Effect of friction on transverse vibration of string under moving load

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A B S T R A C T

Many engineering applications involve exerting moving harmonic load on a string like structure. Usually the interface between these structures and the moving load has some friction. A common example is a pantograph catenary system, which is used in locomotives for power collection. The aim of this paper is to develop a mathematical model of a simplified system consisting of infinitely long axially tensioned continuum and a moving harmonic load with friction acting at the interface. Equation of motion has been derived by resolving forces at that point. Subsequently the basic characteristics of the system are obtained by solving the model numerically. It is observed that the effect of friction obtained is negligibly low higher value of axial tension, but can significantly increase the string response at a particular range of coefficient of friction value when the axial tension is low.

1. Introduction

Moving load problem along an elastic continua has been a research topic for several years due its large scale of practical applications. Railway track, pantograph-catenary coupled system, ropeway etc. are some of those. Literature available in this domain shows that simple model like this can capture the basic dynamics of the system. Charles E. Smith (1964) has presented an "exact" solution for motions of a stretched string carrying a moving mass particle. Krzysztof Marynowski and Tomasz Kapitaniak (2014) in their paper have presented the review of research in the field of dynamics of axially moving continua with particular emphasis on the axially moving plate-like elastic and viscoelastic systems. A brief overview of the most important studies on the dynamics of moving string-like and beam-like systems is also presented and comparative analysis of some results of studies published by the other authors in the field of dynamics of axially moving viscoelastic systems is done. R. Idzikowski et. al (2013) considered the damping dynamical response of a finite string due to a uniformly distributed load and a point force moving with a constant velocity. The classical solution for the transverse displacement of a string has a form of a sum of two infinite series, one of which represents the forced vibrations and the other one represents free vibrations of the beam. In their paper they showed that the series which represents forced vibrations of the string can be presented in a closed, analytial form. J. A. Wickert and C.D. Mote Jr. (1991) have modelled the classical moving load problem in which a mass traverses an elastic structure is e.g. monocable ropeway, (traveling structure-traveling load system), as an axially moving string that transports an attached discrete mass between two supports. The response is calculated and compared to that predicted by the derived asymptotic solution for vanishing discrete mass. Agreement between responses measured in the laboratory and those predicted by the traveling

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structure-traveling load theory supports the analysis. E. Ögzkays and M. Pakdemirli (1999) in their paper have considered transverse vibrations of an axially moving beam. The axial velocity is harmonically varying about a mean velocity. The equation of motion is expressed in terms of dimensionless quantities. The beam effects are assumed to be small. Since, in this case, the fourth order spatial derivative multiplies a small parameter, the mathematical model becomes a boundary layer type of problem. Approximate solutions are searched using the method of multiple scales and the method of matched asymptotic expansions. Results of both methods are contrasted with the outer solution. Akin et. al (1989) have presented an analytical-numerical method to determine the dynamic behavior of beams, with different boundary conditions, carrying a moving mass. In this study, they demonstrated the transformation of a familiar governing equation into a new, solvable series of ordinary differential equations. The correctness of the results has been ascertained by a comparison, using finite element models, and very good agreement has been obtained. The response of structures due to moving mass is also shown.

