are calculated as $-1, -1, -0.9765, -0.9637, 0.4442 \pm j1.2647$ and $-1, -1, -1.0201, -0.9645, 0.4973 \pm j1.2445$ respectively, and the ones at P_1 and P_2 for $k_1 = 0.18$ are calculated as $-1, -1,$

$0.4356 \pm j1.2685$, $-0.9642 \pm j1.2396$ and -1 , -1 , -1.0295 , -0.9651 , $0.5089 \pm j1.4838$ respectively. These equilibrium points all behave unstable saddle-foci (USF), indicating that the spiral chaotic attractors can be formed theoretically according to Shil'nikov theorem [35].

Numerical simulation methods are valid to disclose the dynamical behaviors. When selecting four sets of initial conditions, the max-spike bifurcation diagrams of x_2 and first three Lyapunov exponents (LEs) are plotted in Figs. 4(a) and 4(b), respectively. Note that, ODE23 (built in MATLAB) is employed to simulate the phase portraits, bifurcation diagrams, and local attraction basins. Besides, the Wolf-based algorithm is adopted to simulate the LE spectra in the whole paper.

Fig. 4 Coexisting bifurcation behaviors with respect to memristor coupling strength k_1 when selecting four sets of initial conditions **a** bifurcation diagrams of x_2 **b** first three LE spectra

In Fig. 4(a), with the increase of memristor coupling strength k_1 in the region $[0.08, 0.18]$, the memristive HNN model behaves as the globally periodic states when selecting two sets of initial conditions $(\pm 0.4, 0, 0, 0, 0, 0)$. By contrast, the memristive HNN model has a reverse period-doubling bifurcation route to chaos when considering two other sets of initial conditions. Taking the memristive HNN model with the initial conditions $(0.5, 0, 0, 0, 0, 0)$ as an example, its orbit starts with period-1, enters into chaos at $k_1 = 0.109$ via chaos crisis, and then degrades into period-6 at $k_1 = 0.144$ and period-3 at $k_1 = 0.179$ successively via reverse period-doubling bifurcation. In addition, some periodic windows and chaos crisis scenarios can be also found in the chaotic regions. Furthermore, there are at least three different attractors' states coexisting in the memristive HNN model for a determined memristor coupling strength, demonstrating that the multistable patterns appear in case I [30]. As can be found in Fig. 4(b), the first three Lyapunov exponents (LEs) are precisely matching with the bifurcation diagrams.

For $k_1 = 0.12$, the phase portraits initiated by four sets of initial conditions are plotted in Fig 5. The results show that multiple attractors with different locations and topological structures coexist in the phase space, including the lower period-1 limit cycle, upper period-1 limit cycle, period-8 limit cycle, and spiral chaotic attractor. The local attraction basins can be used to better explore the influences of initial conditions on the model (4) in case I. Two representative examples for $k_1 = 0.12$ and $k_1 = 0.18$ with $(x_3(0), x_4(0), x_5(0), x_6(0)) = (0, 0, 0, 0)$ are shown in Figs. 6(a) and 6(b), where seven colors are used to represent different types of attractors. Here, LP1, UP1, P02, P03, P04, P08, and CH represent lower period-1, upper period-1, period-2, period-3, period-4, period-8, and chaos, respectively, indicating the coexisting multistable patterns. Observed from Fig. 6(a), four types of attractors are revealed when $k_1 = 0.12$ is fixed. The orange, blue, and banded yellow regions

dominate the initial plane. In addition, the riddled basins of attraction are exhibited in a small region, implying that the model in this case is sensitive to the initial conditions and the inconspicuous extreme events are yielded [36, 37]. When k_1 is increased to 0.18, as seen in Fig. 6(b), complex stability evolutions happen, leading to that period-2 along with the riddled domain is embedded in period-4. As a result, the memristive HNN model displays coexisting multistable patterns related to the initial conditions.

