A Novel Dynamic Data Driven Tire Model

Junning Zhang

Shijiazhuang Tiedao University <https://orcid.org/0000-0003-3675-2995>

Shaopu YANG

Shijiazhuang Tiedao University

Yongjie LU (✉ luyj825@163.com)

Shijiazhuang Tiedao University

Original Article

Keywords: Tire dynamics, data driven, Fuzzy algorithm

Posted Date: July 26th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-724346/v1>

License: This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

A Novel Dynamic Data Driven Tire Model

Junning Zhang^{1,2}, Shaopu Yang¹, Yongjie Lu^{1,*}

Abstract: In the study of vehicle dynamics, the accurate description of tire mechanical characteristics is the basis and key of vehicle dynamics simulation. An innovative tire model is proposed based on fuzzy algorithm and a sinusoidal membership function is used to design fuzzy rules. In order to ensure the accuracy of tire behavior calculation, this model is driven by a small amount of experimental data of tire mechanical characteristics. This tire model consists of four layers of fuzzy systems, each of which has a knowledge base. The data in knowledge base I is obtained by experiments, and the data of knowledge base II is computed by the upper system, and so is the later system. Then, the input signal, the change rate of side slip angle and slip rate, is considered to improve the calculation accuracy of the model. The proposed fuzzy tire model can accurately predict the longitudinal force, lateral force and self-aligning torque of the tire under unknown conditions. Finally, by comparing the fuzzy tire model with the experimental data, it is found that the maximum RRMSE (Relative Root Mean Square Error) value is not more than 0.14. It is proved that the model can accurately describe the tire mechanical characteristics under combined conditions.

Key words: Tire dynamics; data driven; Fuzzy algorithm

1 Introduction

In addition to being subjected to gravity and air resistance, acceleration, braking, steering and other movements of the vehicle depend on the force exerted by the ground on the tire. Most of the external forces and moments that vehicles are subjected to come from the contact between tires and road surface^[1,2]. With the development of automotive technology, more and more tire behavior studies have been carried out to further improve the controllability of the vehicle and the predictability of its behavior, which improves the dynamic safety^[1]. The accuracy of the simulation results obtained from the vehicle model depends largely on the accuracy of the tire model used^[3,4,5]. With the continuous in-depth study of tire-road contact mechanism, scholars have studied a variety of different types of tire models, laying the foundation for tire mechanics research^[6]. Generally speaking, tire models can be divided into four categories: theoretical model, empirical model, semi-empirical model, and intelligent model. The theoretical model is derived by simplifying the tire structure^[7,8,9]. The expressions of the mechanical properties of the tire under different working conditions can accurately reflect how the parameters in the tire model affect the mechanical properties of the tire. Typical empirical tire model mainly includes UA model^[10,11], Dugoff model^[12,13], Brush model^[14-16], and LuGre model^[17-20]. Empirical model chooses a certain fitting

function to represent the tire characteristics, and then use the test data as the basis, and then use special functions to fit the curve, to establish the tire mechanical characteristics model. Typical empirical tire model mainly includes magic formula model^[21], polynomial model^[22], Burckhardt model^[23-25], and K-D model^[26]. It has high fitting accuracy and can be used to test other tire models. Semi-empirical model is an empirical model established by setting boundary conditions on the basis of theoretical analysis and experimental data^[27]. The semi-empirical model has the advantages of theoretical model and empirical model, is widely used in vehicle dynamics simulation^[28,29]. The fuzzy tire model described in this paper is a kind of intelligent tire model. It uses fuzzy algorithm and a small amount of experimental data to establish the mapping relationship between input and output. Nowadays, some intelligent tire systems have been proposed, which are equipped with sensors in the tire^[30,31]. And the signals of these sensors are used to predict the dynamic characteristics of the tire. Although the system can accurately predict tire force, it increases the complexity of tire structure. Due to tire complex structure and material features, it has special mechanical characteristics of high nonlinearity. The simulation results and efficiency will be affected by the low accuracy use of general modeling methods. It is a very suitable method to use intelligent algorithm to deal with the modeling of highly non-linear tire mechanical characteristics and then to simulate vehicle handling and stability, which is worthy of vigorous exploration and research in the field of vehicle dynamics^[32,33]. Although the simple tire model has fewer parameters and less difficulty in parameter identification, the model error is large. Therefore, the simple tire model cannot be used to describe the specific characteristics of the research content.

