The field meaning of wave function

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Article

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[Abstract]

Although this paper is the inheritance and development of Copenhagen quantum mechanics theory, it is a brand-new theory, because although quantum mechanics has achieved great success, its physical significance still needs to be explored. In this paper, the concept of generalized field and generalized quantity is introduced. From the behavior of generalized field in potential well, the conclusion that generalized field exists with energy in the form of standing wave in potential well is obtained. In this paper, only one physical model is used: **"the generalized field forms wave, which is wave function"**. With a **basic assumption** of $mv\lambda=h$, the Einstein de Broglie relation is derived, and the wave function has the meaning of generalized field. Every conclusion given in this paper has clear and obvious physical significance, which makes quantum mechanical problems simple and clear. At the same time, the new atom model is established, and the problems of electron transition, electron spin, electron emission and absorption are discussed.

[Introduction]

Quantum mechanics is an important achievement in the research of theoretical physics in the past hundred years, and now it is also an important topic in the research of theoretical physics. Its basic research is the crystallization of collective wisdom. Since the beginning of quantum mechanics, although its conclusions are in good agreement with experiments, the explanation of this theory is still worth exploring. So far, it is generally accepted that born's statistical interpretation of wave function.

However, this interpretation of wave function has been opposed by some people, especially the physicists who founded quantum mechanics, such as Einstein, de Broglie, Schrodinger, etc. Einstein always believed that "quantum mechanics is incomplete". Dirac also said, "I can't accept that the basis of quantum mechanics is correct."

All in all, the argument is not over. In this paper, the reason and essence of wave are found, **generalized fields form waves, which are wave functions**. It is a clear explanation of the physical meaning of wave function by strict mathematical derivation, so as to solve the essential problem of wave function.

Compared with matter wave and light, the problem of light has been studied clearly in Maxwell's equation of electrodynamics. Then the quantum mechanics of matter wave should be studied as follows:

Light→electrodynamics→Maxwell equation→electromagnetic wave→electric field and magnetic field→wave of field→electromagnetic wave itself→real physical existence.

Matter→quantum mechanics→Schrodinger equation→matter wave→field (generalized field)→wave of field→matter wave itself→real physical existence.

It can be seen that matter waves, like light waves, are real physical existence as well as matter itself. Light wave is the alternating wave of electric field and magnetic field in space (that is, the wave of field). Then, matter wave is also the wave of field (generalized field)!

I. The reason for the fluctuation of micro matter

When we think about the energy of the electric field E and the magnetic field B of the electromagnetic wave, we will find that the expressions of various energy density are similar to the form of kinetic energy. Such as electric field energy $w_e = \frac{1}{2}\epsilon\vec{E}^2$, magnetic field energy $w_b = \frac{1}{2}\frac{1}{\mu}\vec{B}^2$, kinetic energy $w_k = \frac{1}{2}m\vec{v}^2$.

For the following description, we use **generalized field** $\Phi(\vec{r},t)$ and **generalized quantity** M to call two of them. The energy density of electric field energy, magnetic

field energy, vibration energy, thermal energy, kinetic energy, etc. and the total energy of the space covered by the generalized field are all in this form, which can be written into these two formulas in a unified way:

$$\text{Energy density: } w = \frac{1}{2}M|\Phi(\vec{r},t)|^2 \quad (1)$$

$$\text{Total energy: } W = \frac{1}{2}M\int|\Phi(\vec{r},t)|^2d\tau \quad (2)$$

By analyzing the **electric field** and **kinetic energy**, we can find out the cause of **the fluctuation of the micro materials**.

There is a potential barrier formed by the energy generated by a generalized field $\Phi(\vec{r},t)$ and its corresponding generalized quantity M in a certain space region. The following is an example of electric field and momentum field to illustrate the relationship between another same (or different) generalized field and potential barrier.

1. The electric charge of Q forms an electric field barrier in space. Another charge of q ($q < Q$) moves closer to Q from a distance, and the electric field of q will affect the electric field distribution of Q . when q and Q are the same sign, the electric field of both ends will become stronger, and the middle area will become weaker, which is equivalent to the superposition effect of the electric field of q passing through the barrier to the other end of Q , and part of it will be reflected back by the barrier, but for the barrier, the electric field of q is "negative". When $Q \rightarrow \infty$ (or $Q \gg q$), the barrier is infinitely high. At this time, the electric field of q does not affect the electric field distribution of Q , that is to say, the electric field cannot pass through the infinitely high barrier, but is all reflected back. The infinite barrier plays a role of shielding the external field.