Fig. 5 For $k_1 = 0.12$, the phase portraits initiated by four different sets of initial conditions

Fig. 6 For two memristor coupling strengths, local attraction basins in the $x_1(0) - x_2(0)$ initial plane **a** $k_1 = 0.12$ **b** $k_1 = 0.18$

3.2. Case II: Memristive HNN model with M_1 and M_2

Two memristors M_1 and M_2 used to link three neurons are taken into account in case II. Hence, the memristor coupling strength matrix can be denoted as

$$\mathbf{K} = \begin{pmatrix} k_1 & 0 & 0 \\ -k_1 & k_2 & 0 \\ 0 & -k_2 & 0 \end{pmatrix} \quad (15)$$

where, k_1 and k_2 are two parameters representing two different memristor coupling strengths. To investigate the bifurcation scenarios with these two parameters, two-dimensional bifurcation diagram is plotted in the regions $k_1 = [0.08, 0.18]$ and $k_2 = [-0.005, 0.065]$ under the determined initial conditions $(-0.4, 0, 0, 0, 0, 0)$, as shown in Fig. 7(a). Besides, when setting four representative values of k_1 as 0.08, 0.11, 0.15, and 0.18, respectively,

one-dimensional bifurcation diagrams are plotted with respect to k_2 , as shown in Fig. 7(b).

Fig. 7 Bifurcation plots under the determined initial conditions $(-0.4, 0, 0, 0, 0, 0)$ in case II **a** two-dimensional bifurcation diagram with respect to k_1 and k_2 **b** one-dimensional bifurcation diagrams with respect to k_2

As can be seen from Fig. 7(a), there exist eight types of attractors. Here the P00, P01, P02, P04, P08, P12, P16, and CH represent the stable point, period-1, period-2, period-4, period-8, period-12, period-16, and chaos respectively. The period-8, period-16, chaos, and period-12 are embedded in successive, resulting in the occurrence of the marvelous bifurcation structure.

Glanced at Fig. 7(b), the phenomena of antimonotonicity appear distinctly. For fixed $k_1 = 0.08$, when increasing k_2 in the region $[-0.05, 0.065]$, the orbit of the memristive HNN model in case II begins with period-1, goes into period-2 and period-4 via the forward period-doubling bifurcations, then degrades into period-2 and period-1 via the reverse period-doubling bifurcations, and finally settle down stable point at $k_2 = 0.0614$. When increasing k_1 from 0.08 to 0.18, the period-4, period-8, period-12, and chaos bubbles are formed respectively. Thus, the forward and reverse period-doubling bifurcation routes are obviously visible in bifurcation processes for the model (4), which are affected by two memristor coupling strengths.

For $k_1 = 0.17$ and $k_2 = 0.005$ with fixed $(x_3(0), x_4(0), x_5(0), x_6(0)) = (0, 0, 0, 0)$, Fig. 8 depicts the local attraction basins related to the coexisting multiple patterns in case II. Here, P01, P02, P03, P05, and CH represent the period-1, period-2, period-3, period-5, and chaos, respectively. There are five different colors distributing in the $x_1(0) - x_2(0)$ initial plane and the purple and yellow regions are relatively sparse. What's more, the red region is riddled and mixed in the blue region, which means that the extreme events also exist in case II.

Fig. 8 For $k_1 = 0.17$ and $k_2 = 0.005$, local attraction basin in the $x_1(0) - x_2(0)$ initial plane

According to the aforementioned analysis, Fig. 9 shows the phase portraits under five sets of initial conditions $(0.4, 0, 0, 0, 0, 0)$, $(-0.4, 0, 0, 0, 0, 0)$, $(0.7, 0, 0, 0, 0, 0)$, $(0.2, 0, 0, 0, 0, 0)$, and $(-1, 1, 0, 0, 0, 0)$ for the model (4) in case II. Five different types of attractors are triggered, including period-1, period-2, period-3, period-5, and chaos.

Fig. 9 For $k_1 = 0.17$ and $k_2 = 0.005$, the phase portraits initiated by five different sets of initial conditions

3.3. Case III: Memristive HNN model with M_1 , M_2 , and M_3

In this case, three memristors from M_1 to M_3 are used to link three neurons end to end. Three memristor coupling strengths from k_1 to k_3 are set as three adjusting parameters. And the matrix $\mathbf{K}$ can be referred to (5).

