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Failure of Rock Slopes with Intermittent Joints: Failure Process and Stability Calculation Models

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Abstract

Rock slopes with intermittent joints in open-pit mines are complex geological bodies composed of intact rock and discontinuous structural planes, and their stability analysis are necessary for mine disaster prevention. In this study, a series of base friction tests were performed to determine the failure process and displacement field evolution of rock slopes with intermittent joints using the speckle technique of a noncontact measurement system. Next, stability calculation models of the slopes were established from the energy perspective using the plastic limit analysis theory, and the effects of the joint inclination angle and coalescence coefficient of rock bridges on the slope stability were evaluated. The four main conclusions are as follows. (1) The failure of rock slopes with intermittent joints shows the feature of collapse-lower traction-upper push. (2) Based on the failure modes of rock bridges in slopes, the failure of rock slopes with intermittent joints could be divided into three types: tensile coalescence (Type A), shear coalescence (Type B), and tensile–shear coalescence (Type C). (3) Among the three slope types, the stability of the Type A slope is significantly influenced by rock cohesion, whereas that the Type B slope is significantly influenced by joint cohesion. The stability of the Type C slope is significantly influenced by the joint inclination angle and joint friction angle. (4) The local-stable slope is unstable while the first through-tensile crack in the zone of the potential sliding body higher than the critical

instability height appeared. This study guides the stability evaluation and instability prediction of jointed rock slopes in open-pit mines.

Keywords: intermittent joints; rock slope; base friction test; failure process; failure criterion

1. Introduction

Rock slopes with intermittent joints in open-pit mines are complex geological bodies composed of intact rock and discontinuous structural planes. The discontinuous joint plays a significant role in controlling the strength and failure mode of slope rock masses (Eberhardt et al., 2003; Sjöberg, 1996). The high-stress concentration of the tips of intermittent joints in open-pit mine slopes is formed under the action of long-term external factors (Fig. 1), which leads to the initiation, propagation, and coalescence of cracks in rock bridges. When the original joints in the slope are interconnected through rock bridges, an overall potential sliding surface will be formed inside the slope (Fig. 2). Because the deformation and failure of rock slopes with intermittent joints are progressive and the failure mode is mostly influenced by the controlling structural plane, the deformation characteristics and failure mechanism of the slopes are complex (Scholtes and Donze, 2015; Zhou and Chen, 2019). An in-depth study of the propagation behavior of internal joints and the macroscopic instability mechanism of rock slopes with intermittent joints is useful for the stability evaluation and engineering treatment of jointed rock slopes in open-pit mines.

The stability of a rock slope with intermittent joints is influenced by two critical factors, i.e., the mechanical properties of rock bridges and joints and the spatial distribution of intermittent joints. The failure of rock bridges in a jointed rock mass is usually investigated through small-scale rock mechanics tests or numerical simulation methods. Several scholars

47 have investigated the mesofailure mechanism of rock masses with joints using experimental
 48 methods. Wong et al. (1997) conducted experimental studies on the compressive strength and
 49 crack coalescence pattern of rock specimens with two or more preexisting flaws under
 50 uniaxial compression (Wong and Chau, 1997; Wong et al., 2019; Wong and Einstein, 2001;
 51 Wong and Einstein, 2006; Yang et al., 2017; Bobeta and Einstein, 1998). Huang et al. (2020)
 52 assessed the mechanical behaviors of rocks containing preexisting flaws under unloading or
 53 dynamic conditions using fracture mechanics principles and an experimental method (Huang
 54 et al., 2020; Zhong et al., 2020; Zhou and Chen, 2019; Zhou and Chen, 2020). Li et al. (2017)
 55 analyzed the cracking processes of rocks with preexisting flaws under dynamic loads (Li et al.,
 56 2017; Zhu et al., 2019). In addition, the numerical modeling of jointed rock-mass failure has
 57 been conducted extensively. Tang et al. (1997) analyzed the generation and propagation
 58 mechanisms of cracks in jointed rock masses and evaluated the deformation and failure
 59 processes of rocks with different intermittent joint distributions using numerical methods
 60 (Tang and Kou, 1998; Tang et al., 2001; Viggiani and Li, 2013; Zhou et al., 2015).

61
 62 Fig. 1 Sketch of the joint distribution of open-pit mine slope

63 Furthermore, scholars have made valuable achievements in the macroscopic failure
 64 mechanism of rock slopes with intermittent joints. Jennings et al. (1970) developed stability

65 calculation methods and established the failure modes of jointed slopes in open-cast mines
66 through theoretical analysis (Jennings, 1970; Baczynski, 2000; Singh and Sun, 1989). Huang et
67 al. (2015) studied different step-path failure modes of rock slopes with different intermittent
68 joints using the 2D particle flow code (Huang et al., 2015; Wang et al., 2003; Wong and Wu,
69 2014)[24-27]. Camones et al. (2013) developed the numerical manifold method for analyzing
70 the progressive failure of rock slopes (Camones et al., 2013; Yang et al., 2020).

71 Most previous studies have been conducted on the failure of rock slopes with
72 intermittent joints through numerical simulation. However, related stability calculation
73 model studies on the failure of rock slopes with intermittent joints are few, and the previous
74 studies ignored the factors that influence the stability of such slope. Because the base friction
75 test method combined with the speckle technique of the noncontact measurement system, in
76 which frictional forces are applied to the model side, can simulate the slope failure process
77 under gravity and the displacement field development. The method is suitable for analyzing
78 the failure of rock slopes with intermittent joints. In this study, a series of base friction tests
79 were first performed to reveal the failure process and displacement field evolution of rock
80 slopes with intermittent joints. Next, stability calculation models and the theoretical failure
81 criterion of the slopes were established based on the plastic limit analysis theory, and the
82 factors influencing the slope stability were evaluated. This study provides theoretical
83 guidance for the stability evaluation and reinforcement design of jointed rock slopes in
84 open-pit mines.