A potential well is formed when there is a low barrier between the two high barriers, such as a charged conductor box (as in the case of electrons in a metal). When a moving charge moves in a potential well, its **electric field will reflect back and forth in the potential well to form an oscillating standing wave, which is equivalent to a resonant cavity. If the depth of the potential well is limited, the oscillating electric field will pass through the wall of the well to form a traveling wave, and a plane wave will be formed in the field free zone. When the micro particles are not emitted, they are moving in a potential well formed by surrounding materials, so the generalized field generated by them oscillates to form a wave, and a plane wave is formed after emission. This is the reason of "micro particle has wave property" in quantum mechanics.**

2. The penetration of momentum field (or velocity field) in kinetic energy barrier can be discussed by the collision phenomenon of two balls. The velocity of two balls M and m are V and v respectively, and the kinetic energy of $M > m$ is large, then M balls form a potential barrier. When two balls collide in the same direction, V increases, v decreases or reverses. When the two balls collide in reverse, V is "negative" relative to the potential barrier. After collision, V decreases and v decreases or reverses. In both cases, the momentum field (or velocity field) of the ball passes through the barrier, and part of the field is reflected by the barrier and overlapped with the original field. When $M \rightarrow \infty$ (or $M \gg m$), the full speed rebound of m ball after collision does not affect the momentum of M ball, which is equivalent to that the momentum field cannot pass through the infinite high kinetic energy barrier. When the oscillator with initial kinetic energy moves in the kinetic energy potential well, it is bounced back and forth by the well wall. The magnitude and direction of its momentum change periodically, and it oscillates to form standing wave or traveling wave.

Obviously, from formula (1) or (2), it can be seen that the barrier can only block or shield the corresponding generalized field (force), but has no effect on other generalized fields. Because only with the corresponding generalized quantity can the generalized field produce the effect of energy and force. In addition, the generalized field $\Phi(\vec{r}, t)$ is a function of space and time. As long as there is movement of matter, all kinds of generalized fields of matter will be excited in a wave state, that is, **the function of generalized field or momentum**. Therefore, "if the mechanical quantity $\vec{F}$ in quantum mechanics has corresponding mechanical quantity in classical mechanics, then the operator $\hat{F}$ representing the mechanical quantity is obtained by replacing $\vec{P}$ with operator $\hat{P}$ in classical expression $\vec{F}(\vec{r}, \vec{P})$." In the above example, the generalized field runs through the high barrier.

On this basis, a unique **quantum mechanical physical model** can be established:

Generalized fields form waves, which are wave functions.

II. Derivation of Einstein de Broglie relation

From this physical model, every conclusion has clear and obvious physical significance. Now, a vibrator with a mass of m is oscillated periodically at the frequency ν_0 in an infinite deep kinetic energy potential well with a width of $\lambda_0/2$. The momentum wave forms a wave packet with a wavelength of λ_0 .

A **basic assumption** is introduced:

$$m\nu\lambda = h \quad (3)$$

Let the kinetic energy of the oscillator be constant, and $\nu = \lambda_0\nu_0$, $c = \lambda\nu$. Then get

1. Energy and momentum of light wave, that is, **Einstein de Broglie relation of light wave**:

$$E_c = mc^2 = (mc\lambda) \frac{c}{\lambda} = h\nu = \hbar\omega \quad (4)$$

$$P_c = mc = (mc\lambda) \frac{1}{\lambda} = \frac{h}{\lambda} = \hbar\vec{k} \quad (5)$$

2. Energy and momentum of matter wave, that is, **Einstein de Broglie relation of matter wave**:

Energy is the sum of kinetic energy and potential energy $E = m_0\nu^2$

$$E_0 = m_0\nu^2 = (m_0\nu\lambda_0) \frac{\nu}{\lambda_0} = h\nu_0 = \hbar\omega_0 \quad (6)$$

$$P_0 = m_0\nu = \frac{E_0}{\nu} = (m_0\nu\lambda_0) \frac{1}{\lambda_0} = \hbar\vec{k} \quad (7)$$

because: $m\lambda = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \lambda_0 \sqrt{1-\frac{v^2}{c^2}} = m_0\lambda_0$

therefore: $h = m_0\nu\lambda_0 = m\nu\lambda$ (3')

That is to say, the Planck constant h is constant in any inertial system. Since the value of Planck constant h is very small, and it is known from formula (3), the wavelength λ is very small in macroscopic, so it does not show volatility unless the mass is very small (such as electromagnetic wave).