Considering the special situation in case III that memristive electromagnetic inductions between three neurons are the same, i.e., $k_1 = k_2 = k_3 = k$, thereby k is chosen as a single adjustable parameter in case III. For this special situation, the bifurcation diagrams with respect to k are plotted in Fig. 10, where three sets of initial conditions are set as $(0.01, 0, 0, 0, 0, 0)$, $(-0.01, 0, 0, 0, 0, 0)$, and $(-0.07, 0, 0, 0, 0, 0)$, respectively. Observed from Fig. 10, when increasing k from 0 to 0.04, the phenomena of cascaded chaotic bubbles, chaos crisis scenarios, and coexisting bifurcations emerge in the memristive HNN model.

Fig. 10 Bifurcation diagrams of x_3 in case III under three sets of initial conditions

Specifically, when setting the initial conditions as $(-0.07, 0, 0, 0, 0, 0)$, the forward and reverse period-doubling bifurcation routes are also visible in bifurcation processes. Accordingly, when we endow four sets of parameter k for the model (4) in case III, the phase portraits of chaotic attractor, periodic limit cycles, and stable point are plotted in Fig. 11.

Fig. 11 For different k with fixed initial conditions $(-0.07, 0, 0, 0, 0, 0)$, the phase portraits of the memristive HNN model under the special situation of case III

In this special situation of case III, to uncover the initial-dependent dynamics, the value of k is kept as 0.005. The local attraction basin is plotted in Fig. 12 and the coexisting tri-stable patterns are displayed, such as the blue period-1 (P01), light cyan period-2 (P02), and red chaos (CH). Note that all the red regions are fully mixed and riddled in the light cyan periodic regions, meaning the occurrence of the rare, recurrent, and irregular dynamics of extreme event.

Fig. 12 For $k = 0.005$, the local attraction basin in the $x_1(0) - x_2(0)$ initial plane, indicating the occurrence of the initial-related extreme event

In summary, the dynamical effects on the memristive HNN model given in (4) are listed in Table 2. As can be seen, when the number of memristors increases from zero to two, the number of coexisting attractors goes from two to four and to five. However, when the memristive HNN model involves three memristors with the aforesaid special situation, the coexisting tri-stable patterns with extreme event appear distinctly. Besides, the anti-monotonicity behavior displays in case II, which is related to two memristor coupling strengths. Notably, we remark that when three memristors utilized in the memristive HNN model are endowed with three different memristor coupling strengths, some more intriguing and intricate dynamical effects need to be further investigated.

Table 2 Dynamical effects under different cases.

Cases	Dimensionality	Coexisting types	Dynamical effects
Example	3	2	Bistability
Case I	4	4	Multistability
Case II	5	5	Antimonotonicity
Case III	6	3	Extreme event

4. PCB-based analog circuit validation

Electronic neuron circuit is nowadays regarded as an excellent artificial block to implement the VLSI applying in neuromorphic computing [38]. It can be achieved in three ways, namely, analog, digital, and hybrid analog/digital circuits [39–42]. In this section, the memristive HNN model is implemented in analog circuit and the PCB-based hardware experiments are carried out to validate the memristive electromagnetic induction effects.

4.1. Circuit synthesis for the memristive HNN model

Based on the unified memristive HNN model given in (4), an electronic neuron circuit can be implemented and its circuit equations are described as

$$\begin{cases} RC \frac{dv}{dt} = -v + \mathbf{R}_W \tanh(v) + \mathbf{R}_K \mathbf{v}_{VM} \tanh(v_\phi) \\ RC \frac{dv_\phi}{dt} = v_M - v_\phi \end{cases} \quad (16)$$

where v and v_ϕ are $n \times 1$ voltage variable matrixes, and the integrating time constant $\tau = RC = 10 \text{ k}\Omega \times 10 \text{ nF} = 0.1 \text{ ms}$. In addition, for $n = 3$, on account of the synaptic weight matrix $\mathbf{W}$, memristor coupling strength weight $\mathbf{K}$, and matrix $\mathbf{V}_M$ in (5), three 3×3 resistance arrays are presented as

$$\mathbf{R}_W = \begin{pmatrix} R/R_1 & -R/R_2 & R/R_3 \\ R/R_4 & 0 & R/R_5 \\ -R/R_6 & R/R_7 & 0 \end{pmatrix} \quad (17a)$$

$$\mathbf{R}_K = \begin{pmatrix} R/R_{k1} & 0 & -R/R_{k3} \\ -R/R_{k1} & R/R_{k2} & 0 \\ 0 & -R/R_{k2} & R/R_{k3} \end{pmatrix} \quad (17b)$$

