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Anbuselvi Mathivanan (✉ anbuselvim@ssn.edu.in)

Sri Sivasubramaniya Nadar college of engineering <https://orcid.org/0000-0001-9541-9856>

Prithviraj Prabhu

Sri Sivasubramaniya Nadar college of engineering

Rishab Venkataraman

Sri Sivasubramaniya Nadar college of engineering

Saravanan Palani

Sri Sivasubramaniya Nadar college of engineering

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Emulation of Ternary and Quaternary Logic Quantum Computers By 2-Dimensional Analog Signal Processing

Anbuselvi Mathivanan^{1*}, Prithviraj Prabhu², Rishab Venkatraman³ and Saravanan Palani⁴

^{1,2,3,4} Sri Sivasubramaniya Nadar College of Engineering, Tamilnadu, India

* Corresponding author, e-mail: anbuselvim@ssn.edu.in

Abstract: In this paper, a novel method for the emulation of qutrit and ququart based quantum computers is proposed. The Hilbert space and Bloch sphere representations of qutrits are initially discussed, preceding the emulation of a single qutrit using a classical system. The signal representing the state of the quantum system is defined as the sum of quadrature and in-phase 2-dimensional space-varying signals, which represent the basis states of the quantum system. Focus is mainly conferred on the frequency-domain representation of the signal, due to an innate bijective mapping found between the dimensions in the Hilbert space and the frequencies of the signal. An m -qutrit addressing scheme is then proposed which is used in the application of quantum ternary gates and measurement operations. The proposed theory is then used to simulate a ternary version of the Deutsch-Jozsa algorithm on MATLAB. This paper also includes a section on the extension of the theory to the case of ququarts. It concludes with a brief introduction to a potential system for emulation, and lists algorithms that can be implemented on the proposed system.

Keywords: Quantum Computing; Multi-dimensional Signal Processing; Analog Signal Processing; Qutrit; Ququart ;

INTRODUCTION

The history of quantum computing starts with Benioff's representation of a Turing machine using a quantum mechanical Hamiltonian model (Benioff, 1980). After a decade and a half, Shor and Grover proposed algorithms for integer factorisation and database searching that heralded an unprecedented change in the computing landscape ((Shor, 1994 & Grover, 1996). They proved to be much more powerful than their classical counterparts, with computation complexities polynomial in $\log(N)$ and $\sqrt{N}$. These algorithms were subsequently implemented on NMR (Vandersypen, 2001 & Chuang, 1998), ion trap (Brickman, 2005) and optical systems (Lu, 2007 & Lanyon, 2007). Once the dynamics of quantum computing were understood better, efforts on the simulation (Quantiki, 2015) and emulation of quantum computers were made. The emulation techniques consisted of FPGAs (Ouchi, 2002 & Fujishima, 2003), radiating quantum wires (Ferry, 2001 & Ferry, 2003), and analog circuits (Kish, 2003, & La Cour, 2015). The analog circuits and signal processing techniques have demonstrated tremendous potential for systems containing upto approximately 50-60 qubits

(La Cour, 2015), thus exhibiting quantum supremacy. Also, modern VLSI/ULSI tools augment the potential of analog emulation circuits by allowing for very small silicon footprints.

Since the inception of quantum computing, quantum mechanical systems and algorithms employing qubits have been paid a great deal of attention, mainly due to their relative simplicity and man's history with binary-logic classical computers. However in recent years, qutrit based quantum computation has gained momentum (Mc Hugh, 2005 & Lanyon, 2008). In this paper, emulation techniques are developed for quantum systems with *three* and *four* basis states; qutrits and ququarts. Throughout the paper, focus is accorded to *qutrit* systems, but all the fundamentals can easily be extended to *ququart* systems.

In section 2, the Hilbert space, Bloch sphere and signal representation of qutrits are discussed. Further, single and multi-qutrit addressing is explained. In section 3, the mechanism by which quantum gate operations are performed in the signal domain is detailed, along with the application of measurement gates to obtain outputs. In the next section, a ternary version of the Deutsch-Jozsa algorithm is emulated using the proposed system. The penultimate section deals with the generalisation of the theory to the case of ququarts. Finally in section 6, a recapitulation of the points in the paper is made along with a brief introduction to a method to implement these theories in real systems.

