A novel method of prescribed constraint control without initial condition of nonlinear systems

Hui Liu, Xiaohua Li, Xiaoping Liu

Abstract—A novel constraint control strategy without initial condition of constrained variables is investigated based on backstepping technique for nonlinear systems. In this paper, the novel constraint control strategy is presented for a class of strict-feedback nonlinear systems with actuator saturation and external disturbances by using a nonlinear mapping and a novel performance constraint function. In this control strategy, there are two prescribed constraint functions, the design of these functions is not related to the initial conditions of the constrained variables. Unlike the existing constraint control method without initial condition, the proposed method gives a new solution. It can guarantee that the constraint variable gets into a prescribed constraint region from any initial value no later than a setting time. And the setting time is a design parameter, it can be set arbitrarily. A prescribed performance constraint tracking controller is designed in this paper. It can make that the tracking error of the nonlinear system is constrained to a given region no later than the given setting time, and the transient and steady state performance of the system are ensured. Finally, the proposed method is compared with the existing method, the effective and superiority of the proposed method are demonstrated by two practical examples.

Index Terms—Nonlinear systems; prescribed constraint control; nonlinear mapping; performance constraint function; without initial condition.

I. INTRODUCTION

BECAUSE the prescribed performance control can guarantee the transient performance of systems, including overshoot, convergence rate and convergence accuracy, it has attracted a great deal of attentions in control field. In fact, the prescribed performance control can be regarded as a kind of constraint control. At present, the existing constraint control methods include prescribed performance control, funnel control, output constraint control, and so on. And the constraint control is usually designed by barrier Lyapunov function (BLF) method. The types of constraint boundary can be summarized as the constant value constraint (CVC) and the time-varying function constraint (TVFC).

In CVC control design approaches, there are mainly four categories at present. The first is the logarithm-type BLF

method [1]–[7]. The logarithm-type BLF was originally investigated for a class of systems with the Brunovsky normal form in [1]. Inspired by [1], [2]–[7] discussed the control problem on tracking error constraints or state constraints based on the logarithm-type BLFs. The second is the tan-type BLF method [8]–[11]. The third is the integral-type BLF method, see [12]–[15]. They were presented to constrain the tracking errors of systems or the full states of systems. The fourth is the nonlinear mapping method, see [16]–[19]. In this method, a one-to-one nonlinear mapping is introduced in order to achieve the constraint performance of systems.

TVFC control is usually called the prescribed performance control. In this control method, the system tracking error is constrained by two time-varying boundaries. The control idea was firstly proposed in [20]. Generally, in this kind of control strategy, an error transformation function is adopted firstly to transform the original inequality constraint for the tracking error into an equality constraint, then the prescribed performance controller is designed according to the transformed error so that the prescribed performance of the system is guaranteed, see [21] and [22]. At present, the prescribed performance control strategy has been widely used in many researches, such as [21]–[24]. Specially, it should be noted that the funnel control method is actually a prescribed performance control method without the error transformation before control design, see [25], [26].

Although the aforementioned constraint control schemes have satisfactory control performance, they have still serious flaws, that is, the choice of prescribed performance function depends on the initial condition of the constrained variable. In other words, the initial condition of the constrained variable must be known before the system controller is designed. But it is difficult to obtain the initial condition in practical systems. Therefore, the developed prescribed performance control design methods will be difficult to be applied in practice. How to remove the effect of initial condition is an extremely challenging and meaningful topic in constraint control design.

Therefore, in [28]–[31], a class of improved prescribed performance control design schemes, which are independent of the initial condition, were proposed. The control schemes in [28]–[31] circumvented the initial condition of the constrained variables by the similar error transformations. The transformations employed a shifting function or a tuning function to make the transformed error being zero at initial time. Compared with the existed methods, the proposed method in this paper presents a new transformation idea. A nonlinear mapping and a new prescribed performance function are adopted, and the constrained variable is transformed to a prescribed scope at

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initial time rather than to zero. At present, the similar work is not still found. However, actuator saturation is not considered in the existing works [28]–[31].

Actuator saturation is a widespread phenomenon in control systems [32]. It should be mentioned that the actuator saturation in practical systems may limit or degrade the system control performance, even destabilize the system, see [33] and [34]. Therefore, the control research for the systems with actuator saturation is meaningful. In this paper, the actuator saturation problem is considered simultaneously while the novel prescribed constraint control scheme is proposed.

Motivated by the above discussions, this paper proposes a low-complexity and novel constraint control design scheme for nonlinear systems with actuator saturation. The main contributions of this paper are summarized as follows:

1) Compared with the traditional constraint control works [5]–[7], [9]–[11] and [22], a novel prescribed constraint control scheme is presented for a class of nonlinear systems with actuator saturation, which circumvents completely the problem that the traditional performance constraint control schemes depend on the initial conditions of the constrained variables. 2) Unlike the existing works [28]–[31], this paper presents a new constraint idea. A nonlinear mapping and a new prescribed performance function are adopted, and the constrained variable is transformed to a defined scope rather than to zero at initial time. 3) The setting time, at which the constrained variable of the system gets into the prescribed constraint region, is a design parameter and can be set arbitrarily.

The rest of this paper is organized as follows. Section II presents the problem formulation. Section III formulates a novel constraint control idea. The design process for adaptive neural network prescribed performance constraint controller is shown in Section IV. The simulation examples are given in Section V. Section VI gives the conclusion of the paper.

II. PROBLEM FORMULATION

Consider the tracking control problem of a class of strict-feedback systems with actuator saturation and unknown nonlinear functions, and external disturbances. The mathematical model of the system is described by the following differential equation.

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + d_i(t), \\ \dot{x}_n = g_n(x)u(v) + f_n(x) + d_n(t), \\ y = x_1, \end{cases} \quad (1)$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ is the state vector of the system, $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$. And $y \in R$ is the output of the system. v is the designed control input, $u(v)$ denotes the plant input with saturation type nonlinearity. $g_i(\cdot)$ and $f_i(\cdot)$, ($i = 1, 2, \dots, n$) are unknown, continuous and smooth nonlinear functions, $d_i(t)$, ($i = 1, 2, \dots, n$) are the bounded external disturbances. The tracking error of the system (1) is defined as $e_1 = x_1 - y_r$, where y_r is a reference signal. Specially, the system (1) satisfies the following assumptions:

Assumption 1 [32] The reference signal $y_r(t)$ and its derivatives $y_r^{(i)}(t)$, $i = 1, 2, \dots, n$ are known and bounded.

Assumption 2 [35]: The sign of $g_i(\bar{x}_i)$ is known and unchanged. Without loss of generality, it is further assumed as $g_i(\bar{x}_i) > 0$ and there exists an unknown constant b_m such that

$$0 < b_m < g_i(\bar{x}_i), \forall \bar{x}_i \in R^i$$

for $i = 1, 2, \dots, n$.

The saturation model in (1) is asymmetric, it is described as [36]

$$u(v) = u_R \times \operatorname{erf}\left(\frac{\sqrt{\pi}}{2u_R}v\right), \quad (2)$$

where $u_R = (u_+ + u_-) + (u_+ - u_-)\operatorname{sign}(v)/2$; $\operatorname{erf}(\cdot)$ denotes a Gaussian error function (GEF) which can be defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (3)$$

To facilitate the control design later, we define

$$\Delta(v) = u(v) - \lambda v, \quad (4)$$

where λ is an unknown positive constant, see [36], [37]. Hence, the asymmetric smooth model (2) can be written as

$$u(v) = \lambda v + \Delta(v). \quad (5)$$

Assumption 3 [37]: There exist positive constants $\Delta_{\max}$ and $\Delta_{\min}$ such that $\Delta_{\min} \leq \Delta(v) \leq \Delta_{\max}$.

Substituting (5) into (1) gives

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + d_i(t), \\ \dot{x}_n = g_n(x)(\lambda v + \Delta(v)) + f_n(x) + d_n(t), \\ y = x_1. \end{cases} \quad (6)$$

In this paper, the following RBFNN is used to approximate a continuous function $\bar{f}_{nn}(Z) : R^q \rightarrow R$.

$$\bar{f}_{nn}(Z) = W^T S(Z), \quad (7)$$

where $Z \in \Omega_Z \subset R^q$ is the input vector, $W = [w_1, w_2, \dots, w_l]^T \in R^l$ is the weight vector, $l > 1$ is the node number of the neural network, $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]^T \in R^l$ means the basis function vector with $s_i(Z)$ being chosen commonly as Gaussian function defined by the following form.

$$s_i(Z) = \exp\left[\frac{-(Z - \mu_i)^T(Z - \mu_i)}{\eta^2}\right], i = 1, 2, \dots, l, \quad (8)$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{iq}]^T$ and η stand for the center of the receptive field and the width of the Gaussian function, respectively.