* Corresponding author. E-mail: luyj825@163.com

¹ Key Laboratory of Mechanical behavior and System Safety of Traffic Engineering Structures, Shijiazhuang Tiedao University, Shijiazhuang, 050043, China

Full list of author information is available at the end of the article

Although the complex tire model improves the calculation accuracy, there are many parameters in the model, which makes parameter identification difficult. Some researchers use magic formula model as reference model and use intelligent algorithm to identify the key parameters to describe the evolution of wheel slip caused by tire-road friction^[34]. Although the identified parameters can ensure the accuracy of the model under the given working conditions, they can't guarantee the adaptability of all working conditions.

In this paper, the fuzzy algorithm is mainly used to accomplish tire mechanical characteristics modeling. The membership function of each fuzzy set in the fuzzy rules is defined by a small amount of experimental data of tire mechanical characteristics. An innovative tire model is established based on fuzzy algorithm. Finally, by comparing with the experimental data, it is verified that the model has a high calculation accuracy.

2 Design of Fuzzy Tire System Based on Experimental Data

The main factors affecting tire characteristics include slip rate, cornering angle, vertical load and roll angle. In this paper, the basic idea is to use a small amount of experimental data to predict the tire longitudinal force, lateral force and self-aligning torque under unknown working conditions. In Fig. 1, fuzzification interface converts the input signal into fuzzy linguistic value of corresponding membership function, while defuzzification interface converts fuzzy calculation results into the output signal. Knowledge base includes two parts: database and rule base. The rule base contains the fuzzy rules related to the problem in this field, while the data base defines the membership functions used in the fuzzy rules. The reasoning mechanism carries out the reasoning process according to these rules and the experimental data and obtains reasonable output or conclusion. In this paper, the slip rate s , slip angle α , roll angle γ and vertical load F_n of tire are taken as input of the model. The output of the model is longitudinal force F_x , lateral force F_y and self-aligning torque M_z .

Fig. 1. Basic fuzzy inference progress

The fuzzy system is of the following form:

Dynamic Part:

Rule l : IF $z_1(t)$ is $M_{1,l}$, and $\dots$, and $z_n(t)$ is $M_{n,l}$,
THEN u is B_l , $l = 1, 2, \dots, r$.

Where l denotes the l th fuzzy rule and r is the number of rules. $M_{i,l}$ ($i = 1, 2, \dots, n$) is fuzzy set of input variable z_i .

R is fuzzy set of output variable u . Each $z_i(t)$ is a measurable time-varying quantity.

There are two functions of $z(t)$ associated with each rule. The first function is called the truth value. The truth value for the l th rule is defined by the equation

$$\omega_l = \prod_{i=1}^n M_{i,l}(z_i(t)), \quad (1)$$

where $M_{i,l}(z_i(t))$ is the membership value of $z_i(t)$ to the fuzzy set $M_{i,l}$. Throughout this paper, we will assume that each ω_l is a non-negative function. At least one of truth values in all rules is always non-zero. The second function is called normalized membership function. The normalized membership value for the l th rule is defined by the Eq. (2)

$$h_l(z(t)) = \frac{\omega_l(z(t))}{\sum_{l=1}^r \omega_l(z(t))}, \quad (2)$$

where r denotes the number of rules in the rule base. Under the previously stated assumptions, it is always a well-defined function taking values between 0 and 1, and the sum of all the normalized membership values is identically equal to 1.

The tire model built in this paper is mainly affected by four parameters: slip rate s , slip angle α , roll angle γ and vertical load F_n . The basic framework of the fuzzy tire model is shown in Fig. 2.