QUANTUM STATE REPRESENTATION

The work begin by representing a single qubit in the two-dimensional complex Hilbert space $H^{(2)}$ (Here, and henceforth, the exponent over H represents the number of basis states of the system), with its two computational basis states $|0\rangle$ and $|1\rangle$ as the two dimensions of the Hilbert space. By definition, the inner product between two vectors (represented by the states $|\psi\rangle$ and $|\phi\rangle$) in this space is denoted by $\langle\phi|\psi\rangle$, and its normalization constraint is $\langle\psi|\psi\rangle=1$ (Griffiths, 2005). The number of dimensions in the Hilbert space of an n -qubit system will scale as 2^n . This is due to the $n-1$ tensor products between the n two-dimensional Hilbert spaces, $H_0^{(2)}, H_1^{(2)} \dots H_{n-1}^{(2)}$ of the individual qubits.

Considering a single qutrit, the Hilbert space $H^{(3)}$ now contains three dimensions corresponding to the three computational basis states $|0\rangle$, $|1\rangle$ and $|2\rangle$. The number of dimensions in the Hilbert space scales as 3^n for a system of n qutrits. This Hilbert space is represented as,

$$H^{(3^n)} = H_{n-1}^{(3)} \otimes \dots \otimes H_1^{(3)} \otimes H_0^{(3)} \quad (1)$$

It is worth noting that transformations in the Hilbert space of the single qubit are defined by the *three* $SU(2)$ matrices, commonly known as the Pauli matrices[20]. However, for a single qutrit,

there exist *eight* $SU(3)$ matrices (Gell-Mann matrices) describing unitary operations on vectors in the 3 dimensional Hilbert space.

The Bloch/Poincaré sphere has grown in popularity as a simple method to visualize a single qubit system (Nielsen, 2010). The pure states that the qubit can exist in, superpositions of the two orthonormal computational basis states of the Hilbert space, $|0\rangle$ and $|1\rangle$, are represented by points on the two-dimensional surface S^2 of the unit sphere in the Euclidean $\Re^3$ space. Operations on these pure states are performed using the matrices in the $SO(3)$ group; distance and origin preserving rotations of the state vectors along the surface of the unit sphere corresponding to geodesics on the S^2 surface. Generalization of the Poincaré sphere to the case of the qutrit results in an S^7 surface in the Euclidean $\Re^8$ space, where points on the surface are subjected to operations belonging to the $SO(8)$ group (Goyal, 2016 & Bertlmann, 2008).

The representation of a single qubit using an analog signal[16] consists of two complex exponentials of frequencies $+\omega$ and $-\omega$, which denote the two computational basis states, $|0\rangle$ and $|1\rangle$.

$$|0\rangle \Rightarrow e^{i\omega t} \quad (2)$$

$$|1\rangle \Rightarrow e^{-i\omega t} \quad (3)$$

This set of frequencies is then shifted by a carrier frequency $\omega_c > \omega$ allowing for real-world implementation of the signal.

$$|0\rangle \Rightarrow e^{i(\omega_c + \omega)t} \quad (4)$$

$$|1\rangle \Rightarrow e^{i(\omega_c - \omega)t} \quad (5)$$

Complex amplitudes are multiplied with these basis frequencies and further normalized to obtain the emulated quantum state of the system.

For a 2-qubit system, a second frequency ω' is chosen such that $\omega' \leq \omega/2$. This frequency is used to produce two signals corresponding to Eq. (2) and (3) as $e^{i\omega't}$ and $e^{-i\omega't}$. Multiplication of these two signals with those of the more significant qubit (represented by $+\omega$) results in 4 frequencies, denoting the 4 basis states of a 2-qubit system ($|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$). A generalization of this theory to n -qubits would result in 2^n frequencies (La Cour, 2015).