Just as [34], [38], the RBFNN (7) can be used to approximate a smooth nonlinear function $\bar{f}(Z)$ over the compact set Ω_Z to arbitrary accuracy as

$$\bar{f}(Z) = W^{*T} S(Z) + \delta(Z), \quad |\delta(Z)| \leq \varepsilon, \quad (9)$$

where $W^* = [w_1, w_2, \dots, w_l]^T \in R^l$ is the ideal constant weight vector. W^* is defined as the value of W that minimizes $\delta(Z)$ for all Z , i.e.,

$$W^* \triangleq \arg \min_{W \in R^l} \left\{ \sup_{Z \in \Omega_Z} |\bar{f}(Z) - W^T S(Z)| \right\},$$

$\delta(Z)$ is the approximation error, ε is an unknown bound constant.

Lemma 1 [40]: Let function $V(t) \geq 0$ be a continuous function defined $\forall t \in R^+$ and $V(0)$ bounded, and $\rho(t) \in L_\infty$ be real-valued function. If the following inequality holds

$$\dot{V}(t) \leq -a_0 V(t) + b_0 \rho(t),$$

where $a_0 > 0$, b_0 are constants, then $V(t)$ is bounded.

The control objective of this paper is summarized as follows.

Design an adaptive neural prescribed constraint tracking controller by using a nonlinear mapping and a novel performance constraint function which make the prescribed performance constraint function be independent of the initial condition of the constrained variable. The designed controller can guarantee that all the signals in the closed-loop system (1) are bounded, and the tracking error $e_1(t)$ is constrained by a prescribed performance constraint function and can converge to a given compact set no later than the given setting time. And the setting time can be set arbitrarily. Specially, the pre-specified constraint performance of the system can't be affected by external disturbances.

III. A NOVEL CONSTRAINT CONTROL IDEA

In the traditional prescribed performance control method, the tracking errors of systems need to satisfy the following constraint:

$$-\mu(t) < e_1(t) < \mu(t), \quad t \geq 0, \quad (10)$$

where $\mu(t) = (\mu_0 - \mu_\infty)e^{-\tau t} + \mu_\infty$ is a prescribed performance function with $\mu_0 > \mu_\infty > 0$, $\tau > 0$. In control design, the achievement of the prescribed performance control requires the initial condition of the controlled variable $e_1(t)$ to satisfy $-\mu(0) < e_1(0) < \mu(0)$. However, the initial conditions of practical systems maybe difficult to be obtained, therefore, for this case, the traditional prescribed performance control method is limited in application.

In order to avoid the defect, a novel prescribed constraint control method is given in this paper. The method is independent of the initial condition of the constrained variable. The novel idea of the method is described as follows.

According to the control objective, the tracking error $e_1(t)$ needs to satisfy $-\rho_1(t) < e_1(t) < \rho_1(t)$ when $t \geq T > 0$ without the limit of the tracking error $e_1(0)$. Here, T is a designable time parameter and it is called the setting time. The constraint function $\rho_1(t)$ is defined as

$$\rho_1(t) = (\rho_0 - \rho_\infty)e^{-\lambda_1(t-T)} + \rho_\infty, \quad (11)$$

where $\rho_1(t)$ is a direct constraint function, $\rho_0 > \rho_\infty > 0$, $\lambda_1 > 0$ and $T > 0$ are the design parameters.

The statement for the novel constraint control idea is clearly illustrated by the graphical representation in Fig. 1.

In order to make the control effect of the tracking error be independent of the initial condition, a new nonlinear mapping of $e_1(t)$ is proposed. The error $e_1(t)$ is mapped to z_1 by the following hyperbolic tangent mapping.

$$z_1 = \tanh(e_1). \quad (12)$$

Remark 1: From (12), it can be observed that $z_1(t)$ and $e_1(t)$ is approximately linear around the origin.

Fig. 1. Illustration of the prescribed constraint control idea

Based on (12), an indirect constraint function $\rho_2(t)$ of $e_1(t)$ is proposed to constrain z_1 into the following prescribed performance range:

$$-\rho_2(t) < z_1(t) < \rho_2(t), \quad t \geq 0, \quad (13)$$

where

$$\rho_2(t) = M e^{-\tilde{\tau} t} \frac{(\check{v}(t) - \check{v}_1)}{\check{v}_0} + \tanh(\rho_1), \quad (14)$$

$$\check{v}(t) = \begin{cases} \left(\check{v}_0 - \frac{t}{T} \right) e^{(1-\frac{T}{t})} + \check{v}_1, & 0 \leq t < T \\ \check{v}_1, & t \geq T \end{cases} \quad (15)$$

and the constants $M \geq 1, \tilde{\tau} \geq 0, \check{v}_0 \geq 1, \check{v}_1 > 0$ are the design parameters.

Remark 2: $\check{v}(t)$ is a smooth function, which has been proven in [41]. In addition, it is easy to prove that $\rho_2(t)$ is a smooth function.

Remark 3: In order to ensure the prescribed performance of $z_1(t)$, $z_1(0)$ must meet the condition $|z_1(0)| < \rho_2(0)$. From (12), it can be seen that $-1 < z_1(t) < 1$ holds for any value of e_1 . Because $-1 < z_1(t) < 1$, the condition can be achieved by choosing $\rho_2(0) \geq 1$. And $\rho_2(0) \geq 1$ can be ensured by $M \geq 1$. Therefore, the proposed method doesn't need the initial value of the constrained variable $e_1(t)$.

Theorem 1: If the inequality (13) holds, then $-\rho_1(t) < e_1(t) < \rho_1(t)$ holds for $t \geq T$.

Proof: From (12) and (13), it follows that

$$-\rho_2(t) < \tanh(e_1(t)) < \rho_2(t). \quad (16)$$

If $t \geq T$, according to (14), the following mathematical relationships can be obtained.

$$0 < \rho_2(\check{v}(t)) = \tanh(\rho_1(t)). \quad (17)$$

From (16) and (17), we have $-\rho_1(t) < e_1(t) < \rho_1(t)$, $t \geq T$, so Theorem 1 is proved.

The control objective of the tracking error $e_1(t)$ can be reduced to constrain $z_1(t)$ according to (13). Next, the constraint control design method for $z_1(t)$ will be presented.

IV. ADAPTIVE NEURAL NETWORK PRESCRIBED PERFORMANCE CONSTRAINT CONTROLLER

Based on the proposed constraint control idea in the above section, an adaptive neural constraint controller will be designed by adopting backstepping technique. The following coordinate transformation is chosen firstly.

$$\zeta_1 = \tan\left(\frac{\pi z_1}{2\rho_2}\right), \quad (18)$$

$$\eta_i = x_i - \alpha_{i-1}(\bar{x}_i, \hat{\beta}, \bar{y}_r^{(i-1)}, \bar{\rho}_2^{(i-1)}), \quad i = 2, 3, \dots, n, \quad (19)$$

where α_{i-1} is the virtual control signal, $\bar{y}_r^{(i-1)} = [y_r, y_r^{(1)}, \dots, y_r^{(i-1)}]^T$, $\bar{\rho}_2^{(i-1)} = [\rho_2, \rho_2^{(1)}, \dots, \rho_2^{(i-1)}]^T$, $\hat{\beta}$ denotes the estimated value of the unknown constant β , which is defined as $\beta = \max\{\|W_i^{*T}\|^2; i = 1, 2, \dots, n\}$, and the estimation error $\tilde{\beta} = \beta - \hat{\beta}$.

Step 1: Choose a Lyapunov functional candidate as

$$V_1 = \frac{1}{2}b_m\zeta_1^2 + \frac{1}{2k_1}b_m^2\tilde{\beta}^2, \quad (20)$$

where k_1 is a positive design parameter.