III. Standing wave condition

Then we consider that the oscillator obtains the kinetic energy from the well wall and increases the frequency. In waves, we have known that only standing waves can exist stably, but **to form standing waves, the number of wave packets must be an integer**. The **standing wave condition** is obtained

$$n \frac{\lambda}{2} = l, (n = 1, 2, 3 \dots) \quad (8)$$

Where l is the width of potential well and $\lambda/2$ is the degree of wave packet linearity (because there are two wave packets in one wavelength). This condition only applies to the system where the generalized field is uniform within the width of potential well, because the energy in each wave packet with the same size is equal. If the generalized field is not uniform, the standing wave condition should be Sommerfeld's **general principle of quantization**:

$$L = \oint P dq = (n - \frac{1}{2})h, (n = 1, 2, 3 \dots) \quad (9)$$

If the width of potential well l is $\lambda/2$ (i.e. the length of a standing wave packet), the wavelength of n wave packets formed after the oscillator absorbs energy is obtained by the formula of standing wave condition (8) :

$$\lambda_n = \frac{\lambda}{n} \quad (10)$$

By substituting the Einstein de Broglie relation, we can get:

$$P_n = \frac{h}{\lambda_n} = n \frac{h}{\lambda} = n\hbar\vec{k} \quad (11)$$

$$E_0 = cP_n = n\hbar\nu = n\hbar\omega \quad (12)$$

That is to say, **the energy and momentum of the oscillator can only be changed by the integral times of $\hbar\omega$ and $\hbar\vec{k}$, and the frequency can also be changed by the integral times of ω** .

That is to say, **energy can only be emitted or absorbed one by one**, which is the essence of the quantum hypothesis of energy, which was first discovered by Planck when he studied black body radiation. This can be regarded as a **generalization of Einstein de Broglie relationship**.

Microscopic particles can be regarded as being in the potential well formed by surrounding materials when they are not emitted, so as long as there is energy, they will form oscillating standing wave and absorb or radiate energy

quantum (in the form of light wave). When the particles are emitted, they form free particles, and form a traveling plane wave with a certain frequency, which carries a certain amount of energy and momentum. Therefore, **volatility is a common phenomenon in the micro system.**

IV. The field meaning of wave function

What is the relationship between generalized field and wave function in this physical model? What is the probability meaning of wave function? What is the wave function?

If the generalized field of a oscillator in the system forms n wave packets, then its energy and momentum are $n\hbar\omega$ and $n\hbar\vec{k}$ respectively. That is to say, each wave packet is equivalent to a quasiparticle with energy and momentum of $\hbar\omega$ and $\hbar\vec{k}$ respectively, which is called an energy quantum, and its range is the volume V of a wave packet.

It is the energy produced by the generalized field of the system in V . Then the total energy of the system and the total energy of the system are as follows:

$$W = \frac{1}{2} M \int |\Phi(\vec{r}, t)|^2 d\tau = n\hbar\omega \quad (13)$$

$$dw = \frac{1}{2} M |\Phi(\vec{r}, t)|^2 d\tau = \hbar\omega dn = \hbar\omega \frac{dn}{d\tau} d\tau \quad (14)$$

The integral of this formula and the comparison of the two formulas are as follows:

$$\left| \frac{\Phi(\vec{r}, t)}{\sqrt{\int |\Phi(\vec{r}, t)|^2 d\tau}} \right|^2 = \frac{w}{W} = \frac{1}{n} \frac{dn}{d\tau} = |\Psi(\vec{r}, t)|^2 \quad (15)$$