Most of the researches have concentrated on the dynamics of the system without considering any friction at the load-continua interface. However, a few researches worked on friction model for inertial mass travelling along a continua. Spelsberg-Korspeter et. al (2008) considerd a moving beam in frictional contact with pads, making the system susceptible for self-excited vibrations. The equations of motion are derived and a stability analysis is performed using perturbation techniques yielding analytical approximations to the stability boundaries. Special attention is given to the interaction of the beam and the rod equations. Daniel Hochlenert et. al (2007) have devoted their paper to the modelling of self-excited vibrations of moving continua generated by frictional forces. Special regard has given to an accurate formulation of the kinematics of the frictional contact in two and three dimensions. On the basis of a travelling Euler– Bernoulli beam and a rotating annular Kirchhoff plate with frictional point contact the essential properties of the contact kinematics leading to self-excited vibrations are worked out. A Ritz discretization is applied and the obtained approximate solution is compared to the exact one of the traveling beam. V. N. Pilipchuk and C. A. Tan (2004) in their paper have investigated friction-induced vibration of a two-degree-of-freedom mass-damper-spring system interacting with a decelerating rigid strip. The friction law is approximated by an analytical function to facilitate the analyses and numerical integrations. It is shown that, after a quasi-harmonic transient period, accompanied by viscous energy dissipation, a short period of intensive 'creep–slip' vibration occurs, which generates a series of 'micro-impacts' on the strip. Because of the impulsive character of such kind of loading, its Fourier spectrum is rich and quite broadband. Using an averaging technique, the 'normal form' equations of motion show that the out-of-phase vibration mode absorbs more energy from the decelerating strip when its natural frequency satisfies certain resonance conditions. The study is then applied to an automotive disc brake model to gain useful insight into the generation of squeal. It is shown that the out-of-phase creep–slip vibration (in the longitudinal direction) of the brake pads generates an impulsive bending moment on the decelerating strip (disc rotor). This impulsive load may be considered as a possible source for brake squeal. H. Ouyang and J. E. Mottershead (2004) in their paper have investigates the instability of the transverse vibration of a disk excited by two corotating sliders on either side of the disk. Each slider is a mass-spring-damper system traveling at the same constant speed around the disk. There are friction forces acting in the plane of the disk at the contact interfaces between the disk and each of the two sliders. The equation of motion of the disk is established by taking into account the bending couple acting in the circumferential direction produced by the different friction forces on the two sides of the disk. The normal forces and the friction couples produced by the rotating sliders are moving loads and are seen to bring about dynamic instability. Regions of instability for parameters of interest are obtained by the method of state space. It is found that the moving loads produced by the sliders are a mechanism for generating unstable parametric resonances in the subcritical speed range.

In this present work we have added the friction force at the interface of the load and resolved it to suitable components along the normal and the tangential direction of the string. Discontinuity of the slope at the point of application is dealt with the specific model (D. Griffiths and S. Walborn,1999). To the author's knowledge there is scarcity of literature which deals with friction in this type of problem and the effect of friction on the response of the string is also not well documented in the small number of available papers. Therefore a very basic study in this regard becomes obvious. In this paper, mainly, the modelling aspect is emphasised and the approach taken is completely different from the available literature.

In the following sections, we introduce the mathematical modelling related to the problem, which is followed by a detailed solution in section 3. Numerical results obtained are discussed in result and discussion with possible explanation to those results. Finally, the application of this problem along with the drawbacks of the model is discussed in conclusion.

This problem may have applications in engineering, where the considerable long axially tensioned string has an contact with the moving load or mass system e.g. ropeway system.

2. Mathematical Modelling

An infinitely long string is considered which is being supported by a visco-elastic layer. The string is axially tensioned. A massless block which is in continuous contact with the cable as shown in the Fig. 1-(a) is travelling with a uniform speed, uplift force F_V and horizontal force F are acting on the block. Now the forces can be written in terms of friction and normal forces acting at the interface. As shown in the Fig.1-(b), the normal force N and friction force $\mu_k N$ are acting normal and tangential to the string, respectively, at the point of contact. Due to the application of the force the string is deflected by an angle θ and the vertical deflection is considered as W.

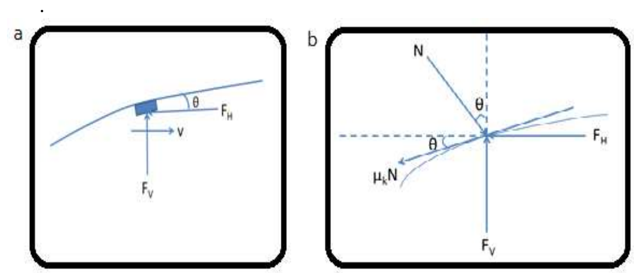

Fig. 1 - (a) force diagram for string; (b) resolution of forces on string.