The derivative of V_1 is obtained as

$$\begin{aligned} \dot{V}_1 &= b_m\zeta_1\dot{\zeta}_1 - \frac{1}{k_1}b_m^2\tilde{\beta}\dot{\tilde{\beta}} \\ &= \frac{b_mQ}{\rho_2} \left(\frac{4}{(e^{e_1} + e^{-e_1})^2} g_1(\eta_2 + \alpha_1) + \frac{4}{(e^{e_1} + e^{-e_1})^2} (f_1 \right. \\ &\quad \left. + d_1 - \dot{y}_r) - \frac{z_1\dot{\rho}_2}{\rho_2} \right) - \frac{1}{k_1}b_m^2\tilde{\beta}\dot{\tilde{\beta}}, \end{aligned} \quad (21)$$

where $Q = \frac{\pi}{2} \tan\left(\frac{\pi z_1}{2\rho_2}\right) / \cos^2\left(\frac{\pi z_1}{2\rho_2}\right)$.

The following inequalities can be obtained by invoking Young's inequality.

$$\frac{b_mQ}{\rho_2} \frac{4}{(e^{e_1} + e^{-e_1})^2} g_1\eta_2 \leq \frac{b_mQ^2}{\rho_2^2} \frac{16}{(e^{e_1} + e^{-e_1})^4} g_1 + b_mg_1\eta_2^2, \quad (22)$$

$$\frac{b_mQ}{\rho_2} \frac{4}{(e^{e_1} + e^{-e_1})^2} d_1 \leq \frac{b_m^2Q^2}{\rho_2^2} \frac{16}{(e^{e_1} + e^{-e_1})^4} + d_1^2. \quad (23)$$

Substituting (22) and (23) into (21) gives

$$\begin{aligned} \dot{V}_1 &\leq \frac{b_mQ}{\rho_2} \left(\frac{4}{(e^{e_1} + e^{-e_1})^2} g_1\alpha_1 + \frac{Q}{\rho_2} \frac{16}{(e^{e_1} + e^{-e_1})^4} g_1 \right. \\ &\quad \left. + \frac{4}{(e^{e_1} + e^{-e_1})^2} (f_1 - \dot{y}_r) - \frac{z_1\dot{\rho}_2}{\rho_2} \right. \\ &\quad \left. + \frac{b_mQ}{\rho_2} \frac{16}{(e^{e_1} + e^{-e_1})^4} \right) - \frac{1}{k_1}b_m^2\tilde{\beta}\dot{\tilde{\beta}} + b_mg_1\eta_2^2 + d_1^2. \end{aligned} \quad (24)$$

Step i ($2 \leq i \leq n-1$): Consider the following Lyapunov function

$$V_i = \frac{1}{2}b_m\eta_i^2. \quad (25)$$

The derivative of V_i is

$$\begin{aligned} \dot{V}_i &= b_m\eta_i \left(g_i(\eta_{i+1} + \alpha_i) + f_i + d_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (g_j x_{j+1} + f_j + d_j) \right. \\ &\quad \left. - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\hat{\beta}} - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right). \end{aligned} \quad (26)$$

Applying Young's inequality yields

$$b_m\eta_i g_i \eta_{i+1} \leq b_m g_i \eta_i^2 + b_m g_i \eta_{i+1}^2. \quad (27)$$

$$b_m\eta_i d_i \leq b_m^2 \eta_i^2 + d_i^2. \quad (28)$$

$$b_m\eta_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} d_j \leq b_m^2 \eta_i^2 \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 + \sum_{j=1}^{i-1} d_j^2. \quad (29)$$

With the help of (27)~(29), (26) can be rewritten as

$$\begin{aligned} \dot{V}_i &\leq b_m\eta_i \left(g_i\alpha_i + g_i\eta_i + f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (g_j x_{j+1} + f_j) \right. \\ &\quad \left. - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\hat{\beta}} - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right. \\ &\quad \left. + b_m\eta_i + b_m\eta_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 \right) + \sum_{j=1}^i d_j^2 + b_m g_i \eta_{i+1}^2. \end{aligned} \quad (30)$$

Step n: Similarly to Step i , we take a Lyapunov functional as:

$$V_n = \frac{1}{2\lambda} b_m \eta_n^2. \quad (31)$$

Differentiating (31) with (1) and (19) yields

$$\begin{aligned} \dot{V}_n &= \frac{1}{\lambda} b_m \eta_n \left(g_n(\lambda v + \Delta(v)) + f_n + d_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} \right. \\ &\quad \left. + f_j + d_j) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\beta}} \dot{\hat{\beta}} - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right). \end{aligned} \quad (32)$$

By Young's inequality, the following formulas hold.

$$\frac{1}{\lambda} b_m g_n \eta_n \Delta(v) \leq \frac{1}{\lambda^2} b_m^2 g_n^2 \eta_n^2 + \Delta_{\max}^2, \quad (33)$$

$$\frac{1}{\lambda} b_m \eta_n d_n \leq \frac{1}{\lambda^2} b_m^2 \eta_n^2 + d_n^2, \quad (34)$$

$$\frac{1}{\lambda} b_m \eta_n \sum_{j=1}^{i-1} \frac{\partial \alpha_{n-1}}{\partial x_j} d_j \leq \frac{1}{\lambda^2} b_m^2 \eta_n^2 \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 + \sum_{j=1}^n d_j^2. \quad (35)$$

Substituting (33)~(35) into (32) yields

$$\begin{aligned} \dot{V}_n &\leq \frac{1}{\lambda} b_m \eta_n \left(\lambda g_n v + \frac{1}{\lambda} b_m \eta_n g_n^2 + f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} \right. \\ &\quad \left. + f_j) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\beta}} \dot{\hat{\beta}} - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right. \\ &\quad \left. + \frac{1}{\lambda} b_m \eta_n + \frac{1}{\lambda} b_m \eta_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 \right) + \Delta_{\max}^2 + \sum_{j=1}^n d_j^2. \end{aligned} \quad (36)$$

Then, according to (20), (25) and (31), Lyapunov function candidate for the system (1) can be defined as

$$V = \sum_{i=1}^n V_i. \quad (37)$$

From (24), (30), (36) and (37), the following result holds.

$$\begin{aligned} \dot{V} \leq & b_m \eta_1 \left(\frac{4}{(e^{e_1} + e^{-e_1})^2} g_1 \alpha_1 + \bar{f}_1(Z_1) \right) - \frac{b_m^2 \eta_1^2}{4} - \frac{1}{k_1} b_m^2 \tilde{\beta} \dot{\beta} \\ & + \sum_{i=2}^{n-1} \left(b_m \eta_i (g_i \alpha_i + \bar{f}_i(Z_i)) - \frac{b_m^2}{4} \eta_i^2 \right) + \frac{1}{\lambda} b_m \eta_n (\lambda g_n v \\ & + \bar{f}_n(Z_n)) - \frac{b_m^2 \eta_n^2}{4\lambda^2} + \Delta_{\max}^2 + \sum_{k=1}^n \sum_{j=1}^k d_j^2 + \Lambda. \end{aligned} \quad (38)$$

where $\eta_i = Q/\rho_2$ and

$$\begin{aligned} \bar{f}_1(Z_1) &= \frac{16g_1\eta_1}{(e^{e_1} + e^{-e_1})^4} + \frac{4}{(e^{e_1} + e^{-e_1})^2} (f_1 - \dot{y}_r) \\ &\quad - \frac{z_1 \dot{\rho}_2}{\rho_2} + \frac{16b_m\eta_1}{(e^{e_1} + e^{-e_1})^4} + \frac{b_m\eta_1}{4}, \\ \bar{f}_i(Z_i) &= g_i \eta_i + f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (g_j x_{j+1} + f_j) - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} \\ &\quad - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} + b_m \eta_i + b_m \eta_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 \\ &\quad + g_{i-1} \eta_i - \xi_i(Z_i) + \frac{b_m \eta_i}{4}, \\ \bar{f}_n(Z_n) &\leq \frac{1}{\lambda} b_m \eta_n g_n^2 + f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + f_j) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} \\ &\quad - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} + \frac{1}{\lambda} b_m \eta_n + \frac{1}{\lambda} b_m \eta_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 \\ &\quad + \lambda g_{n-1} \eta_n - \lambda \xi_n(Z_n) + \frac{b_m \eta_n}{4\lambda}, \end{aligned}$$