In Fig. 2, the knowledge base I represents the synthesis of rule base and database, and the data in the database are obtained by experiments. The experimental data in this paper are measured from the figures of the reference^[37] using the software Origin. The data mainly include the tire longitudinal force, lateral force and self-aligning torque under the combined action of tire vertical load force, roll angle, slip angle and slip rate. The red part of the figure indicates the domain in the data base when the vertical load is F_n . The output of defuzzification interface I provides data for knowledge base II to build database and rule base. Similarly, the data in current fuzzy system knowledge are provided by the previous fuzzy system. The output of the fuzzy tire model is longitudinal force F_x , lateral force F_y and self-aligning torque M_z . In this paper, it is assumed that the roll angle γ of tire have little influence on the calculation accuracy of tire model, so the influence of their variation on the calculation accuracy is not considered. Sinusoidal membership function not only retains the simplicity of the commonly-used triangular and trapezoid membership functions, but also is well-defined since sine basis function has been widely used in Fourier series and function approximation theory^[35,36]. Therefore, the membership function used in this paper is a sinusoidal membership function, and its detailed expression is shown in Eq. (3). It can be defined by a triad(a, b, c), as in Fig. 3.

Fig. 2. Fuzzy tire model through data driven

Fig. 3. A sinusoidal membership function

$$M_{i,l}(z_i) = \begin{cases} \frac{1}{2} \left(1 - \cos \left(\frac{\pi(z_i - a)}{b - a} \right) \right) & \text{if } a \leq z_i < b \\ \frac{1}{2} \left(1 + \cos \left(\frac{\pi(z_i - b)}{c - b} \right) \right) & \text{elseif } b \leq z_i < c \\ 0 & \text{else} \end{cases} \quad (3)$$

The fuzzy set B_{l_1} ($l_1 = 1, 2, \dots, r_1$) of Knowledge base I is derived from experimental data. In the first layer system, the membership value corresponding to the input variable F_n is expressed as $M_{l_1, l_1}(F_n)$ ($l_1 = 1, 2, \dots, n_1$), and the fuzzy set corresponding to the output variable is $B_{l_1}(F_{n, j_1}, \gamma_{j_2}, \alpha_{j_3}, s_{j_4})$ ($j_1 = 1, 2, \dots, m_1$; $j_2 = 1, 2, \dots, m_2$; $j_3 = 1, 2, \dots, m_3$; $j_4 = 1, 2, \dots, m_4$). For accurate input $F_n(t)$,

through fuzzy reasoning, the output of defuzzification is Eq. (4)

$$\mu(F_n) = \frac{\sum_{l_1=1}^{r_1} B_{l_1}(F_{n, j_1}, \gamma_{j_2}, \alpha_{j_3}, s_{j_4}) h_{l_1}(F_n)}{\sum_{l_1=1}^{r_1} h_{l_1}(F_n)}. \quad (4)$$

The set $\mu(F_n)$ obtained from the above Equation (4) is regarded as fuzzy set B_{l_2} ($l_2 = 1, 2, \dots, r_2$) of Knowledge base II. In the second layer system, the membership value corresponding to the input variable γ is expressed as $M_{l_2, l_2}(\gamma)$ ($l_2 = 1, 2, \dots, n_2$), and the fuzzy set corresponding to the output variable is $B_{l_2}(\gamma_{j_5}, \alpha_{j_6}, s_{j_7})$ ($j_5 = 1, 2, \dots, m_5$; $j_6 = 1, 2, \dots, m_6$; $j_7 = 1, 2, \dots, m_7$). For accurate input $\gamma(t)$, through fuzzy reasoning, the output of defuzzification is

$$\mu(\gamma) = \frac{\sum_{l_2=1}^{r_2} B_{l_2}(\gamma_{j_5}, \alpha_{j_6}, s_{j_7}) h_{l_2}(\gamma)}{\sum_{l_2=1}^{r_2} h_{l_2}(\gamma)}, \quad B_{l_2}(\gamma_{j_5}, \alpha_{j_6}, s_{j_7}) = \mu(F_n). \quad (5)$$