The equation of motion for the transverse dynamics of the string is given by-

$$\mu\frac{\partial^2 W}{\partial t^2} - T\frac{\partial^2 W}{\partial x^2} + \bar{K}W + \bar{C}\frac{\partial W}{\partial t} = F_V\delta(x - vt) \quad \text{.......................... (1)}$$

where,

$F_V = (F_0 + F_a e^{i\Omega t})$

$\delta(.)$= Dirac delta function

μ= Linear mass density of string

T = Axial tension in the string

$\bar{K}$= Stiffness property of viscoelastic layer

$\bar{C}$= Damping property of viscoelastic layer

F_0= Constant vertical force acting on the string

F_a= Amplitude of fluctuating force acting on the string

Ω= Angular frequency of the force acting on the string

x= Spatial co-ordinate

t= Time

$W(x,t)$ = Transverse displacement of string

v = Uniform speed of the force.

Now, a new local co-ordinate system ξ is considered which is moving with the force F_V. So,

$$\xi = x - vt$$

Equation (1) can be rewritten in local co-ordinate system as,

$$\mu\frac{\partial^2 W}{\partial t^2} - 2\mu v\frac{\partial^2 W}{\partial\xi\partial t} + (\mu v^2 - T)\frac{\partial^2 W}{\partial\xi^2} + \bar{C}\frac{\partial W}{\partial t} - \bar{C}v\frac{\partial W}{\partial\xi} + \bar{K}W = F_V\delta(\xi) \quad \text{...(2)}$$

The forces acting on the string are resolved along the tangent and normal at the point of application of the force. This gives the following relationships-

$$F_V = \mu_k N\sin\theta + N\cos\theta$$

For small value of θ,

$$F_V = \mu_k N\frac{\partial W}{\partial\xi} + N \quad \text{....................................... (3)}$$

Now, replacing equation (3) in equation (2), and integrating it from $\xi = 0 - \varepsilon$ to $= 0 + \varepsilon$, one can get

$$(\mu v^2 - T)\left[\frac{\partial W}{\partial\xi}\bigg|_{\xi=0+\varepsilon} - \frac{\partial W}{\partial\xi}\bigg|_{\xi=0-\varepsilon}\right] = \int_{\xi=0-\varepsilon}^{\xi=0+\varepsilon} F_V\delta(\xi)d\xi \quad \text{............... (4)}$$

where, ε is a small positive quantity.

The symbol ε is dropped thereafter. The integration of the right hand side of eq. (4) gives the following result

$$\int_{\xi=0^-}^{\xi=0^+} F_V\delta(\xi)d\xi = N + \mu_k N\int_{\xi=0^-}^{\xi=0^+}\frac{\partial W}{\partial\xi}\delta(\xi)d\xi$$

where, $\frac{\partial W}{\partial\xi}$ is itself discontinuous at $\xi = 0$.

The integration of dirac delta function with a discontinuous function at the neighbourhood of discontinuity yields (D. Griffiths and S. Walborn,1999),

$$(\mu v^2 - T)\left[\frac{\partial W}{\partial\xi}\bigg|_{\xi=0^+} - \frac{\partial W}{\partial\xi}\bigg|_{\xi=0^-}\right] = N + \frac{1}{2}\mu_k N\left[\frac{\partial W}{\partial\xi}\bigg|_{\xi=0^+} + \frac{\partial W}{\partial\xi}\bigg|_{\xi=0^-}\right] \quad \text{......(5)}$$

Now, the displacement field is written as,

$$W(\xi,t) = W_0(\xi) + W_a(\xi)e^{i\Omega t} \quad \text{...................................... (6)}$$

where, $W_0(\xi)$ and $W_a(\xi)$ are the displacements corresponds to F_0 and F_a respectively.