with $\Lambda = b_m \sum_{i=2}^{n-1} \eta_i (\xi_i(Z_i) - \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta}) + b_m \eta_n \frac{1}{\lambda} (\lambda \xi_n(Z_n) - \frac{\partial \alpha_{n-1}}{\partial \beta} \dot{\beta})$, $Z_1 = [x_1, \bar{y}_r^{(1)}, \bar{\rho}_2^{(1)}]^T \in \Omega_{Z_1} \subset R^5$, $Z_i = [\bar{x}_i, \hat{\beta}, \bar{y}_r^{(i)}, \bar{\rho}_2^{(i)}]^T \in \Omega_{Z_i} \subset R^{3(i+1)}$, $Z_n = [x, \hat{\beta}, \bar{y}_r^{(n)}, \bar{\rho}_2^{(n)}]^T \in \Omega_{Z_n} \subset R^{3(n+1)}$. $\xi_i(Z_i)$ ($2 \leq i \leq n$) are the auxiliary functions to compensate the term $\frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta}$ in (38), they will be given in Appendix I. Here, each unknown function $\bar{f}_i(Z_i)$ is approximated by a RBFNN in this control design, namely, $\bar{f}_i(Z_i) = W_i^{*T} S_i(Z_i) + \delta_i(Z_i)$, ($1 \leq i \leq n$) and $|\delta_i(Z_i)| \leq |\varepsilon_i|$, ε_i is an unknown positive constant.

With the aid of Young's inequality, it follows that

$$\begin{aligned} b_m \eta_1 \bar{f}_1(Z_1) &= b_m \eta_1 (W_1^{*T} S_1(Z_1) + \delta_1) \\ &\leq \frac{b_m^2 \eta_1^2}{2b_1} W_1^{*T} W_1^* S_1^T(Z_1) S_1(Z_1) + \frac{b_1}{2} + \frac{b_m^2 \eta_1^2}{4} + \delta_1^2 \\ &\leq \frac{b_m^2 \eta_1^2}{2b_1} \beta S_1^T(Z_1) S_1(Z_1) + \frac{b_1}{2} + \frac{b_m^2 \eta_1^2}{4} + \eta_1^2, \end{aligned} \quad (39)$$

$$\begin{aligned} b_m \eta_i \bar{f}_i(Z_i) &= b_m \eta_i (W_i^{*T} S_i(Z_i) + \delta_i) \\ &\leq \frac{b_m^2 \eta_i^2}{2b_i} W_i^{*T} W_i^* S_i^T(Z_i) S_i(Z_i) + \frac{b_i}{2} + \frac{b_m^2 \eta_i^2}{4} + \delta_i^2 \\ &\leq \frac{b_m^2 \eta_i^2}{2b_i} \beta S_i^T(Z_i) S_i(Z_i) + \frac{b_i}{2} + \frac{b_m^2 \eta_i^2}{4} + \varepsilon_i^2, \end{aligned}$$

$$\begin{aligned} \frac{1}{\lambda} b_m \eta_n \bar{f}_n(Z_n) &= \frac{1}{\lambda} b_m \eta_n (W_n^{*T} S_n(Z_n) + \delta_n) \\ &\leq \frac{b_m^2 \eta_n^2}{2b_n} W_n^{*T} W_n^* S_n^T(Z_n) S_n(Z_n) + \frac{b_n}{2\lambda^2} + \frac{b_m^2 \eta_n^2}{4\lambda^2} + \delta_n^2 \\ &\leq \frac{b_m^2 \eta_n^2}{2b_n} \beta S_n^T(Z_n) S_n(Z_n) + \frac{b_n}{2\lambda^2} + \frac{b_m^2 \eta_n^2}{4\lambda^2} + \varepsilon_n^2, \end{aligned} \quad (40)$$

where $b_i > 0$ ($1 \leq i \leq n$) are design parameters. Based on (38)~(40), it can be obtained that

$$\begin{aligned} \dot{V} \leq & b_m \eta_1 \left(\frac{4}{(e^{e_1} + e^{-e_1})^2} g_1 \alpha_1 + \frac{b_m \eta_1}{2b_1} \beta S_1^T(Z_1) S_1(Z_1) \right) \\ & - \frac{1}{k_1} b_m^2 \tilde{\beta} \dot{\beta} + \sum_{i=2}^{n-1} b_m \eta_i \left(g_i \alpha_i + \frac{b_m \eta_i}{2b_i} \beta S_i^T(Z_i) S_i(Z_i) \right) \\ & + \frac{1}{\lambda} b_m \eta_n \left(\lambda g_n v + \frac{\lambda b_m \eta_n}{2b_n} \beta S_n^T(Z_n) S_n(Z_n) \right) \\ & + \sum_{i=1}^n \varepsilon_i^2 + \sum_{i=1}^{n-1} \frac{b_i}{2} + \frac{b_n}{2\lambda^2} + \Delta_{\max}^2 + \sum_{k=1}^n \sum_{j=1}^k d_j^2 + \Lambda. \end{aligned} \quad (41)$$

According to (41), we can design the virtual control laws

$$\begin{aligned} \alpha_1 &= \frac{(e^{e_1} + e^{-e_1})^2}{4} \left(-c_1 \xi_1 \cos^2 \left(\frac{\pi z_1}{2\rho_2} \right) \rho_2 \right. \\ &\quad \left. - \frac{\eta_1}{2b_1} \hat{\beta} S_1^T(Z_1) S_1(Z_1) \right), \end{aligned} \quad (42)$$

$$\alpha_i = -c_i \eta_i - \frac{\eta_i}{2b_i} \hat{\beta} S_i^T(Z_i) S_i(Z_i), \quad 2 \leq i \leq n-1, \quad (43)$$

the real control law

$$v = -c_n \eta_n - \frac{\eta_n}{2b_n} \hat{\beta} S_n^T(Z_n) S_n(Z_n), \quad (44)$$

and the adaptive law

$$\dot{\hat{\beta}} = \sum_{i=1}^n \frac{k_i}{2b_i} \eta_i^2 S_i^T(Z_i) S_i(Z_i) - d_0 \hat{\beta}, \quad (45)$$

where c_i ($i = 1, 2, \dots, n$) and d_0 are positive constants.

Based on the above deduction, we give the following theorem.

Theorem 2: Consider the nonlinear system (1) satisfying Assumptions 1-3, if the direct constraint function ρ_1 and indirect constraint function ρ_2 are chosen respectively in (11) and (14), the virtual control laws, actual control law and adaptive laws of the system are designed according to (42)~(45), then the tracking error of the controlled system (1) can be driven into the prescribed scope given by ρ_1 and $-\rho_1$ when $t \geq T$. And all the signals in the closed loop system (1) are bounded.

Proof: 1) On stability for the closed-loop system (1)

By substituting (42)~(45) into (41), the following inequalities can be obtained

$$\begin{aligned} \dot{V} \leq & -\frac{\pi}{2} c_1 b_m^2 \xi_1^2 - \sum_{i=2}^n c_i b_m^2 \eta_i^2 + \frac{d_0}{k_1} b_m^2 \tilde{\beta} \dot{\beta} + \sum_{i=1}^n \varepsilon_i^2 \\ & + \sum_{i=1}^{n-1} \frac{b_i}{2} + \frac{b_n}{2\lambda^2} + \Delta_{\max}^2 + \sum_{k=1}^n \sum_{j=1}^k d_j^2 + \Lambda. \end{aligned} \quad (46)$$

Remark 4: According to (45), $\hat{\beta}(t) \geq 0$ always holds for all $t \geq 0$ if $\hat{\beta}(0) \geq 0$. $\hat{\beta}(t) \geq 0$ has been used in the deduction of (46).

