Finite Element Modelling and Simulating Effects of Hairiness Performance on Hairiness Entanglement During Fabric Pilling

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Finite element modelling and simulating effects of hairiness performance on hairiness entanglement during fabric pilling

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Abstract: For analyzing behaviors of hairiness entanglement during fabric pilling, nonlinear dynamic motion equations are deduced based on the elastic thin rod element, combined with the moving characteristics of hairiness, which follow the principles of mechanical equilibrium and energy conservation. The finite element simulation model of the effects of hairiness performance on behaviors of hairiness entanglement was established by ABAQUS. The analysis solution values of nonlinear dynamics were compared with the finite element simulation results. The results showed that hairiness elastic modulus, hairiness friction coefficient and hairiness diameter have significant effects on frictional dissipation energy, strain energy and kinetic energy produced by hairiness entanglement during pilling. Compared the finite element simulation results with analysis solution values, they are in good agreement. The fitness is greater than 0.96, which verifies the validity of finite element method.

Keywords: polyester hairiness; entanglement; pilling; finite element method; nonlinear dynamic

1 Introduction

Pilling is a classic problem in textile materials^[1]. It has not yet been substantively solved. Therefore, controlling pilling continues to be a significant challenge for the designer and manufacturers of a wide range of textile materials. Understanding of pilling mechanisms can give quantitative information on the importance of factors involved in pilling. A number of different perspectives modelling pilling mechanisms have been tackled, such as chemical reaction kinetics model, mechanical dynamic model, and artificial neural network model and so on.

Chemical reaction kinetic model by using principle of chemical reaction kinetic to quantitatively pilling process was developed^[2]. The foundation of this work comes from the research of Gintis^[3], who qualitatively described pilling process as three stages. This model was very complex with several differential equations, which include seven parameters. Fuzzing and entanglement had a greater impact than falling off on pilling. Based on mechanism of pilling, a simplified dynamic model with only three parameters was established^[4]. This model is the basis for development of pilling mechanisms. An extended kinetic model was developed^[5]. It extended the simplified kinetic model. The model was too complex to verify it through experiment data. Naylor et al.^[6] developed optimized chemical reaction kinetics model. This model considered the fact that the rate of pill formation was less than the rate of pill falling off. The analysis indicated that pillable hairiness is 75% of the total hairiness lost. And the loss of non-pillable hairiness only occurred at the beginning of pilling process. The loss was very small. The previous dynamic model of pilling ignored micro mechanical and micro-dynamical processes. Hearle et al.^[7] analyzed mechanical mechanism of hairiness diffusion, entanglement, pulling out, fatigue fracture, and falling off. Mechanical dynamic model of pilling was established. However, calculation of the model is relatively cumbersome. And the model is not verified by experiment.

Hairiness is an elongated body with a certain degree of elasticity, large aspect ratio, flexibility, and continuity. During pilling, hairiness will exhibit complex dynamic behaviors, such as

translating and rotating, and flexible deformations, such as bending, twisting, rotation and stretching. Therefore, there is an urgent need to study nonlinear dynamic behavior of hairiness entanglement according to general nonlinear mechanic theory. Hairiness is regarded continuous elastic thin rod to study their nonlinear dynamic behavior. In recent years, analytical mechanics with two independent variables of space and time based non thin elastic rods has made remarkable development. An elastic thin rod dynamic model based on Gauss principle was proposed^[8]. Compared with Kirchhoff model, cross sectional shear deformation, center-line expansion, and volume force were considered in this model. However, the research has not yet been verified by experiments. Moreover, the nonlinear dynamic method mostly adopts simultaneous partial differential equations to express the small deformation mechanics behavior of the elastic thin rod. It makes the actual calculation of hairiness entanglement become difficult. With the development of computer simulation and finite element method^[9] (FEM), the actual calculation of fiber entanglement has been effectively solved. Fibers are considered as thin elastic rod to propose a finite element model during twisting process of air-jet vortex spinning^[10]. This model simulated the motion trajectory of free end fibers and realized the numerical simulation and calculation of nonlinear dynamic behavior of elastic thin rod. However, the effects of fiber properties on fiber motion behavior were not systematically studied.

