A Novel Methodology for Simultaneous Minimization of Manufacturing Objectives in Tolerance Allocation of Complex Assembly

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Abstract

A novel methodology is presented in this work to minimize the manufacturing objectives, namely tolerance cost and machining time for allocating optimum tolerance to the components of complex assemblies with considering the proper process and machine selections from the given alternatives. A methodology that provides a two-step solution is developed for this work. First, a heuristic approach is applied to determine the best machine for each process, and then a combined Whale optimization algorithm with a univariate search method is used to allocate optimum tolerances with the best process selection for each sub-stage/operation. The efficiency of the proposed novel methodology is validated by solving two typical tolerance allocation problems of complex assemblies: a wheel mounting assembly and a knuckle joint assembly. Compared with previous approaches, the proposed methodology showed a considerable reduction in tolerance cost and machining time in relatively less computation time.

Keywords: Tolerance allocation; Machine and process selection; Heuristic approach; Univariate search method; Whale optimization algorithm.

1. Introduction

All aspects of manufacturing like machine investment cost, manufacturing cost, the functionality of the product, quality of manufacturing, and the reliability of the product directly connect with tolerance allocation. Hence, most of the researchers are still focusing on this research topic to improve manufacturing efficiency. It involves the allocation of critical dimensions of an assembly, known from the product's functional requirements. As per the literature, more equations are available based on the combinations of component's tolerance values; however, few equations provide better results. Tolerance allocation requires discovering the best possible combination of component's tolerances by considering the manufacturing objective(s) and the associated constraints. The researchers proposed different tolerance allocation strategies in different period. The summary of those strategies is presented in Table 1. Details of the terminology included in Table 1 are described in the following paragraphs.

MHM: In this method, the tolerance accumulation model, namely the root sum square (RSS) method [Vasseur (1997), Cheng and Tsai (2011), Cheng and Tsai (2013), Feng and Kusiak (1997), Chase (1999), Jayaprakash et al. (2012)], was used to calculate the assembly tolerance value by summing the randomly picked discrete tolerance values. Initially, the discrete tolerance values were identified by splitting the process tolerance limits. Most of the researchers had applied the MHM such as simulated annealing algorithm [Zhang and Wang (1993)] and genetic algorithm [Ji et al. (2000), Uttam (2001), Singh et al. (2003), Prabhakaran et al. (2004), Singh et al. (2004a), Etienne et al. (2008), Wua et al. (2009), Sivakumar et al. (2009), Sivakumar et al. (2011), Sivasubramanian and Sivakumar (2010)] for this purpose.

Table 1: Summary of literature survey

Author's Name	Methodology/Tools used				Type of cost function		Example product			Objective function			APS	AMS	Tolerance allocation model	
	MHM	HM	LMM	OM	DCF	CCF	SA	CA	NA	MC	QC	MT			WC	RSS
Zhang and Wang (1993)	√					√		√		√					√	
Zhang and Wang (1993a)	√					√	√			√			√		√	
Vasseur <i>et al.</i> (1997)	√					√	√			√	√					√
Zhang and Wang (1997)	√					√		√		√					√	
Ji <i>et al.</i> (2000)	√					√	√			√	√				√	
Carfagni <i>et al.</i> (2001)	√					√	√			√					√	
Uttam (2001)	√					√	√			√			√	√	√	
Ming and Mak (2001)	√			√		√	√			√			√		√	
Singh <i>et al.</i> (2003)	√					√		√		√			√		√	
Singh <i>et al.</i> (2004)	√					√		√	√	√					√	
Prabhakaran <i>et al.</i> (2004)	√					√	√		√	√					√	
Singh <i>et al.</i> (2004a)	√					√		√	√	√			√		√	
Singh <i>et al.</i> (2005)	√					√		√		√			√		√	
Prabhakaran <i>et al.</i> (2005)	√					√	√		√	√					√	
Etienne <i>et al.</i> (2008)	√					√	√			√			√		√	
Wua <i>et al.</i> (2009)	√			√		√			√	√					√	
Sivakumar <i>et al.</i> (2009)	√					√	√			√			√		√	
Sivakumar <i>et al.</i> (2011)	√					√	√			√			√		√	
Sivasubramanian and Sivakumar (2010)	√					√		√		√		√	√	√	√	
Sampath Kumar <i>et al.</i> (2010)	√					√	√			√	√				√	

Rao (2011)	√				√	√		√	√			√
Sivakumar et al. (2012)	√				√	√	√	√				√
Geetha et al. (2013)	√				√		√	√	√	√	√	√
Kim <i>et al.</i> (1999)		√			√	√		√		√		√
Gadallah (2011)		√			√	√		√				√
Chase <i>et al.</i> (1990)			√	√	√	√		√				√
Chen (1996)			√		√	√		√				√
Christopher <i>et al.</i> (2003)			√		√		√	√				√
Cheng and Tsai (2011)			√		√		√	√				√
Cheng and Tsai (2013)			√		√	√		√				√
Liu and Jin (2013)			√		√	√		√	√			√
Ostwald <i>et al.</i> (1997)				√		√		√				√
Loosli (1987)			√		√	√		√		√		√
Wu <i>et al.</i> (1988)			√		√	√		√	√			√
Lee and Woo (1989)			√		√	√	√	√				√
Dong <i>et al.</i> (1990)			√		√		√	√				√
Nagarwala et al. (1995)			√		√	√		√		√		√
Feng and Kusiak (1997)			√	√		√		√	√			√
Chase (1999)			√	√		√		√				√
Feng and Kusiak (2000)			√		√	√		√		√		√
Diplaris and Sfantsikopoulos (2001)			√		√	√		√				√
Choi <i>et al.</i> (2000)			√		√		√	√	√			√
Chou and Chang (2001)			√		√	√		√	√			√
Jeang and Chang (2002)			√		√		√	√				√
Ye and Salustri (2003)			√		√	√		√	√			√

