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Research

Keywords: Transient pulse test, Fractional derivative, Permeability, Mining disturbance, Coal

Posted Date: August 10th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-746642/v1>

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Estimation and verification of fractional derivative-based permeability of coals considering mining-induced stresses

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Abstract To overcome the inaccuracy of the traditional transient pulse test, a new fractional derivative-based permeability estimation formula based on the transient pulse test is proposed to describe the pressure difference decay of a coal body subjected to mining-induced stresses. The permeability of coal specimens under mining disturbance conditions is measured using the MTS815 rock mechanics test system. The experimental results show that the transient pulse test based on the fractional derivative model provides a much better estimation of the coal specimen's permeability than the conventional exponential decay model. Analyzing the evolution of the coal's permeability shows that the permeability tends to decrease in the pre-peak compaction stage, following which it gradually increases in the plastic phase, and then increases sharply in the post-peak phase. The significance of the fractional derivative order γ is discussed, and its analysis shows that the solid-liquid interaction inside the specimen becomes complicated when the stress within the coal specimen changes.

Keywords Transient pulse test, Fractional derivative, Permeability, Mining disturbance, Coal

1 Introduction

With the continuous increase in the depth of coal mining in China, deep mining is becoming common (Xie et al. 2015). The environment in which deep coal is present and mined is complex, making coal mining at depth a difficult

undertaking. In addition, there is a greater potential for disasters, such as mine water inrush and gas outbursts (which are associated with seepage phenomena in coal and rock bodies) to occur (Xie et al. 2012). Accurately measuring permeability, as a critical parameter of the seepage behavior of coal and associated rock

masses, is of great theoretical and practical importance for coal mining. In general, permeability measurement methods can be classified into two categories, i.e., steady-state methods and unsteady-state methods (Sander et al. 2017).⁰ In the steady-state method, a constant osmotic pressure difference is applied at both ends of the specimen, and the flow rate of the fluid through the specimen per unit time is calculated when the fluid percolation in the specimen reaches a steady state. Thus, the permeability of the specimen can be calculated according to Darcy's law. The steady-state method is mainly suitable for measuring certain materials with high permeability. However, most coal and associated rock bodies have low permeability, and it takes a long time for the flow in the specimens to reach a steady state. Therefore, the transient pulse method, which is one of the commonly used unsteady-state methods, is often used to measure the permeability of low-permeability rocks (Brace et al. 1968). Although Hsieh et al. (1981) and Neuzil et al. (1981) have provided an analytical solution of the governing equation corresponding to the transient pulse method, it is difficult to apply it in practice because it contains infinite series summations. The simplified solution provided by Brace et al. (1968) is widely used to evaluate the permeability of various geological materials due to its simple formulation (Fedor et al. 2008; Ghabezloo et al. 2009; Metwally et al. 2011; Chen et al. 2011; Mokhtari et al. 2015; Zhao et al. 2017).

Many studies have shown that microfracture variations within rocks caused by hydraulic coupling may affect the overall connectivity of the pore structure as well as the permeability of rocks (Zoback et al. 1975; Shao et al. 2005; Mitchell et al. 2008). Fluid flow through a specimen has been found to be characterized by non-Darcy flow when the

evolution of rock permeability under triaxial compression is determined by the transient pulse method (Chen et al. 2015; Liu et al. 2016; Wang et al. 2019). In fact, the presence of a large number of microfractures enhances the connectivity inside the rock, resulting in more complex rough contact and seepage paths; the enhanced roughness and flow tortuosity of the fracture network inside the rock also make the fluid-solid interaction stronger, leading to non-Darcy flow (Liu et al. 2016; Zimmerman et al. 1996; Ranjith et al. 2011; Yang et al. 2019). In mining engineering, after a coal mass is affected by mining disturbance, the vertical stress in the coal mass rises continuously to the peak stress value and then gradually decreases with the destruction of the coal mass; at the same time, the horizontal stress in the coal mass continues to decrease until it falls to zero, and the stress state of the coal mass in front of the working face under mining disturbance is shown in Fig. 1 (σ_1 is the axial stress; σ_2 and σ_3 are the horizontal stresses; γ is the overlaying rock gravity; H is the burial depth of the coal seam; α and β are the stress concentration factors of the axial stress and horizontal stress, respectively; the subscripts 1, 2, and 3 represent the different mining methods; R_c' is the residual strength of the coal bodies) (Xie et al. 2011). Due to mining disturbance, a large number of micro-cracks appear in the coals, which causes the seepage characteristics of the mining-affected coal body to not agree with Darcy flow. In recent years, fractional calculus has been widely used in geohydrology (Riccardo et al. 2015; Zhang et al. 2017), time-dependent rheology (Zhou et al. 2011, 2013, 2017; Wu et al. 2020a, 2020b), anomalous diffusion (Chen et al. 2016; Liang 2018; Liang et al. 2018; Zhou et al. 2018a), and non-Darcy flow (Zhou et al. 2018b, 2019; Yang et al. 2018; Wei et al. 2020) with great results due to the nonlocal feature of fractional derivatives (Kilbas et al. 2006; Mainardi 2010).