In this paper, hairiness is treated as elastic thin rod. According to nonlinear dynamics characteristics of hairiness during fabric pilling, following the principles of mechanical balance and energy conservation, the nonlinear dynamic mechanics model is established by using D'Alembert theory^[11]. At the same time, the finite element method is adopted to simulate hairiness entanglement during fabric pilling. The effects of hairiness properties on frictional dissipation energy, strain energy and kinetic energy are numerically studied. Finally, the finite element results are compared with the analysis solution results to verify the finite element model. The research can provide a new theoretical basis and idea for studying the mechanisms of pilling.

2 Experimental Materials and Method

2.1 Experimental Material

Polyester hairiness is used. The technical parameters of polyester hairiness are shown in Table 1. The yarn count is Ne 24 s. Yarn twist is 950 T/m. Warp density is 390 N/10cm, weft density is 185 N/10cm. And fabric structure is Twill.

Table 1. Technical parameters of polyester hairiness

Index	value
Density, (ρ , kg/m ³)	1380
Length, (L, m)	5×10 ⁻⁴
Diameter, (d , μ m)	12
Elastic modulus (E , GPa)	4
Poisson's ratio (ν , Pa)	0.3
Friction coefficient	0.2

2.2 Experimental Method

2.2.1 Pilling test

The fabric was cut into 6 pieces. The shape of samples is circle. And the diameter is 140 mm. The specimens were conditioned for 24 h at 20 °C and 65 % relative humidity prior to testing. And

then they were mounted on YG(B)401T Martindale wear resistant tester. According to ISO12945-2:2000, the specimen is rubbed with a Lisharu trajectory. Each group of samples is rubbed with 125, 500, 1000, 2000, and 5000 pilling rubs. And then the number of pills on the sample is counted. The degree of pilling was estimated by the average number of pills.

2.2.2 Scanning electron microscopy

The specimen was extracted with acetone and ethanol and then dried in a vacuum oven at room temperature for 12 h. Then they were conditioned for 24 h at 20 °C and 65 % relative humidity prior to testing. SEM images with different pilling rubs were carried out with TM3030 microscope, typically operating at a 15 KV acceleration voltage, after gold coating.

3 Nonlinear dynamic theory analysis of hairiness pilling

3.1 Nonlinear dynamic equation of pilling

In previous studies, these existing fiber models can obtain the position and direction of fibers, but cannot describe fiber deformation such as bending, twisting, rotating, and stretching^[12]. Fiber is mostly described as a series of spheres connected in contact with each other. The deformation of fiber tensile, bending and torsion can be represented by changes in the distance between spheres, connection angle, and twist angle, respectively. The common points of these models are that external forces only act on spheres, and connectors are only used to transmit the internal force and maintain hairiness structure. These models can provide a method to determine fiber internal force and discrete fiber. However, they are too simple to describe the precise force acting on fiber or the physical properties of fiber.

In this paper, hairiness on the fabric is considered as elastic thin rod^[13]. An elastic thin rod is composed of many elements. Each element is connected by nodes. The external forces acting on hairiness are assumed to act on the nodes. The position variables on hairiness such as displacement, strain, and stress are solved by each element. The unknown variables on the entire hairiness can be solved by combining the solutions of all the elements. The displacement of each element is solved by the displacement difference of element nodes, as shown in equations (1) and (2).

$$\mathbf{D} = \mathbf{N}_e \mathbf{d}_e = \sum_{i=1}^n \mathbf{N}_i \mathbf{d}_i \quad (1)$$

$$\mathbf{d}_i = [u_i, v_i, \omega_i]^T \quad (2)$$

Where $\mathbf{D}$ is the overall displacement of a hairiness, $\mathbf{N}_e$ is the interpolation function matrix of element nodes, $\mathbf{N}_i$ is the i th component of $\mathbf{N}_e$, $\mathbf{d}_i$ is the displacement of the i th node of the element. u_i , v_i and ω_i are the component of the matrix $\mathbf{d}_i$ in the direction of x-axis, y-axis and z-axis, respectively.