Rao and Wu (2005)	√	√	√			√
Savage <i>et al.</i> (2005)	√	√	√	√		√
Yuan and Shiau (2006)	√	√	√	√		√
Gopalakrishnan et al. (2007)	√	√	√	√	√	√
Li et al. (2008)	√	√	√	√		√
Janakiraman and Saravanan (2010)	√	√	√	√		√
Jayaprakash et al. (2012)	√	√	√	√		√
Hung and Chan (2013)	√	√	√	√		√
Barari (2013)	√	√	√	√		√

MHM – Meta-Heuristic Method; HM – Heuristic Method; LMM – Lagrange’s Multiplier Method; OM – Other Methods; DCF – Discrete Cost Function; CCF – Continuous Cost Function; SA – Simple / Linear Assembly; CA – Complex Assembly; NA – Non-linear Assembly; MC – Manufacturing Cost/Tolerance Cost; QC – Quality Cost; MT – Machining Time; APS – Alternative Process Selection; AMS – Alternative Machine Selection; WC – Worst-Case; RSS – Root Sum Square method;

HM: The application of this method is seldom found in literature, since the usage of thumb rules, previous experiences and standards [Kim et al. (1999), Gadallah (2011)] in solving the tolerance allocation problems. Nevertheless, the alternative methods, namely the Branch and Bound algorithm [Lee and Woo (1989)] and Design of Experiments [Feng and Kusiak (2000)], were used to identify the effective tolerance allocation models. Further, a new method was developed by integrating HM with Tabu Search for optimal tolerance allocation and subsequent manufacturing cost reduction [Kumar et al. (2009)].

DCF & CCF Models: The researchers have used different cost function models [Singh et al. (2004), Chase et al. (1990), Speckhart (1972), Sutherland and Roth (1975), Spotts (1978), Chase and Greenwood (1988)] in the various period to evaluate the manufacturing cost. These models were classified as DCF & CCF based on their nature. However, the researchers mainly used the CCF models, since it yields closed-form solutions to the tolerance allocation problems. On the other side, the DCF models [Ostwald and Huang (1977), Chase et al. (1990), Feng and Kusaik (1997), Chase (1999)] were not preferred due to the model fitting errors during the manual formulation. Further, several studies have been carried out by considering the objective function as the sum of quality loss (as per Taguchi quality loss concept) and manufacturing cost [Vasseur (1997), Ji et al. (2000), Sampath Kumar et al. (2010), Rao (2011), Geetha et al. (2013), Liu and Jin (2013), Hallmann et al. (2020), Wu et al. (1988), Feng and Kusaik (1997), Choi et al. (2000), Chou and Chang (2001), Ye and Salustri (2003), Gopalakrishnan (2007)].

Simple, Complex and Non-Linear assembly: The researchers used different tolerance allocation methods to obtain a solution concerning product type. For instance, the LMM was used for only simple products [Zhang and Wang (1993a), Vasseur et al. (1997), Ji et al. (2000), Ming and Mak (2001)], which has only two mating components. Several works have been reported on finding the optimum allocated tolerance values for the components of complex assembly [Li et al. (2008), Hung and Chan (2013), Barari (2013), Rao and More (2014), Ramesh Kumar et al. (2016), Natarajan et al. (2018), Vignesh Kumar et al. (2019)]. A limited number of authors have concentrated on non-linear assembly products that consist of more than two components [Lee and Woo (1989), Chase (1999), Choi et al. (2000)].

APS: In practice, it is possible to produce components using more than one alternative process. It is necessary to select the proper process for the correct component to reduce the manufacturing cost. Some authors have considered alternative process selection (i.e., every combination of the process has a feasible tolerance range, and for a given process combination, the cost of machining is the function of the tolerance value) for optimum tolerance allocation of both simple and complex assemblies [Feng and Kusaik (2000), Singh et al. (2003), Etienne et al. (2008), Sivakumar et al. (2009)]. Kumar et al. (2009a) dealt with the distribution of tolerance on the component dimension of a complex assembly with alternative process selection. Authors have attempted to reduce the manufacturing cost of a product using the Lagrange multiplier method for complex assemblies with the bottom curve follower method.

AMS: It is possible to reduce the manufacturing cost of a product by choosing the suitable machine for the correct process. Several researchers have considered machine selection as one of the criteria for minimizing manufacturing costs in recent years. However, very few authors [Uttam (2001), Sivasubramanian, Siva Kumar (2010), Geetha et al. (2013)] have discussed alternative machine selection for optimum tolerance allocation, and even fewer studies [Geetha et al. (2013)] on minimizing both cost and machining time with process and machine selection for optimum allocation of tolerance have been reported in the literature.

From this literature review, it appears that no significant effort has been made to consider machine time as an objective in optimum tolerance allocation, even though machine time is a crucial manufacturing parameter. Therefore, in the present study, both tolerance cost and

machining time are optimized. Realizing the complexity of the problem, a combined heuristic and univariate search method is introduced to select the best machine for each process and the optimum tolerance for each component using alternative process and machine selection, with available machine time as a constraint.

2. Problem Definition

The biggest challenge facing today's manufacturing companies is to reduce production costs while maintaining better quality and higher productivity. Tolerance allocation plays a vital role in achieving those goals. The production of a component involves selecting processes as well as selecting machines. These decisions directly influence the allocated tolerance values of components. The optimum allocated tolerance values govern the manufacturing costs and machining time of the product. Therefore, an operation may be possible with multiple alternatives, and as such, is treated as a non-polynomial hard problem.