The set $\mu(\gamma)$ obtained from the above Equation (5) is regarded as fuzzy set B_{l_3} ($l_3 = 1, 2, \dots, r_3$) of Knowledge base III. In the third layer system, the membership value corresponding to the input variable α is expressed as $M_{l_3, l_3}(\alpha)$ ($l_3 = 1, 2, \dots, n_3$), and the fuzzy set corresponding to the output variable is $B_{l_3}(\alpha_{j_8}, s_{j_9})$ ($j_8 = 1, 2, \dots, m_8$; $j_9 = 1, 2, \dots, m_9$). For accurate input $\alpha(t)$, through fuzzy reasoning, the output of defuzzification is

$$\mu(\alpha) = \frac{\sum_{l_3=1}^{r_3} B_{l_3}(\alpha_{j_8}, s_{j_9}) h_{l_3}(\alpha)}{\sum_{l_3=1}^{r_3} h_{l_3}(\alpha)}, \quad B_{l_3}(\alpha_{j_8}, s_{j_9}) = \mu(\gamma). \quad (6)$$

The set $\mu(\alpha)$ obtained from the above Eq. (6) is regarded as fuzzy set B_{l_4} ($l_4 = 1, 2, \dots, r_4$) of Knowledge base IV. In the fourth layer system, the membership value corresponding to the input variable s is expressed as $M_{l_4, l_4}(s)$ ($l_4 = 1, 2, \dots, n_4$), and the fuzzy set corresponding to the output variable is $B_{l_4}(s_{j_{10}})$ ($j_{10} = 1, 2, \dots, m_{10}$). For accurate input $s(t)$, through fuzzy reasoning, the output of defuzzification is

$$\mu(s) = \frac{\sum_{l_4=1}^{r_4} B_{l_4}(s_{j_{10}}) h_{l_4}(s)}{\sum_{l_4=1}^{r_4} h_{l_4}(s)}, \quad B_{l_4}(s_{j_{10}}) = \mu(\alpha). \quad (7)$$

The final output of the model is

$$[F_x, F_y, M_z]^T = \mu(s) \quad (8)$$

3 Verification of Fuzzy Tire Model

This paper mainly analyses the changing trend of

longitudinal force F_x , lateral force F_y and self-aligning torque M_z under different combined conditions. Due to the small effect of roll angle γ on the longitudinal force F_x , so this paper does not analyze the effects on the longitudinal force of roll angle^[37]. The reason why this paper does not analyze the influence of the tire force of LuGre tire model with the change of the roll angle is that the tire roll angle γ on the tire force is not considered in the classic LuGre tire model.

3.1 Longitudinal force under combined conditions

(a) $F_n = 1600 \text{ N}, \gamma = 0^\circ$

(b) $F_n = 5000 \text{ N}, \gamma = 0^\circ$

(c) $F_n = 8000 \text{ N}, \gamma = 0^\circ$

(d) $F_n = 11000 \text{ N}, \gamma = 0^\circ$

Fig. 4. Curves of longitudinal force with slip rate under different vertical loads

Table 1. RRMSE value between simulation results and experimental data

F_n/N \ $\alpha/^\circ$	1	4	8	14	20
1600	0.1029	0.0458	0.0523	0.0316	0.1035
5000	0.1302	0.1344	0.1209	0.1356	0.0877
8000	0.1174	0.1287	0.1103	0.1242	0.1046
11000	0.0559	0.0752	0.0602	0.0462	0.0354

Compare the fuzzy tire model with experimental data; Fig. 4 shows the comparison of longitudinal forces F_x , which varies with slip rate s , between fuzzy tire model and experimental data under different vertical loads F_n and slip angles α . And the roll angles γ are 0° . The results show that the new model has high accuracy to predict longitudinal force F_x . RRMSE is calculated by

$$RRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left| \frac{x_{si} - x_{ei}}{x_{ei}} \right|^2}, \quad (9)$$

which is used to measure the accuracy of the new model, as shown in Table 1. It can be seen from the table that the maximum RRMSE value is not more than 0.14, which further verifies the accuracy of the fuzzy tire model. It can also be seen from Fig.4 that when the parameters of the tire model remain unchanged, it is difficult to ensure that the longitudinal force F_x of the LuGre model can meet multiple simulation conditions at the same time. Where x_{si} and x_{ei} are the simulation data and the experimental data respectively.

In Fig. 5, the trend of longitudinal force F_x with slip rate s and slip angle α in three-dimensional coordinates, which shows the continuity in different slip rate s and slip angle α . The model can be used to predict the longitudinal force in different combined conditions.