$$\mathbf{v}_{VM} = \begin{pmatrix} v_{VM1} & 0 & 0 \\ 0 & v_{VM2} & 0 \\ 0 & 0 & v_{VM3} \end{pmatrix} \quad (17c)$$

where $\mathbf{R}_W$ represents the determined resistance array, $\mathbf{R}_K$ represents the adjustable resistance array whose values change with different cases, and $\mathbf{v}_{VM}$ represents the voltage matrix of membrane potential difference. Notice that, because the resistances are positive in the real circuit, the negative signs in (17) are adjusted by changing the connection way of the circuit. According to (17), the resistances in $\mathbf{R}_W$ are configured as $R_1 = R/3.8 = 2.6316 \text{ k}\Omega$, $R_2 = R/1.9 = 5.2632 \text{ k}\Omega$, $R_3 = R/0.7 = 14.2857 \text{ k}\Omega$, $R_4 = R/2.8 = 3.5714 \text{ k}\Omega$, $R_5 = R/1 = 10 \text{ k}\Omega$, $R_6 = R/6.6 = 1.5152 \text{ k}\Omega$, and $R_7 = R/1.3 = 7.6923 \text{ k}\Omega$, respectively.

Circuit schematic is designed synthetically using Multisim 12.0 software, the screenshot of which can be

seen in Fig. 13, where three cases of circuit modules can be controlled by six-pin DIP switch with $S_1 \sim S_3$ keys. Therefore, the states of these three keys for the electronic neuron circuit and theoretical resistances for R_K are summarized in Table 3. Besides, three different functional circuits denoted by fourteen hierarchical blocks (HBs) are shown in Fig. 13, where six HBs from ‘-T1’ to ‘-T6’ represent the hyperbolic tangent function circuit modules with negative output, five HBs from ‘-H1’ to ‘-H5’ represent the inverting operation circuits, and three HBs from ‘I1’ to ‘I3’ represent the subtraction operation circuits. Notice that, global connectors are employed to connect the common port for simplifying the circuit connection.

Fig. 13 Schematic diagram for implementing the memristive HNN model given in (16)

Table 3 Four cases of circuit modules controlled by a DIP switch and theoretical resistances for R_W and R_K .

S_1	S_2	S_3	Cases of memristive HNN model	Theoretical resistances in R_W and R_K
off	off	off	An example of the HNN model	$R_W = R/ w_{ij} $, $W = 1, \dots, 7$
on	off	off	Case I: Memristive HNN model with M_1	$R_{k1} = R/k_1 = 83.33 \text{ k}\Omega$
on	on	off	Case II: Memristive HNN model with M_1 and M_2	$R_{k1} = R/k_1 = 58.82 \text{ k}\Omega$, $R_{k2} = R/k_2 = 2000 \text{ k}\Omega$
on	on	on	Case III: Memristive HNN model with three M	$R_k = R/k = 2\text{M}\Omega$, $500 \text{ k}\Omega$, $333.33 \text{ k}\Omega$, and $250 \text{ k}\Omega$

4.2. PCB-based hardware circuit validation

The designed PCB is made using the Altium designer 10.0 software. The off-the-shelf commercial components contain bipolar junction transistor MPS2222, operational amplifier TL082CP, operational amplifier AD633JNZ, chip resistor, precision potentiometer, and monolithic ceramic capacitor. And the phase portraits are captured by Tektronix digital oscilloscope in the X-Y mode.

The photograph of the electronic neuron circuit is displayed in Fig. 14, where the HNN circuit module is in the left, and six same and independent $-\tanh(\cdot)$ modules are in the right dotted box. ‘S’ is a six-pin DIP switch that is used to control the circuit connection states. For instance, when S_1 and S_2 are on and S_3 is off in this

figure, the circuit is in the state of case II.