3. Mathematical Formulation

The proposed work aims to simultaneously minimize tolerance cost and machining time, represented in Equation (1). Tolerance cost (TC_i) and machining time (MT_{ik}) are calculated using Equations (2) and (3). Equations (4) and (5) respectively determine the critical dimension of the sub-assembly (Y) and its tolerance (t_Y). Machine engagement time (met) is estimated using Equation (6). The constraints considered in this work are expressed by Equations (7)–(9). The allocated tolerance (t_i) should be within the process limits, represented by Equation (7). The calculated sub-assembly tolerance within the given sub-assembly tolerance; this constraint is given by Equation (8). The total individual machine engagement time to manufacture the product should be less than the given available machine time (amt) represented in Equation (9). In addition, it is assumed that the following data are known well in advance before manufacturing the product:

- Number of sub-assemblies and their dimensional chain details
- Allowable tolerance for each sub-assembly (at_Y)
- Number of parts in the assembly (nc)
- Number of sub-stages required to make the parts (no)
- Number of possible processes for each sub-stage of the components (np)
- Number of possible machines for each process (nm)
- Number of hours available for each machine (amt)
- Possible process—machine combination details ($opno$)
- Mathematical model to compute tolerance cost and machining time
- Constants to compute tolerance cost (A and B) and machining time (X1 and Y1)
- Process tolerance limits (t_{min} and t_{max})

$$Z = \min(TC, MT) \quad (1)$$

$$TC_{ijk} = \eta_{jk} \left(A_j + \frac{B_j}{t_{ijk}} \right) \quad (2)$$

$$MT_{ijk} = \eta_{jk} \left(X1_j + \frac{Y1_j}{t_{ijk}} \right) \quad (3)$$

$$Y = \sum_{l=1}^{nc} \pm d_l \quad (4)$$

$$t_Y = \sum_{i=1}^{no} t_{ijk} \quad (5)$$

$$met_k = \sum_{i=1}^{no} MT_{ijk} \quad (6)$$

$$t_{\min} \leq t_i \leq t_{\max} \quad (7)$$

$$t_{aY} \geq t_Y \quad (8)$$

$$amt_k \geq met_k \quad (9)$$

Z	Objective function
TC_{ijk}	Tolerance cost of i^{th} sub-stage using j^{th} process on k^{th} machine
MT_{ijk}	Machining time of i^{th} sub-stage using j^{th} process on k^{th} machine
t_{ijk}	Allocated tolerance of i^{th} sub-stage in j^{th} process and k^{th} machine
A_j, B_j	Tolerance cost function constants for j^{th} process
XL_j, YL_j	Tolerance machining time function constants for j^{th} process
η_{jk}	Efficiency factor using j^{th} process on k^{th} machine
d_l	Dimension of l^{th} component
Y	Critical dimension of sub-assembly
t_y	Calculated tolerance of critical dimension
t_{ay}	Specified tolerance of critical dimension
i	Sub-stage number index
j	Process number index
k	Machine number index
l	Component number index
met_k	k^{th} machine engagement time
amt_k	Available time of k^{th} machine

4. Methodology

The proposed method consists of two stages: (i) selection of the best machine for each process by applying a heuristic approach, (ii) selection of the best process and optimum allocated tolerance for each component using combined whale optimization algorithm and univariate search method. In the first stage, the process tolerance is divided into nd number of discrete values using Equation (10) and the allocated tolerance (t_{ejk}) is calculated using Equation (11). The tolerance cost (TC_{ejk}) and machining time (MT_{ejk}) for t_{ejk} are calculated using Equations (2) and (3), respectively. Nagarajan et al. (2018) explained the distance method is used to combine the two different objective functions into a single one. For each discrete value, points are plotted on a graph where the x-axis and y-axis represent tolerance cost (TC_{ejk}) and machining time (MT_{ejk}), respectively. Assuming point $(x1, y1)$ as the origin and point $(x2, y2)$ as discrete tolerance cost and machining time, and substituting $(x1, y1)$ as $(0,0)$ and $(x2, y2)$ as (TC_{ejk}, MT_{ejk}) in Equation (12), then the distance equation becomes Equation (13). The detailed steps of the heuristic and univariate search methods are shown in Figures 1 and 2. The pseudocode for the combined whale optimization algorithm and a univariate search method is presented in section 5.

$$td_j = \frac{t_{\max_j} - t_{\min_j}}{nd} \quad (10)$$

$$t_{ejk} = t_{\min_j} + (e-1)td_j \quad (11)$$

where e is the index for discrete point of tolerance and takes from $1, 2, 3, \dots, nd$

$$dis = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \quad (12)$$

$$dis_{ejk} = \sqrt{(TC_{ejk} - 0)^2 + (MT_{ejk} - 0)^2} = \sqrt{TC_{ejk}^2 + MT_{ejk}^2} \quad (13)$$

Fig. 1: Heuristic approach to determine the best machine for each process

Fig. 2: Flow chart of univariate search method

5. Numerical Illustration

The proposed method is initially implemented in the existing problem (wheel mounting assembly) discussed by Geetha et al. (2013) to show the method's effectiveness in case study 1. Later, it is implemented on knuckle joint assembly in case study 2.

Case study 1: Wheel Mounting Assembly (WMA)

The components of the wheel mounting assembly are shown in Figure 3, and its manufacturing details are presented in Tables 2 and 3. The first stage of the proposed work, the procedure to determine the best machine for each process, is discussed below.