Fig. 5. Curved surface of longitudinal force with slip angle and slip rate under different vertical loads ($\gamma=0^\circ$)

3.2 Lateral force under combined conditions

Fig. 6 shows the comparison of lateral forces F_y , which varies with slip rate s , between tire model and experimental data under different vertical loads F_n and slip angles α . And the roll angles γ are 0° . The results show that the model has high accuracy to predict lateral force F_y . The RRMSE value is used to measure the accuracy of the model, as shown in Table 2. It can be seen from the table that the maximum RRMSE value is not more than 0.05, which further verifies the accuracy of the fuzzy tire model. It can also be seen from Fig.4 that when the parameters of the tire model remain unchanged, it is difficult to ensure that the longitudinal force F_y of the LuGre model can meet multiple simulation conditions at the same time.

(a) $F_n=1600$ N, $\gamma=0^\circ$

(b) $F_n=5000$ N, $\gamma=0^\circ$

(c) $F_n=8000$ N, $\gamma=0^\circ$

(d) $F_n=11000$ N, $\gamma=0^\circ$

Fig. 6. Curves of lateral force with slip rate under different vertical loads

Table 2. RRMSE value between simulation results and experimental data

$\alpha/^\circ \backslash F_n/N$	1	4	8	14	20
1600	0.0414	0.0079	0.0050	0.0030	0.0022
5000	0.0277	0.0075	0.0047	0.0028	0.0020
8000	0.0266	0.0061	0.0049	0.0021	0.0015
11000	0.0347	0.0064	0.0044	0.0026	0.0020

In Fig. 7, the trend of lateral force F_y with slip rate s and slip angle α in three-dimensional coordinates, which shows the continuity in different slip rate s and slip angle α . The model can be used to predict the lateral force in different combined conditions.

Fig. 7. Curved surface of lateral force with slip angle and slip rate under different vertical loads ($\gamma=0^\circ$)

Fig. 8 shows the comparison of lateral forces F_y , which varies with slip angle α , between tire model and experimental data under different vertical loads F_n and roll

angle γ . And the slip rates s are 0. The results show that the model has high accuracy to predict lateral force F_y . The RRMSE value is used to measure the accuracy of the model, as shown in Table 3. It can be seen from the table that the maximum RRMSE value is not more than 0.13, which further verifies the accuracy of the fuzzy tire model.

Fig. 8. Curves of lateral force with slip angle under different vertical loads

Table 3. RRMSE value between simulation results and experimental data

$\gamma/^\circ$ F_n/N	-8	-4	0	4	8
1600	0.1206	0.0487	0.1015	0.0922	0.0346
5000	0.0959	0.0374	0.0410	0.1213	0.0576
8000	0.0323	0.0313	0.0337	0.0556	0.1145
11000	0.0301	0.0317	0.0347	0.0530	0.0970

In Fig. 9, the trend of lateral force F_y with roll angle γ and slip angle α in three-dimensional coordinates, which shows the continuity in different roll angle γ and slip angle α . It can be used to predict the lateral force in different combined conditions.

Fig. 9. Curved surface of lateral force with slip angle and roll angle under different vertical loads ($s = 0^\circ$)

3.3 Self-aligning torque under combined conditions

Fig. 10 shows the comparison of self-aligning torque M_z , which varies with slip angle α , between tire model and experimental data under different vertical loads F_n and roll angle γ . And the slip rates s are 0. The results show that the model has high accuracy to predict self-aligning torque M_z . The RRMSE value is used to measure the accuracy of the model, as shown in Table 4. And it can be seen from the table that the maximum RRMSE value is not more than 0.13, which further verifies the accuracy of the fuzzy tire model.

Fig. 10. Curves of self-aligning torque with slip angle under different vertical loads

Table 4. RRMSE value between simulation results and experimental data

$\gamma/^\circ \backslash F_n/\text{N}$	-8	-4	0	4	8
1600	0.0591	0.0698	0.0527	0.0398	0.1086
5000	0.0774	0.0689	0.0646	0.0115	0.0806
8000	0.0446	0.0739	0.0571	0.0480	0.1276
11000	0.1002	0.1093	0.0588	0.0159	0.0457

In Fig. 11, the trend of self-aligning torque M_z with roll angle γ and slip angle α in three-dimensional coordinates, which shows the continuity in different roll angle γ and slip angle α . It can be used to predict the self-aligning torque in different combined conditions.