In (46), it can be seen that V is bounded if $\Lambda \leq 0$ holds. In fact, $\Lambda \leq 0$ can be proven. In order to clarify the derivation, the proof of $\Lambda \leq 0$ is placed in Appendix I. Therefore, (46) can be rewritten as

$$\begin{aligned} \dot{V} \leq & -\frac{\pi}{2}c_1b_m^2\zeta_1^2 - \sum_{i=2}^n c_ib_m^2\eta_i^2 + \frac{d_0}{k_1}b_m^2\tilde{\beta}\hat{\beta} + \sum_{i=1}^n \varepsilon_i^2 \\ & + \sum_{i=1}^{n-1} \frac{b_i}{2} + \frac{b_n}{2\lambda^2} + \Delta_{\max}^2 + \sum_{k=1}^n \sum_{j=1}^k d_j^2. \end{aligned} \quad (47)$$

For the term $\frac{d_0}{k_1}b_m^2\tilde{\beta}\hat{\beta}$ in (47), the following inequality is verified easily by using Young's inequality.

$$\begin{aligned} \frac{d_0}{k_1}b_m^2\tilde{\beta}\hat{\beta} &= -\frac{d_0}{k_1}b_m^2\tilde{\beta}^2 + \frac{d_0}{k_1}b_m^2\tilde{\beta}\beta \\ &\leq -\frac{d_0}{2k_1}b_m^2\tilde{\beta}^2 + \frac{d_0}{2k_1}b_m^2\beta^2. \end{aligned} \quad (48)$$

Then, substituting (48) into (47) yields

$$\begin{aligned} \dot{V} \leq & -\frac{\pi}{2}c_1b_m^2\zeta_1^2 - \sum_{i=2}^n c_ib_m^2\eta_i^2 - \frac{d_0}{2k_1}b_m^2\tilde{\beta}^2 + \sum_{i=1}^n \varepsilon_i^2 \\ & + \sum_{i=1}^{n-1} \frac{b_i}{2} + \frac{b_n}{2\lambda^2} + \Delta_{\max}^2 + \sum_{k=1}^n \sum_{j=1}^k d_j^2 + \frac{d_0}{2k_1}b_m^2\beta^2 \\ & \leq -aV + b\rho(t), \end{aligned} \quad (49)$$

where $a = \min\{\pi c_1 b_m, 2c_i b_m, 2\lambda c_n b_m, d_0; i = 2, \dots, n-1\}$, $b = 1$, $\rho(t) = \sum_{i=1}^n \varepsilon_i^2 + \sum_{i=1}^{n-1} \frac{b_i}{2} + \frac{b_n}{2\lambda^2} + \Delta_{\max}^2 + \sum_{k=1}^n \sum_{j=1}^k d_j^2 + \frac{d_0}{2k_1}b_m^2\beta^2$.

From the formula (49) and Lemma 1, we can know that V is bounded, which means that ζ_1, η_i ($i = 2, \dots, n$) and $\tilde{\beta}$ are bounded. From (43) and (44), we also know α_i ($i = 2, \dots, n-1$) and v are bounded, so $x_3, x_4, \dots, x_n$ are bounded due to the coordinate transformation (19). However, the boundedness of x_1 and x_2 can not be obtained from the above analysis. The proof of the boundedness of them will be given in the part 3).

2) On constraint of z_1

Considering $\zeta_1 = \tan\left(\frac{\pi z_1}{2\rho_2}\right)$ is bounded, $\tan(\pm\frac{\pi}{2}) = \infty$ and $z_1(0) < \rho_2(\check{v}(0))$, then the following inequality always holds that

$$-\rho_2(\check{v}(t)) < z_1(t) < \rho_2(\check{v}(t)). \quad (50)$$

3) On constraint performance and boundedness of the tracking error $e_1(t)$

From (50) and Theorem 1, it can be known that the control object has been achieved, that is, $-\rho_1(t) < e_1(t) < \rho_1(t)$ when $t \geq T$. With the help of the continuity of the tracking error $e_1(t)$, it is easy to be known that $e_1(t)$ is bounded in whole time domain by mathematical logic. Thus, x_1 is bounded. Furthermore, α_1 can be proved to be bounded easily, so x_2 is also bounded. Therefore, the control objective of this paper is achieved.

The proof of Theorem 2 has been completed.

Remark 5: There are the design parameters $\rho_0, \rho_\infty, \lambda_1$ and T in the performance function $\rho_1(t)$. From (11), it can be found that increasing the design parameter λ_1 and decreasing the design parameters ρ_0, ρ_∞, T can bring better tracking performance. The smaller the parameter ρ_∞ , the smaller the steady state error. Increasing λ_1 can accelerate the convergence rate of the tracking error. However, the shorter setting time

T , the smaller ρ_∞ and the larger λ_1 can induce the larger control input. Therefore, the design parameter T and the function $\rho_1(t)$ should be chosen depending on the physical limitations and performance requirements of the output in practical applications. For the design parameters $\check{\tau}, M, \check{v}_0$ and $\check{v}_1$ in $\rho_2(t)$, we find that increasing the value of $\check{\tau}$ can achieve a better tracking performance, but the design parameters $M, \check{v}_0$ and $\check{v}_1$ are only need to satisfy the design condition given in this paper. The larger the parameter c_i , the larger the control input. The smaller the parameter b_i , the larger the control input. The design parameters k_1 and d_0 slightly affect the control performance.

V. SIMULATION STUDIES

In order to illustrate the effectiveness and superiority of the proposed control method, the developed adaptive neural prescribed constraint controller is applied to two practical examples.

Example 1: Consider a rigid robot manipulator described in [42], and input saturation and external disturbance are added to the system, then the dynamic model of the system is

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f_2 + g_2 u(v) + d_2, \\ y = x_1, \end{cases} \quad (51)$$

where $g_2 = \frac{1}{J_0}$, $f_2 = -\frac{1}{J_0}m_r g_v l_r \cos(x_1)$. x_1, x_2, m_r, g_v and l_r are the angular position of manipulator, the relative angular velocity, the load mass, the gravity and the length of manipulator, respectively. And $J_0 = 4m_r l_r^2/3$ is the inertia coefficient, $m_r = 5\text{mg}$, $g_v = 9.8\text{m/s}^2$ and $l_r = 0.25\text{m}$.

The simulations is conducted with the system initial condition $[x_1(0), x_2(0)]^T = [0, 5]^T$, and $\hat{\beta}(0) = 2$. The reference signal is $y_r = \cos(t)$. The parameters of the asymmetric smooth saturation nonlinearity model are $u_+ = 20$ and $u_- = -10$. The neural network $W_1^{*T}S_1(Z_1)$ contains $l_1 = 7^5$ nodes with centers

μ_l ($l = 1, \dots, l_1$) evenly spaced in $\overbrace{[-3, 3] \times \dots \times [-3, 3]}^{5 \text{ terms}}$, and width $\eta_l = 2$ ($l = 1, \dots, l_1$). The neural network $W_2^{*T}S_2(Z_1)$ contains $l_2 = 7^9$ nodes with centers μ_l ($l = 1, \dots, l_2$) evenly

spaced in $\overbrace{[-3, 3] \times \dots \times [-3, 3]}^{9 \text{ terms}}$, and width $\eta_l = 2$ ($l = 1, \dots, l_2$). The design parameters are chosen as $c_1 = 4, c_2 = 4, d_0 = 4, b_1 = 1, b_2 = 1, k_1 = 0.4, \rho_0 = 0.1, \rho_\infty = 0.01, \lambda_1 = 0.7, \check{v}_0 = 5, \check{v}_1 = 3, M = 10, \check{\tau} = 20$ and the setting time $T = 3\text{s}$. The simulation of the rigid robot manipulator is carried out after the controller is designed according to Theorem 2. The simulation results are given in Figs. 2-7.

In order to compare with the method in [30], according to the constraint control idea in [30], an adaptive constraint tracking controller is designed for the rigid robot manipulator system. The designed controller (70)~(72) can be founded in Appendix II. For the purpose of comparison, the same constraint function is chosen as the same as the prescribed performance function in this paper for the tracking error e_1 , that is, $F_1 = F_2 = 0.07e^{-0.4(t-3)} + 0.01$. The other design parameters and neural networks of the method in [30] are as the same as the proposed method in this paper. The designed

controller is applied to the rigid robot manipulator, and the simulation results are also given simultaneously in Figs. 2-7.

Fig. 2 shows the tracking effect of the output y . Fig. 3 gives the control effect of the tracking error e_1 . Fig. 4 illustrates the actual controller input v and the asymmetric saturation output u . The boundedness of the system state x_2 is shown in Fig. 5. Fig. 6 shows the boundedness of the adaptive parameter $\hat{\beta}$. Fig. 7 gives the external disturbance d_2 .