Fig. 3: Wheel Mounting Assembly (WMA) [Geetha et al. (2013)]

Table 2: Feasibility Matrix for WMA [Geetha et al. (2013)]

		Process numbers				
		<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>	<i>P5</i>
Operation numbers	<i>O1</i>	1	1	0	1	0
	<i>O2</i>	1	0	1	0	1
	<i>O3</i>	0	1	0	1	0
	<i>O4</i>	0	0	1	0	1
	<i>O5</i>	1	0	0	1	0
	<i>O6</i>	0	1	0	0	1
	<i>O7</i>	0	0	1	1	0
	<i>O8</i>	1	1	0	0	0
Machine numbers	<i>M1</i>	1	0	1	0	1
	<i>M2</i>	0	1	0	1	1
	<i>M3</i>	1	1	1	1	0
	<i>M4</i>	0	0	1	0	1

Table 3: Cost and Time Function Constants for WMA [Geetha et al. (2013)]

		Process number				
		P1	P2	P3	P4	P5
Cost function constant	a	1.4	1.5	0.9	2.5	1.9
	b	0.24	0.22	0.18	0.23	0.15
Time function constant	$X1$	2	5	3	4.5	3
	$Y1$	0.4	0.2	0.8	0.5	0.2
Process capability limits in mm	t_{min}	0.01	0.03	0.02	0.03	0.009
	t_{max}	0.08	0.09	0.07	0.13	0.1
Cost and Time manipulating Factor Machine Numbers	$M1$	0.8	0	0.85	0	1.08
	$M2$	0	0.85	0	1.11	1.01
	$M3$	1.15	1	0.9	0.95	0
	$M4$	0	0	1.02	0	0.8

Figure 4 represents the graphical representation of possible alternative processes and machines to carry out all the sub-stage operations to complete the manufacturing of the product. In the first stage, the best machine is selected for each process by implementing the heuristic approach.

Fig. 4: Possible process and machines for each operation/substage

Using Equations (10), (11), (2), (3), and (13), discrete tolerance (td_j), tolerance (t_{ejk}), tolerance cost (TC_{ejk}), machining time (MT_{ejk}), and distance values (dis_{ejk}) are calculated for each machine and are shown in Table 4. Figure 5 is constructed for a discrete tolerance value of

0.01mm ($t_{ejk}=0.01\text{mm}$), assuming TC_{ejk} on the x -axis and MT_{ejk} on the y axis. The distance between the origin and the discrete point of the machine represents the critical factor that decides selecting the best machine. The value of less distance is more likely to be the best machine for the process.

Table 4: Best machine for process number 1

T_{ejk}	TC_{e11}	MT_{e11}	dis_{e11}	TC_{e13}	MT_{e13}	dis_{e13}	$\min(dis_{ejk})$	Z_{e11}	Z_{e13}
0.0100	20.32	33.60	39.27	29.21	48.30	56.45	39.27	0.00	17.18
0.0147	14.21	23.42	27.39	20.43	33.66	39.38	27.39	0.00	11.98
0.0193	11.05	18.15	21.25	15.89	26.09	30.55	21.25	0.00	9.30
0.0240	9.12	14.93	17.50	13.11	21.47	25.15	17.50	0.00	7.66
0.0287	7.82	12.76	14.97	11.24	18.35	21.51	14.97	0.00	6.55
0.0333	6.88	11.20	13.14	9.89	16.10	18.90	13.14	0.00	5.75
0.0380	6.17	10.02	11.77	8.87	14.41	16.92	11.77	0.00	5.15
0.0427	5.62	9.10	10.70	8.08	13.08	15.37	10.70	0.00	4.68
0.0473	5.18	8.36	9.83	7.44	12.02	14.14	9.83	0.00	4.30
0.0520	4.81	7.75	9.13	6.92	11.15	13.12	9.13	0.00	3.99
0.0567	4.51	7.25	8.53	6.48	10.42	12.27	8.53	0.00	3.73
0.0613	4.25	6.82	8.03	6.11	9.80	11.55	8.03	0.00	3.51
0.0660	4.03	6.45	7.60	5.79	9.27	10.93	7.60	0.00	3.33
0.0707	3.84	6.13	7.23	5.52	8.81	10.39	7.23	0.00	3.16
0.0753	3.67	5.85	6.90	5.27	8.41	9.92	6.90	0.00	3.02
0.0800	3.52	5.60	6.61	5.06	8.05	9.51	6.61	0.00	2.89

TC_{e11} – Tolerance cost of e^{th} discrete tolerance using process number 1 on machine number 1;
 MT_{e13} – Machining time of e^{th} discrete tolerance using process number 1 on machine number 3

Fig. 5: Selection of the best machine for discrete tolerance

As shown in Figure 5, it is clear that $dis-M1$ is less than $dis-M3$; therefore, machine 1 is considered the best machine for process 1 for achieving a discrete tolerance value of 0.01mm. The sum of the difference between minimum discrete distance [$\min(dis_{ijk})$] and discrete distance (dis_{ijk}) to that particular machine is calculated to select the best machine suitable for

all discrete tolerances of the process tolerance. As shown in Table 4, it can be concluded that machine number 1 is the best machine for process number 1, shown graphically in Figure 6. Similarly, the best machine for the other processes is calculated and shown in Figure 6. It is clearly understood that machine numbers 2, 1, 3, and 4 are the best machines for process numbers 2, 3, 4, and 5, respectively.

Fig. 6: Best machine for process number 1 to 5

Fig. 7: Best machine for the process of each operation/substage after implementation of the first stage

After implementing the heuristic approach, alternative machines are removed for each process; instead, the best machine is selected, which is shown in Figure 7.