When considering the frequency domain representation of the signal, there exist three discrete collinear points $-\omega$, 0 and $+\omega$ on the frequency axis, ω . These can also be viewed as three collinear points on the 2-dimensional frequency plane consisting of the two frequency axes, u and v . Transforming this Cartesian co-ordinate system to a polar co-ordinate system, it can be

observed that there is an angular spacing of π radians between the two basis states, as shown in Fig. 1.

Fig.1: Frequency domain representation of a 1-qubit system showing the three collinear points, $-\omega$, 0 and $+\omega$

In the case of the qutrit, there exists an additional basis state ($|2\rangle$). The extra basis state can be accommodated for by changing the angle between basis states to $2\pi/3$. As the number of basis states increases from 2 to 3, and onwards to 4, the angle between basis states, considering the center point 0 as the origin, varies from π to $2\pi/3$ and finally to $\pi/2$, under the constraint that $u_{\max} = v_{\max}$, where $u_{\max}$ and $v_{\max}$ are the maximum frequencies of the system in the x and y directions respectively.

This paper employs a frequency spacing scheme similar to the one used by Cour and Ott[16], called the octave spacing scheme, but adapted to qutrits. The carrier frequency is chosen as half of the maximum frequency,

$$(u_c, v_c) = \frac{1}{2}(u_{\max}, v_{\max}) \quad (6)$$

The three frequency points (u_0, v_0) , (u_1, v_1) and (u_2, v_2) , representing the coordinates of the three basis states of the qutrit, are displaced from (u_c, v_c) by a distance,

$$r = \frac{1}{2}(u_c, v_c) \quad (7)$$

as illustrated in Fig. 2.

Fig. 2: Frequency domain representation of a **1**-qutrit system. Frequency component (u_c, v_c) represents the carrier frequency of the signal. Frequency components (u_0, v_0) , (u_1, v_1) and (u_2, v_2) represent the $|0\rangle$, $|1\rangle$ and $|2\rangle$ bases.

Conventionally, 1-D signals are represented in the time domain. But due to the absence of a second manipulable time dimension, two spatial dimensions are used to represent these signals. Hence the spatial-domain signal representing the 1-qutrit state $|\psi\rangle$ is defined as

$$\psi(x, y) = \alpha_0 e^{i(u_0 x + v_0 y)} + \alpha_1 e^{i(u_1 x + v_1 y)} + \alpha_2 e^{i(u_2 x + v_2 y)} \quad (8)$$

where $\alpha_0 = a_0 + ib_0$; $\alpha_1 = a_1 + ib_1$; $\alpha_2 = a_2 + ib_2$ are the complex amplitudes of the three basis states. For arbitrarily chosen values of α_0 , α_1 and α_2 as $\alpha_0 = 0.143 + i0.636$, $\alpha_1 = 0.752 + i0.235$ and $\alpha_2 = 0.356 + i0.912$, the spatial domain signal is shown in Fig. 3.

Fig. 3: Spatial-domain representation of a 1-qutrit system with the following parameters: (i) $(u_c, v_c) = (2, 2)$ (ii) $(u_0, v_0) = (2, 3)$ (iii) $(u_1, v_1) = (2.866, 1.5)$ (iv) $(u_2, v_2) = (1.134, 1.5)$ (v) $\alpha_0 = 0.143 + i0.636$ (vi) $\alpha_1 = 0.752 + i0.235$ (vii) $\alpha_2 = 0.356 + i0.912$

The real-valued signal, containing the real and imaginary components of $\psi(x, y)$ can be obtained from either one of the two equations,

$$s(x, y) = \text{Re}[\psi(x, y)e^{i(u_c x + v_c y)}] \quad (9)$$

$$s(x, y) = \text{Im}[\psi(x, y)e^{i(u_c x + v_c y)}] \quad (10)$$

These two equations can also be written as,

$$s(x, y) = \psi_R(x, y) \cos(u_c x + v_c y) - \psi_I(x, y) \sin(u_c x + v_c y) \quad (11)$$

$$s(x, y) = \psi_R(x, y) \sin(u_c x + v_c y) + \psi_I(x, y) \cos(u_c x + v_c y) \quad (12)$$

Following convention[16], Eq. (9) and (11) will be chosen to represent the real-valued signals.