Fig. 2 Failure of rock slope with intermittent joints of Xiaowan hydroelectric station, China (Huang et al., 2015)

2. Laboratory base friction tests

2.1 Principle of base friction and devices

The base friction test method uses the drag forces acting along the base to represent the volume force (gravity) based on the principle that the distribution of drag forces in the drag direction is similar to that of the actual gravitational field (He et al., 2012). The prefabricated simulation model is laid on the test bench, and the model bottom is in close contact with the belt (Fig. 3). As the rotating roller rotates counterclockwise, friction between the model bottom and the belt is generated by the frame constraint. If the rotation speed is uniform, the test model bottom is subjected to constant frictional stress $F = f\gamma t$, where f is the friction coefficient between the model and moving belt, γ is the unit weight of the model material (N/m^3), and t is the thickness of the model material (m) (Wang et al., 2020). According to Saint-Venant's principle, the frictional stress can simulate gravitational stress with the model if the model is sufficiently thin (Wang et al., 2020; Bray and Goodman, 1981). Hence, the failure of the model caused by the belt rotation can be equivalent to the failure of the

prototype under gravity. However, similar geometric and stress conditions need to be satisfied for the test results to be valid (He et al., 2012; Wang and Zhang, 2014).

Fig. 3 Base friction tester

Base friction tests were performed using a tester at China University of Mining and Technology, Beijing (Fig. 3). The length, width, and height of the tester were 1900, 1400, and 700 mm, respectively, and those of the red model container with a wooden frame were 1000, 920, and 50 mm, respectively. The motor speed was 1.5-12 RPM, and the seamless belt was driven using a rotating roller with a diameter of 300 mm. During the base friction tests, a GoPro HERO 7 Black motion camera with inbuilt Wi-Fi and Bluetooth and determined to be compatible with the GoPro capture app was used to capture and record the images (Fig. 4). Furthermore, an image-correlation-analysis software, Vic-2D, was used to determine the total displacement field of the slope, and the deformation of any monitoring point in the model could be tracked using GIPS software to obtain the history curves of displacement and time of the tracking points.

Fig. 4 Experimental procedure. (a) Base friction tester (b) GoPro HERO 7 Black motion camera
(c) Mobile phone (d) Computer

2.2 Models and materials

Because of the complexity of actual slopes in open-pit mines, the slope simulation models established in this study were simplified to analyze the typical failure modes of rock slopes with intermittent joints under gravity based on the non-contact measurement results of in-situ joints in Qidashan open-pit iron mine (Fig. 5). The designed thickness of the model was 15 mm, which satisfied the condition of "the sufficiently thin model" in Saint-Venant's principle. Based on several test results, six models of slopes with different joint distributions were developed (Fig. 6). Each slope model had a height of 600 mm and a width of 1000 mm. The geometry factors of the models used for the tests were 100 each, and the corresponding prototype had a height and width of 60 and 100 m, respectively. In addition, the dynamic friction coefficient between the model and the belt was 0.7, and the unit weight scaling factor was 1.

Fig. 5 Joint distribution of Qidashan open-pit iron mine

Fig. 6 Model slopes with different joint distributions adopted in test (cm). Slopes 1, 3, 4 and 5 contain a low-dip joint set (inclination angle of 25°), which are parallel but not coplanar, and the bridging angles γ (the inclination angle of the ligament between adjacent tips of joints) increases from 45°, 90°, and 115° to 135°. Slope 2 contains low- and high-dip joints (inclination angles of 25° and 60°, respectively), and the bridging angle γ is 62°. Slope 5 contains three joints with different dips and rock bridges longer than those of other slopes.

Previous studies established the deformation and failure behavior of soft foundation waste dumps through base friction tests and satisfactorily replicated the failure process of waste dumps (Wang et al., 2019; Chen et al., 2018). Therefore, the composition and proportion of similar materials used in this study were determined through several material proportion

tests, referring to materials selected in the previous studies (Table 1). The physicomaterial properties of the similar materials are listed in Table 2.

Table 1 Weight percentage of the model materials (%)

Type	Similar Materials					
	Sand	Gypsum	Lime	Cement	Clay soil	Water
Bedrock	60	14	12	5	--	10
Joints	--	--	--	--	80	20

Table 2 Physicomaterial parameters of the model materials

Type	Density / kg·m ⁻³	Cohesion / kPa	Internal friction angle / °	Elastic modulus / MPa	Poisson's ratio
Bedrock	2500	300	40	20	0.2
Joints	2010	25	10	0.07	0.45

2.4 Results and analysis

2.4.1 Failure process analysis

Figs. 7–10 depict the deformation and failure patterns of the models at different test times. Here, the test time (t) refers to the accumulated time when the belt starts to run. The potential sliding body consisted of several blocks cut from a few tensile cracks originating from around the tips of the joints and propagating to the slope surface (Figs. 7–10). It was found that the tensile cracks were generally perpendicular to the original joints. The detailed analysis of the failure process of each slope model is described in the subsequent paragraphs. Because of space limitations, and the failure processes of Slopes 3 and 4 were similar to that of Slope 5, only Slope 5 is described in this paper.