In the second stage, since the same allocated tolerances reported in Geetha et al. (2013) have been used for demonstration purposes, only univariate search method is implemented to get the best process, minimum tolerance cost, and minimum machining time for each sub-stage/operation of WMA. The univariate search method proposed in this work provides the best results in relatively less computation time than the existing method. The search space of the existing method proposed in Geetha et al. (2013) contains 403,200 combinations, whereas, in the proposed method, there are only 11 possible combinations in the search space. Equations (14) and (15) compute the possible combinations in the existing and proposed methods, respectively. Table 5 represents the 11 possible combinations obtained using the univariate search approach, and the first combination is shown as yellow shaded text in figure 7.

$$n_{cs_e} = \prod_{i=1}^{no} \left(\sum_{j=1}^{np} nm_{ij} \right) \quad (14)$$

$$= (2+2+2)(2+3+2)(2+2)(3+3)(2+2)(2+3)(3+2)(2+2)$$

$$= 6 \times 7 \times 4 \times 6 \times 4 \times 5 \times 5 \times 4 = 403200$$

$$n_{cs_u} = 1 + \sum_{i=1}^{no} np_i - no \quad (15)$$

$$= 1 + 3 + 3 + 2 + 2 + 2 + 2 + 2 + 2 - 8 = 11$$

Using Equations (2) and (3), the tolerance cost and machining time are calculated for each operation with alternative processes and its corresponding best machine is shown in figure 7. In all 11 combinations of alternative processes, the same tolerance mentioned in case 1 and case 2 by Geetha et al. (2013) is assumed for each operation. The alternative process and its corresponding machine allocated tolerance of each sub-stage/operation and its tolerance cost

and machining time are presented in Tables 5 and 6. The best result is shown as shaded text in both tables 5 and 6.

Table 5: Result of Univariate search method for the existing problem

Process no. (Machine no.) Table 10 in Geetha et al.(2013) case 1									TC	MT	Dist
O1	O2	O3	O4	O5	O6	O7	O8				
0.060784	0.061882	0.039412	0.041831	0.055569	0.070471	0.062745	0.056824				
1	1(1)	1(1)	2(2)	3(1)	1(1)	2(2)	3(1)	1(1)	35.15	75.64	83.41
2	1(1)	1(1)	2(2)	3(1)	1(1)	2(2)	3(1)	2(2)	35.22	75.66	83.45
3	1(1)	1(1)	2(2)	3(1)	1(1)	2(2)	4(3)	1(1)	37.80	74.10	83.19
4	1(1)	1(1)	2(2)	3(1)	1(1)	5(4)	4(3)	1(1)	37.10	72.11	81.09
5	1(1)	1(1)	2(2)	3(1)	4(3)	5(4)	4(3)	1(1)	38.83	77.58	86.75
6	1(1)	1(1)	2(2)	5(4)	1(1)	5(4)	4(3)	1(1)	37.06	59.53	70.13
7	1(1)	1(1)	4(3)	5(4)	1(1)	5(4)	4(3)	1(1)	38.96	67.29	77.76
8	1(1)	3(1)	2(2)	5(4)	1(1)	5(4)	4(3)	1(1)	36.08	66.30	75.48
9	1(1)	4(3)	2(2)	5(4)	1(1)	5(4)	4(3)	1(1)	36.30	57.74	68.21
10	2(2)	4(3)	2(2)	5(4)	1(1)	5(4)	4(3)	1(1)	36.37	57.93	68.40
11	4(3)	4(3)	2(2)	5(4)	1(1)	5(4)	4(3)	1(1)	37.99	62.97	73.54

Figures 8 and 9 show a comparison of the tolerance cost and machining time for the existing problem presented in Geetha et al. (2013) and the proposed method; it can be seen that the proposed method works well, considering tolerance cost, machining time and both as objective functions.

Fig. 8: Tolerance cost comparison with the existing problem presented in Geetha et al. (2013) MC: Tolerance cost; EM10 & EM11: Existing method, as per data presented in Table 10 & 11 of Geetha et al.; PM10 & PM11: Proposed method, as per data presented in Table 10 & 11 of Geetha et al. (2013)

Table 6: Details of individual sub-stages of the existing problem presented in Table 11 Geetha et al. (2013) case 2

O.No.	O1					O2				O3				O4				O5				O6				O7				O8				Cmfg	Tmc	Dist
Tol	0.0688235					0.0440392				0.0824706				0.0511098				0.0583137				0.0685882				0.0617647				0.0589412						
T.No.	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT	P.No.	M.No.	TC	MT				
1	1	1	3.9	6.2	1	1	5.5	8.9	2	2	3.5	6.3	3	1	3.8	15.9	1	1	4.4	7.1	2	2	4.0	6.7	3	1	3.2	13.6	1	1	4.4	7.0	32.7	71.7	78.8	
2	1	1	3.9	6.2	1	1	5.5	8.9	2	2	3.5	6.3	3	1	3.8	15.9	1	1	4.4	7.1	2	2	4.0	6.7	3	1	3.2	13.6	2	2	4.4	7.1	32.8	71.8	78.9	
3	1	1	3.9	6.2	1	1	5.5	8.9	2	2	3.5	6.3	3	1	3.8	15.9	1	1	4.4	7.1	2	2	4.0	6.7	4	3	5.9	12.0	1	1	4.4	7.0	35.4	70.1	78.5	
4	1	1	3.9	6.2	1	1	5.5	8.9	2	2	3.5	6.3	3	1	3.8	15.9	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	34.7	68.1	76.4	
5	1	1	3.9	6.2	1	1	5.5	8.9	2	2	3.5	6.3	3	1	3.8	15.9	4	3	6.1	12.4	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	36.4	73.4	81.9	
6	1	1	3.9	6.2	1	1	5.5	8.9	2	2	3.5	6.3	5	4	3.9	5.5	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	34.8	57.8	67.4	
7	1	1	3.9	6.2	1	1	5.5	8.9	4	3	5.0	10.0	5	4	3.9	5.5	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	36.3	61.5	71.4	
8	1	1	3.9	6.2	3	1	4.2	18.0	2	2	3.5	6.3	5	4	3.9	5.5	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	33.5	66.9	74.8	
9	1	1	3.9	6.2	5	4	4.2	6.0	2	2	3.5	6.3	5	4	3.9	5.5	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	33.5	54.9	64.4	
10	2	2	4.0	6.7	5	4	4.2	6.0	2	2	3.5	6.3	5	4	3.9	5.5	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	33.6	55.4	64.8	
11	4	3	5.5	11.2	5	4	4.2	6.0	2	2	3.5	6.3	5	4	3.9	5.5	1	1	4.4	7.1	5	4	3.3	4.7	4	3	5.9	12.0	1	1	4.4	7.0	35.2	59.9	69.4

Fig. 9: Machining time comparison with the existing problem presented in Geetha et al. (2013)

Table 7 represents the % of savings in both tolerance cost and machining time as compared to the existing method and the proposed method.