Fig. 2. Tracking effect of the system output y

Fig. 3. Control effect of the tracking error e_1

Fig. 4. The control input v and the asymmetric saturation output u

From Figs. 2-3, we can observe that the tracking performance of the system output y is satisfactory, e_1 is constrained by the constraint function ρ_1 no later than the given setting time $T = 3s$ when $e_1(0)$ isn't between $\rho_1(0)$ and $-\rho_1(0)$ in the two design methods. Figs 2-7 demonstrate clearly that the

Fig. 5. System state x_2

Fig. 6. Adaptive parameter $\hat{\beta}$

proposed control method is effective. From Fig. 3, it can be observed that the tracking performance of our method is better than the method in [30].

Example 2: To further illustrate the effectiveness of the proposed constraint control scheme, consider a inverted pendulum system taken from [4], and actuator saturation and external disturbance are added to the inverted pendulum system. The

Fig. 7. External disturbance d_2

dynamic model of the system is

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{g \sin(x_1) - \frac{ml_0 x_2^2 \cos(x_1) \sin(x_1)}{M_0+m}}{l_0 \left(\frac{4}{3} - \frac{m \cos^2(x_1)}{M_0+m} \right)} \\ \quad + \frac{\cos(x_1)}{M_0+m} u(v) + d_2, \\ y = x_1, \end{cases}$$

where $g = 9.8\text{m/s}^2$, $M_0 = 1\text{kg}$, $m = 0.1\text{kg}$, $l_0 = 0.5\text{m}$ are the acceleration of gravity, mass of cart, the mass of pole and the half length of pole, respectively.

After the controller is designed according to Theorem 2, the simulation is conducted with the initial condition $[x_1(0), x_2(0), \hat{\beta}(0)]^T = [1, 0.3, 0.1]^T$. The reference signal is $y_r(t) = \sin(t)$. The parameters of the asymmetric smooth saturation nonlinearity model are $u_+ = 20$ and $u_- = -30$. The design parameters are chosen as $c_1 = 2$, $c_2 = 15$, $d_0 = 0.4$, $b_1 = 1$, $b_2 = 1$, $k_1 = 0.4$, $\rho_0 = 0.12$, $\rho_\infty = 0.01$, $\lambda_1 = 0.6$, $\check{v}_0 = 5$, $\check{v}_1 = 2$, $M = 3$, $\check{\tau} = 3$ and the setting time $T = 1\text{s}$. The neural networks are the same as in this paper.

The simulation results for the inverted pendulum system are shown in Figs. 8-13. Fig. 8 shows the tracking effect of the output y . Fig. 9 exhibits the control effect of the tracking error e_1 , and $e_1(t)$ converges to the prescribed constraint region no later than the given setting time $T = 1\text{s}$ when $e_1(0)$ isn't between $\rho_1(0)$ and $-\rho_1(0)$. Fig. 10 gives the actual controller input v and the asymmetric saturation output u . The boundedness of the system state x_2 and the adaptive parameter $\hat{\beta}$ is depicted in Fig. 11 and 12, respectively. Fig. 13 presents the external disturbance d_2 . These figures verify the effectiveness and feasibility of the proposed method.

Fig. 8. Tracking effect of the system output y

VI. CONCLUSION

In this paper, a novel adaptive neural network prescribed constraint control design scheme is presented based on backstepping technique, which is independent of the initial condition of the constrained variable. Be different from the existing constraint control method, in this paper, the purpose of constraint control is achieved by using a nonlinear mapping and a novel performance constraint function. In this method,

Fig. 9. Control effect of the tracking error e_1

Fig. 10. The control input v and the asymmetric saturation output u

Fig. 11. System state x_2

Fig. 12. Adaptive parameter $\hat{\beta}$

Fig. 13. External disturbance d_2

the setting time T , at which the constrained variable of the system gets into the prescribed constraint region, is a design parameter. The designed controller not only guarantees the system stability but also achieves the desired constraint performance no later than the setting time for any initial condition even if the system is affected by external disturbance. The effectiveness of the proposed control method has been demonstrated via the simulations of two practical systems. Therefore, the proposed constraint control strategy is more practical in application. Our future work will focus on the application research of the proposed method to stochastic systems, switched systems, large-scale systems and multi-agent systems.

APPENDIX I

Proof of $\Lambda \leq 0$

Proof: As we know, it always holds that

$$\begin{aligned}
 \Lambda &= b_m \sum_{i=2}^n \eta_i (\xi_i(Z_i) - \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta}) + b_m \eta_n \frac{1}{\lambda} (\lambda \xi_n(Z_n) - \frac{\partial \alpha_{n-1}}{\partial \hat{\beta}} \dot{\beta}) \\
 &= b_m \sum_{i=2}^n \eta_i \xi_i(Z_i) - b_m \sum_{i=2}^n \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} - b_m \frac{\eta_n}{\lambda} \frac{\partial \alpha_{n-1}}{\partial \hat{\beta}} \dot{\beta} \\
 &\quad - b_m \frac{1}{\lambda} \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} + b_m \frac{1}{\lambda} \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \\
 &= b_m \sum_{i=2}^n \eta_i \xi_i(Z_i) - b_m \frac{1}{\lambda} \sum_{i=2}^n \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \\
 &\quad - b_m \frac{1}{\lambda} (\lambda - 1) \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta}. \tag{52}
 \end{aligned}$$

To clarify the deducing procedure of the proof, the following two inequalities are given firstly by using the formula (45) and Lemma 1 in [39].

The first inequality is

$$\begin{aligned}
 & - \frac{b_m}{\lambda} \sum_{i=2}^n \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \\
 &= - \frac{b_m}{\lambda} \sum_{i=2}^n \left(-d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \left(\sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. + \sum_{k=i}^n \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right) \Bigg) \\
 &\leq - \frac{b_m}{\lambda} \sum_{i=2}^n \left(-d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right. \\
 &\quad \left. - \frac{k_1}{2b_i} \eta_i^2 S_i^T(Z_i) S_i(Z_i) \sum_{k=2}^i \left| \eta_k \frac{\partial \alpha_{k-1}}{\partial \hat{\beta}} \right| \right) \\
 &\leq - \frac{b_m}{\lambda} \sum_{i=2}^n \left(-d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right. \\
 &\quad \left. - \frac{k_1}{2b_i} \eta_i^2 s^2 \sum_{k=2}^i \left| \eta_k \frac{\partial \alpha_{k-1}}{\partial \hat{\beta}} \right| \right) \\
 &= -b_m \sum_{i=2}^n \eta_i \Xi_i, \tag{53}
 \end{aligned}$$

where

$$\begin{aligned}
 \Xi_i &= \frac{1}{\lambda} \left(-d_0 \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right. \\
 &\quad \left. - \frac{k_1}{2b_i} \eta_i^2 s^2 \sum_{k=2}^i \left| \eta_k \frac{\partial \alpha_{k-1}}{\partial \hat{\beta}} \right| \right), i = 2, \dots, n.
 \end{aligned}$$

The second inequality is

$$\begin{aligned}
 & - \frac{b_m(\lambda - 1)}{\lambda} \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \\
 &= - \frac{b_m(\lambda - 1)}{\lambda} \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \left(\sum_{k=1}^n \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) - d_0 \dot{\beta} \right) \\
 &= - \frac{b_m(\lambda - 1)}{\lambda} \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \left(-d_0 \dot{\beta} + \sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right. \\
 &\quad \left. + \sum_{k=i}^n \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right) \\
 &\leq -b_m \sum_{i=2}^{n-1} \left(-\frac{d_0(\lambda - 1)}{\lambda} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \right. \\
 &\quad + \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \frac{(\lambda - 1)}{\lambda} \sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \\
 &\quad \left. - \frac{|\lambda - 1|}{\lambda} \left| \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \right| \sum_{k=i}^n \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \right) \\
 &\leq -b_m \sum_{i=2}^{n-1} \left(-\frac{d_0(\lambda - 1)}{\lambda} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \right. \\
 &\quad + \frac{(\lambda - 1)}{\lambda} \eta_i \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \sum_{k=1}^{i-1} \frac{k_1}{2b_k} \eta_k^2 S_k^T(Z_k) S_k(Z_k) \\
 &\quad \left. + b_m \frac{1}{\lambda} |\lambda - 1| \sum_{i=2}^n \frac{k_1}{2b_i} \eta_i^2 s^2 \sum_{k=2}^i \left| \eta_k \frac{\partial \alpha_{k-1}}{\partial \hat{\beta}} \right| \right) \\
 &= -b_m \sum_{i=2}^n \eta_i \check{\Xi}_i, \tag{54}
 \end{aligned}$$

where

$$\check{\Xi}_i = -\frac{d_0(\lambda - 1)}{\lambda} \left| \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \dot{\beta} \right| + \frac{(\lambda - 1)}{\lambda} \frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} \sum_{k=1}^{i-1} \frac{k_1}{2b_k} s^2$$

$$-\frac{1}{\lambda} |\lambda - 1| \frac{k_1}{2b_i} \eta_i s^2 \sum_{k=2}^i \left| \eta_k \frac{\partial \alpha_{k-1}}{\partial \hat{\beta}} \right|, i = 2, \dots, n-1,$$

$$\ddot{\Xi}_n = -\frac{1}{\lambda} |\lambda - 1| \frac{k_1}{2b_n} \eta_n \sum_{k=2}^n s^2 \left| \eta_k \frac{\partial \alpha_{k-1}}{\partial \hat{\beta}} \right|.$$

Here, s is the an upper bound of $\|S_k(Z_k)\|$.

