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Iterative Standard Strategy for Non-Linear Optimization in Melting Furnace Charge Calculations

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ABSTRACT

This paper describes an approach to furnace charge calculation for melting processes. Some complexities of the problem, including non-homogeneous element loss, non-metal contaminations, and, melt correction have been considered in the model. The contribution of the model is that it examines a rearrangement of the non-linear complexities into an iterative standard linear programming framework. The performance of the model was evaluated and demonstrated on an industrial scale test problem. The results revealed that the non-linear phenomenological complexities can be effectively fitted in the framework of an iterative standard LP model. This feature provides a backbone for reliable and fast optimization in melting which is of significant benefit for industrial automation.

KEYWORDS: Furnace Charge Calculation; Optimization; Melting; Non-Linear programming; Material loss; Mass balance

1. INTRODUCTION

Melting is not only changing a solid to a liquid. In almost all of the basic metal productions and foundry processes, melting is a major process operation dealing with different phenomena, several parameters, and a variety of input/output objects like metal scraps, ingots, contaminations, refractory ceramics, gas, dust, slags, fuels, energies, radiations, and finally the melt. Melting is widely considered to be responsible for a couple of critical issues. First, melting operations are highly energy-intensive, taking more than 55% of the total energy consumed in the metallurgical sector (~20%) of the global industry [1–3]. Second, melting is the origin of the formation of the chemical composition of metals which directly affects the quality of products. A proper optimization model for furnace charge calculation would be a solution to both of the issues. Despite this crucial importance, melting

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has been poorly appreciated and received inadequate mathematical modeling investigations compared to the other industrial operations.

Melting furnaces are often considered as non-reacting systems when smelting is not the primary goal of the process. For furnace charge calculations, current models are based on the conservation of mass of chemical elements. A set of mass balance equations is raised with each equation tracking a main alloying element of the target melt [4]. The solution of the model is expected to give a set of weights for all charge burdens by which the composition of the final melt would fall inside the specified range according to the grade of the alloy. The model is also required to minimize the cost of melting, i.e. it should find an optimum combination of the weights by minimizing an objective function. Such a charge calculation model is recognized as a standard linear programming (LP) problem [5]. Solving a standard linear programming problem is straightforward by any means, e.g. using MATLAB, Excel solver, or a piece of code with the SIMPLEX algorithm. However, the mass balance models are faced with some serious uncertainties; (1) the melting operation is entangled with unwanted physio-chemical reactions such as oxidations, evaporations, melt-slag, and melt-refractory interactions. Those reactions cause sensible amounts of material losses; (2) It is usually supposed that the chemical compositions of the charge materials are known as input parameters before the calculations are started. Contrary to expectations, there is always a significant uncertainty in the composition of input charge materials, since the charge materials, especially metal scraps, are contaminated and recycled from different sources. (3) A straightforward charge calculation is guaranteed with a predefined total mass of the melt as an input parameter. However, most of the practical problems are mainly concerned with a melting furnace that holds an initial amount of melt with a composition that falls out of the specified range. Modeling of the correction of an initial melt introduces other unknowns to the problem. It means the final weight of the melt will not be known anymore. It also puts uncertainty on the feasibility of the solution. Researchers have pointed out a few of these complexities in the problem. Li et al [6] developed a model to investigate the kinetics of melting of solid scrap in the liquid metal. Ziółkowski [7,8] has addressed the uncertainties in chemical compositions and proposed a fuzzy optimization model to solve the issue. Omole and Raymond [9] studied carbon and silicon loss in a rotary furnace for cast iron melting, aiming to minimize the loss and to keep the alloy in the range. Capuzzi and Timelli [10] provided a review of the process of melting aluminum scraps. They also pointed out the issue of the accumulation of undesired elements in metal recycling and discussed possible technological opportunities to redress the issue. As a solution for the correction of the initial out-of-range melt, Ziółkowski [11] suggested setting a second-order compromise objective function to minimize the total mass of the final melt as well as minimizing the cost function. Collecting all these ambiguities into consideration would make the model complex and highly non-linear which lies out of the standard LP problems and could be