Table 7: % Savings in Tolerance cost and Machining Time

Details	Case 1		Case 2	
Exsiting method	37.32	68.21	35.72	68.90
Proposed Method	36.30	57.74	33.50	54.90
% Savings	2.73	15.35	6.22	20.32

Case Study 2: Knuckle Joint Assembly

In practice, the availability of machine time will restrict the selection of the machine for performing a process, which will influence the tolerance cost and machining time. Therefore, in this work, the available machine time is considered as a constraint in selecting the machine. The methodology is demonstrated using a knuckle joint assembly (Figure 10), consisting of six components performed in ten sub-stages. Two critical dimensions are considered for the proper functioning of the product. It is assumed that nine processes are performed using ten machines. Equations (16) and (17) are used to determine the critical dimension, and the tolerances of the critical dimensions are estimated using Equations (18)–(21). Table 8 shows the dimensions, sub-stages, tolerance symbols, and tolerance stake-up of the knuckle joint assembly.

Fig. 10: Knuckle joint assembly

$$Y1 = d5 - d1 - d2 - d3 - d4 \quad (16)$$

$$Y2 = d2 - d6 \quad (17)$$

$$t_{Y1} = t_{d5} + t_{d1} + t_{d2} + t_{d3} + t_{d4} \quad (18)$$

$$t_{Y1} = t_{O7} + t_{O8} + t_{O1} + t_{O2} + t_{O3} + t_{O4} + t_{O5} + t_{O6} \quad (19)$$

$$t_{Y2} = t_{d2} + t_{d6} \quad (20)$$

$$t_{Y2} = t_{O2} + t_{O3} + t_{O9} + t_{O10} \quad (21)$$

$$C = \sum_{i=1}^{n0} TC_i \quad (22)$$

$$T = \sum_{i=1}^{n0} MT_i \quad (23)$$

Table 8: Dimension and tolerance symbol of knuckle joint assembly

Name of the Component	Dimension No.	Operation / Substage No.	Tolerance symbol	Tolerance stack-up
Width of fork top	$d1$	$O1$	t_{d1}	t_{O1}
Dimension of fork space	$d2$	$O2$ and $O3$	t_{d2}	$t_{O2} + t_{O3}$
Width of bottom fork	$d3$	$O4$	t_{d3}	t_{O4}
Center distance of hole in spacer	$d4$	$O5$ and $O6$	t_{d4}	$t_{O5} + t_{O6}$
Center distance of hole in pin	$d5$	$O7$ and $O8$	t_{d5}	$t_{O7} + t_{O8}$
Width of collar	$d6$	$O9$ and $O10$	t_{d6}	$t_{O9} + t_{O10}$
Critical dimension 1	$Y1$		t_{Y1}	
Critical dimension 2	$Y2$		t_{Y2}	

Table 9 shows the cost and time function constants of each process of the knuckle joint assembly. The possible operation–process and process–machine feasibility matrix and the available time of the individual machines are presented in Tables 10 and 11, respectively.

Table 9: Cost and time function constants of each process of knuckle joint assembly

P.No.	A	B	X1	Y1	tmin	tmax	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
P1	2.5	0.22	4.7	1.3	0.030	0.09	1.3	1.2	0	0.8	0.9	0	1.3	0	0.7	0
P2	3.9	0.18	3.8	0.8	0.020	0.07	0.9	0	0.75	0.82	0	1.2	1	0	1.8	1.6

<i>P3</i>	2.9	1.23	4.1	1.5	0.030	0.13	0	1.3	1.1	0	1.4	0	1.4	0	0	0.9
<i>P4</i>	3.9	2.15	3.9	2	0.009	0.10	0	1.1	0	1.7	0	1.6	1	1.7	0	0
<i>P5</i>	1.7	1.09	3.1	0.6	0.020	0.14	0	0.7	1.1	0	0	1.8	0	0.8	0	1.5
<i>P6</i>	2.3	1.22	2.1	0.9	0.010	0.13	1.8	0	1.7	0	0.9	0	1.3	0	0.9	0
<i>P7</i>	1.7	0.4	4.2	1.2	0.007	0.15	1.7	0	1.2	0	1	1.8	1.7	1.1	0.9	0
<i>P8</i>	3.2	1.1	2.9	1.1	0.010	0.15	0.8	0	1	1.7	0	1.4	1.1	0	1	1.6
<i>P9</i>	2.1	0.08	5.2	1.7	0.030	0.17	0	1.1	0	0.7	1.8	0	1.1	0	0	0.8

Table 10: Possible Operation – Process and Process – Machine Feasibility Matrix of knuckle joint assembly