The constitutive relationship between the stress and strain of hairiness follows Hooke's law. For simplification, hairiness is assumed as isotropic materials. The constitutive equations are obtained, which are showed in (3), (4) and (5).

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (3)$$

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad (4)$$

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (5)$$

Where E is elastic modulus of hairiness, ν is Poisson's ratio, σ is the stress, ε is the strain,
The stress-strain constitutive relationship of an element can be expressed by equation (6).

$$\boldsymbol{\sigma} = \mathbf{A} \boldsymbol{\varepsilon} \quad (6)$$

Where $\mathbf{A}$ is the constitutive matrix as expressed in equation (3).

Hairiness has nonlinear dynamic behavior characteristics during pilling. Therefore, large nonlinear deformation occurs. Green-Lagrange strain tensor is adopted to describe the strain-displacement relationship of hairiness element, as exhibited in equations (7) and (8).

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \boldsymbol{\varepsilon}_e = \mathbf{B} \mathbf{d}_e = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right] \\ \frac{\partial v}{\partial y} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \\ \frac{\partial w}{\partial z} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \\ \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) \right] \\ \frac{1}{2} \left[\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} \right) + \left(\frac{\partial u}{\partial z} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \frac{\partial w}{\partial y} \right) \right] \\ \frac{1}{2} \left[\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right) + \left(\frac{\partial u}{\partial z} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} \frac{\partial w}{\partial x} \right) \right] \end{Bmatrix} \quad (7)$$

$$\mathbf{d}_e = (u, v, w) \quad (8)$$

Where, $\mathbf{B}$ is the matrix of strain-displacement.

Substitute equations (6) and (7) into the strain energy expression to obtain equation (9).

$$U = \frac{1}{2} \iint_V \boldsymbol{\varepsilon}_e^T \boldsymbol{\sigma}_e dV = \frac{1}{2} \iint_V \mathbf{B}^T \mathbf{A} \mathbf{B} dV \mathbf{d}_e = \frac{1}{2} \mathbf{d}_e^T \mathbf{K} \mathbf{d}_e \quad (9)$$

Where $\mathbf{K}$ is the stiffness matrix and V is the volume of the hairiness element.

Calculate the partial derivative of equation (9) to obtain equation (10).

$$\mathbf{K} \mathbf{d}_e = \mathbf{F}_e \quad (10)$$

Where, $\mathbf{F}_e$ is the force acting on the element nodes.

When a body force is applied on the element, its equivalent nodal force is shown in equation (11).

$$\mathbf{F} = \iint_V \mathbf{N}_e^T p dV \quad (11)$$

Where $\mathbf{F}$ is the equivalent nodal force, p is the volume force exerted on the element.

According to D'Alembert's principle, the control equation of a moving hairiness is expressed by equation (12).

$$p = p_s - \rho \frac{\partial^2 D}{\partial t^2} - \mu \frac{\partial D}{\partial t} \quad (12)$$

Where p_s is the static force, D is the hairiness displacement, ρ is the density of the hairiness, μ is friction coefficient of the hairiness.

Substitute equation (1) into equation (12) to obtain equation (13).

$$p = p_s - \rho N_e \ddot{d}_e - \mu N_e \dot{d}_e \quad (13)$$

Substituting equation (13) into equation (11), the equivalent nodal force equation (14) is obtained.

$$\mathbf{F} = \iint_e \mathbf{F}_e^T p dV = \mathbf{F}_{s_e} - \mathbf{M}_e \ddot{d}_e - \mathbf{C}_e \dot{d}_e \quad (14)$$

$$F_{S_e} = \iiint_V N_e^T p_s dV \quad (15)$$

$$M_e = \iiint_V N_e^T \rho N_e dV \quad (16)$$

$$C_e = \iiint_V N_e^T \mu N_e dV \quad (17)$$

Where F_{se} is the element nodal force equivalent to the static force, M_e is the mass matrix of the hairiness element, C_e is the damping matrix of the hairiness element.