The real and imaginary components of $\psi(x, y)$ include the real and imaginary components of the complex amplitudes, and are represented as,

$$\psi_R(x, y) = \sum_{j=0}^2 [a_j \cos(u_j x + v_j y) - b_j \sin(u_j x + v_j y)] \quad (13)$$

$$\psi_I(x, y) = \sum_{j=0}^2 [b_j \cos(u_j x + v_j y) + a_j \sin(u_j x + v_j y)] \quad (14)$$

The above two equations represent the quadrature (cos) and in-phase(sin) modulating signals. The demodulation can be obtained by multiplying $s(x, y)$ by the quadrature and in-phase signals and in

turn, low-pass filtering them. The low-pass filtering operation can be performed by taking a spatial average over integer-multiples of the largest horizontal and vertical fundamental periods, $X = 2\pi N / u$ and $Y = 2\pi N / v$, as illustrated below.

$$\left(\frac{1}{XY}\right) \int_{x-X}^x \int_{y-Y}^y 2 \cos(u_c x' + v_c y') s(x', y') dy' dx' = \psi_R(x, y) \quad (15)$$

$$\left(\frac{1}{XY}\right) \int_{x-X}^x \int_{y-Y}^y -2 \sin(u_c x' + v_c y') s(x', y') dy' dx' = \psi_I(x, y) \quad (16)$$

If $|\phi\rangle$ represents any other arbitrary 1-qutrit state, where the complex amplitudes of the three basis states are $\beta_0 = a'_0 + ib'_0$; $\beta_1 = a'_1 + ib'_1$; $\beta_2 = a'_2 + ib'_2$; then to complete the Hilbert space description, the inner product between the two states is defined as,

$$\langle \phi | \psi \rangle = \beta_0^* \alpha_0 + \beta_1^* \alpha_1 + \beta_2^* \alpha_2 \quad (17)$$

This inner product is represented in the spatial-domain by a spatial average over the periods X and Y on the product of the two signals $\phi^*(x, y)$ and $\psi(x, y)$,

$$\left(\frac{1}{XY}\right) \int_0^X \int_0^Y \phi^*(x, y) \psi(x, y) dy dx = \beta_0^* \alpha_0 + \beta_1^* \alpha_1 + \beta_2^* \alpha_2 \quad (18)$$

Extending the above system to n-qutrits, it can be observed that for an axis in the Hilbert space $|q_{n-1} q_1 q_0\rangle$, for which the decimal value is $q \in \{0, 1, 3^n - 1\}$, the basis signal is represented as,

$$\phi_q(x, y) = \phi_{q_{n-1}}^{u_{n-1}, v_{n-1}}(x, y) \cdot \dots \cdot \phi_{q_1}^{u_1, v_1}(x, y) \cdot \phi_{q_0}^{u_0, v_0}(x, y) \quad (19)$$

where, $\phi_0^{u_k, v_k}(x, y) = e^{i(u_{k0}x + v_{k0}y)}$; $\phi_1^{u_k, v_k}(x, y) = e^{i(u_{k1}x + v_{k1}y)}$; $\phi_2^{u_k, v_k}(x, y) = e^{i(u_{k2}x + v_{k2}y)}$.

Multiplying a complex amplitude to each of these basis signals and summing them together, the signal representing the state of the system is obtained as,

$$\psi(x, y) = \sum_{q=0}^{3^n-1} \alpha_q \phi_q(x, y) \quad (20)$$

It should be noted that the amplitudes must be normalized to ensure that the normalization constraint is preserved. To obtain a physically implementable signal, the above signal containing all the basis states is multiplied with the carrier frequency using Eq. (9) or (10).

For the system of n qutrits, the value r from Eq. (7) is recursively halved n times and the new frequency points are derived from the frequency points of the (n-1) qutrit system. Following this procedure, one may observe the frequency domain representation of a 2-qutrit system in Fig. 3.

Fig. 4: Frequency domain representation of a 2-qutrit system. The frequencies representing the 9 basis states are $\{1, 2, 9\}$. The angular spacing between the basis states of each qutrit is $2\pi/3$.