<i>P.N</i> <i>o.</i>	Operation / Sub stage number										Machine number									
	<i>O1</i>	<i>O2</i>	<i>O3</i>	<i>O4</i>	<i>O5</i>	<i>O6</i>	<i>O7</i>	<i>O8</i>	<i>O9</i>	<i>O10</i>	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>M5</i>	<i>M6</i>	<i>M7</i>	<i>M8</i>	<i>M9</i>	<i>M10</i>
<i>P1</i>	1	0	0	1	1	1	0	0	0	1	1	1	0	1	1	0	1	0	1	0
<i>P2</i>	0	1	1	0	1	0	0	0	1	0	1	0	1	1	0	1	1	0	1	1
<i>P3</i>	1	1	0	0	1	0	1	1	1	1	0	1	1	0	1	0	1	0	0	1
<i>P4</i>	1	1	0	1	1	1	1	0	1	1	0	1	0	1	0	1	1	1	0	0
<i>P5</i>	0	0	0	0	0	1	0	1	1	1	0	1	1	0	0	1	0	1	0	1
<i>P6</i>	1	1	0	1	1	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0
<i>P7</i>	0	0	0	1	1	0	0	1	0	1	1	0	1	0	1	1	1	1	1	0
<i>P8</i>	0	0	1	0	1	1	1	1	0	0	1	0	1	1	0	1	1	0	1	1
<i>P9</i>	0	0	0	0	1	0	1	1	0	0	0	1	0	1	1	0	1	0	0	1

Table 11: Available machine time of each machine in the knuckle joint assembly

Machine .Number	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>M5</i>	<i>M6</i>	<i>M7</i>	<i>M8</i>	<i>M9</i>	<i>M10</i>
Available machine time (AMT)	75	80	60	40	30	70	30	30	90	50

As per stage 1, the best machine for each process is determined using the heuristic method and presented in Table 12 and Figure 11.

Table 12: Best machine for each process of the knuckle joint assembly

Process number	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>	<i>P5</i>	<i>P6</i>	<i>P7</i>	<i>P8</i>	<i>P9</i>
Best machine number	9	3	10	7	2	5	9	1	4

Fig. 11: Best Machine for each process

In the second stage, the whale optimization algorithm is implemented with a univariate search method. The optimum allocated tolerance for each sub-stage/operation, tolerance cost, machining time, and best process are obtained by assuming 100 whales, 100 iterations as stopping criteria with 20 runs. The pseudocode and the parameters (Table 13) involved in the algorithm are given below.

Pseudocode of Whale Optimization Algorithm with Univariate search method

Initialize no. of Whales' position as tolerance of each sub-stage/opteation (t_{ij} , where $i = 1, 2, \dots, nw$ and $j = 1, 2, \dots, no$)

While ($it < nitr$)

For each whale $i=1$ to nw

Determine the assembly tolerance (t_{ay}) using equations (19) and (21)

Check t_{ay} within the specified tolerance value t_y

For each operation $j=1$ to no

For each alternative process of operation $k=1$ to $npo(j)$

Calculate TC_{ijk} and MT_{ijk} using equations (2) and (3)

Calculate dis_{ejk} using equation (13)

End

Select the process corresponding to minimum value of dis_{ijk}
as $bp_i=j$, its TC_{ij} as bTC_i and MT_{ij} as bMT_i

End

Compute C and T using equations (22) and (23)

Calculate fitness function F_i using equation (13)

Calculate machine engage time (met) and check with machine available time (amt)

End

Sort F_i in ascending order and store the first whale's fitness value as F_{it} , position as X_{it} , and save its corresponding bp , bTC , bMT , C , T and F

Using the sorted data, assign $X_p = X_l$.

Compute $a = 2 - it * (\frac{2}{nitr})$ and $a2 = -1 + it * (\frac{-1}{nitr})$

For each whale

Update the position using

$A = 2 * a * rand() - a$ and $G = 2 * rand()$ and $l = (a2 - 1) * rand() + 1$ and $p = rand()$

If $p < 0.5$
 If $abs(A) \geq 1$
 $D_i = abs(G * X_r - X_i)$ and $X_i = X_r - A * D_i$
 If $abs(A) < 1$
 $D_i = abs(G * X_p - X_i)$ and $X_i = X_p - A * D_i$
 If $p \geq 0.5$
 $D_i = abs(X_p - X_i)$ and $X_i = X_p + D_i * Exp^{b * l} * cos(2\pi l)$
 Check X_i within bounds
 End
 End
 Sort F_{it} in ascending order and display the first data's position as the optimal allocated tolerance, bP as the best process, bTC and bMT as minimum tolerance cost and machining time of each operation and C , T as optimum manufacturing cost and machining time.

Table 13: Terms involved in Whale Optimization Algorithm

Whale Optimization Algorithm	Optimization Problem
Number of whales ($i=1,2,...nw$)	Number of solutions = 100
Position of a whale (X_i)	Combination of allocated tolerance of each sub-stage/operation
Number of dimensions involved in defining the position of whale ($j=1,2,...nd$)	Number of operations = 10
Position of Prey (X_p)	Value of optimum tolerance of each operation ($Tol.$)
Fitness of whale (F_i)	Combined optimum tolerance cost (TC) and machining time (MT) in terms of distance (dis)
Stopping criteria	Maximum number of iteration = 100

The convergence plot for tolerance cost and machining time for stopping criteria considered as 100 iterations are shown in figure 12. The Pareto optimal solution for a sample run is shown in figure 13. The tolerance cost and machining time for 31 runs are presented in Table 14. Out of these 31 runs, the best value is calculated using EDAS and CODAS multi-criteria decision-making techniques implemented by Adali & Tuş (2019). The appraisal score (APS) and the assessment score (AS) obtained by implementing EDAS and CODAS method are presented in Table 14. Run number ($R.No.$) 11 has the highest APS value in EDAS, and run number 7 has the highest AS value in CODAS. The optimum allocated tolerances of each sub-stage are presented for both methods are listed in Table 15, along with the tolerance cost and machining time for the required sub-assembly tolerances of 0.55 mm and 0.22 mm. The optimum total tolerance cost and total machining time are presented in Table 16 for the given sub-assembly tolerance values with the constraint of available machine time.