This formula can also be derived from the derivation in turn, and can also be obtained from the statistical (probability) interpretation of the wave function. n test particles (one energy quantum) with energy and momentum of $\hbar\omega$ and $\hbar\vec{k}$ are put into the system to test the probability distribution of particles in the system. According to the probability knowledge, n particles will determine the probability distribution according to the wave function. That is to say, the number of particles allocated in unit volume element $d\tau$ at time $\vec{r}$ of t is:

$$dn = n |\Psi(\vec{r}, t)|^2 d\tau \quad (16)$$

If the energy of each test particle is, the internal energy is:

$$dw = \hbar\omega dn = n\hbar\omega \cdot \frac{1}{n} \frac{dn}{d\tau} \cdot d\tau = W |\Psi(\vec{r}, t)|^2 d\tau \quad (17)$$

By integrating this formula, we can get:

$$|\Psi(\vec{r}, t)|^2 = \frac{1}{n} \frac{dn}{d\tau} = \frac{w}{W} = \left| \frac{\Phi(\vec{r}, t)}{\sqrt{\int |\Phi(\vec{r}, t)|^2 d\tau}} \right|^2 \quad (15')$$

Where $\frac{1}{n} \frac{dn}{d\tau}$ is the probability of the occurrence of the unit volume internal oscillator at $\vec{r}$ at t time. This is the significance [2] the square $|\Psi(\vec{r}, t)|^2$ of the wave function in the probability interpretation of the wave function. $\Psi(\vec{r}, t)$ is the normalized wave function of the system. From this important relation (15), we can draw the following conclusions:

$|\Psi(\vec{r}, t)|^2$ is the ratio w/W of the energy of the system to the total energy of the system in the unit volume at the

time of t , $\vec{r}$. The generalized field $\Phi(\vec{r}, t)$ is the wave function of the system without normalization.

Let Φ (or ϕ) be the wave function without normalization, then there will be $\Psi = \frac{1}{\sqrt{\int |\Phi|^2 d\tau}} \Phi = \sqrt{\frac{M}{2W}} \Phi$, where $\frac{1}{\sqrt{\int |\Phi|^2 d\tau}} = \sqrt{\frac{M}{2W}}$ is the normalization constant and Ψ is the wave function after normalization. Obviously: $\int |\Psi(\vec{r}, t)|^2 d\tau = 1$.

It can be seen from the above important relations that the statistical significance of wave function is to treat energy quantum as a particle particle, in fact, it has volume and size, **which is naturally seen that its volume is the volume of a wave packet.**

In the micro world, the scale of matter and space is already very small, and the speed of motion is very fast. If we still deal with it in a macro way and ignore its volume size as a particle, then the position of each particle in the wave packet range is "uncertain", and the degree of uncertainty is the range of "uncertainty relationship".

In addition, when the oscillator with the total energy of the system is regarded as a particle, its position is in n wave packets, which is only the significance of probability. If it is regarded as the energy generated by the generalized field dispersed in the whole space and its corresponding generalized quantity, its volume is the space of the generalized field distribution of the system, and its mass is obtained from the mass energy equation. Therefore, **matter can be regarded as energy or generalized field.** In this way, the concepts of "material density" and "energy density" can have practical significance. If matter is regarded as a particle, these concepts are meaningless, and only the concept of "probability density" can be replaced. Now we see that the wave function is the field (generalized field), and half of the product of the square of the wave function and the generalized quantity is the energy density!

V. Superposition, orthogonality and equivalence of wave functions and their physical significance

1. The generalized field is not only a strength quantity, but also a directional extensional quantity, so it has the property of superposition. The generalized field is a wave function, and the wave function also has the property of superposition. **If the system has multiple similar generalized field sources, the generalized field at any point is the vector superposition of multiple generalized field sources at this point,** that is

$$\Phi = c_1 \phi_1 + c_2 \phi_2 + \dots + c_n \phi_n = \sum_n c_n \phi_n \quad (18)$$

In fact, the above phenomenon of barrier penetration of two charges and two spheres is the manifestation of superposition of generalized fields, rather than particles passing through the high barrier.