Now, replacing $W(\xi,t)$ in equation (5), one gets –

$$(\mu v^2 - T)[(W_0'(0^+) + W_a'(0^+)e^{i\Omega t}) - (W_0'(0^-) + W_a'(0^-)e^{i\Omega t})] = N + \frac{1}{2}\mu_k N[(W_0'(0^+) + W_a'(0^+)e^{i\Omega t}) + (W_0'(0^-) + W_a'(0^-)e^{i\Omega t})]$$

Now, by comparing the coefficients of $e^{i\Omega t}$ of the above identity, the following results are obtained.

$$(\mu v^2 - T)\big(W_0'(0^+) - W_0'(0^-)\big) = N + \tfrac{1}{2}\mu_k N\big(W_0'(0^+) + W_0'(0^-)\big) \quad \text{..(7)}$$

$$(\mu v^2 - T)\big(W_a'(0^+) - W_a'(0^-)\big) = \tfrac{1}{2}\mu_k N\big(W_a'(0^+) + W_a'(0^-)\big) \quad \text{......... (8)}$$

Replacing the value of N from equation (8) into equation (7), one can obtain the final relationship as,

$$W_0'(0^+) - W_0'(0^-) = \left[\frac{2}{\mu_k}\frac{\big(W_a'(0^+) - W_a'(0^-)\big)}{\big(W_a'(0^+) + W_a'(0^-)\big)}\right]\left[1 + \frac{1}{2}\mu_k\big(W_0'(0^+) + W_0'(0^-)\big)\right] \quad \text{............... (9)}$$

The solution of equation (2) can be obtained by replacing the displacement field equation (6) into the same. The coefficient of $e^{i\Omega t}$ is compared on both sides of the identity to get,

$$(\mu v^2 - T)W_0''(\xi) - \bar{C}vW_0'(\xi) + \bar{K}W_0(\xi) = F_0\delta(\xi) \quad \text{........................(10)}$$

and,

$$(\mu v^2 - T)W_a''(\xi) - (2i\mu v\Omega + \bar{C}v)W_a''(\xi) + (\bar{K} - \mu\Omega^2 + i\bar{C}\Omega)W_a'(\xi) = F_a\delta(\xi) \quad \text{.......... (11)}$$

The boundary conditions of these equations are $W_0|_{|\xi|\to\infty} = W_a|_{|\xi|\to\infty} = 0$.

3. Solution

The solution of these ODE's can be obtained by Green's function method under the same boundary conditions and which are given by-

$$W_0(\xi) = \int_{-\infty}^{+\infty} G_0(\xi,\eta)F_0\delta(\eta)d\eta = F_0G_0(\xi,0) \quad \text{.............................. (12)}$$

and,

$$W_a(\xi) = \int_{-\infty}^{+\infty} G_a(\xi,\eta)F_a\delta(\eta)d\eta = F_aG_a(\xi,0) \quad \text{.............................. (13)}$$

Where $G_0(\xi,\eta)$ and $G_a(\xi,\eta)$ are Green's functions of corresponding ODE's. The green's function represents the string response in the ξ section in the moving reference frame, due to a unit force applied in the section of the same moving reference frame. It has to be observed the fact that the wire response is defined by the Green's function and this function is the receptance.

The Green's function can be built as a linear combination of the eigen functions of the differential operator of the equation (10) and (11). To find this function, the starting point is the homogenous equation from equation (10) and rewriting it in terms of Green's function $[G_0(\xi;\eta)]$ as,

$$(\mu v^2 - T)G_0''(\xi) - \bar{C}vG_0'(\xi) + \bar{K}G_0(\xi) = 0$$

The characteristic equation of this ODE can be given as,

$$(\mu v^2 - T)k^2 - \bar{C}vk + \bar{K} = 0 \quad \text{..........................(14)}$$

and k_1 , k_2 are its roots which are obtained as-

$$k_{1,2} = \frac{\bar{C}v \pm \sqrt{\Delta}}{2T(\alpha^2 - 1)}$$

where,

$\alpha = \frac{v}{c}$, $c = \sqrt{\frac{T}{\mu}}$, $\Delta = \bar{C}^2v^2 - 4(\mu v^2 - T)\bar{K}$t

The solution to this depends on the relative position of k_1 , k_2 in the complex plane. These positions of roots change with the variation in speed ratio, α. Therefore,

when,

(i) for $\alpha < 1$ & $\Delta > 0$, $\Rightarrow k_1 < 0, k_2 > 0$

So, the Green's function can be written as,

$$G(\xi,\eta) = Ae^{k_1(\xi-\eta)} \qquad 0 \le \xi - \eta < \infty$$
$$G(\xi,\eta) = Be^{k_2(\xi-\eta)} \qquad -\infty < \xi - \eta \le 0$$

which follows the boundary conditions of equation (10) according to the properties of Green's function.