Fig. 12: Convergence Plot for Tolerance cost and machining time

Fig. 13: Optimum paerto front solutions

Table 14: Appraisal (APS) and Assesement (AS) Scores of EDAS and CODAS

R.No.	EDAS			CODAS		
	TC	MT	APS	TC	MT	AS
1	68.08	153.97	0.5009	68.01	154.60	-0.0245
2	68.83	149.68	0.5346	67.92	155.39	-0.0106
3	69.75	147.60	0.5308	67.92	158.54	-0.0275
4	69.24	149.31	0.5185	67.93	155.59	-0.0147
5	69.07	149.32	0.5272	67.91	155.64	-0.0109
6	68.83	149.18	0.5433	67.82	155.70	0.0111
7	69.11	149.15	0.5283	67.90	156.88	-0.018
8	82.51	129.70	0.519	67.84	155.22	0.0113
9	68.85	149.45	0.5375	67.81	156.30	0.0085
10	69.45	148.01	0.5367	67.90	159.17	-0.0222
11	68.47	150.20	0.5452	67.83	156.08	0.0062
12	68.57	151.45	0.5181	67.90	158.74	-0.0233
13	68.18	151.55	0.5379	67.87	154.75	0.0075
14	69.23	148.28	0.5417	67.82	158.41	-0.0036
15	68.25	152.04	0.5256	67.92	156.45	-0.0197

16	69.29	149.66	0.5093	68.01	157.39	-0.0453
17	68.49	152.20	0.5093	68.03	155.22	-0.0348
18	68.74	149.58	0.5414	67.89	157.46	-0.018
19	69.20	148.34	0.5418	67.83	157.84	-0.0046
20	69.14	148.82	0.5326	67.82	158.67	-0.0045
21	68.30	151.69	0.5287	67.94	154.87	-0.0095
22	68.81	149.56	0.5379	67.81	157.83	0.0004
23	69.00	149.17	0.5337	67.89	158.09	-0.0191
24	69.28	148.94	0.5224	67.88	157.48	-0.0145
25	68.94	149.36	0.5341	67.89	158.21	-0.0192
26	69.58	148.35	0.5201	67.89	156.61	-0.0134
27	69.07	149.18	0.5302	67.90	157.29	-0.0193
28	69.00	149.61	0.5262	67.98	154.50	-0.0152
29	68.23	151.46	0.5367	67.80	156.40	0.0095
30	68.14	152.32	0.5266	67.94	154.21	-0.0031
31	68.24	152.13	0.5245	67.93	155.66	-0.0167

Table 15: The optimum allocated tolerance of knuckle joint assembly

O.No.	EDAS					CODAS				
	Tol.	P.No.	M.No.	TC	MT	Tol.	P.No.	M.No.	TC	MT
1	0.074144	1	9	4.58	16.97	0.074438	1	9	4.57	16.93
2	0.040709	2	9	7.22	18.54	0.036949	2	9	7.55	20.04
3	0.039105	2	9	7.35	19.14	0.041246	2	9	7.17	18.35
4	0.081597	1	9	4.39	15.85	0.069137	1	9	4.73	17.86
5	0.065967	2	3	5.95	12.90	0.062934	2	3	6.05	13.33
6	0.089941	5	10	10.18	7.77	0.09	5	10	10.18	7.77
7	0.050332	9	7	3.21	28.84	0.045295	9	7	3.34	31.47
8	0.108207	5	3	8.75	6.98	0.13	5	3	7.57	6.33
9	0.051985	2	9	6.50	15.34	0.051806	2	9	6.51	15.38
10	0.088202	5	7	10.35	7.86	0.089999	5	7	10.18	7.77

O.No. – Operation/Substage number; P.No. – Process number; M.No. – Machine number, Tol. – Allocated tolerance; TC – Tolerance cost; MT – Machining time

Table 16: Sub-assembly tolerance, total cost and machining time

Details	EDAS	CODAS
<i>ta1</i>	0.55	0.55
<i>ta2</i>	0.22	0.22
TC	68.47	67.84
MT	150.20	155.22

ta1-Calculated sub-assembly 1 tolerance; *ta2*-Calculated sub-assembly 2 tolerance

Table 17 shows the machine engagement times of the individual machines to manufacture the knuckle joint assembly within the available machine time.

Table 17: Machine engagement time (*met*) of individual machine

M.No.	1	2	3	4	5	6	7	8	9	10
EDAS	0	0	19.88	0	0	0	36.70	0	85.85	7.77
CODAS	0	0	19.66	0	0	0	39.24	0	88.55	7.77

For supporting the proposed method, the statistical analysis for distance and CODAS methods are presented in figure 14. Since the value of probability greater than 0.005 in both cases, it is understood that the results obtained in 31 runs are from normally distributed data.

6. Concluding Remarks

Most previous studies on tolerance allocation problems concentrated on minimizing manufacturing costs, quality loss, or combining the two. Machining time, a vital manufacturing objective, has barely been contemplated. In this paper, the machining time was considered along with manufacturing cost in optimum tolerance allocation of complex assemblies, representing a more realistic product development scenario. Alternative machine and process selections with available machine time make this problem cumbersome and complicated. Therefore, a new methodology was developed that applies a heuristic approach and combined whale optimization algorithm with a univariate search method. The total manufacturing cost and machining time of 36.3 \$ and 57.74 min reported in this paper for wheel mounting assembly is 2.73% and 15.35% less than the problem dealt with in case 1 by Geetha et al. (2013). Similarly in case 2, 6.22% and 20.32% of savings in the tolerance cost and machining time reported by implementing the proposed method. The results presented in this paper demonstrate that the proposed method can reduce tolerance cost and machining time simultaneously with less computation time. The proposed method is also suitable for solving two and three-dimensional problems. As a further extension of this work, the operation sequence, machine sequence, or both may be considered with additional objectives such as total investment cost of machines, idle time of machines, idle cost of machines, and the number of machines required to manufacture the product.

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