2. Orthonormalization: two wave functions ψ_k and ψ_l satisfy the relation:

$$\int \psi_k^* \psi_l d\tau = \delta_{kl} = \begin{cases} 1, & (\text{When } k=l) \\ 0, & (\text{When } k \neq l) \end{cases} \quad (19)$$

Here, its physical meaning is very obvious: **only the generalized field and its corresponding generalized quantity can produce the effect of energy.**

The proof is as follows, because only when $k=l$ can

there be

$$\begin{aligned} W &= \frac{1}{2} M_k \int \phi_k^* \phi_k d\tau = \frac{1}{2} M_l \int \phi_l^* \phi_l d\tau = \frac{1}{2} M \int \phi^* \phi d\tau \\ &= \frac{1}{2} M \cdot \frac{2W}{M} \int \left(\sqrt{\frac{M_k}{2W}} \phi_k^* \right) \cdot \left(\sqrt{\frac{M_l}{2W}} \phi_l \right) d\tau \\ &= W \int \psi_k^* \psi_l d\tau \end{aligned}$$

That is to say, when $k=l$, $W=W$, $\int \psi_k^* \psi_l d\tau = 1$ (can form energy); when $k \neq l$, $W=0$, $\int \psi_k^* \psi_l d\tau = 0$ (can't form energy).

3. Equivalence principle: when different generalized fields act on a system at the same time, the total energy can be equivalent to the energy formed by any generalized field and its corresponding generalized quantity. Its expression:

$$W = \frac{1}{2} M \int \Phi^* \Phi d\tau = \sum_n W_n = \sum_n \frac{1}{2} M_n \int \phi_n^* \phi_n d\tau \quad (20)$$

Normalize ϕ to ψ :

$$\frac{M}{2W} \int \Phi^* \Phi d\tau = \sum_n \frac{M_n}{2W} \int \phi_n^* \phi_n d\tau = \sum_n \frac{W_n}{W} \cdot \frac{M_n}{2W_n} \int \phi_n^* \phi_n d\tau$$

$$\text{Namely: } \int \Psi^* \Psi d\tau = \sum_n \frac{W_n}{W} \int \psi_n^* \psi_n d\tau = \sum_n \frac{W_n}{W} = \sum_n |C_n|^2 = 1$$

In this paper, n integral terms in the above formula are combined by quadratic term theorem, where k , $l=1,2,3,\dots,n$, and $k \neq l$ (that is, the 0 added in the following formula):

$$\begin{aligned} \int \Psi^* \Psi d\tau &= \sum_n \int |C_n \psi_n|^2 d\tau + 0 \\ &= \sum_n \int |C_n \psi_n|^2 d\tau + \sum_n 2 |C_k^* C_l| \int \psi_k^* \psi_l d\tau \\ &= \int (\sum_n |C_n \psi_n|)^2 d\tau \end{aligned}$$

Another expression of the equivalence principle can be obtained: **ψ can be expanded into series $\Psi = \sum_n C_n \psi_n$ according to ψ_n .** This is the essence of completeness. The relations $|C_n|^2 = \frac{W_n}{W}$, $\sum_n |C_n|^2 = 1$ and $C_n = \int \psi_n^* \Psi d\tau$ can also be obtained.

This formula indicates: $|C_n|^2$ represents the proportion of energy generated by the n generalized field in the total energy of the system.

Here, its square is also the proportion of energy formed by one of the generalized fields in the total energy of the system w/W ; it also represents the proportion of energy in the total energy of the system w/W in the unit volume at time $t \vec{r}$.

VI. Typical application examples

By using standing wave condition, only a few algebraic operations are needed to solve some typical problems and other parameters.

1. One dimensional infinite potential well

The oscillator^[3] in a one-dimensional infinite potential well with a width of $2a$ obviously has only kinetic energy and no energy change. The wavelength $\lambda_n = \frac{4a}{n}$ can be obtained from the standing wave condition (8), then the energy level $E_n = \frac{P^2}{2m} = \frac{4\pi^2 \hbar^2}{2m \lambda_n^2} = \frac{\pi^2 n^2 \hbar^2}{8ma^2}$, the velocity

$$v_n = \pm \sqrt{\frac{2E}{m}} = \pm \frac{\pi n \hbar}{2ma} \quad \text{and} \quad \text{the frequency}$$

$$v_n = \frac{v}{\lambda} = \frac{\pi n^2 \hbar}{8ma^2} \quad \text{can be obtained. Then the amplitude}$$

$A_n = \frac{4a}{\pi n} = \frac{1}{\pi} \lambda_n$ can be obtained from the vibration energy $E_n = \frac{1}{2} m \omega^2 A_n^2$. The oscillator can be regarded as a rotating body of sinusoidal vibration, then its volume $V = \int_0^\pi \pi (A_n \sin \theta)^2 d\theta = \frac{\pi^2 A_n^2}{2} = \frac{8a^2}{n^2}$ can be obtained. Because the potential well is infinitely deep, the energy level is independent of the potential well depth, so there is no orbital radius, and the energy level can be stable at any depth.