(1) Slope 1: When the test time was 4.3s, shear coalescence first occurred in the lower-part slope at rock bridge ① and shear cracks were formed at the right tip of Joint 2 and the left tip of Joint 4 (Fig. 7b). In addition, tensile cracks appeared perpendicular to the joints at the right

tips of Joints 1–4 and the left tip of Joint 5 and then gradually propagated along the slope surface direction. However, the tensile cracks did not extend to the slope surface. When the test time increased to 8.2 s, shear coalescence first occurred at rock bridge ② and then at rock bridge ③ and a shear crack appeared at the right tip of Joint 4 (Fig. 7c). Additionally, new tensile cracks were formed on the slope and intertwined in the middle-part slope, which caused several tensile cracks to cut the potential sliding body. As the test time increased further, many tensile cracks extending to the slope surface were formed in the sliding body located on the potential sliding surface composed of intermittent joints and connected rock bridges. This behavior caused the sliding body to disintegrate into several blocks gradually (Fig. 7d).

Because of the large free face of the slope toe, the sliding body at the slope toe tended to collapse under gravity and caused the middle-part slope to slide. Next, the separation of the low-part slope from the main slope decreased the front support force of the middle-part slope. Finally, the middle-part slope slid to the slope toe under the push action of the high-part slope. Thus far, the progressive deformation and failure mode starting from the low-part slope and developing toward the high-part slope was formed.

(2) Slope 2: The failure process of Slope 2 was similar to that of Slope 1 (Fig. 8). When the test time was 4.7 s, shear coalescence initially occurred in the lower-part slope at rock bridge ① and shear cracks appeared at the right tips of Joints 2 and 3 (Fig. 8b). Simultaneously, tensile cracks were formed perpendicular to the joints at the right tips of Joints 1 and 3, and the left tip of Joint 2 gradually propagated along the slope surface direction. However, the tensile cracks did not extend to the slope surface. When the test time was 8.3 s, the tensile

crack formed at the left tip of Joint 2 extended to the slope surface, and the local sliding block below the tensile crack tended to slide (Fig. 8c). As the test time increased further, shear coalescence occurred at other rock bridges, and multiple tensile cracks extending to the slope surface were formed, resulting in the gradual disintegration of the sliding body into several blocks (Fig. 8d). Finally, the slope formed a progressive deformation and failure mode, starting from the low-part slope and developing toward the high-part slope, similar to the failure process of Slope 1.

Fig. 7 Failure process of Slope 1. (a) Initial state (b) Crack coalescence at rock bridge ① (c) Crack coalescence at other rock bridges (d) Final failure state. The red points a–f are the tracking points of the test.

(3) Slope 5: Tensile coalescence failure occurred at the rock bridges of Slope 5 with increasing test time, different from those of Slopes 1 and 2. However, the failure process of Slope 5 was similar to those of Slopes 1 and 2, i.e., the progressive deformation and failure mode started from the low-part slope and spread to the high-part slope (Fig. 9d).

Fig. 8 Failure process of Slope 2. (a) Initial state (b) Crack coalescence at rock bridge ① (c) Crack coalescence at other rock bridges (d) Final failure state. Red points a–f indicate the tracking points of the test.

Fig. 9 Failure process of Slope 5. (a) Initial state (b) Crack coalescence at rock bridge ① (c) Crack coalescence at other rock bridges (d) Final failure state

(4) Slope 6: Numerous tensile cracks were formed at the tips of the joints (Fig. 10), and shear cracks propagated along the joint direction at the right tips of Joints 1 and 2 (Fig. 10b). Finally, tensile–shear failure appeared at the rock bridges of Slope 6 with an increase in the test time, and the progressive deformation and failure mode of Slope 6 were similar to those

207 of other models (Fig. 10d).

208
209 Fig. 10 Failure process of Slope 6. (a) Initial state (b) Crack coalescence at Rock bridge ① (c) Crack coalescence at other
210 rock bridges (d) Final failure state

211 Based on the above findings, several key conclusions could be summarized as follows:

212 (1) The failure of a rock slope with intermittent joints occurs progressively, and the rock
213 bridges are cut one by one under gravity from the bottom upward. Because of the large free
214 face of the slope toe, the sliding body at the slope toe initially tended to collapse under
215 gravity, causing the middle-part slope to slide. Next, the separation of the low-part slope
216 from the main slope reduced the front support force of the middle-part slope. Finally, the
217 middle-part slope slid to the slope toe under the push action of the high-part slope (Fig. 11).
218 Thus far, the progressive deformation and failure mode of the rock slope with intermittent
219 joints showed collapse-lower traction-upper push.

Fig. 11 Sketch of failure process of a rock slope with intermittent joints

Fig. 12 Variation of vertical displacement difference of tracking points of Slopes 1 and 2. v_{ab} , v_{cd} , and v_{ef} represent the vertical misalignments between two points. The points in the colorful shadow area represent the corresponding time when v_{ab} , v_{cd} , and v_{ef} increase sharply.

Fig. 12 shows the variation in the vertical displacement difference between Slopes 1 and 2 tracking points with increasing test time, which indicates the change in the dislocation distance between different slope parts. The vertical displacement difference of the tracking points in the lower-part slope (v_{ab}) first increased sharply under gravity. Next, those of the middle-part slope (v_{cd}) and upper-part slope (v_{ef}) also increased successively. This trend indicated that when a tensile-coalescence crack was formed near the slope toe, the sliding

body at the toe tended to collapse, and the separation of the low-part slope from the main slope could provide sufficient space for the failure of the middle and upper sliding-body parts. Therefore, the reinforcement of potential failure of a rock slope with intermittent joints should be applied to the lower-part slope near the toe.

(2) Based on the failure modes of rock bridges in slopes, the failure of rock slopes with intermittent joints could be divided into three cases.