The addressing of a single qutrit is relatively straightforward; individual basis states can be accessed by multiplying the conjugate of respective basis signals, and low-pass filtering. When addressing a qutrit in a multi-qutrit environment, certain peculiarities arise that are absent in the single-qutrit environment. Let us define a simple 2-qutrit system, $|AB\rangle$, where A is the most significant qutrit, and B is the least significant qutrit. The frequency domain representation of this system is illustrated in Fig. 3. In order to address qutrit A , the components of the system are rearranged as

$$|\psi\rangle = |0\rangle_A \otimes |\psi_0^{(A)}\rangle + |1\rangle_A \otimes |\psi_1^{(A)}\rangle + |2\rangle_A \otimes |\psi_2^{(A)}\rangle \quad (21)$$

$$\text{where, } |\psi_i^{(A)}\rangle = \sum_{j=0}^2 \alpha_{ij} |j\rangle_B.$$

Thus the sets of frequencies $\{1,2,3\}$, $\{4,5,6\}$ and $\{7,8,9\}$ from Fig. 3 correspond to $|0\rangle_A$, $|1\rangle_A$ and $|2\rangle_A$. Following the case of the single qutrit, one could multiply the conjugates of the respective signals denoting the different basis states, to obtain the 9 complex amplitudes. But this would lead to an exponentially large frequency isolation and recombination process as the number of qutrits increases. Instead, an implementation-optimized method of accessing these sets would be to use a low-pass filter centred at the *centroid* of the sets, with a radius in the frequency plane equivalent to the distance from the centroid to any of the frequencies, as illustrated in Fig. 4. Another method would be to first multiply the conjugate of the signal representing the $|a\rangle_A$ basis, and then apply a low-pass filter centred at the origin.

Fig. 5: Frequency domain representation of qutrit-addressing in a 2-qutrit system.

The circles represent the radii of the low-pass filters that are used to address the $|0\rangle_A$, $|1\rangle_A$ and $|2\rangle_A$ states of qutrit A . The addressing of qutrit B is counter-intuitive to the addressing of qutrit A . The components of the system are now re-arranged as,

$$|\psi\rangle = |0\rangle_B \otimes |\psi_0^{(B)}\rangle + |1\rangle_B \otimes |\psi_1^{(B)}\rangle + |2\rangle_B \otimes |\psi_2^{(B)}\rangle \quad (22)$$

where, $|\psi_i^{(B)}\rangle = \sum_{j=0}^2 \alpha_{ji} |j\rangle_A$.

Now the sets of frequencies corresponding to the three computational basis states $|0\rangle_B$, $|1\rangle_B$ and $|2\rangle_B$ are $\{1,4,7\}$, $\{2,5,8\}$ and $\{3,6,9\}$. Hence to address these 9 frequencies, the following method is used. First, the centroid of the points corresponding to the $|a\rangle_B$ basis of qutrit B is computed. Next, the conjugate of the signal at this frequency is multiplied. Finally, low-pass filters are used to isolate the $|0a\rangle$, $|1a\rangle$ and $|2a\rangle$ states. While performing gate-based operations on qutrits, one must note that individual basis states must be accessed. Hence operations on individual basis states of qutrit B will be restricted to the above mentioned sets. For example, a phase change operation on the $|1\rangle_B$ basis would require an isolation of the $\{2,5,8\}$ set. After isolating this set, the phase change operation is performed, modifying the magnitude of the frequencies in these sets.

Let us now generalize the above to an m -qutrit addressing scheme for a system in the state $|\psi\rangle$ containing n -qutrits, where $m \in \{1, 2, n-1\}$. For $a \in \{0, 1, 2\}$ and $i \in \{0, 1, n-1\}$, the subspace of basis state a of qutrit i is defined as [16]

$$\pi_a^{(n,i)} : \otimes_{j=0}^{n-1} \mathbf{H}_j^{(3)} \rightarrow \otimes_{j=i+1}^{n-1} \mathbf{H}_j^{(3)} \otimes \otimes_{j=0}^{i-1} \mathbf{H}_j^{(3)} \quad (23)$$

In the frequency domain, this subspace will require 3^{n-i-1} low-pass filters for each of the basis states, leading to a total of 3^{n-i} filters for the addressing of one qutrit.