Substitute equation (10) into equation (14) to obtain the dynamic equation of the element nodes, as shown in (18).

$$M_e \ddot{d}_e + C_e \dot{d}_e + K_e d_e = F_{S_e} \quad (18)$$

The kinetic equation is given by combing the kinetic equations of all the elements, as illustrated in equation (19).

$$M \ddot{d} + C \dot{d} + K d = F_{S_e} \quad (19)$$

Where M , C and K are mass matrix, damping matrix and stiffness matrix of the hairiness, respectively. d is the node displacement vector of the entire hairiness.

3.2 Frictional dissipation energy of hairiness

Gralen and Coworkers found that friction between hairiness is calculated according to equation (20). It can be seen from equation (20) that friction between hairiness is positively correlated with friction coefficient, contact pressure, contact length and hairiness radius.

$$f = \mu N + \beta l R \quad (20)$$

Where f is the friction between hairiness, μ is friction coefficient of hairiness, N is the contact pressure between hairiness, β is a constant, l is the contact length between hairiness, and R is the radius of hairiness.

According to the definition of friction dissipation energy, the friction dissipation energy of the entire hairiness can be calculated as shown in equation (21). It can be seen from equation (21) that friction dissipation energy generated during pilling process is affected by factors such as hairiness friction coefficient, contact pressure, hairiness radius, and contact length between hairiness.

$$E_f = \int_V f g l V \quad (21)$$

3.3 Strain energy of hairiness

According to equation (9), strain energy of the entire hairiness can be obtained as shown in equation (22). It can be seen from equation (22) that strain energy of the hairiness is related to the displacement of the entire hairiness during pilling and the stiffness of the hairiness.

$$U = \frac{1}{2} d_e^T K d_e \quad (22)$$

3.4 Kinetic energy of hairiness

According to kinetic energy theory of the particle system, kinetic energy of the entire hairiness is obtained by combing the dynamic equation (19), as shown in equation (23). It can be seen from equation (23) that kinetic energy of the hairiness during entanglement and pilling is mainly related to the hairiness displacement, the quality of the hairiness, friction coefficient and the stiffness. The quality of hairiness is expressed by the linear density of the hairiness. Friction coefficient is

expressed by the dynamic friction coefficient. And the stiffness of the hairiness is expressed by the hairiness elastic modulus of the hairiness.

$$\Delta E_k = \int_V \mathbf{F}_{S_e} \cdot d\mathbf{r} = \int_V (\mathbf{M} \cdot d\mathbf{r} + \mathbf{C} \cdot d\mathbf{r} + \mathbf{K} \cdot d\mathbf{r}) \quad (23)$$

4 Finite Element Models of Hairiness Entanglement and Pilling

4.1 Geometrical analysis

This model mainly simulates entanglement of hairiness on the fabric to form pills. Therefore, 3D hairiness parts and fabric part should be created. For a 3D fabric part, yarn width, yarn height, yarn spacing, cross-sectional shape and fabric thickness are the basic information. The micro-image of the fabric was depicted in Figure 1 by using SEM. Based on micro-image in Figure 1, the width, height and distance between warp and weft yarns of the fabric were obtained by using Nano Measurer 1.2. The basic parameters of fabric and yarns are got, as shown in Table 2. For 3D hairiness parts, it can be considered as an entity with the diameter being 0.012mm and height being 0.66mm.