impractical for industrial applications. Ignoring the uncertainties, on the other hand, gives oversimplistic calculations which lead to multiple melt corrections, extensions of melting time, and further increases in the cost. In this context, the present study tries to re-examine the formulation of the problem. It aims to presents a modified modeling approach that takes the materials loss and the initial melt into account, and simultaneously, preserves the model in the reliable framework of standard linear programming. Therefore, the main question of the paper is how to carry out the non-linear complexities of the problem into a linear framework. The following section of the paper begins with problem identification. The concept of the modeling approach is described with formulations. A new algorithm is presented which integrates the model formulations and completes the answer to the main question raised by this study. The next following section provides a test case demonstrating the implementation and calculations of the model. Then, the results are shown, evaluated, and discussed. The conclusion is drawn in the final section.

2. MATHEMATICAL MODEL

2.1. PROBLEM IDENTIFICATION

A melting operation is considered on an industrial scale as a batch (discontinuous) process equipped with a melting furnace with a capacity of m_F regarding its maximum allowable mass content (in kg or ton). The geometry and dimensions of the furnace can be different depending on the process requirements. The desired output of the process is a target melt with a specified range of chemical compositions with a number of E alloying elements. The last element of the alloy (number E) is supposed to be the base metal which is taken as the reminder element. The furnace probably contains an initial amount m_0 of melt with a known chemical composition. A selection of N different materials is available and waiting to be charged into the furnace; e.g. scraps, ingots, returns, hardeners, etc. Each of the charge materials has been characterized by its purchase price, its chemical composition, and its percentage of non-metallic contaminations. However, inevitable phenomena occur during the process. Evaporation, oxidation, splashing, adsorption, and other reactions lead to an undesired element loss in the form of gas, dust, fumes, slag, and refractory build-up. In detail, those phenomena may be neither investigated nor interested within the context of the present problem. Nevertheless, element loss is important and experimental data on the percentage of element loss in each charge material are likely to be available. Finally, the problem asks us to determine the weight (mass) of each of the charge materials in such a way that two following conditions are satisfied; (1) the chemical composition of the final melt falls within the specified range; (2) Total cost of the consumed charge materials are kept at the minimum value.

To establish a model, an open system is defined on the boundaries of the furnace. Figure 1 illustrates the melting furnace and input/output of the system. The input charge materials are denoted by j and have an unknown mass m_j . The output includes two parts; the molten metal m_M , and the total of the material loss evaluated by m_L . The following assumptions are made:

Figure 1. Schematic representation of the melting furnace with input, output, and materials loss.

(1) The melting furnace is considered an open system. Accumulations of mass, e.g. refractory buildups, are regarded as a part of output mass.

(2) The output melt is assumed to be entirely metal. The input charge, however, includes both metal parts and non-metallic contaminations. The percentage of the weight of contaminations in each charge is known as L_j^C . The contaminations are considered as a portion of the lost material.

(3) Chemical composition of each charge material is denoted by C_{ij} only refer the metallic part of the charge, i.e. the constituents of the non-metallic portion are not included in the chemical composition. It should be reminded that semi-metallic elements, like carbon and silicon that exist as an alloying elements in the metallic charge, are considered in the composition.

(4) A portion of alloying elements of the metallic part of the charge materials will be lost through oxidations, evaporations, dust, deslagging, etc. Each material could have a different amount of loss of every constituent element. Every element loss in each material is characterized by L_{ij} which denotes the weight percentage of i element loss in charge j corresponding to the total weight of the element i in that charge. The values L_{ij} are assumed to be already known from experimental data or any other knowledge on the matter.

(5) The only cost, which will be taken into account for optimization, is the purchase cost of raw materials. Thus, all other costs, like energy, labor, machinery, and processing expenses will not be included in this optimization. It is also assumed that there is no value for any of the lost materials. Therefore, the total cost of the final melt will be the cost of consumed charge materials.