Substituting (53) and (54) into (52), one has

$$\begin{aligned} \Lambda &\leq b_m \sum_{i=2}^n \eta_i \xi_i(Z_i) - b_m \sum_{i=2}^n \eta_i \Xi_i - b_m \sum_{i=2}^n \eta_i \ddot{\Xi}_i \\ &= b_m \sum_{i=2}^n \eta_i (\xi_i(Z_i) - \Xi_i - \ddot{\Xi}_i). \end{aligned}$$

If the auxiliary functions $\xi_i(Z_i)$, $i = 2, \dots, n$ are chosen as $\xi_i(Z_i) = \Xi_i + \ddot{\Xi}_i$, then

$$\Lambda \leq 0.$$

So far, the proof has been completed.

APPENDIX II

The controller design based on the method in [30]

According to the design idea in [30], the design process is presented as follows:

According to (4), the rigid robot manipulator in (51) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = g_2(\lambda v + \Delta(v)) + f_2 + d_2, \\ y = x_1, \end{cases} \quad (55)$$

Define the coordinate transformation

$$e_1 = x_1 - y_r, \quad e_2 = x_2 - \alpha_1,$$

where e_1 is the tracking error, y_r is the desired signal, α_1 is the virtual control.

Adopting the idea of [30], the shifting function is chosen as

$$\varphi(t) = \begin{cases} 1 - \left(\frac{T-t}{T}\right)^4, & 0 \leq t < T \\ 1, & t \geq T \end{cases} \quad (56)$$

then

$$\zeta_1(t) = \varphi(t)e_1(t). \quad (57)$$

Step 1: Choose a barrier Lyapunov function candidate as

$$V_1 = \frac{b_m \zeta_1^2}{(F_1 + \zeta_1)(F_2 - \zeta_1)} + \frac{1}{2k_1} b_m^2 \tilde{\beta}^2, \quad (58)$$

where b_m , k_1 and $\tilde{\beta}$ are as the same as the definition in this paper, F_1 and F_2 are the prescribed constraint function of ζ_1 .

Then, the time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= b_m \check{M}_1 \zeta_1 (\dot{\varphi} e_1 + \varphi \alpha_1 + \varphi e_2 \\ &\quad - \varphi \dot{y}_r + \varphi e_1 \eta_1) - \frac{1}{k_1} b_m^2 \tilde{\beta} \dot{\hat{\beta}}, \end{aligned}$$

where

$$\check{M}_1 = \frac{2F_1 F_2 + \zeta_1 F_2 - F_1 \zeta_1}{(F_1(t) + \zeta_1(t))^2 (F_2(t) - \zeta_1(t))^2},$$

$$\eta_1 = \frac{(-\dot{F}_{11} F_{12} - F_{11} \dot{F}_{12} + (\dot{F}_{11} - \dot{F}_{12}) \zeta_1)}{(2F_{11} F_{12} + \zeta_1 F_{12} - F_{11} \zeta_1)}.$$

The following inequality can be derived by using Young's inequality.

$$\begin{aligned} -b_m \check{M}_1 \zeta_1 \dot{\varphi} e_1 &\leq \frac{1}{4} k_1 b_m^2 \check{M}_1^2 \dot{\varphi}^2 \zeta_1^2 e_1^2 + \frac{1}{k_1} \\ &= \frac{1}{4} k_1 b_m^2 \check{M}_1^2 \dot{\varphi}^2 \zeta_1 \varphi e_1^3 + \frac{1}{k_1}. \end{aligned} \quad (59)$$

Substituting (59) into (58) generates

$$\begin{aligned} \dot{V}_1 &\leq b_m \check{M}_1 \zeta_1 \varphi (\alpha_1 + \bar{f}_1(Z_1)) + b_m \check{M}_1 \zeta_1 \varphi e_2 \\ &\quad - \frac{1}{4} (b_m \check{M}_1 \zeta_1 \varphi)^2 + \frac{1}{k_1}, \end{aligned} \quad (60)$$

where

$$Z_1 = [x_1, y_r, \dot{y}_r, F_1, \dot{F}_1, F_2, \dot{F}_2]^T,$$

$$\bar{f}_1(Z_1) = \frac{1}{4} k_1 b_m \check{M}_1 \dot{\varphi}^2 \zeta_1^3 - \dot{y}_r + e_1 \eta_1 + \frac{1}{4} b_m \check{M}_1 \zeta_1 \varphi.$$

Step 2: Consider a Lyapunov function candidate as

$$V_2 = \frac{1}{2\lambda} b_m e_2^2, \quad (61)$$

then

$$\begin{aligned} \dot{V}_2 &= \frac{1}{\lambda} b_m e_2 (g_2 \lambda v + \Delta v + f_2 + d_2 - \dot{\alpha}_1) \\ &= \frac{1}{\lambda} b_m e_2 \left(g_2 \lambda v + \Delta v + f_2 + d_2 - \xi_1 - \frac{\partial \alpha_1}{\partial \hat{\beta}} \dot{\hat{\beta}} \right), \end{aligned} \quad (62)$$

where

$$\begin{aligned} \xi_1 &= \frac{\partial \alpha_1}{\partial x_1} x_2 + \sum_{j=0}^{i-1} \frac{\partial \alpha_1}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=0}^1 \frac{\partial \alpha_1}{\partial \varphi^{(j)}} \varphi^{(j+1)} \\ &\quad + \sum_{k=1}^2 \sum_{j=0}^1 \frac{\partial \alpha_1}{\partial F_k^{(j)}} F_k^{(j+1)}. \end{aligned}$$

Applying Young's inequality, the following inequalities can be obtained.

$$\frac{1}{\lambda} b_m e_2 d_2 \leq \frac{1}{4\lambda^2} b_m^2 e_2^2 + d_2^2, \quad (63)$$

$$\frac{1}{\lambda} b_m e_2 \Delta v \leq \frac{1}{4\lambda^2} b_m^2 e_2^2 + \Delta_{\max}^2. \quad (64)$$

Now, substituting (63)~(64) into (62), it follows that

$$\begin{aligned} \dot{V}_2 &\leq \frac{1}{\lambda} b_m e_2 (g_2 \lambda v + \bar{f}_2(Z_2)) + \Delta_2 - b_m \check{M}_1 \zeta_1 \varphi e_2 \\ &\quad - \frac{1}{4} \left(\frac{1}{\lambda} b_m e_2 \right)^2 + \frac{1}{\lambda} b_m e_2 \left(\xi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\beta}} \dot{\hat{\beta}} \right), \end{aligned} \quad (65)$$

where

$$Z_2 = [x_1, x_2, \hat{\beta}, y_r, \dot{y}_r, \ddot{y}_r, F_1, \dot{F}_1, \ddot{F}_1, F_2, \dot{F}_2, \ddot{F}_2]^T,$$

$$\bar{f}_2(Z_2) = f_2 - \xi_1 + \frac{1}{2\lambda} b_m e_2 + \lambda \check{M}_1 \zeta_1 \varphi + \frac{1}{4\lambda} b_m e_2 - \xi_2(Z_2),$$

$$\begin{aligned} \xi_2(Z_2) &= -\frac{\partial \alpha_1}{\partial \hat{\beta}} \frac{k_1}{2b_1} \check{M}_1^2 \varphi^2 e_1^2 S_1^T(Z_1) S_1(Z_1) \\ &\quad - \left| \frac{\partial \alpha_1}{\partial \hat{\beta}} \right| \frac{k_1}{2b_2} e_2^2 s^2 + \frac{\partial \alpha_1}{\partial \hat{\beta}} d_0 \dot{\hat{\beta}}. \end{aligned} \quad (66)$$