In summary, the fuzzy tire model has high calculation accuracy. It can predict the longitudinal force F_x , lateral force F_y and self-aligning torque M_z of tire under different combined conditions. Comparing the fuzzy model with the experimental data, the maximum RRMSE value is not more than 0.14.

Fig. 11. Curved surface of self-aligning torque with slip angle and roll angle under different vertical loads

3.4 Comparison with the computational overhead of traditional LuGre model

In order to show that the model proposed in this paper has small computational overhead and meets the use requirements of the model. In this paper, the computational overhead comparison between Fuzzy tire model and traditional LuGre tire model is given. The computational overhead concerns the CPU (Central processing unit) time for obtaining the output F_x , F_y or M_z from the Fuzzy tire model compared with using the LuGre tire model. When the input roll angle γ , slip rate s , slip angle α and vertical load F_n of the two models are 0° , 0.3, 8° and 8000 N respectively, the comparison results are shown in Table 5.

Table 2. The computational overhead comparison between Fuzzy tire model and traditional LuGre tire

Items	Longitudinal Force F_x	Lateral Force F_y	Self-aligning Torque M_z	All of the three
Fuzzy tire model	0.010649	0.010208	0.010733	0.010879
LuGre tire model	0.007630	0.008052	0.007893	0.009051
Errors	0.003019	0.001146	0.002840	0.001828

It can be seen from Table 5 that the computational overhead error level of the two models is 10-3 s. Compared to the LuGre tire model, the CPU time for Fuzzy tire model is longer, but the Fuzzy model takes into account the effect of the roll angle γ on the tire forces. In the next work, a Fuzzy tire model based on the 3-Dimensional membership function was designed, which reduced the computational overhead of the model and further shortened the CPU calculation time.

4 Conclusions

In this paper, the method of fuzzy algorithm is applied to propose an innovative tire model driven by a small amount of data. It avoids the influence of parameters in existing models on the physical characteristics of tire. The key point of the tire model is the fuzzy model which can describe the

nonlinearity of tire force characteristics. By comparing the fuzzy model with the experimental data, the maximum RRMSE is not more than 0.14, which proves that the model can accurately describe the longitudinal force, lateral force and self-aligning torque of the tire under combined conditions. It also can be used to predict longitudinal force, lateral force and self-aligning torque in different combined conditions.

(1) The data-driven model abandons the physical structure and working process of the object of study.

(2) This model can be used to describe the specific characteristics of the research content, which cannot be achieved by simple models.

(3) Although the complex tire model improves the calculation accuracy, there are many parameters in the model, which makes parameter identification difficult. The model proposed in this paper does not need to consider the difficulty of identifying too many parameters.

Availability of data and materials

Not applicable.

Authors' Contributions

Y-SP provided the idea; Z-JN and L-YJ wrote the manuscript.

All authors read and approved the final manuscript.

Author Details

¹ Key Laboratory of Mechanical behavior and System Safety of Traffic Engineering Structures, Shijiazhuang Tiedao University, Shijiazhuang, 050043, China. ² School of Traffic and Transportation, Shijiazhuang Tiedao University, Shijiazhuang, 050043, China

Authors' Information

Shao-Pu Yang, born in 1962, is currently a professor at Shijiazhuang Tiedao University, China. He is the winner of National Science Fund for Distinguished Young Scholars, Chief Scientist of "973", National Outstanding Professional and Technical Talents and Yanzhao Scholar of Hebei Province. His research interests include nonlinear vibration and control and vehicle-road coupled system dynamics.

ZHANG Junning, born in 1989, received his PhD degree from Shijiazhuang Tiedao University, China, in 2021. E-mail: zhangtinian@163.com

LU Yongjie, born in 1981, is currently a professor at Shijiazhuang Tiedao University, China. Her research interests include vehicle active safety control, vehicle road rigid flexible coupling dynamics and intelligent tire dynamics. E-mail: lu-yongjie@163.com

Competing Interests

The authors declare that they have no competing interests.