As, the Green's function has to be continuous at $\xi = \eta$

So,

$$A = B$$

By satisfying the jump condition of Green's function at $\xi = \eta$, one can get

$$(\mu v^2 - T)\left[\frac{\partial G_0}{\partial\xi}\bigg|_{\xi=\eta^+} - \frac{\partial G_0}{\partial\xi}\bigg|_{\xi=\eta^-}\right] = 1 \quad \text{..........................(15)}$$

After simplifying, one obtains-

$$A = B = \frac{1}{T(k_1 - k_2)(\alpha^2 - 1)} \quad \text{...............................(16)}$$

So, now the Green's function becomes-

$$G_0(\xi,\eta) = \frac{e^{k_2(\xi-\eta)}}{T(k_1-k_2)(\alpha^2-1)} \qquad 0 \le \xi-\eta < \infty \ldots\ldots (17a)$$
$$G_0(\xi,\eta) = \frac{e^{k_1(\xi-\eta)}}{T(k_1-k_2)(\alpha^2-1)} \qquad 0 \le \xi-\eta < \infty \ldots\ldots (17b)$$

So, the solution for displacement can be given by equation (9) as,

$$W_0(\xi) = \frac{F_0 e^{k_2\xi}}{T(k_1-k_2)(\alpha^2-1)} \qquad -\infty < \xi \le 0 \ldots\ldots\ldots\ldots (18a)$$
$$W_0(\xi) = \frac{F_0 e^{k_1\xi}}{T(k_1-k_2)(\alpha^2-1)} \qquad 0 \le \xi < \infty \ldots\ldots\ldots\ldots (18b)$$

Similarly,

(ii)- for $\alpha^2 > 1$ & $\Delta > 0$,

$$W_0(\xi) = \frac{F_0\left[e^{k_1\xi} - e^{k_2\xi}\right]}{T(k_1-k_2)(\alpha^2-1)} \qquad -\infty < \xi \le 0 \ldots (19a)$$
$$W_0(\xi) = 0 \qquad 0 \le \xi < \infty \ldots\ldots (19b)$$

(iii)- for $\alpha^2 > 1$ & $\Delta = 0$,

$$W_0(\xi) = \frac{F_0 e^{K_1\xi}}{-T(\alpha^2-1)} \qquad -\infty < \xi \le 0 \ldots\ldots\ldots\ldots (20a)$$
$$W_0(\xi) = 0 \qquad 0 \le \xi < \infty \ldots\ldots\ldots\ldots (20b)$$

(iv)- for $\alpha^2 > 1$ & $\Delta < 0$,

$$W_0(\xi) = \frac{F_0 e^{\alpha\xi}\sin(\beta\xi)}{-T(\alpha^2-1)\beta} \qquad -\infty < \xi \le 0 \ldots\ldots\ldots\ldots (21a)$$
$$W_0(\xi) = 0 \qquad 0 \le \xi < \infty \ldots\ldots\ldots\ldots (21b)$$

where, $k_{1,2} = \alpha \pm i\beta$

The displacement field can be similarly obtained for equation (11) as-

(i)- for $\alpha<1$,

$$W_a(\xi) = \frac{F_a e^{k_2{}^r\xi}}{T(k_1{}^r - k_2{}^r)(\alpha^2-1)} \qquad -\infty < \xi \le 0 \ldots\ldots\ldots (22a)$$
$$W_a(\xi) = \frac{F_a e^{k_1{}^r\xi}}{T(k_1{}^r - k_2{}^r)(\alpha^2-1)} \qquad 0 \le \xi < \infty \ldots\ldots\ldots\ldots (22b)$$