The unknown function $\bar{f}_i(Z_i)$ can be approximated by a neural network, that is, $\bar{f}_i(Z_i) = W_i^{*T} S_i(Z_i) + \delta_i(Z_i)$, $i = 1, 2$. By applying the Young's inequality, it produces

$$\begin{aligned} b_m \check{M}_1 \zeta_1 \varphi \bar{f}_1(Z_1) &= b_m \check{M}_1 \zeta_1 \varphi (W_1^{*T} S_1(Z_1) + \delta_1(Z_1)) \\ &= b_m \check{M}_1 \zeta_1 \varphi W_1^{*T} S_1(Z_1) + b_m b_1 \check{M}_1 \zeta_1 \varphi \delta_1(Z_1) \\ &\leq \frac{1}{2b_1} (b_m \check{M}_1 \zeta_1 \varphi)^2 W_1^{*T} W_1 S_1^T(Z_1) S_1(Z_1) + \frac{b_1}{2} \\ &\quad + \frac{1}{4} (b_m \check{M}_1 \zeta_1 \varphi)^2 + \varepsilon_1^2 \\ &\leq \frac{1}{2b_1} (b_m \check{M}_1 \zeta_1 \varphi)^2 \beta S_1^T(Z_1) S_1(Z_1) + \frac{b_1}{2} \\ &\quad + \frac{1}{4} (b_m \check{M}_1 \zeta_1 \varphi)^2 + \varepsilon_1^2, \end{aligned} \quad (67)$$

$$\begin{aligned} \frac{1}{\lambda} b_m e_2 \bar{f}_2(Z_2) &= \frac{1}{\lambda} b_m e_2 (W_2^{*T} S_2(Z_2) + \delta_2(Z_2)) \\ &= \frac{1}{\lambda} b_m e_2 W_2^{*T} S_2(Z_2) + \frac{1}{\lambda} b_m e_2 \delta_2(Z_2) \\ &\leq \frac{b_m^2}{2b_2} e_2^2 W_2^{*T} W_2 S_2^T(Z_2) S_2(Z_2) + \frac{b_2}{2\lambda^2} + \frac{1}{4\lambda^2} b_m^2 e_2^2 + \varepsilon_2^2 \\ &\leq \frac{b_m^2}{2b_2} e_2^2 \beta S_2^T(Z_2) S_2(Z_2) + \frac{b_2}{2\lambda^2} + \frac{1}{4\lambda^2} b_m^2 e_2^2 + \varepsilon_2^2, \end{aligned} \quad (68)$$

where b_1 and b_2 are positive design parameters.

The total Lyapunov function of the system (55) is

$$V = V_1 + V_2.$$

With the aid of (60), (65), (67) and (68), one has

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 \\ &\leq b_m \check{M}_1 \zeta_1 \varphi (\alpha_1 + \bar{f}_1(Z_1)) \\ &\quad + \frac{1}{\lambda} b_m e_2 (g_2 \lambda v + \bar{f}_2(Z_2)) + \frac{1}{k_1} + d_2^2 + d_1^2 + \Delta_{\max}^2 \\ &\quad + \frac{1}{\lambda} b_m e_2 \left(\xi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\beta}} \dot{\hat{\beta}} \right) - \frac{1}{k_1} b_m^2 \tilde{\beta} \dot{\hat{\beta}}. \end{aligned} \quad (69)$$

According to (69), design the virtual control

$$\alpha_1 = -c_1 e_1 - \frac{\zeta_1}{2b_1} \check{M}_1 \varphi \hat{\beta} S_1^T(Z_1) S_1(Z_1), \quad (70)$$

the control input

$$v = -c_2 e_2 - \frac{e_2}{2b_2} \hat{\beta} S_2^T(Z_2) S_2(Z_2), \quad (71)$$

and the adaptive law

$$\dot{\hat{\beta}} = \frac{k_1}{2b_1} (\check{M}_1 \zeta_1 \varphi)^2 S_1^T(Z_1) S_1(Z_1) + \frac{k_1}{2b_2} e_2^2 S_2^T(Z_2) S_2(Z_2) - d_0 \hat{\beta}, \quad (72)$$

where c_1 , c_2 , k_1 and d_0 are positive design parameters.

By means of (66) and (72), one has

$$\begin{aligned} &\frac{1}{\lambda} b_m e_2 \left(\xi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\beta}} \dot{\hat{\beta}} \right) \\ &= \frac{1}{\lambda} b_m e_2 (\xi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\beta}} \frac{k_1}{2b_1} (\check{M}_1 \zeta_1 \varphi)^2 S_1^T(Z_1) S_1(Z_1) \\ &\quad - \frac{\partial \alpha_1}{\partial \hat{\beta}} \frac{k_1}{2b_2} e_2^2 S_2^T(Z_2) S_2(Z_2) + \frac{\partial \alpha_1}{\partial \hat{\beta}} d_0 \hat{\beta}) \\ &\leq \frac{1}{\lambda} b_m e_2 (\xi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\beta}} \frac{k_1}{2b_1} \check{M}_1^2 \varphi^2 e_1^2 S_1^T(Z_1) S_1(Z_1) \\ &\quad - \left| \frac{\partial \alpha_1}{\partial \hat{\beta}} \right| \frac{k_1}{2b_2} e_2^2 s^2 + \frac{\partial \alpha_1}{\partial \hat{\beta}} d_0 \hat{\beta}) \\ &= 0, \end{aligned}$$

that is,

$$\frac{1}{\lambda} b_m e_2 \left(\xi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\beta}} \dot{\hat{\beta}} \right) \leq 0. \quad (73)$$

By substituting (70)~(73) into (69), it is derived that

$$\begin{aligned} \dot{V} &\leq -c_1 b_m^2 \check{M}_1 \zeta_1^2 - c_2^2 b_m^2 e_2^2 + \frac{d_0}{k_1} b_m^2 \tilde{\beta} \dot{\hat{\beta}} + \Delta_1 + \Delta_2 \\ &\leq -c_1 b_m^2 \frac{\zeta_1^2}{(F_1 + \zeta_1)(F_2 - \zeta_1)} - c_2^2 b_m^2 e_2^2 \\ &\quad - \frac{d_0}{2k_1} b_m^2 \tilde{\beta}^2 + \frac{d_0}{2k_1} b_m^2 \beta^2 + \frac{1}{k_1} + d_2^2 + d_1^2 + \Delta_{\max}^2 \\ &\leq -aV + b, \end{aligned} \quad (74)$$

where $a = \min\{c_1 b_m, \lambda c_2 b_m, d_0\}$, $b = \frac{d_0}{2k_1} b_m^2 \beta^2 + \frac{1}{k_1} + d_2^2 + d_1^2 + \Delta_{\max}^2$.

According to Lemma 1, V is bounded, furthermore,

$$-F_1(t) < \zeta_1(t) < F_2(t). \quad (75)$$

When $t \geq T$, it follows from (56), (57) and (75) that

$$-F_1(t) < e_1(t) < F_2(t).$$

which means that F_1 and F_2 will become the prescribed performance function of the tracking error e_1 .

So far, the design process is completed.

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Code availability Not applicable.

Authors contributions 1) Compared with the traditional constraint control works [5]–[7], [9]–[11] and [22], a novel prescribed constraint control scheme is presented for a class of nonlinear systems with actuator saturation, which circumvents completely the problem that the traditional performance constraint control schemes depend on the initial conditions of the constrained variables. 2) Unlike the existing works [28]–[31], this paper presents a new constraint idea. A nonlinear mapping

and a new prescribed performance function are adopted, and the constrained variable is transformed to a defined scope rather than to zero at initial time. 3) The setting time, at which the constrained variable of the system gets into the prescribed constraint region, is a design parameter and can be set arbitrarily.

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