Funding

Supported by National Natural Science Foundation of

China (Grant Nos. 12072204, 11972238, 11572207, 11472180), Natural Science Foundation of Hebei Province(Grant Nos. A2016210103, A2020210039)

References

- [1] LINDSLEY N J. *A new tire model for aircraft landing gear dynamics*[M]. 1999:83-95.
- [2] KUIPER E, OOSTEN J J M V. The PAC2002 advanced handling tire model[J]. *Vehicle System Dynamics*, 2007, 45(sup1):153-167.
- [3] LIU Z, GAO Q. Analytical investigation on tire dynamics by rigid-elastic coupled tire model with nonlinear sidewall stiffness[J]. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 2018, 40(2):80.
- [4] LIANG W, MEDANIC J, RUHL R. Analytical dynamic tire model[J]. *Vehicle System Dynamics*, 2008, 46(3):197-227.
- [5] LACOMBE J. Tire model for simulations of vehicle motion on high and low friction road surfaces[C]// *Simulation Conference Proceedings, 2000. Winter*. Society for Computer Simulation International, 2000:1025-1034.
- [6] DENG L, CAO R, WANG W, et al. A multi-point tire model for studying bridge-vehicle coupled vibration[J]. *International Journal of Structural Stability & Dynamics*, 2016, 16(08):1550047.
- [7] LU Y, ZHENG W, CHEN E, et al. 2387. Research on ride comfort and safety of vehicle under limited conditions based on dynamical tire model[J]. *Journal of Vibroengineering*, 2017, 19(2):1241-1259.
- [8] DING N, TAHERI S. A modified Dugoff tire model for combined-slip forces[J]. *Tire Science and Technology*, 2010, 38(3): 228-244.
- [9] SAHA A, WAHI P, WIERCIGROCH M, et al. A modified LuGre friction model for an accurate prediction of friction force in the pure sliding regime[J]. *International Journal of Non-Linear Mechanics*, 2016, 80:122-131.
- [10] GIM G, NIKRAVESH PE. An analytical model of pneumatic tires for vehicle dynamic simulations. part 3: validation against experimental data[J]. *International Journal of Vehicle Design*, 1991,12(2):217-228.
- [11] GIM G, NIKRAVESH PE. A three dimensional tire model for steady-state simulations of vehicles[J]. *SAE Inc*, 1993, 102(2): 150-159.
- [12] DUGOFF H, FANCHER P, SEGEL L. An analysis of tire traction properties and their influence on vehicle dynamic Performance. *SAE Transactions*, 1970,79:341-366.
- [13] AHMADI J, SEDIGH A, KABGANIAN M. Adaptive vehicle lateral-plane motion control using optimal tire friction forces with saturation limits consideration[J]. *IEEE Transactions on Vehicular Technology*, 2009,58(8):4098-4107.
- [14] HAHN J, RAJAMANI R, ALEXANDER L. GPS based real-time identification of tire-road friction coefficient[J]. *IEEE Transactions on Control Systems Technology*, 2002,10 (3):331-343.
- [15] CHO W, YOON J, YIM S, et al. Estimation of tire forces for application to vehicle stability control[J]. *IEEE Transactions on Vehicular Technology*, 2010,59(2):638-649.
- [16] HSU Y, LAWS S AND GERDES J. Estimation of tire slip angle and friction limits using steering torque[J]. *IEEE Transactions on Control Systems Technology*, 2010,18(4):896-907.
- [17] CARLOS C, TSIOTRAS P. Dynamic tire friction models for vehicle traction control[C]// *Proceedings of the 38th Conference on Decision & Control*, Phoenix, AZ, USA, 1999:3746-3751.
- [18] CLAEYS X, YI J, ALVAREZ L, et al. A dynamic tire/road friction model for 3D vehicle control and simulation[C]// *Proceedings of the 2001 IEEE Intelligent Transportation Systems Conference*, Oakland, CA, USA, 2001: 483-488.
- [19] CARLOS C, TSIOTRAS P VELENIS E, et al. Dynamic friction models for road/tire longitudinal interaction[J]. *Vehicle System Dynamics*, 2003,39(3):189-226.