2. One dimensional linear harmonic oscillator

The equation of motion of one-dimensional linear harmonic oscillator is $\ddot{X} + \omega^2 X = 0$, and its solutions are $X = A \cos(\omega t + \delta)$ and $\dot{X} = -\omega A \sin(\omega t + \delta)$.

There are many solutions. Since the generalized field X or $\dot{X}$ is variable, the general rule of quantization (9) is used, and the maximum kinetic energy is the total energy

$$\oint P dq = m \oint \dot{X}^2 dt = \frac{1}{2} m \omega^2 A^2 \cdot 2 \int_0^{\frac{2\pi}{\omega}} \sin^2(\omega t + \delta) dt = (n - \frac{1}{2}) \hbar,$$

The energy level $E_n = \frac{1}{2} m \omega^2 A^2 = (n - \frac{1}{2}) \hbar \omega$ can be obtained by integrating in $0 \rightarrow \frac{2\pi}{\omega}$ period a , and the position (wave function turning point coordinate) $X_n = \pm \sqrt{\frac{2(n - \frac{1}{2}) \hbar}{m \omega}}$ and velocity $v_n = \dot{X}_n = \pm \sqrt{\frac{2(n - \frac{1}{2}) \hbar \omega}{m}}$ can be obtained by other solutions.

It can be seen that the **amplitude, energy level radius and position are equal** to $v_n = \omega X_n = \omega r_n$, where X_n is equal to the energy level radius r_n . Combining the energy level $E_n = \frac{1}{2} m \omega^2 A^2 = (n - \frac{1}{2}) \hbar \omega$, the amplitude

$$A_n = r_n = X_n = \pm \sqrt{\frac{2(n - \frac{1}{2}) \hbar}{m \omega}}$$

wavelength $\lambda_n = \frac{2\pi}{\omega} v_n = 2\pi A_n = \pm 2\pi \sqrt{\frac{2(n - \frac{1}{2}) \hbar}{m \omega}}$ can be

obtained. If a harmonic oscillator can be regarded as a rotating body with sinusoidal vibration, its volume is

$$V = \int_0^\pi \pi (A_n \sin \theta)^2 d\theta = \frac{\pi^2 A_n^2}{2} = \frac{\pi^2 (n - \frac{1}{2}) \hbar}{m \omega}.$$

It can be seen that the wavelength is 2π times of the amplitude $\frac{\lambda_n}{A_n} = 2\pi$.

3. Hydrogen like atom

Hydrogen like atoms are nuclei or ions with only one electron outside the nucleus^[4]. If the potential energy is zero at infinity, and the reduced mass of the system is $\mu = \frac{mM}{m+M}$, and the number of nuclear charges is Z , then the

equation of motion $\mu \frac{v^2}{r} = \frac{Ze_s^2}{r^2}$, (其中 $e_s = \frac{e}{\sqrt{4\pi\epsilon_0}}$), potential

energy $U = -\frac{Ze_s^2}{r}$ and total energy

$$E = \frac{1}{2} \mu v^2 + U = -\frac{Ze_s^2}{2r} \quad \text{are obtained.}$$

The oscillating electric field propagates along the outer space of the nucleus to form a closed standing wave. The

wavelength $\lambda_n = \frac{2\pi r}{n}$ can be obtained from the standing wave condition (8), and the energy level $E_n = -\frac{\mu Z^2 e_s^4}{2n^2 \hbar^2}$ and the orbital radius (i.e. position) $r_n = \frac{n^2 \hbar^2}{\mu Z e_s^2} = \frac{n^2}{Z} a_0$ can be obtained by substituting the relation $\hbar = \frac{\mu v \lambda}{2\pi}$ into the above equation, then the velocity $v_n = \pm \sqrt{\frac{Z e_s^2}{\mu r_n}} = \pm \frac{Z e_s^2}{n \hbar}$ and the wavelength $\lambda_n = \frac{2\pi n \hbar^2}{\mu Z e_s^2} = \frac{2\pi}{n} r_n$ can be obtained, and then the amplitude $A_n = \frac{2n \hbar^2}{\mu Z e_s^2} = \frac{2}{n} r_n = \frac{1}{\pi} \lambda_n$ can be obtained from the vibration $E_n = \frac{1}{2} \mu \omega^2 A_n^2$.