Considering $\{q_c : q_c \subseteq n\}$, as the set of m qutrits that need to be addressed, q_a is the basis state of each addressed qutrit. q_d is then defined as $\{q_d : q_d = n - q_c\}$, the set of unaddressed qutrits. The addressing of the m qutrits is defined by,

$$\left[\prod_{c=0}^{m-1} \pi_{a_{q_c}}^{(n-c, q_c)} \right] |\psi\rangle = \left| \psi_{q_{d_{n-m-1}}} \psi_{q_{d_1}} \psi_{q_{d_0}} \right\rangle \left(a_{q_{c_{m-1}}} a_{q_{c_1}} a_{q_{c_0}} \right) \quad (24)$$

OPERATIONS ON QUTRITS

The operation of an arbitrary m -qutrit gate on a system of n -qutrits, where $m \leq n$ can be modelled as a manipulation of the amplitudes of state vectors in the Hilbert space. Since each basis state maps bijectively onto a particular frequency in the frequency plane, the operation involves a direct manipulation of the complex magnitude of that frequency. A simple mechanism would be to use the qutrit addressing technique, wherein repeated 2-dimensional low-pass filters are used to isolate the frequency sets of the corresponding qutrits.

Let an arbitrary 1-qutrit state $|\psi\rangle$ be defined as,

$$|\psi\rangle = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} \quad (25)$$

Consider, also, an arbitrary unitary 1-qutrit gate A ,

$$A = \begin{bmatrix} A_{0,0} & A_{0,1} & A_{0,2} \\ A_{1,0} & A_{1,1} & A_{1,2} \\ A_{2,0} & A_{2,1} & A_{2,2} \end{bmatrix} \quad (26)$$

If A is assumed to be one of the three qutrit NOT gates(X)[23], then the application of A on $|\psi\rangle$ is described as,

$$X|\psi\rangle = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ \alpha_1 \\ \alpha_0 \end{bmatrix} \quad (27)$$

In our signal representation scheme, this operation would involve a few steps, namely: [(i)]

1. frequency-set isolation, using the qutrit addressing techniques from Sec. 2.3
2. multiplication of gate coefficients to respective frequency sets
3. signal reconstruction by recombination of modified frequency sets.

It would finally result in a manipulation of the complex magnitudes of the respective frequency points. The ternary quantum logic gates that can be implemented by this method include

those specified in (Di, 2011) and (Muthukrishnan, 2000), and any other unitary ternary quantum logic gate.

Measurement has influenced quantum computing in two fundamental ways: one is the extraction of useful information from a quantum state, and the other is the measurement-based model of quantum computing (Briegel, 2009). This paper restricts itself to the specification of the former to the case of qutrits.

A model for making measurements on the system is required which does not involve the complete breakdown of the signal into individual frequencies. This is because it requires a large amount of resources to separate all the individual frequencies and measure the amplitudes, scaling exponentially with the number of qutrits as $O(3^n)$. A different strategy must be chosen, by which observations are made on the density matrix of each qutrit. This would allow us to approximate the value of that qutrit to one of the computational basis states, scaling only as $O(n)$. Examples of various strategies include the Ternary tree, Simulation using hidden variables and Threshold detection.

SIMULATION OF THE TERNARY DEUTSCH-JOZSA ALGORITHM

To verify the proposed theory, a suitable algorithm was required to be emulated. Considerations began with the Deutsch algorithm (Deutsch, 1985), for which a model had been proposed (Tamir, 2008), modifying the algorithm for ternary logic. A simpler algorithm (Gedik, 2015), which still provided exponential speed-up over classical algorithms was also considered. The above algorithms were discarded in favour of the Deutsch-Jozsa algorithm (Deutsch, 1992) due to the fact that they employed just one or two qutrits, whereas the Deutsch-Jozsa algorithm could be emulated for an arbitrary number of qutrits. Although the Deutsch-Jozsa algorithm has already been emulated on analog electronic circuits (Dyson, 2011 & La Cour, 2016), this method restricted itself to using just real components for the quantum states. The proposed approach emulates both the real and imaginary components of the quantum state. A simulation of the processes in the course of this emulation was performed on MATLAB on a personal computer, for a system involving 4 qutrits. Additional qutrits can be added, with the trade-off of lower accuracy and higher run-time. The quantum circuit used for the implementation of this algorithm is given in Fig. 6