Case 1: Shear coalescence failure (Type A). In this case, the rock bridge length was relatively short, and the bridging angle was generally lower than 90° . Type A slopes corresponded to Slope 1 (Fig. 6a) and Slope 2 (Fig. 6b) in this study.

Case 2: Tensile coalescence failure (Type B). Here, the rock bridge length was relatively short, and the bridging angle was generally higher than 90° . Type B slopes corresponded to Slopes 3–5 (Fig. 6c).

Case 3: Tensile–shear coalescence failure (Type C). The rock bridge length was relatively long. In this study, Type C slopes corresponded to Slope 6 (Fig. 6d).

2.4.2 Deformation analysis

The speckle technique of the noncontact measurement system was used to investigate the deformation behavior and analyze the total displacement field of the slopes. Fig. 13 shows the contours of the total displacement of the different models.

The displacement of the sliding body above the step-path slip surface was significantly larger than that of the bedrock, and the displacement fields were somewhat partitioned into several blocks by the tensile cracks on the main body. The maximum displacement was

observed near the toe of the low-part slope, the center of the middle-part slope, and the base of the crown. These indicated that the slope toe tended to slide first under gravity, inducing the progressive failure of the upper-sliding body due to the loss of support, which further verified the conclusion stated in Section 2.4.1.

Fig. 13 Contours of total displacement of different model slopes. (a) Slope 1 (b) Slope 2 (c) Slope 5 (d) Slope 6

3. Stability calculation models of rock slopes with intermittent joints

From the above test results, it was observed that the failure of rock slopes with intermittent joints started from the low-part slope and steadily developed towards the high-part slope. The rock bridges in the slopes were successively cut under gravity from the bottom upward, indicating that the mechanical properties of rock bridges and joints and the spatial distribution of joints in the slope significantly influenced the slope stability. Therefore, based on the plastic limit analysis theory, the theoretical mechanical models of three slope types (Types A, B and C) were established in this study, considering the failure characteristics

of rock slopes with intermittent joints under gravity. The two assumptions are as follows.

(1) The intermittent joints of the slope tended toward the free-face direction. And the potential sliding body tended to slide along the direction of the low-dip joints, and it satisfied the small deformation hypothesis, i.e., the potential sliding body is regarded as a rigid-plastic body with small deformation, and the geometric dimension of which is the same as that before deformation.

(2) For Type A slopes (e.g., Slopes 1 and 2), the rock bridges were fractured, owing to shear extension under gravity. For the Type B slopes (e.g., Slope 5), the rock bridges were fractured by tensile cracks. For the Type C slope (e.g., Slope 6), the rock bridges were fractured, owing to tensile–shear extension.

3.1 Stability calculation model of A-type slope

Based on the slope model shown in Fig. 6a, a two-dimensional mechanical model of the Type A slope was established (Fig. 14).

According to the above test analysis, the rock bridges of Type A slopes were fractured, owing to shear extension under gravity. As shown in Fig. 14, H is the slope height, and H_0 is the height of the local surface exposure of the joint from the slope bottom. AB and BC are the joint distribution ranges along the joint trace direction and the direction perpendicular to the joint trace, respectively. a_i is the length of the joint parallel to AB , d_i is the projected length of an intact segment i of the rock bridge to AB , and s_i is the projected length of expanded crack i to AB in the rock bridge. ΔACD is the range of the potential sliding body after simplification. θ is the inclination angle of the slope surface, W is the weight of the overlying

288 potential sliding body, and x-A-y is the local coordinate system.

Fig. 14 Mechanics model of Type A slope

291 From the geometric relationship depicted in Fig. 14, the weight of the overlying potential
292 sliding body (W) can be calculated using Eq. (1).

$$W(h) = \frac{\gamma}{2} \cdot (H - H_0) \cdot L_1 \quad (1)$$

294 The coalescence coefficient of the joints along the joint trace direction (p) can be defined
295 as follows (Jennings, 1970; Huang et al., 2015; Cen et al., 2014).

$$p = \frac{\sum_{i=1}^n a_i}{L_{AB}} \quad (0 \leq p \leq 1) \quad (2)$$

297 where n is the number of the intermittent joints, and L_{AB} is the length of the joint distribution
298 range along the joint trace direction.

299 The coalescence coefficient of rock bridges perpendicular to the joint trace direction (q)
300 can be defined as follows (Jennings, 1970; Huang et al., 2015).

$$q = \frac{\sum_{i=1}^m s_i}{\sum_{i=1}^{n-1} d_i} = \frac{\sum_{i=1}^m s_i}{(1-p) \cdot L_{AB}} \quad (3)$$

302 where m is the number of expanded shear cracks at the tips of the joints, and $n-1$ is the
303 number of rock bridges.

304 Because the sliding body analysis is based on the small deformation assumption, the

sliding body size is always the same. According to the plastic limit analysis theory, assuming that the speed of the sliding body during the limit sliding is v and the direction is as shown in Fig. 14, the internal energy dissipation of the sliding body during failure (P_D) could be calculated as follows (Donald and Chen, 1991; Mccook, 1975).

$$P_D = P_{D1} + P_{D2} + P_{D3} = c_1 \cdot p \cdot L_{AB} \cdot v \cdot \cos \varphi_1 + c_2 \cdot q \cdot (1-p) \cdot L_{AB} \cdot v \cdot \cos \varphi_1 + c_3 \cdot (1-q) \cdot (1-p) \cdot L_{AB} \cdot v \cdot \cos \varphi_1 \quad (4)$$

where P_{D1} , P_{D2} , and P_{D3} are the internal energy dissipation values of the intermittent joints, expanded shear cracks at the joint tips, and intact rock segments at the rock bridges during the failure of Type A slope, respectively. c_1 , c_2 and c_3 are the cohesion values of the intermittent joint, expanded shear crack at the joint tip, and intact rock at the rock bridge, respectively, and φ_1 is the internal friction angle of the joints.