2.2. MODEL FORMULATIONS

With the aforementioned assumptions, the open system of melting furnace can be examined starting from an overall mass balance:

$$m_{\text{Total}} = m_0 + \sum_{j=1}^N m_j = m_{\text{Output Melt}} + m_{\text{Loss}} \quad (1)$$

Initial Melt + Input Charge

For convenience in formulations, the initial melt is considered an input charge with the index zero. The initial melt however exerts some extra constraints on the problem which will be discussed later in this section. The total material loss includes loss of allowing elements plus non-metal contaminations:

$$m_L = m_{\text{Total element losses (Metal loss)}}^E + m_{\text{Contaminations (Non-Metals)}}^C = \sum_{i=1}^E m_i^E + \sum_{j=0}^N m_j^C \quad (2)$$

The contamination of each charge burden is evaluated by a coefficient (fraction) of contamination:

$$m_j^C = L_j^C m_j \quad (3)$$

The loss of each metal element is obtained by summation of the loss of the same element in all charges:

$$m_i^E = \sum_{j=0}^N L_{ij} C_{ij} (1 - L_j^C) m_j \quad (4)$$

Substituting equations (3) and (4) into the equation (2) gives the total mass of materials loss:

$$\begin{aligned} m_L &= \sum_{i=1}^E \sum_{j=0}^N L_{ij} C_{ij} (1 - L_j^C) m_j + \sum_{j=0}^N L_j^C m_j \\ &= \sum_{j=0}^N \left((1 - L_j^C) \sum_{i=1}^E L_{ij} C_{ij} + L_j^C \right) m_j \end{aligned} \quad (5)$$

The mass balance of each element would take the form $\sum C_{ij} m_j = C_{iM} m_M$ if there was no material loss in the system. However, the presence of materials loss requires two corrections; First, the contaminations m_j^C should be subtracted from the left side, since the input contaminations are assumed to have no metallic elements. Second, the loss of the element m_i^E should be considered on the right (output) side in addition to the melt content. Therefore, the element mass balance equation is corrected as the following form:

$$\sum_{j=0}^N (1 - L_j^C) C_{ij} m_j = C_{iM} m_M + \sum_{j=0}^N (1 - L_j^C) L_{ij} C_{ij} m_j \quad (6)$$

To have a normalized formulation, we would rather define mass fractions instead of masses, and rewrite the mass balance equations in reduced form:

$$x_j = \frac{m_j}{m_{\text{Total}}}, \quad x_M = \frac{m_M}{m_{\text{Total}}}, \quad x_L = \frac{m_L}{m_{\text{Total}}} \quad (7)$$

$$\sum_{j=0}^N x_j = x_M + x_L = 1 \quad (8)$$

$$\sum_{j=0}^N (1 - L_j^C) (1 - L_{ij}) C_{ij} x_j = C_{iM} x_M \quad (9)$$

$$x_L = \sum_{j=0}^N \left((1 - L_j^C) \sum_{i=1}^E L_{ij} C_{ij} + L_j^C \right) x_j \quad (10)$$

In the case of the presence of an initial melt in the above set of relations, the weight of the target melt m_M is usually unknown. Moreover, x_M and m_{Total} are also unknown, which makes the system to be non-linear. To arrange the equations in a standard linear programming form, equations (9) can be written with $x_M = 1 - x_L$ as:

$$\sum_{j=0}^N \frac{(1 - L_j^C) (1 - L_{ij})}{(1 - x_L)} C_{ij} x_j = \sum_{j=0}^N C_{ij}^o x_j = C_{iM} \quad (11)$$

Where the coefficients C_{ij}^o are the modified compositions defined as:

$$C_{ij}^o = \frac{(1 - L_j^C) (1 - L_{ij})}{(1 - x_L)} C_{ij} \quad (12)$$

Since the modified coefficients of the equation (12) are depending on x_j 's, this balance equation is a non-linear relation, but now, it matches the form of standard linear programming. Another variable to be expressed for the optimization model is the objective function. The cost function is defined to be the total price of the target melt (see assumption 4) which is equal to the total costs of the consumed charge materials:

$$P_M m_M = \sum_{j=0}^N P_j m_j \quad (13)$$

To derive the objective function in the standard form, the mass of the melt $m_M = m_{\text{Total}} - m_L$ is substituted and the sides of the equation are divided by total mass to obtain the following form:

$$P_M = \sum_{j=0}^N \frac{P_j}{(1 - x_L)} x_j = \sum_{j=0}^N \beta_j x_j \quad (14)$$