Fig. 6: 4 qutrit Deutsch-Jozsa algorithm, where the query register consists of 3 qutrits, and the answer register contains one qutrit.

For better understanding, the algorithm is split into four parts: [(i)]

1. *State initialization* ($|\psi_0\rangle$): A grid of size 499x499 is chosen to represent the state and its corresponding magnitude spectrum. To create the initial state $|0001\rangle$, Eq. (19) and Eq. (20) were used. This resulted in a signal of frequency (264,461). The corresponding magnitude spectrum is shown in Fig. 6.

Fig. 7: Real and Imaginary components of the magnitude spectrum of $|\psi_0\rangle$, where the peak at (264,461) implies that the state is $|0001\rangle$. This positive peak is shown by the yellow dot.

2. *Preparation of input state* ($|\psi_1\rangle$): Next, to prepare the initial state, the ternary Hadamard gate, H, is used to put all the three qutrits in the query register and the one qutrit in the answer register in a superposition of all the possible input states.

$$H = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \quad (28)$$

Where $\omega = e^{i2\pi/3}$, is one of the third roots of unity.

For each individual Hadamard gate, the respective qutrit's basis states are addressed and the gate coefficients are multiplied. Using these modified basis states, the state of the system is then reconstructed by a simple recombination process. The state $|\psi_1\rangle$ is shown in Fig. 7

Fig. 8: Real and Imaginary components of the magnitude spectrum of $|\psi_1\rangle$. The yellow dots signify positive peaks and the blue dots signify negative peaks in the respective spectra.

3. *Implementation of oracle ($|\psi_2\rangle$):* Three matrices were chosen to simulate the implementation of the oracle U_f . They are given below as $I0$, $I1$ and $I2$, and will be implemented on the y qutrit.

$$I0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (29)$$

$$I1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (30)$$

$$I2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (31)$$

Given an input qutrit y , the multiplication of one of these matrices produces $y \otimes f(x)$, where $f(x)$ is 0 for $I0$, 1 for $I1$, and 2 for $I2$. The $\otimes$ operation is performed *modulo 3*. In this implementation of

the Deutsch-Jozsa algorithm, it is promised that the output of the oracle is either constant or balanced, and hence there exist two cases for the oracle.

(a) *Constant*: In this case, any one of the above three matrices is applied to all the states of the y qutrit.

(b) *Balanced*: When the oracle has a balanced output, $f(x)$ must yield 0, 1, and 2 in equal proportion. To simplify the implementation of the oracle, I_0 is applied to $|0\rangle_y$, I_1 is applied to $|1\rangle_y$ and I_2 is applied to $|2\rangle_y$.

4. *Output state ($|\psi_3\rangle$)*: To obtain the final state before measurement, a Hadamard gate is applied to each of the three query register qutrits. In the constant case, measurement on the query register should yield all 0s on the three qutrits. The above implementation of the algorithm does in fact indicate that all the power in the magnitude spectrum lies in the frequency range of $|000\rangle$, as shown in Fig. 8.

Fig. 9: Real and Imaginary components of the magnitude spectrum of $|\psi_3\rangle$, where the oracle produces *constant* output.

The three non-zero frequency points in the above illustration denote the $|0000\rangle$, $|0001\rangle$ and $|0002\rangle$ basis states. As before, the yellow dots signify positive peaks and the blue dots signify negative peaks in the respective spectra.

In the balanced case, the power in the $|000\rangle$ frequency region vanishes, and is distributed across the rest of the spectrum. Hence a measurement will now yield a value other than $|000\rangle$. In Fig. 9, a residual amount of power is left in the $|000\rangle$ frequency region. This can be attributed to inaccuracy in the performance of the signal to frequency domain conversion process.