The power generated by gravity (P_G) is the product of the weight of the sliding body and the vertical component of the velocity, and it could be calculated as follows.

$$P_G = \frac{\gamma}{2} \cdot (H - H_0) \cdot L_1 \cdot v \cdot \sin(\alpha - \varphi_1) \quad (5)$$

When the entire sliding body of Type A slope tends to slide, the energy stability coefficient (E_{slz}) is defined as the ratio of the internal energy dissipation to the gravity power.

The energy stability coefficient (E_{slz}) could be obtained using Eq. (6).

$$E_{slz} = \frac{2 \cdot [c_1 \cdot p + c_2 \cdot q \cdot (1-p) + c_3 \cdot (1-q) \cdot (1-p)] \cdot L_{AB} \cdot \cos \varphi_1}{\gamma \cdot (H - H_0) \cdot L_1 \cdot \sin(\alpha - \varphi_1)} \quad (6)$$

The higher the value of E_{sl} , the better the slope stability. When $E_{sl} > 1$, the slope is stable.

The slope is in a critical state when $E_{sl} = 1$ and tends to be unstable when $E_{sl} < 1$.

3.2 Stability calculation model of Type B slope

Based on the slope model depicted in Fig. 6c, a two-dimensional mechanical model of Type B slope was established (Fig. 15).

Fig. 15 Mechanics model of Type B slope

As shown in Fig. 15, the coalescence coefficient of rock bridges perpendicular to the joint trace direction (q') can be obtained as follows (Jennings, 1970; Huang et al., 2015).

$$q' = \frac{\sum_{i=1}^m u_i}{\sum_{i=1}^m d_i} = \frac{\sum_{i=1}^m u_i}{L_{BC}} \quad (7)$$

According to the plastic limit analysis theory, the internal energy dissipation of the sliding body during the failure of Type B slope (P_D) could be calculated as follows (Donald and Chen, 1991; Mccook, 1975).

$$P_D = P_{D1} + P_{D2} = c_1 \cdot L_{AB} \cdot v \cdot \cos \phi_1 + (1 - q') \cdot L_{BC} \cdot \sigma_{t,BC} \cdot v \cdot \cos \phi_1 \quad (8)$$

where P_{D1} and P_{D2} are the internal energy dissipation coefficients of the intermittent joints and intact rock segments at the rock bridges during the failure of Type B slope, respectively, and $\sigma_{t,BC}$ is the tensile strength of the intact rock. The definitions of other parameters in Eq. (8) are the same as those in Eq. (4).

Therefore, when the sliding body of Type B slope tended to slide as a whole, the energy stability coefficient (E_{s2z}) could be expressed as follows.

$$E_{s2z} = \frac{2 \cdot [c_1 \cdot L_{AB} + (1 - q') \cdot L_{BC} \cdot \sigma_{t,BC}] \cdot \cos \phi_1}{\gamma \cdot (H - H_0) \cdot L_2 \cdot \sin(\alpha - \phi_1)} \quad (9)$$

3.3 Stability calculation model of Type C slope

$$P_D = P_{D1} + P_{D2} = p \cdot c_1 \cdot L_{AB} \cdot v \cdot \cos \varphi_1 + q \cdot (1-p) \cdot c_2 \cdot L_{AB} \cdot v \cdot \cos \varphi + (1-p) \cdot (1-q) \cdot c_3 \cdot L_{AB} \cdot v \cdot \cos \varphi_1 + (1-q') \cdot (1-k') \cdot L_{BC} \cdot \sigma_{t,BC} \cdot v \cdot \cos \varphi_1 \quad (12)$$

The energy stability coefficient (E_{s3z}) of Type C slope can be obtained by Eq. (13).

$$E_{s3z} = \frac{2 \cdot \{ [p \cdot c_1 + q \cdot (1-p) \cdot c_2 + (1-p) \cdot (1-q) \cdot c_3] \cdot L_{AB} + (1-q') \cdot (1-p') \cdot L_{BC} \cdot \sigma_{t,BC} \} \cdot \cos \varphi_1}{\gamma \cdot (H - H_0) \cdot L_1 \cdot \sin(\alpha - \varphi_1)} \quad (13)$$

4. Discussion

4.1 Effects of joint inclination angle and rock cohesion on slope stability

The effects of the joint inclination angle (α) and rock cohesion (c_3) on the stability of Types A, B, and C slopes were analyzed, and the following physical and mechanical parameters were assumed: $H = 24m$, $H_0 = 2m$, $\varphi_1 = 20^\circ$, $c_1 = 0.1\text{MPa}$, $\varphi_3 = 42^\circ$, $q = q' = 0$ and $p = p' = 0.8$. The relevant parameters above were substituted into Eqs. (6), (9) and (13). Variation curves of the energy stability coefficients (E_{sz}) of the three slope types with different rock cohesions and joint inclination angles were obtained (Figs. 17–19).