Figure.1 SEM image of cross-sectional shape of the fabric

Table.2 The basic parameters of fabric and yarns

Index	Yarn width (warp/weft), mm	Yarn height (warp/weft), mm	Yarn spacing(warp/weft), mm	Cross-sectional shape(warp/weft), mm	Fabric thickness, mm
value	0.28/0.24	0.24/0.15	0.29/0.39	ellipse/ellipse	0.38

TexGen software was used to reconstruct the fabric with basic yarn parameters in Table 1. The geometric part of the fabric is imported into software ABAQUS. Hairiness part was created in ABAQUS. The parts of fabric and hairiness were assembled in ABAQUS. The geometric structure of fabric with hairiness is shown in Figure 2

Figure 2 The geometric structure of fabric with hairiness

4.2 Material properties

Hairiness is simulated by the type of three-dimensional entity. Hairiness elastic modulus and Poisson's ratio are set according to the parameter values in Table 1. In practical applications, fabric refers to a sheet-like object made of hairiness, which have the anisotropy characteristics. This model assumes that the fabric is homogeneous and isotropic ^[14]. The fabric is set up with a three dimensional homogeneous solid.

4.3 Interaction

Explicit dynamic algorithm is adopted to simulate hairiness entanglement and pilling. Hairiness and fabrics are assembled. Hairiness ends exposed on the fabric surface are completely fixed on the fabric surface along the direction of x-axis, y-axis, and z-axis. The hairiness on the fabric surface interacts with each other. The hairiness is entangled with each other under the action of cyclic body force. Therefore, there is mutual contact between hairiness. In the process of constant friction, there is friction force between hairiness. The friction formula of the tangential behavior of the contact attribute is set as the penalty function. The friction coefficient is set according to Table 1. The pressure interference of the normal behavior is set as "hard" contact. To achieve an economical solution and ensure the convergence of the simulation, it is necessary to shorten the analysis step time. The time period of the analysis step is 5E-05s, which is much shorter than the actual experimental time. When the time is 1E-05s, it is equivalent to 1000 pilling rubs. When the time is 2E-05s, it is equivalent to 2000 pilling rubs. When the time is 3E-05s, it is equivalent to 3000 pilling rubs. When the time is 4E-05s, it is equivalent to 4000 pilling rubs. When the time is 5E-05s, it is equivalent to 5000 pilling rubs.

4.4 Boundary conditions and loads

According to fabric pilling test of the Martindale method, the fabric rotates in a Lisa curve on the platform. Therefore, the hairiness on fabric surface is subjected to pressure along vertical downward and force along horizontal. These two forces are combined. Volume forces whose value is 0.415N/cm² is applied on the hairiness. The fabric is completely fixed. The nodes where hairiness contact fabric is always kept on fabric surface, and will not leave fabric surface.

4.5 Meshing

The density of mesh is not as dense as possible. Appropriate mesh density can help the calculation convergence. Element division should be as uniform as possible. Meshing with larger sizes should be applied when the element is subjected to an obvious force, or the structure of the

element is very fine. This operation can ensure the stability of calculations. The fabric and hairiness are both divided by hexahedral unit type, namely C3D8R.

5 Results and Discussion

The energy applied on the fabric is mainly absorbed in three ways: (i) frictional dissipation energy generated by friction between hairiness, (ii) strain energy generated by entanglement of hairiness to deform hairiness, (iii) kinetic energy generated by the relative movement between hairiness. The effects of hairiness properties on hairiness entanglement and pilling are mainly manifested by friction dissipation energy, strain energy and kinetic energy. In this paper, the effects of hairiness elastic modulus, friction coefficient, and diameter on frictional dissipation energy, strain energy and kinetic energy during hairiness entanglement and pilling are mainly studied.

5.1 Effect of hairiness elastic modulus on hairiness entanglement and pilling

5.1.1 Effect of hairiness elastic modulus on frictional energy dissipation

Finite element software ABAQUS is applied to simulate entanglement and pilling of hairiness with different hairiness elastic modulus. The influence law of hairiness elastic modulus on frictional dissipation energy is obtained, as depicted in Figure 3(a). It can be seen from Figure 3(a) that frictional dissipation energy produced by hairiness entanglement and pilling gradually increases with the increase of hairiness elastic modulus. The explanations are as follows. Hairiness is deformed by bending, twisting, and entanglement during pilling. The larger hairiness elastic modulus of the hairiness is, the more likely it is to fatigue and fracture. According to equations (20) and (21), the analysis solution values of friction dissipation energy were derived. FEM value and analysis solution value were compared in Figure 3(b). From Figure 3(b), it can be concluded that FEM value agreed very well with analysis solution value. When hairiness elastic modulus is 4GPa, 8GPa, and 40GPa, the fitness is 0.986, 0.985, and 0.994, respectively.