Fig. 10: Real and Imaginary components of the magnitude spectrum of $|\psi_3\rangle$, where the oracle produces *balanced* output.

Measurement now yields at least one qutrit in the state $|1\rangle$ or $|2\rangle$. As before, the yellow dots signify positive peaks and the blue dots signify negative peaks in the respective spectra.

QUQUARTS

The Bloch sphere representation of the ququart will require an S^{14} sphere in an $\mathfrak{R}^{15}$ Euclidean space. The generalization of the Hilbert space for a ququart will contain 4 computational basis states, viz., a 4-dimensional Hilbert space, with transformations U belonging to $SU(4)$. It is important to note here that a single ququart system has the same transformations ($SU(4)$) as a 2-qubit system ($SU(2^2)$), whose Hilbert space also contains 4 dimensions. A single ququart can be represented in the frequency domain in a manner similar to the qutrit representation, by taking the angle between the frequency points as $2\pi/4$.

The single frequency axis in the qubit is considered to contain g discretized frequency points, with the ability to emulate g basis states. It may hence be observed that the frequency-plane (2 dimensions) now contains space for g^2 frequency points, directly implying that it is possible to emulate quantum systems containing g^2 orthogonal basis states, under maximal packing of points in the frequency plane. But this can only exist in ideal conditions.

The suggested frequency spacing scheme fails in the cases of qudits where d is greater than 5, and the number of qudits is greater than 2. Here, qudit addressing fails because low-pass filters cannot be used to access the frequencies of a particular basis state. This is because frequencies corresponding to undesired basis states also get coupled into the low-pass filter's passband region. In the special case where d is a multiple of 6, multiple basis states lie on the same frequency coordinates, and so the information associated with the complex amplitudes is lost.

CONCLUSIONS

To recapitulate, this paper discussed the representation of qutrits in the Hilbert space, the Bloch sphere, and using two dimensional analog signals. It has also illustrated the isolation and recombination of the signals representing qutrits, and has discussed its results; in the application of gates, and measurement of final states. Finally the theory has been generalized to ququarts, elucidating the scaling of the Hilbert space and the usable frequency points in the frequency plane. Due to the extreme parallelism offered by simultaneous operation on all the basis states in the Hilbert space, this technique can effectively be used to perform faster calculations than the digital computer in use today. If a mere 10 qubits can offer performance comparable to that of a 1GHz personal computer[31], the potential of qutrit and qudit based implementations implies a quicker end to the era of supercomputers. Considering spatially varying signals, if one could acquire a screen of radius 3 metres, discretizing it with a pixel density of approximately 173.622 pixels/centimetre(441 ppi), a system of 14 qutrits can effectively be emulated. If the maximum dimensions of the screen are increased to the radius of the observable universe($46.5e+9$ light years), and the maximum resolution is increased to one Planck length($1.6e-35$ metres), the number of qutrits that can be emulated rises to 204. The implications of this are that this method can be used to show quantum supremacy. Quantum supremacy requires a system of 50 qubits, which in essence implies approximately $2^{50}=1.126e+15$ basis states. If ternary logic is used, a mere 32 qutrits are required, and if quaternary logic is used, a paltry 25 ququarts are required.

Now, a potential physically implementable emulation model is introduced. Consider a cathode ray tube, which deflects an accelerated beam of electrons in two dimensions as a result of the application of electric and magnetic fields. When alternating currents are applied to the drivers of the electric and magnetic fields, an output image that represents the state of the quantum system is produced. Potential areas for further research are in the implementation of gates on the above system, and the application of measurement techniques to obtain verifiable outputs.

Finally, it is worth noting that this model allows for the emulation of non-linear quantum mechanics, suggesting polynomial-time solution for NP-complete and #P algorithms (Abrams, 1998). Other examples of algorithms that can be run on our emulated quantum computer include ternary versions of Shor's quantum factoring algorithm (Bocharov, 2016 & Wei, 2015).

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