Fig. 17 Variations of E_{sz} with α and c_3 . (a) Type A (b) Type B (c) Type C

The energy stability coefficients (E_{sz}) of the three slope types increased with increasing c_3 and decreasing α (Fig. 17), indicating that c_3 and α significantly influenced the stability of rock slopes with intermittent joints. Moreover, the possibility of slope failure increased with decreasing c_3 and increasing α . When c_3 was constant and α increased from 40° to 70° , the rate of decrease in E_{sz} of Type C slope was the highest, followed by those of Types A and B

slopes, i.e., $\Delta E_{s3z} > \Delta E_{s1z} > \Delta E_{s2z}$ (Fig. 18). This trend signified that the stability of the Type C slope was the most sensitive to the variation in α , followed by that of Type A, and Type B stability was the least sensitive. Furthermore, the energy stability coefficients (E_{sz}) of Types A and C slopes sharply increased with c_3 when α was constant (Lines I, II, V, and VI) (Fig. 19). However, E_{sz} values of the Type B slope were unchanged (Lines III and IV), suggesting that the stabilities of Types A and C slopes were significantly influenced by c_3 , whereas that of the Type B slope were unaffected by c_3 . The major reason was that the rock bridges in the Type B slope were in a tensile state during failure, and the coalescence mainly depended on the tensile strength of the intact rock ($\sigma_{t,BC}$).

Fig. 18 Variation of ΔE_{siz} with α ($c_3 = 3 \text{ MPa}$).

Fig. 19 Variation of E_{sz} with c_3 ($\alpha = 45^\circ, 50^\circ$).

4.2 Effects of joint cohesion and friction angle on slope stability

The following physical and mechanical parameters were assumed: $H = 24m$,
 $H_0 = 2m$, $\beta = 45^\circ$, $\alpha = 30^\circ$, $c_3 = 3\text{MPa}$, $q = q' = 0$ and $p = p' = 0.8$. These parameters were
substituted into Eqs. (6), (9) and (13), and the variation curves of the energy stability
coefficients (E_{sz}) of the three slope types with the joint cohesion (c_1) and friction angle (φ_1)
were plotted (Figs. 20–22).

The energy stability coefficients (E_{sz}) of the three slope types increased with c_1 and φ_1
(Fig. 20), showing that c_1 and φ_1 significantly influenced the stability of rock slopes with
intermittent joints, and the probability of slope failure increased with decreasing c_1 and φ_1 .
When c_1 was constant and φ_1 increased from 10° to 28° , the increasing amplitude of E_{sz} of the
Type C slope was the largest and significantly higher than those of Types A and B, i.e.,
 $\Delta E_{s3z} > \Delta E_{s1z} > \Delta E_{s2z}$. This trend suggested that the sensitivity of Type C slope stability to the
variation in φ_1 was significantly higher than those Types A and B slopes, and the stability of
the Type B slope was the least sensitive to the change in φ_1 . Furthermore, the increase rate of
 E_{sz} was more rapid when $\varphi_1 > 24^\circ$ than when $\varphi_1 < 24^\circ$ (Fig. 21). This difference indicated that
 E_{sz} was more sensitive to the variation in φ_1 when $\varphi_1 > 24^\circ$. When c_1 increased from 0.1 to 1.0
MPa, the increase in the amplitude of E_{sz} of the Type B slope was the highest, followed by
those of Types A and C slopes, i.e., $\Delta E_{s2z} > \Delta E_{s1z} \approx \Delta E_{s3z}$. This behavior signified that the
stability of the Type B slope was more significantly influenced by the change in φ_1 than with
those of Types A and C slopes.

Fig. 20 Variation of E_{sz} with c_1 and φ_1 . (a) Type A (a) Type B (a) Type C

4.3 Effect of coalescence coefficient of rock bridges on slope stability

The calculated physical and mechanical parameters were assumed as follows: $H = 24m$, $H_0 = 2m$, $\beta = 45^\circ$, $\alpha = 30^\circ$, $c_1 = 0.2 \text{ MPa}$, $c_2 = 0.5 \text{ MPa}$, $c_3 = 6 \text{ MPa}$ and $p = p' = 0.8$. The parameters above were substituted into Eqs. (6), (9), and (13), respectively, and variation curves of the energy stability coefficients (E_{sz}) of the three slope types with the coalescence coefficient of

rock bridges (q and q') were plotted (Fig. 23–25).

Fig. 23 Variation of E_{sz} with q and q' . (a) Type A (a) Type B (a) Type C

477

Fig. 24 Variation of E_{sz} with q/q' ($q=0.5$ or $q'=0.5$).

The energy stability coefficients (E_{sz}) of the three slope types decreased with the increase in the coalescence coefficients of rock bridges along the joint trace direction (q) and perpendicular to the joint trace direction (q') (Fig. 23). These variations indicated that q and q' significantly influenced the stability of rock slopes with intermittent joints, and the probability of slope failure increased with increasing q and q' . In addition, E_{sz1} of the Type A slope decreased with an increase in q , but it remained constant with an increase in q' (Fig. 19a and 19b). On the contrary, E_{sz2} of the Type B slope decreased with increasing q' but remained unchanged with increasing q . The main reason was that the rock bridges of the Type A slope were in a shear state during failure, and the coalescence mainly depended on the shear

488 strength of the intact rock; however, Type B rock bridges were in a tensile state, and the
 489 coalescence mainly depended on the intact-rock tensile strength. Furthermore, E_{sz3} of the Type
 490 C slope was related to both q and q' .

Fig. 25 Variation in E_{sz3} of C-type slope with respect to q/q'

492 Fig. 24 shows that Lines I and III change more significantly with an increase in q/q' than
 493 Lines II and IV, and Line IV shows the most gentle variation. These trends indicate that the
 494 stability sensitivities of Types A and C slopes to the change in q are similar and higher than
 495 that of the Type B slope to q' . The stability of the Type C slope is the least sensitive to
 496 variation in q' . As can be seen from Fig. 25, Lines I, II, and III change more sharply than Lines
 497 IV, V, and VI. These trends suggest that the stability of the Type C slope was more
 498 significantly influenced by q than q' .