Figure 3 Friction dissipation energy during pilling with different hairiness elastic modulus: (a) effect of hairiness elastic modulus on friction dissipation energy; (b) FEM value of Friction dissipation energy vs analysis solution value

Figure 4 plots the fatigued and fractured hairiness. After the hairiness is fatigued and fractured, it is more likely to bend and entangle with the surrounding hairiness. And then it participates in

pilling. Finally, frictional energy dissipation increases.

Figure 4 Hairiness fatigue and fracture during entanglement and pilling

SEM image of pilling is illustrated in Figure 5(a), and the finite element simulation result of hairiness entanglement and pilling is depicted in Figure 5(b). Compared with Figure 5(a) and 5(b), the hairiness entanglement and pilling obtained by finite element simulation are very consistent with actual pilling test.

Figure 5 images of a pill: (a) SEM image of an actual pill; (b) image of a pill by FEM

5.1.2 Effect of hairiness elastic modulus on strain energy of hairiness entanglement and pilling

Hairiness entanglement and pilling with different hairiness elastic modulus were simulated by using FEM. The influence law of hairiness elastic modulus on strain energy is obtained, as shown in Figure 6(a). It can be indicated from Figure 6(a) that strain energy increase when hairiness elastic modulus becomes larger. The reason may be that strain energy is positively related to the stiffness and displacement of hairiness. The larger hairiness elastic modulus makes the stiffness of hairiness greater. Therefore, strain energy increases. According to equations (22), the analysis solution values of strain energy were derived. FEM value and analysis solution value were compared in Figure 6(b). It can be indicated from Figure 6(b) that FEM value agreed very well with analysis solution value. When hairiness elastic modulus is 4GPa, 8GPa, and 40GPa, the fitness is 0.977, 0.957, and 0.966, respectively.

Figure 6 Effect of hairiness elastic modulus on strain energy: (a) effect of hairiness elastic modulus on strain energy; (b) FEM value of strain energy vs analysis solution value

5.1.3 Effect of hairiness elastic modulus on kinetic energy of hairiness entanglement and pilling

To investigate the effect of hairiness elastic modulus on strain energy of hairiness entanglement and pilling, the finite element simulation was adopted. The result is displayed in Figure 7(a). Figure 7(a) is the effect of hairiness elastic modulus strain energy of hairiness entanglement and pilling according to FEM. It can be seen from Figure 7(a) that kinetic energy of hairiness decreases with the increase of hairiness elastic modulus. The explanations are as follows. The total energy of hairiness pilling is the same. And the total energy is composed of three ways, which include friction dissipation energy, strain energy and kinetic energy. When hairiness elastic modulus becomes larger, friction dissipation energy and strain energy also increase. Therefore, the kinetic energy decreases. According to equations (23), the analysis solution values of kinetic energy were derived. FEM value and analysis solution value were compared in Figure 7(b). It can be indicated from Figure 7(b) that FEM value agreed very well with analysis solution value. When hairiness elastic modulus is 4GPa, 8GPa, and 40GPa, the fitness is 0.994, 0.976, and 0.977, respectively.