Fig. 14 PCB-based electronic neuron circuit for implementing the memristive HNN model

On account of Table 3, when three keys are all off, the example of the HNN model given in (1) can be denoted by a third-order analog circuit. Besides, when case I and case II are classified, the adjustable resistance arrays $\mathbf{R}_K$ are configured as

$$\mathbf{R}_K = \begin{pmatrix} R/R_{k1} & 0 & 0 \\ -R/R_{k1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (18a)$$

$$\mathbf{R}_K = \begin{pmatrix} R/R_{k1} & 0 & 0 \\ -R/R_{k1} & R/R_{k2} & 0 \\ 0 & -R/R_{k2} & 0 \end{pmatrix} \quad (18b)$$

respectively, where R_{k1} and R_{k2} are two adjustable resistances for each case. When configuring the two resistances in turn, the phase portraits for the first two cases are experimentally captured, as shown in Fig. 15. Note that due to the difficulty in achieving specific capacitor initial voltages in analog circuit, power supply should be on-and-off switched to endow the initial conditions in the real circuit [31, 43]. In Fig. 15, three types of attractors coexisted in case I and three types of attractors coexisted in case II can be captured. And the initial-related coexisting behaviors can be available realized by circuit simulations [27, 30].

Fig. 15 The phase portraits captured by adjusting the states of keys and the resistances $\mathbf{R}_K$ a experimental resistance $R_{k1} = 83.33 \text{ k}\Omega$ in case I b experimental resistances $R_{k1} = 58.82 \text{ k}\Omega$ and $R_{k2} = 2012 \text{ k}\Omega$ in case II

Furthermore, when three keys are all on, the adjustable resistance array $\mathbf{R}_K$ is configured as (17b). For the special situation in case III, the adjustable resistances need to be the same, i.e., $R_k = R_{k1} = R_{k2} = R_{k3} = R/k_1 = R/k_2 = R/k_3$. Thus, the experimental phase portraits can be captured and shown in Fig. 16. As can be seen, dynamical

evolutions from chaos to period-2, to period-1, and to stable point can be readily observed and the experimental results validate the simulation results given in (11). Besides, the photograph of PCB-based hardware circuit for case III is shown in Fig. 16(b), in which the chaotic attractor is cropped by Tektronix digital oscilloscope.

Fig. 16 When $S_1 \sim S_3$ are all on, experimental phase portraits and photograph **a** four captured phase portraits by setting $R_{k1} = R_{k2} = R_{k3} = 2000 \text{ k}\Omega, 600 \text{ k}\Omega, 430 \text{ k}\Omega$, and $250 \text{ k}\Omega$, respectively **b** photograph for PCB-based hardware circuit linked with a digital oscilloscope

Due to the parasitic resistances and inner interferences in the real circuit, PCB-based experiment results are consistent with the numerical results basically though possessing some errors. In general, we can say that the electronic neuron circuit for implementing the memristive HNN model well validates the memristive electromagnetic induction effects on HNN. And in the next step, we may try to use simple activation function in HNN, or find some alternative schemes for the complex activation functions so as to investigate the unified HNN model with n neurons.

5. Conclusions

Using the threshold hyperbolic-type memristors to link the interconnected neurons, this paper presented a unified memristive HNN model to simulate electromagnetic induction effects. The uniform boundedness of the unified memristive HNN model was deduced in theory, proving that all the motions are trapped into a bounded region. With the consideration of three cases, the dynamical effects were revealed in succession using multifold dynamical analysis methods. In case I, stability analysis proved that the equilibrium points are all unstable saddle-foci, and numerical simulations disclosed the coexisting bifurcation behaviors and coexisting multistable patterns herein. In case II, the antimonotonicity phenomena appeared with the creation and annihilate of periodic and chaotic bubbles, which was controlled by two memristor coupling strengths. Besides, coexisting five-stable patterns were induced by the initial conditions. In the special situation of case III, when three memristor coupling strengths were the same, the extreme event with complex riddled basins taken place distinctly, meaning that the memristive HNN model is increasingly sensitive to the initial conditions.

The electronic neuron circuit of the memristive HNN model was constructed by a PCB hardware platform, and the results beautifully validated the numerical simulations for three cases. Accordingly, the study of the memristive electromagnetic induction effects on HNN not only reveals the interactions of neurons, but also provides a potential application in the neuromorphic computation. Furthermore, we emphasize that, based on the other examples of memristive HNN models, the electromagnetic induction effects may not the same and need to be explored in the future.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflicts of interest. These authors contribute equally to this work.

Availability of data and materials

The datasets supporting the conclusions of this article are included within the article and its additional files.

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