(ii)- for $\alpha>1$,

$$W_a(\xi) = \frac{F_a\left[e^{k_1{}^r\xi} - e^{k_2{}^r\xi}\right]}{T(k_1{}^r - k_2{}^r)(\alpha^2-1)} \qquad -\infty < \xi \le 0 \ldots\ldots\ldots (23a)$$
$$W_a(\xi) = 0 \qquad 0 \le \xi < \infty \ldots\ldots\ldots (23b)$$

where,

$$k_1{}^r, k_2{}^r = \frac{\omega_0}{c}\left[\frac{\alpha(\zeta+ir) \pm \sqrt{(1-r^2) - \alpha^2(1-\zeta^2) + 2i\zeta r}}{(\alpha^2-1)}\right], \qquad r = \frac{\Omega}{\omega_0},$$
$$\zeta = \frac{\bar{C}}{2\sqrt{\bar{K}\mu}}$$

4. Results and Discussion

All The responses of the string are calculated. By differentiating the different expressions of displacement obtained in equations(18)-(23) and then replacing those in equation (9) for ξ=0 and after simplification one gets,

$$F_0 = \frac{\sqrt{(1-r^2) - \alpha^2(1-\zeta^2) + 2i\zeta r}}{\alpha(\zeta+ir)}\left[\frac{2T(\alpha^2-1)}{\mu_k} + \frac{cv}{\sqrt{c^2v^2 - 4T(\alpha^2-1)\bar{K}}}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

The values for different parameters for pantograph catenary system is given as,

$F_0 = 180$ N, $C = 50$ N-sec/m, $K = 1500$ N/m, $\mu = 2.231$ kg/m

$\bar{K} = \frac{K}{d}$ & $\bar{C} = \frac{C}{d}$

Where, d = distance between droppers=7.1428 m.

The dependency between r and α is plotted in Fig. 2(a) and 2(b) for different values of tension T.

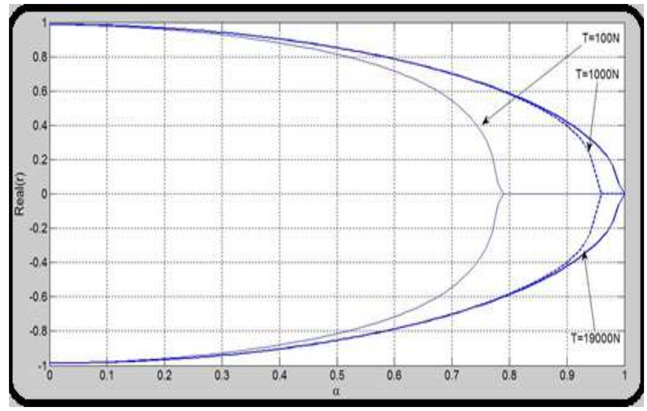

Fig. 2(a) Fig. 2(a): Real(r) v/s α for different values of T.

Figure 2(a) shows the real part of r goes to 0 after a particular value of α. This value of α increases as we increase the value of axial tension T. The real value of r indicates fluctuation in the string, while the imaginary part gets multiplied with the amplitude of string displacement. The variation of imaginary part with α is shown in fig. 2(b).

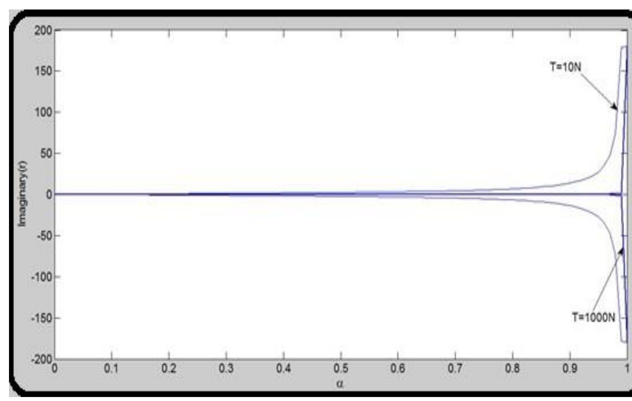

Fig. 2(b): Imaginary(r) v/s α for different values of T.