The ratio of light speed $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ to electron speed $v_n = \frac{Z e_s^2}{n \hbar}$ in vacuum is the fine structure constant $\frac{c}{v_n} = \frac{4\pi n \hbar}{Z e^2} \cdot \sqrt{\frac{\epsilon_0}{\mu_0}} = 137$.

The meaning of r_n is that only when the generalized field with energy E_n (frequency and wavelength are also determined) is in the region with radius r_n , its wave can form standing wave and its frequency can be constant. Such a wave packet can not "gradually expand and disappear", and can form a stable "electron". The generalized field formed by the nucleus and the generalized field are superposed each other, and the motion law of the generalized field should also satisfy the principle of the least energy and the principle of the least action. When the energy changes (i.e. transitions between energy levels), it is emitted and absorbed in the form of a fluctuating electromagnetic field.

In these three examples, the system has symmetry. From the velocity expression, we can see that **the wave formed by the generalized field can propagate in two directions, thus forming the standing wave.**

The standing wave length of matter wave is π times of the amplitude $\frac{\lambda_n}{A_n} = \pi$.

[References]

- [1] Zhou Shixun, quantum mechanics course (Second Edition), higher education press, 2009, 7 pages.
- [2] Zhou Shixun, quantum mechanics course, higher education press, 1979, 20 pages.
- [3] Same as [2], page 34.
- [4] Same as [2], page 64.

VII. Velocity of matter motion and propagation of matter waves

In the above examples, all are analyzed from the perspective of energy, where v_n is the speed of matter. We then analyze the velocity v_n from the wave point of view.

From the point of view of wave, $v = \lambda \nu$ is the wave velocity. In the above examples, matter is in a bound state, and its waves are standing waves. There are two wave packets in a wavelength, each wave packet is a quantum, and the energy of each quantum should be half of the energy of the quantum of two wave packets in a wavelength $\frac{1}{2} h v_n$. Now let's substitute the velocity v_n of matter in the above examples as the propagation velocity of wave into $E_n = \frac{1}{2} h v_n$, and find out the energy of each quantum in each example, and see what we can get.

1. One dimensional infinite potential well:

$$E_n = \frac{1}{2} h v_n = \frac{1}{2} \cdot 2\pi \hbar \cdot \frac{v_n}{\lambda_n} = \frac{1}{2} \cdot 2\pi \hbar \cdot \frac{\pi n \hbar}{2ma} \cdot \frac{n}{4a} = \frac{\pi^2 n^2 \hbar^2}{8ma^2}$$

2. One dimensional linear harmonic oscillator:

$$E_n = \frac{1}{2} h v_n = \frac{1}{2} h \cdot \frac{v_n}{\lambda_n} = \frac{1}{2} h \cdot v_n \cdot \frac{m v_n}{h} = \frac{1}{2} m v_n^2 = \frac{1}{2} m \cdot \frac{2(n-\frac{1}{2})\hbar\omega}{m} = (n-\frac{1}{2})\hbar\omega$$

3. Hydrogen like atom:

$$E_n = \frac{1}{2} h v_n = \frac{1}{2} h \cdot \frac{v_n}{\lambda_n} = \frac{1}{2} \cdot 2\pi \hbar \cdot v_n \cdot \frac{n}{2\pi r_n} = \frac{1}{2} n \hbar \cdot \frac{v_n}{r_n} = \frac{1}{2} n \hbar \cdot \frac{Z e_s^2}{n \hbar} \cdot \frac{m Z e_s^2}{n^2 \hbar^2} = \frac{m Z^2 e_s^4}{2n^2 \hbar^2}$$

This is the energy level in each case, that is, the energy of each quantum. Therefore, **the velocity of matter is the velocity of matter wave.**

In deriving the Einstein de Broglie relation, the wave velocity relations $c = \lambda \nu$ and $v = \lambda_0 \nu_0$ are used, in which c and v are not only the velocity of wave, but also the velocity of matter. This conclusion shows that the **wave described in quantum mechanics is an objective and real physical wave**, not only a "probability wave" with mathematical significance.

Due to the limitation of space, the discussion and deduction of new atomic model and quantum mechanics theory are omitted.