Where the term β_j is defined as the gross price of each charge material which is a bit more than the purchase price of it regarding the cost of contaminations and metal losses. Attention should be paid to the point that the gross price is depending on the materials losses in the process, which is also unknown and more non-linearity to the problem.

$$\beta_j = \frac{P_j}{1 - x_L} \quad (15)$$

A set of constraints should be applied to the problem including the limit range of chemical composition of the target melt, the conservation of overall mass, and the limitation of the capacity of the furnace for correction of initial melt. Since the equations have been arranged in the standard form, the chemical composition and overall mass conservation can be simply implemented. The limitation of the capacity of the furnace puts a constraint on the total mass of the charged materials which entails $m_0 \leq m_T \leq m_F$. Dividing m_0 by the terms of this inequality gives:

$$\frac{m_0}{m_F} = x_0 \leq x_0 \leq x_0 = \frac{m_0}{m_0} \quad (16)$$

It should be noticed that there are two special cases for the constraint limits x_0 and x_0 according to the status of the initial melt and demand of the melting process. One special case occurs when there is no initial melt. In that case both of x_0 and x_0 take zero value to force x_0 to be zero in the calculations. Another special case occurs when not only there is an initial melt to be corrected, but also the melting process demands to reach a certain weight of the target melt, i.e. m_M is determined.

In this case, the total mass could be evaluated from the equation (1) and both of the constraint limits x_0^L and x_0^U would be equal to m_0/m_T which force x_0 to that certain amount in the calculations.

2.3. SUMMARY

The mathematical model is now ready to be summarized in the standard linear programming form. The problem is to find mass fractions x_j to minimize the below objective function, with the following set of constraints:

$$\begin{aligned} \text{Minimize } P_M &= \sum_{j=0}^N P_j^o x_j \\ \left\{ \begin{aligned} \sum_{j=0}^N x_j &= 1 & 0 \leq x_j \leq 1, & x_0^L \leq x_0 \leq x_0^U \\ \sum_{j=0}^N C_{ij}^o x_j &= C_{iM} & C_{iM}^L \leq C_{iM} \leq C_{iM}^U \end{aligned} \right. \end{aligned} \quad (17)$$

The modified compositions C_{ij}^o are expressed in the equation (12) and the modified prices P_j^o are defined in the equation (15). The terms C_{iM}^L and C_{iM}^U are the minimum and maximum limits of the specified range for the target melt.

2.4. ITERATIVE ALGORITHM

Although the developed model summarized in the equation (17) has been arranged in a standard linear programming format, it is a non-linear system of relations. That is because the modified prices P_j^o and the modified coefficients C_{ij}^o are depending on the solutions of mass fractions x_j . To solve the non-linear system an iterative algorithm is proposed according to the following steps:

Step (1). Set the initial values to be $x_j = 0$, $x_0^L = 0$ and $x_0^U = 1$ (if there is no initial melt $m_0 = 0$ set $x_0^L = 0$); Ignore the loss coefficients L_j^C and L_{ij} ; Thus $x_L = 0$, $P_j^o = P_j$, $C_{ij}^o = C_{ij}$. In these conditions, the problem is initialized so that the melting process would have no materials loss.

Step (2) Solve the optimization problem equation (17) using the most recent coefficients and parameters. Find mass fractions x_j and record the relative change in each one from their previous values.