It is observed that with the increase in α, imaginary part of r increases very slowly. At critical velocity (α=1) it is very much high as shown in Fig. 2(b), due to resonance. As we decrease the value of axial tension T the point of resonance shifts towards the lower value of α, which may be happening due to the effect of friction. This concludes that higher value of tension eliminates the effect of friction at the interface.

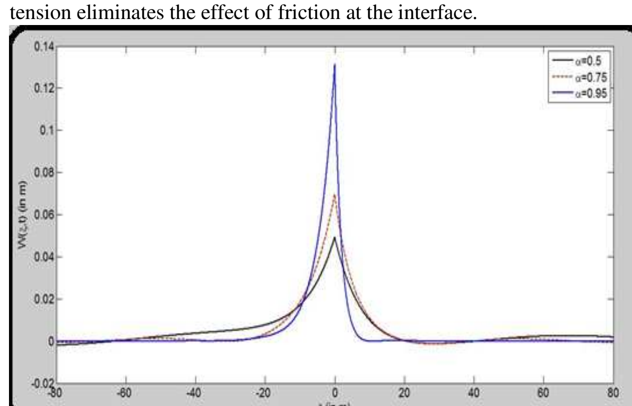

Fig. 3: W(ξ,t) v/s ξ for different values of α (T=19000N).

Figure 3 shows the variation of transverse displacement along with the local co-ordinate system ξ for different value of α and a particular value of T. The displacement is maximum at the point where the moving load is touching the infinitely long taut string. The plot shows that there are two waves propagating ahead and behind of the load. The wave which is propagating ahead of the load is of higher frequency than that of the wave propagating behind the load. This can be verified from the displacement field functions obtained in Eqs. (22)-(23). As we increase the value of α the value of transverse displacement increases and the string profile at the right side of the load becomes more and more stiff. The increase in the string displacement with the increase in α is probably due to dependency between r and α. As α moves towards the critical value, imaginary part of frequency ratio dramatically increases, which subsequently result in high value in string displacement.

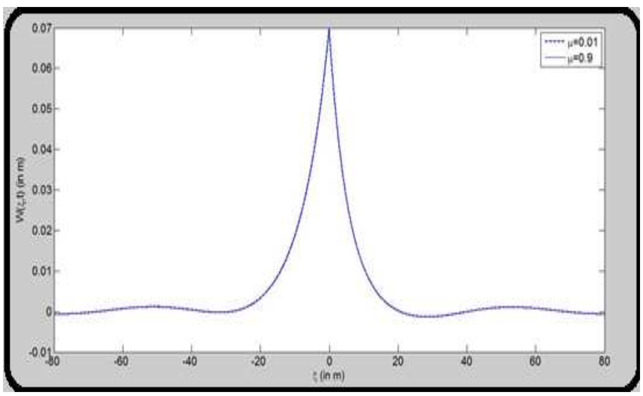

Fig. 4: W(ξ,t) v/s ξ for different values of μ (T=19000N).

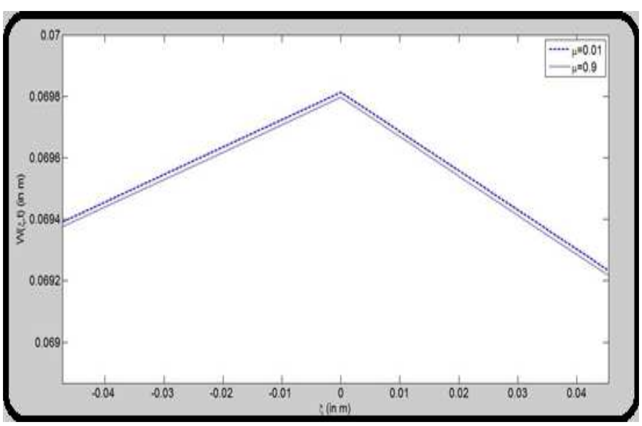

Fig. 5: W(ξ,t) v/s ξ for different values of μ (at T=19000N) (zoomed view).

In Fig. 4 one can see the effect of friction on infinitely long taut string under the action of moving load for a particular axial tension value of 19000N. For two different values of coefficient of friction (μ=0.01 & μ=0.9) the transverse displacement variation is negligibly low which can be seen in Fig. 5 by zooming Fig. 4. This very much approves the conclusion what we set from Fig. 2(b).