500 4.4 Failure criterion of slope

501 The Type A slope is used as an example. The difference function between the internal
 502 energy dissipation and gravity power was derived as follows.

$$f(H) = P_D - P_G = [c_1 \cdot p + c_2 \cdot q \cdot (1-p) + c_3 \cdot (1-q) \cdot (1-p)] \cdot L_{AB} \cdot v \cdot \cos \varphi_1 - \frac{\gamma}{2} \cdot (H - H_0) \cdot L_1 \cdot v \cdot \sin(\alpha - \varphi_1) \quad (14)$$

504 Furthermore, from Eq. (14), the $f(H)$ equation can be re-expressed as follows.

$$f(H) = M_1 \cdot H + N_1 - M_1 \cdot H \quad (15)$$

$$M_1 = \frac{c_3 \cdot \cos(\beta - \alpha) \cdot v \cdot \cos \varphi_1}{\sin \beta} - \frac{\gamma \cdot L_1 \cdot v \cdot \sin(\alpha - \varphi_1)}{2} \quad (16)$$

$$N_1 = [c_1 \cdot \sum_{i=1}^n a_i + c_2 \cdot \sum_{i=1}^m s_i - c_3 \cdot (\sum_{i=1}^n a_i + \sum_{i=1}^m s_i)] \cdot v \cdot \cos \varphi_1 \quad (17)$$

It can be observed from Eq. (15) that $f(H)$ is a linear function of H . The relevant parameters in Eqs. (16) and (17) are constant at a specific time during the creeping of the sliding body of each slope. Based on the quantitative relationship between M_1 and N_1 , the curve pattern of the function can be divided into the following three types (Fig. 26).

(1) Type I: Stable slope

When $M_1 > 0$, and $T_1 = N_1 - M_1 \cdot H_0 > 0$, the curve pattern of $f(H)$ exists as Curve I of Fig. 26. In this case, regardless of the value of H , the slope is always stable. This behavior signifies that the potential sliding body at the lower part of any through-tensile crack is stable under gravity.

Fig. 26 Curve patterns of $f(H)$ and stability zoning sketch of potential sliding body of local stable slope. T_I , T_{II} , and T_{III} are the intercepts of Curves 1, 2, and 3, respectively, H_1 is the height of the potential sliding body, and H_{cr} is the critical instability height of the slope.

(2) Type II: Local-stable slope

When $M_1 < 0$, and $T_1 = N_1 - M_1 \cdot H_0 > 0$, the curve pattern of $f(H)$ assumes Curve II of Fig.

26. H_{cr} is defined as the critical point of slope instability, i.e., the critical instability height (CIH). When $0 < H < H_{cr}$, the slope is stable, indicating that when the through-tensile cracks are within the height range of zero to H_{cr} of the slope, local sliding failure will not start from these cracks. The main reason is that the sliding force of the potential sliding body at the lower part of these cracks is lower than the antisliding force generated by the rock bridges and joints, which does not satisfy the necessary and sufficient conditions for instability. When $H = H_{cr}$, the slope exists in a critical state. However, when $H_{cr} < H < H_1$, the slope is unstable while the first through-tensile crack in the zone of the potential sliding body higher than the CIH appears. This behavior occurs because the sliding force of the potential sliding body at the lower part of these cracks is higher than the antisliding force provided by the rock bridges and joints, which satisfies the necessary and sufficient conditions for instability. The upper-part sliding body then tends to slide, owing to the loss of the front support force. Therefore, the penetration of the first tensile crack (the initial failure position) in the zone of the potential sliding body higher than the CIH can be regarded as the failure criterion of the local-stable slope. In addition, with increasing H_{cr} , the unstable zone (red) area in Fig. 26 decreases, while the stable zone (green) area increases. These trends show that the probability of slope instability decreases with the CIH.

Let $f(H) = 0$. Hence, the CIH of the local-stable Type A slope (H_{cr1}) can be determined as follows.

$$H_{cr1} = \frac{M_1 \cdot H_0 - N_1}{M_1} \quad (18)$$

The relevant geometric and mechanical parameters should satisfy the following conditions.

$$\begin{cases} 2c_3 \cdot \cos(\beta - \alpha) \cdot v \cdot \cos \varphi_1 < \gamma \cdot L_1 \cdot \sin(\alpha - \varphi_1) \cdot \sin \beta \\ \frac{[c_1 \cdot \sum_{i=1}^n a_i + c_2 \cdot \sum_{i=1}^m s_i - c_3 \cdot (\sum_{i=1}^n a_i + \sum_{i=1}^m s_i)] \cdot v \cdot \cos \varphi_1}{H_0} > \frac{c_3 \cdot \cos(\beta - \alpha) \cdot v \cdot \cos \varphi_1}{\sin \beta} - \frac{\gamma \cdot L_1 \cdot v \cdot \sin(\alpha - \varphi_1)}{2} \end{cases} \quad (19)$$

where M_1 and N_1 can be calculated using Eqs. (16) and (17), respectively.

(3) Type III: Unstable slope

When $M_1 < 0$, and $T_1 = N_1 - M_1 \cdot H_0 < 0$, the $f(H)$ curve follows the Curve III pattern shown in Fig. 26. In this case, the slope is always unstable irrespective of the H value. This suggests that the potential sliding body at the lower part of any through-tensile crack is unstable under gravity, and the separation of the low-part slope from the main slope decreases the front support force of the middle-part slope. The middle-part slope tends to slide toward the slope toe under the push action of the high-part slope.