Step (3) Check whether the last solution is feasible; In the case of initial melt $m_0 > 0$, calculate total mass $m_{\text{Total}} = m_0/x_0$; Check if the condition $m_{\text{Total}} \leq m_F$ is satisfied; If the total mass exceeds

the capacity of the furnace, i.e. $m_{\text{Total}} > m_F$, then update $m_0 = x_0 m_F$ and report that the initial melt should be decreased to the new m_0 for a feasible solution in the capacity of the furnace.

Step (4) Check if convergence is reached; i.e. if the summation of the absolute change of all x_j is less than a small value $\sum |\Delta x_j| \leq \varepsilon$ then stop calculations and report the solution.

Step (5) Correct the parameters and coefficients; Calculate x_L from the equation (10), $P_j^{\%}$ equations (15), and $C_{ij}^{\%}$ equation (12) respectively, using the most recent mass fractions x_j .

Step (6) Return to step 2 and repeat the calculations.

3. TEST PROBLEM

To evaluate the proposed model, a test problem was drawn. All the input data of the problem has been summarized in Table I. The problem involves a 10-ton industrial induction furnace for melting brass alloys. An initial melt has been already prepared in the furnace which has been measured to be 7 tons. The initial melt has been achieved from C26000 brass scraps [12]. The composition of the initial melt has been available by chemical analysis. The target melt, however, has been demanded to be in the specifications of C47940 alloy [12], i.e. the initial melt is required to be corrected with a minimum cost so that it falls in the range of the specified target alloy. The final weight of the melt is not known, and we do not know even if this melt correction is feasible or not. Available charge materials with their prices, non-metal contaminations, all alloying element contents, and percentages of element loss are listed in Table I. The prices have been roughly estimated from different sources on the internet. The data of contaminations and element losses in materials have been collected from experimental estimations and observations made in industrial sites. There is about $\pm 20\%$ or more uncertainty on the collected values. For some element losses, e.g. evaporations of zinc, there is some information available in the literature [13]. Taken from any source of information, the coefficients of loss for materials elements will have some uncertainties, which should be investigated out of the scope of the present paper. Finally, It is noticed that the test problem (and the presented parameters in Table I) has only been drawn to test the performance of the model. Thus, this is not a real test and the parameters do not represent any real case melting process. Therefore, careful attention should be paid to the realistic evaluation of the model parameters for implementation in a real process.

Table I. The specification of charge materials and parameters of the test problem.

Materials, j	Spec. No.	Price ¹ (\$/kg) P_j	Contamination (wt%) L_j^c	Composition and loss of elements								
				Ele- ments, i	Pb (wt%)	Fe (wt%)	Sn (wt%)	P (wt%)	Ni (wt%)	Zn (wt%)	Total others (wt%)	Cu (wt%)
Initial melt	C26000	0.00	0.5	C_{ij} L_{ij}	0.040 5	0.050 0.5	0.001 2	0.001 0.5	0.001 0.1	35.80 5	0.50 0	rem 0.5
Architectural Bronze	C38500	1.91	1.5	C_{ij} L_{ij}	16.00 5	0.300 0.5	0.001 3	0.002 0.5	0.000 0.1	1.00 4	0.50 0	rem 0.5
Copper-Phosphorous master alloy	CuP	2.80	0.5	C_{ij} L_{ij}	0.050 2	0.100 0.5	0.001 3	8.00 1	0.001 0.1	0.010 3	0.40 0	rem 0.5
CuSn trims	C52100	2.35	0.5	C_{ij} L_{ij}	0.050 2	0.100 0.5	10.00 4	0.020 0.5	0.001 0.1	0.200 3	0.40 0	rem 0.5
High Strength Copper trims	C19400	2.21	0.5	C_{ij} L_{ij}	0.100 3	2.60 1	0.001 2	0.100 0.5	0.001 0.1	0.150 3	0.40 0	rem 0.5
Zinc	-	2.50	0.1	C_{ij} L_{ij}	0.015 2	0.001 0	0.012 3	0.005 0.5	0 0	99.95 6	0.01 0	rem 0.2
Copper-Nickel alloy	C70100	4.00	0.5	C_{ij} L_{ij}	0.001 2	0.050 0	0.001 2	0.005 0.5	3.50 0.5	0.250 3	0.40 0	rem 0.5
Copper anode	-	6.20	0.1	C_{ij} L_{ij}	0 1	.0005 0	.0003 2	0.001 0.5	0.001 0	0.020 2	0.005 0	rem 0.5
Target Melt	C47940	?	?	C_{iM}	≥ 1.00 ≤ 2.00	≥ 0.1 ≤ 1.0	≥ 1.2 ≤ 2.0	≥ 0.02 ≤ 0.04	≥ 0.1 ≤ 0.5	≥ 27.96 ≤ 34.58	≥ 0 ≤ 0.5	rem ²