Figure 7(a) Effect of hairiness elastic modulus on kinetic energy according to FEM; (b) FEM value of kinetic energy vs analysis solution value of kinetic energy

5.2 Effect of hairiness friction coefficient on hairiness entanglement and pilling

5.2.1 Effect of hairiness friction coefficient on frictional energy dissipation

Hairiness entanglement and pilling with different friction coefficient is simulated by finite element method. The effect of friction coefficient on hairiness entanglement and pilling by FEM is depicted in Figure 8(a). It can be concluded from Figure 8(a) that when hairiness coefficient friction increases, friction dissipation energy during entanglement and pilling process gradually increases. This may be associated with the component of friction dissipation energy. According to equation (20) and (21), the greater friction coefficient leads to the bigger frictional force. And friction dissipation energy increases with the increase of frictional force. Effect of friction coefficient on friction dissipation energy is verified by comparing with analysis solution values. The analysis solution value of friction dissipation energy with different friction coefficient is calculated by using equation (20) and (21). The compared results are illustrated in Figure 8(b). It can be indicated from Figure 8(b) that FEM value of friction dissipation energy consists very well with analysis solution value. When friction coefficient is 0.2, 0.3, and 0.4, the fitness is 0.986, 0.992, and 0.994, respectively.

Figure 8(a) Effect of friction coefficient on friction dissipation energy; (b) FEM value of friction dissipation energy vs analysis solution value

5.2.2 Effect of hairiness friction coefficient on strain energy

Effect of hairiness friction coefficient on strain energy during entanglement and pilling is studied by FEM. The simulated value of strain energy is showed in Figure 9(a). It can be seen from Figure 9(a) that when friction coefficient becomes bigger, strain energy during pilling process increases. This can be attributed to the hairiness deformation. Friction resistance increases with the increase of friction coefficient. And the hairiness deformed more easily. Therefore, strain energy of hairiness increases. FEM value of strain energy with different friction coefficient is compare with analysis solution value. The compared curves are illustrated in Figure 9(b). From Figure 9(b), it can be indicated that FEM values are verified to be consistent with analysis solution values. When friction coefficient is 0.2, 0.3, and 0.4, the fitness is 0.976, 0.985, and 0.981, respectively.

Figure 9 Effect of friction coefficient on strain energy: (a) effect of friction coefficient on strain energy; (b) FEM value of strain energy vs analysis solution value of strain energy

5.2.3 Effect of hairiness friction coefficient on kinetic energy

Hairiness entanglement and pilling with different friction coefficient is simulated by FEM to obtain the influence law. The curves of kinetic energy with hairiness friction coefficient are plotted in Figure 10(a). It can be indicated from Figure 10(a) that the larger friction energy makes kinetic energy smaller. The reason may be that if friction coefficient is larger, resistance force of relative movement between hairiness is also greater. Under the same external force, the speed of hairiness movement is lower. Therefore, kinetic energy is smaller. Analysis solution value of kinetic energy is calculated by equation (23). FEM value of kinetic energy is verified by comparing with analysis solution value. The curves of kinetic energy are depicted in Figure 10(b). It can be concluded from Figure 10(b) that FEM values of kinetic energy consist very well with analysis solution value. When friction coefficient is 0.2, 0.3, and 0.4, the fitness is 0.994, 0.982, and 0.983, respectively.

Figure 10 Effect of friction coefficient on kinetic energy: (a) effect of friction coefficient on kinetic energy; (b) FEM value of kinetic energy and analysis solution value of kinetic energy

5.3 Effect of hairiness diameter on hairiness entanglement and pilling

5.3.1 Effect of hairiness diameter on frictional dissipation energy

Figure 11(a) shows the effect of hairiness diameter on friction dissipation energy by using FEM. It is speculated that friction dissipation energy decreases with the increase hairiness diameter. The explanation may be that when the hairiness diameter is larger, the probability contact between hairiness is lower. Therefore, the contact length between hairiness is smaller. According to equation (20), friction force is positively related to the contact length. Finally, the frictional dissipation energy decreases. The comparison of friction dissipation energy solved by analytical method and finite element method is illustrated in Figure 11(b). It can be indicated from Figure 11(b) that friction dissipation energy solved by FEM is similar to that by analytical method. The fitness of these two methods is 0.990, 0.992, and 0.986, respectively. Therefore, finite element model is verified.