Similar to the Type A slope, Types B and C slopes can also be divided into three types based on the quantitative relationship between the correlation coefficients, i.e., stable slope (Type II), local-stable slope (Type I), and unstable slope (Type III). Because of space limitations, the analysis will not be explained in detail in this paper. The CIH of the Type B slope (H_{cr2}) can be calculated as follows.

$$\begin{cases} H_{cr2} = \frac{M_2 \cdot H_0 - N_2}{M_2} \\ M_2 = [2 \cdot c_1 \cdot \cos(\beta - \alpha) + \frac{2 \cdot \sin(\beta - \alpha) \cdot \sigma_{t,BC}}{\sin \beta}] \cdot v \cdot \cos \varphi_1 - \gamma \cdot L_2 \cdot v \cdot \sin(\alpha - \varphi_1) \\ N_2 = -2 \cdot v \cdot \cos \varphi_1 \cdot \sigma_{t,BC} \cdot \sum_{i=1}^m u_i \end{cases} \quad (20)$$

The relevant geometric and mechanical parameters of the Type B slope should meet the following conditions.

$$\begin{cases} 2 \cdot c_1 \cdot \cos(\beta - \alpha) + \frac{2 \cdot \sin(\beta - \alpha) \cdot \sigma_{t,BC}}{\sin \beta} > \frac{\gamma \cdot L_2 \cdot \sin(\alpha - \varphi_1)}{\cos \varphi_1} \\ \frac{-2 \cdot \sigma_{t,BC} \cdot \cos \varphi_1 \cdot \sum_{i=1}^m u_i}{H_0} < [2 \cdot c_1 \cdot \cos(\beta - \alpha) + \frac{2 \cdot \sin(\beta - \alpha) \cdot \sigma_{t,BC}}{\sin \beta}] \cdot \cos \varphi_1 - \gamma \cdot L_2 \cdot \sin(\alpha - \varphi_1) \end{cases} \quad (21)$$

Similarly, the CIH of the Type C slope (H_{cr3}) can be calculated using the following equation.

$$\begin{aligned} & \left(\sum_{i=1}^m a_i + \sum_{j=1}^n b_{lj} \right) \cdot c_1 + (c_2 + c_3) \cdot \sum c_i - \frac{c_3 \cdot \sin \beta \cdot (\sum c_i)^2}{(H_{cr3} - H_0) \cdot \cos(\beta - \alpha) - (\sum_{i=1}^m a_i + \sum_{j=1}^n b_{lj}) \cdot \sin \beta} \\ & + \left(\sigma_{t,BC} - \frac{\sigma_{t,BC} \cdot \sin \beta \cdot \sum l_i}{(H_{cr3} - H_0) \cdot \sin(\beta - \alpha) - \sum_{i=1}^{m+n} b_{li} \cdot \sin \beta} \right) = \frac{1}{2} \cdot \gamma \cdot (H_{cr3} - H_0) \cdot L_1 \cdot \sin(\alpha - \varphi_1) \end{aligned} \quad (22)$$

In summary, the stability discrimination process of rock slopes with intermittent joints can be depicted as follows (Fig. 27).

Fig. 27 Stability discrimination process of rock slopes with intermittent joints. Step 1: Classify the slope according to the joint distribution (Types A, B, and C); Step 2: Further classify the slope according to the quantitative relationship of the correlation coefficient of the difference function (Types I, II, and III); Step 3: Evaluate the stability of the slope.

5. Conclusions

In this study, the failure behavior of rock slopes with intermittent joints under the influence of gravity was investigated through base friction tests and plastic limit theory

analysis. The failure criterion was proposed. The four main conclusions of this study are as follows.

(1) The deformation and failure of rock slopes with intermittent joints showed collapse-lower traction-upper push, and they started from the lower-part slope and developed toward the higher-part slope. The slopes exhibited sliding failure characteristics along the direction of the low-dip joints. The displacement fields were somewhat partitioned into several blocks by the tensile cracks on the main body, and the maximum displacement occurred at the low-part slope near the toe, the center of the middle-part slope, and the base of the crown. The reinforcement of potential failure in a rock slope with intermittent joints is suggested to be applied to the lower-part slope near the toe.

(2) Based on the failure modes of rock bridges in slopes, the failure of rock slopes with intermittent joints could be divided into three cases: Case 1, known as shear coalescence failure (Type A), in which the rock bridge length was relatively short, and the bridging angle was typically less than 90° ; Case 2, called tensile coalescence failure (Type B), in which the rock bridge length was relatively short, and the bridging angle generally exceeded 90° ; Case 3, known as tensile–shear coalescence failure (Type C), in which the rock bridge length was relatively long.

(3) Among the three rock slope types, the stability of the Type A slope was significantly influenced by the coalescence coefficient of rock bridges along the joint trace direction and rock cohesion. The stability of the Type B slope was significantly influenced by the coalescence coefficient of rock bridges perpendicular to the joint trace direction and joint cohesion but was unaffected by rock cohesion, which was different from the other slopes. The

stability of the Type C slope was significantly influenced by the joint inclination angle, joint friction angle, and coalescence coefficient of rock bridges along the joint trace direction.

(4) When $M_1 > 0$, and $T_1 = N_1 - M_1 \cdot H_0 > 0$, the slope was always stable, regardless of the value of H . When $M_1 < 0$, $T_1 = N_1 - M_1 \cdot H_0 > 0$, and $0 < H < H_{cr}$, the slope was stable, and when $H = H_{cr}$, the slope existed in a critical state. However, when $H_{cr} < H < H_1$, the slope was unstable while the first through-tensile crack in the zone of the potential sliding body higher than the CIH appeared. When $M_1 < 0$, and $T_1 = N_1 - M_1 \cdot H_0 < 0$, the slope was unstable while the first through-tensile crack in the slope appeared, irrespective of the H value.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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