Other parametersFurnace capacity m_F 10,000 (kg)Initial melt m_0 7,000 (kg)¹ The prices have been estimated from averaging on data collected from the internet. These numbers were only used to evaluate the performance of the present model, and they do not represent a real case study.² The specification of the alloy C47940 requires that copper (the reminder element) falls between 63 and 66 wt%.

Based on the standard model summarized in the equation (17), the test problem was prepared in a spreadsheet. A matrix of the coefficients was built with $(E + 2) \times (N + 1)$ rows and columns. The problem is then solved through iterative optimization processing according to the proposed algorithm. With each iteration, the matrix of coefficient and cost parameters were updated based on the values obtained from the previous iteration. The SIMPLEX LP algorithm of the Excel solver was used to reach optimal conditions in each iteration. The convergence criteria being $\sum |\Delta x_j| \leq 10^{-4}$ was reached after four iterations.

4. RESULTS AND DISCUSSION

The solution of the test problem resulted in the mass fractions of the charge materials shown in Figure 2. The bar chart demonstrates the solutions for three iterations. The first iteration yields the results as the melting process would have no materials loss. In the next two iterations, the mass

fractions have been updated taking the loss of elements and contaminations into accounts. A fast convergence has been achieved with the standard model. As indicated in the legend of Figure 2, the total weight of the calculated charge has been predicted to be about 9909 kg. The total mass fraction of material loss has also been predicted to be $x_L = 0.02624$ for the test problem which corresponds to about 260 kg of material loss. The weight of the final melt is about 9649 kg consequently from the relation $m_M = x_M m_T = (1 - x_L) m_T$. Regarding the capacity of the furnace, the solution of the total charge is feasible. However, if the furnace had less capacity than this, the solution would not be feasible. In that case, the algorithm would report the issue and would ask the operator to decrease the amount of initial melt. Figure 3 shows the converged solution for input mass fractions of charge materials in conjunction with output fractions of material loss related to each material.

Figure 2. Calculated mass fraction of charge materials with the proposed model.

Figure 3. Calculated mass fractions of charge materials with the output mass fractions of materials loss.

The chemical composition of the output melt has been displayed in Figure 4. Specified upper and lower limits for the alloying elements of the target melt have been indicated on the plot. The relative amount of each element loss has also been depicted on the diagram. The weight percentage of each element loss can be multiplied by the total input mass to obtain the mass of element loss. It should be noticed that the reminder element has a considerable amount of loss as well as other elements. Therefore, it is necessary to take the reminder element into account to be able to evaluate the loss of the total material. However, the constraints on the reminder element might not be applied in the standard model. Attention should be paid to the point that the constraint of the overall mass balance $\sum x_j = 1$ is equivalent to the constraint of the reminder element. Thus, only one of these two constraints should be applied in the model, otherwise, the model would become ill-conditioned. That would cause numerical instability in fine precisions. Nevertheless, some alloy specifications require that the amount of the reminder element falls in a particular range, like the target melt C47940 in the present test problem that requires $63 \leq C_{Cu} \leq 66$. In this case, there is probably another unconstrained element, zinc for instance, or the value of total other elements, which could be conditioned with upper and lower limits so that the reminder element falls within the specified range. Therefore a general rule can be stated for charge calculations; Always take “total other” and “reminder” elements into account, but do not put constraints on the reminder element. It is only used for the calculation of total element loss.