- [20] YI J, ALVAREZ L, CLAEYS X, et al. Emergency braking control with an observer based dynamic tire/road friction model and wheel angular velocity information[C]// *Proceedings of the 2001 American Control Conference*, Arlington, VA: 2001:19-24.
- [21] PACEJKA, Hans B.; BAKKER, Egbert. The magic formula tyre model[J]. *Vehicle system dynamics*, 1992, 21.S1: 1-18.
- [22] GERMANN S, DAISS A. Monitoring of the friction coefficient between tyre and road surface[C]// *IEEE Conference on Control Applications*. IEEE, 1994:613-618.
- [23] STEPHANT J, BAFFET G, CHARARA A. Sideslip angle, lateral tire force and road friction estimation in simulations and experiments[C]// *IEEE International Symposium on Computer Aided Control System Design*, IEEE International Conference on Control Applications. IEEE, 2009:903-908.
- [24] ALVAREZ L. Adaptive emergency braking control in automated highway systems[C]// *IEEE Conference on Decision & Control*. IEEE, 1999:3740-3745.
- [25] Baffet G, Charara A, Dherbomez G. An Observer of Tire-Road Forces and Friction for Active Security Vehicle Systems[J]. *IEEE/ASME Transactions on Mechatronics*, 2007, 12(6):651-661.
- [26] KIENCKE U, DAISS A. Estimation of Tyre Friction for enhanced ABS-Systems[C]// *International Symposium on Anec*. 1994.
- [27] BESSELINK I. J.M., SCHMEITZ A. J.C., PACEJKA H. B. An improved Magic Formula/Swift tire model that can handle inflation pressure changes[J]. *Vehicle System Dynamics*, 2010, 48(sup1): 337-352.
- [28] Guo K, Lu D. UniTire: unified tire model for vehicle dynamic simulation[J]. *Vehicle System Dynamics*, 2007, 45(sup1):79-99.
- [29] TEZUKA Y, ISHII H, KIYOTA S. Application of the magic formula tire model to motorcycle maneuverability analysis[J]. *Jsaе Review*, 2001, 22(3):305-310.
- [30] DANIEL G P, JORGE Y, OLUREMI O, et al. A strain-based method to estimate slip angle and tire working conditions for intelligent tires using fuzzy logic[J]. *Sensors*, 2017, 17(4):874.
- [31] KIM S S J, KIM K K S, YOON Y S. Development of a tire model based on an analysis of tire strain obtained by an intelligent tire system[J]. *International Journal of Automotive Technology*, 2015, 16(5):865-875.
- [32] CHEN C. A criterion of robustness intelligent nonlinear control for multiple time-delay systems based on fuzzy Lyapunov methods[J]. *Nonlinear Dynamics*, 2014, 76(1):23-31.
- [33] GENG C, MOSTEFAI L, HORI Y. Body slip angle observer for electric vehicle stability control based on empirical tire model with fuzzy logic approach[C]// *Conference of the IEEE Industrial Electronics Society*. IEEE Xplore, 2008.
- [34] LI K, CAO J, YU F. Nonlinear tire-road friction control based on tire model parameter identification[J]. *International Journal of Automotive Technology*, 2012, 13(7):1077-1088.
- [35] JIN L, GUPTA M M. *Adaptive fuzzy systems with sinusoidal membership functions*[M]//Uncertainty Analysis in Engineering and Sciences: Fuzzy Logic, Statistics, and Neural Network Approach. Springer, Boston, MA, 1998: 273-289.
- [36] XU X, ZHANG Y, LIANG Y, et al. An improved fuzzyneural network for ultrasonic motors control[C]// *Advances in Neural Networks - ISNN 2007, 4th International Symposium on Neural Networks*, ISNN 2007, Nanjing, China, June 3-7, 2007, Proceedings, Part I. Springer-Verlag, 2007.
- [37] XU NAN. *Study on the steady state tire model under combined conditions*[D]. Jilin University, 2012:77-109. (in Chinese)