Figure 11 Effect of hairiness diameter on friction dissipation energy: (a) effect of hairiness diameter on friction dissipation energy; (b) FEM value of friction dissipation energy and analysis solution value of friction dissipation energy

5.3.2 Effect of hairiness diameter on strain energy

Figure 12(a) describes strain energy as function of hairiness diameter. Strain energy shows a slight increasing tendency with the increase of hairiness diameter. Larger hairiness diameter leading to higher strain energy may be related to the larger hairiness deformation at larger levels of hairiness diameter. Larger hairiness diameter will enable hairiness to be distributed onto a larger contact area efficiently. Since strain energy is associated with hairiness deformation, the larger hairiness diameter would give a larger hairiness deformation, thus higher strain energy. Figure 12(b) shows strain energy solved by finite element method and analytical method with different hairiness diameters. The strain energy and time plots captured using finite element method is similar to that of analytical method, although the slopes of the curves are slightly different. The fitness is 0.978, 0.965, and 0.977. Therefore, finite element model is validated by comparing with analytical method.

Figure 12 Effect of hairiness diameter on strain energy: (a) effect of hairiness diameter on strain energy; (b) FEM value of strain energy and analysis solution value of strain energy

5.3.3 Effect of hairiness diameter on kinetic energy

Kinetic energy as function of hairiness diameter is depicted in Figure 13(a). Kinetic energy firstly climbs to the peak and then decreases with the increase of pilling time. And kinetic energy is positively related to hairiness diameter. The reason may be that larger hairiness diameter make hairiness contact to be distributed on a larger area. This leads the probability of relative moving between hairiness to be improved. Since kinetic energy is associated with the involved area at each contact point, the higher hairiness diameter would increase the involved area and provide a higher velocity of the entire hairiness. Therefore, kinetic energy gradually increases.

Kinetic energy simulated by finite element model has been validated by comparing with that of analytical method, as showed in Figure 13(b). From the plots developed using the analytical method, it can be observed that kinetic energy is increased with the increase of hairiness diameter and time. Similar behavior is also observed from the plots developed using finite element model. The predicted value of strain energy by finite element model and analytical method is very closely. The fitness is 0.992, 0.993 and 0.994, respectively.

Figure 13 Effect of hairiness diameter on kinetic energy: (a) effect of hairiness diameter on kinetic energy; (b) FEM value of kinetic energy and analysis solution value of kinetic energy

6 Conclusions

In this paper, the effects of hairiness properties on hairiness entanglement and pilling have been analyzed using finite element simulation. The finite element model is established based on nonlinear dynamics theory. Mechanical equilibrium state of nonlinear large deformation of hairiness during hairiness entanglement and pilling process is studied. And the nonlinear dynamic equation of hairiness elastic thin rod is built. The effects of hairiness elastic modulus, friction coefficient, and hairiness diameter on friction dissipation energy, strain energy and kinetic energy of hairiness during pilling process have been figured out by using finite element software ABAQUS. The following conclusions can be drawn from the research:

Hairiness elastic modulus has a significant effect on hairiness entanglement and pilling. Larger Hairiness elastic modulus gives rise to higher friction dissipation energy, and strain energy of hairiness, but lower kinetic energy. Increasing friction coefficient enhances friction dissipation, and strain energy of hairiness. However, kinetic energy decreases with the increase of friction coefficient. Hairiness diameter also has an important effect on hairiness entanglement and pilling. Increasing hairiness diameter can decrease friction dissipation energy, but enhance strain energy and kinetic energy.

Comparing finite element simulation with theoretically derived energy, the fitness is as high as 0.96, which validates finite element model. The energy-time behaviors predicted by finite element model and analytical method are observed to be similar for various parameters of hairiness property investigated in this study. Therefore, finite element simulation has solved the problem that hairiness entanglement and pilling is difficult to be quantitatively analyzed. It provides a theoretical basis for studying the essential mechanisms of hairiness pilling from a microscopic perspective.

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