Figure 4. Calculated chemical composition of the final melt and weight percentage of each element loss from the total input mass.

To evaluate the optimality of the obtained solution, it can be checked that whether the objective function, i.e. the calculated price, has found its minimum value with the obtained solution. In another

word, if any of the obtained x_j is changed, the solution of the problem is expected to lead to a more cost value. Therefore, other rounds of calculations were performed using the model with separate increments in the obtained solution. In each calculation, an extra constraint was introduced into the model which held one x_j at an incremental deviation from its solution. Then, the calculation was repeated. If another feasible solution was obtained, the price was recorded. Figure 5 shows the variations of cost value concerning the deviations of each mass fraction x_j . The diagram reveals that any deviation in mass fractions will increase the cost function or will lead to an infeasible solution. This is a reason to the point that the objective function has reached its minimum feasible value. It is worth mentioning that since the proposed model has been organized in a standard LP form, it will be convenient for the user to investigate the optimality conditions and to evaluate the numerical behavior of the model in various cases of industrial application. That is of great importance in online industrial charge calculations where a numerically fast and stable model is required. Furthermore, the standard form of the model in conjunction with the iterative solution algorithm is capable of being generalized for more complexities and non-linearities in applications.

Figure 5. Variations of the optimized price of melt versus deviations of mass fraction of charge materials.

An important point to be discussed is considering the element loss in each material. The coefficient of loss, or sometimes the coefficients of absorption (the complementary of loss) of the alloying elements are usually evaluated based on industrial and experimental estimations. For instance, zinc has a relatively high amount of loss, taking about 4-6% on average, because of its low boiling point regarding the melting temperature of the brass alloys. Nevertheless, the element loss in different charge materials depends on a variety of parameters, e.g. specific surface of scraps.

Therefore the value of element loss is not the same in different charge materials. It has been well substantiated in melting that materials in the shape of swarfs and chips exhibit much more element loss than ingots and thick parts. An average element loss however will strongly depend on how much weight of swarfs, foils, chips, and sheets is present in the charged burden. In the proposed model, the loss coefficients L_{ij} provide flexibility for considering more realistic element loss in different materials. Another major source of the unreliability of the charge calculation models is the presence of contaminations in charge material. In the present model, the coefficients of non-metal contamination L_j^C have been considered to help to reduce the uncertainties in the chemical composition of the final melt. In this context, the methodology of the evaluation of these coefficients is also important, which falls out of the scope of the present paper and needs more investigation in future studies.

5. CONCLUSIONS

In this paper, a charge calculation model for melting furnaces has been presented. The model is based on the mass balance of the alloying element and optimization of the cost of the charge materials. The model considers non-metallic contaminations in charge materials and element losses due to unwanted evaporations, oxidations, dust, deslagging, etc. The model also considers the likely case of an initial out-of-range melt in the furnace which should be corrected according to a target alloy specification. The main contribution of the presented model is that all of the non-linear formulations, emerged from considering contaminations, material loss, and melt corrections, have been arranged into a standard linear-programming framework of optimization problems, which has been summarized in the equation (17). The model was tested and the performance of the model showed that non-linear phenomenological complexities can be effectively considered through an iterative algorithm of the standard LP model. This feature provides a backbone for reliable and fast optimization which is of significant benefit for industrial automation.

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LIST OF ABBREVIATIONS

Letters		Superscripts	
C	Composition	C	Contamination
E	Number of elements	E	Elements
L	Coefficient of loss	$\cap$	Maximum limit
m	Mass	$\cup$	Minimum limit
N	Number of charge materials	$:$	Modified value
P	Price	Subscripts	
x	Mass fraction	0	Initial melt
Greek Letters		i	element
Δ	Difference	j	charge material
Abbreviations		L	Loss
LP	Linear Programming	T	Total
		M	Melt
		F	Furnace

DECLERATIONS

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable

Availability of data and materials

All data generated or analyzed during this study are included in this published article.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

AJ developed the model, collected data and analyzed the results. DY and MM implemented the model in the test problem. All authors read and approved the final manuscript.

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