Since the effect of friction is very less when high axial tension is applied, the axial tension is reduced to a much lower value to show how string behaves for different friction values. As one increases the value of coefficient of friction, the value of transverse displacement increases as shown in Fig. 6. The value of transverse displacement increases very slowly up to the value of coefficient of friction 0.375 but after that the transverse displacement increases drastically with the increase in co-efficient of friction. The system may become unstable in low axial tension values, and this requires further analysis.

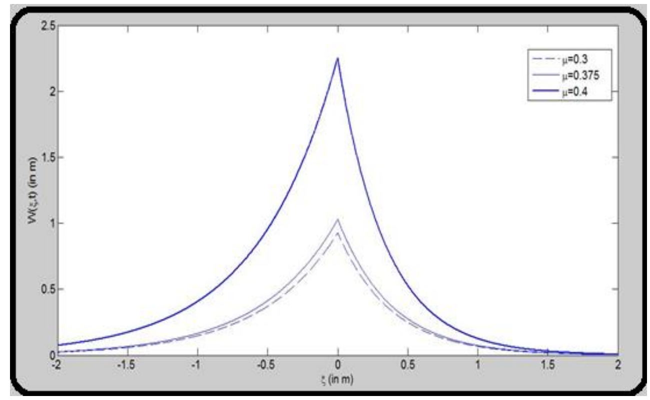

Fig. 6: W(ξ,t) v/s ξ for different values of μ.(at T=100N).

Two waves propagating both sides of the load are not very clear in figure 4 due to high contact load (F_0). Therefore, the figure has been zoomed and plotted for different value of α in Fig. 7. As discussed earlier, the wave numbers of these waves are different. From Eqs. (22)-(23) it can be observed that with the increase in α , the wave number of both waves also increases. The wave propagating ahead of the load generally remains always of higher frequency than the one prapagating behind. This is kind of Poppler effect which is very common in moving sourse problems. However, this well known phenomenon is not observed in Fig. 7(b). This is probably due to dependency of r and α . Since with the increase in α , real part of r decreases (see Fig. 2(a)), the Doppler effect is not clearly observed.

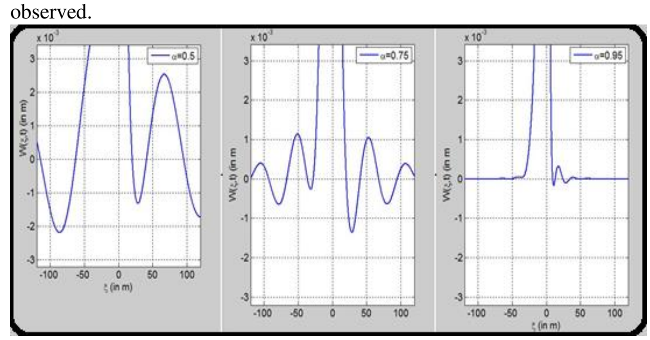

Fig. 7: Wave number representation for different values of α.

5. Conclusion

The interaction between a string on a viscoelastic layer and a uniformly moving load under the action of friction at the interface is modeled in this work. The string is considered as an infinitely long and axially tensioned continuum supported by visco-elastic layer. The forces acting on the string are resolved into friction and normal forces acting tangential and normal to the string, respectively. The equation of motion for the transverse dynamics of string is solved using Green's Function Method. It is found that the frequency ratio is related to the speed ratio. The effect of friction is significantly low when the tension value is high. However, at low axial tension, friction adversely affects the dynamics of the string. The amplitude of displacement suddenly jumps to a higher value after a certain value of friction. This model certainly has some drawbacks. The bending stiffness which plays an important role is absent in the string model. As a result the slope of the continuum at the point of application of the load has become discontinuous. It has also restricted the analysis to subcritical speed regime only. However, this simplicity of the model is indeed advantageous also because it is able to provide us with basic insight to the friction problem which is not emphasized till now, and may help in understanding the couple dynamics of pantograph-catenary system in better way.

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