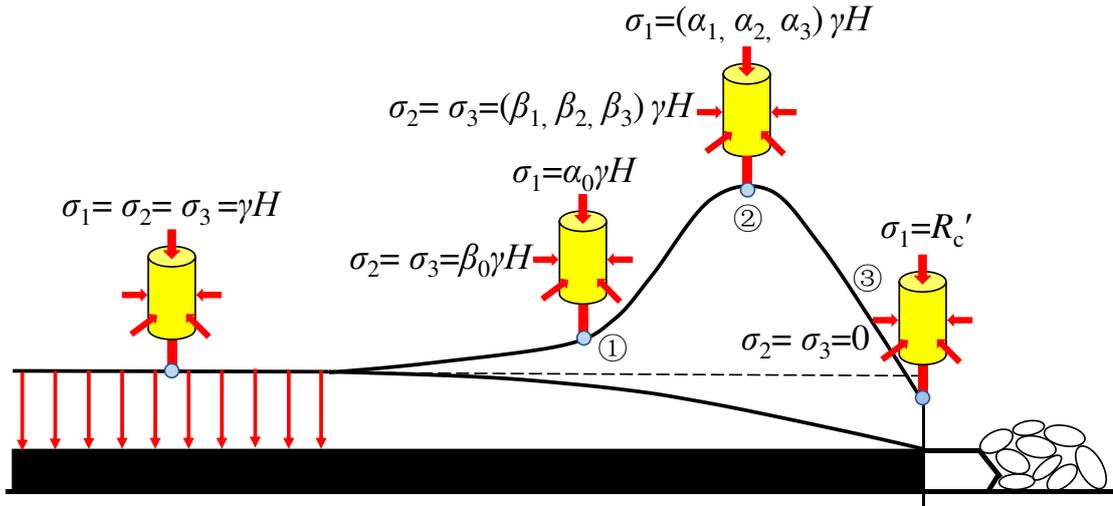


Fig. 1 Stress state of the coal mass in front of the working face under mining disturbance (Xie et al. 2011)

To precisely describe the pressure difference decay curves of a coal body affected by mining disturbance, a new permeability estimation formula using fractional derivatives considering mining-induced stresses based on the transient pulse test is proposed. Meanwhile, to verify the new permeability estimation formula, the MTS 815 rock mechanics test system is used to measure the permeability of a coal body, and the new fractional derivative-based permeability estimation formula is adopted to calculate the permeability of the coal body and compare it with the permeability obtained through the conventional transient method. The evolution of coal permeability is briefly analyzed. In addition, a set of pressure difference curves at different fractional order γ are obtained and the significance of the parameter γ is discussed.

2 Transient pulse method for permeability measurements

2.1 Conventional permeability formula based on the transient pulse test

The transient pulse method has been widely

used by many domestic and foreign scholars to determine the permeability of tight and low-permeability rocks due to the short time required for testing through this method (Fedor et al. 2008; Ghabezloo et al. 2009; Metwally et al. 2011; Chen et al. 2014; Mokhtari et al. 2015; Zhao et al. 2017). Its basic principle is shown in Fig. 2 (Escoffier et al. 2005). Two reservoirs are connected at the upper and lower ends of the specimen, and when the pressure in the reservoirs at both ends of the specimen is balanced, a pressure pulse is suddenly applied to the upstream reservoir at the moment of t_0 to form a pressure difference between the two ends of the specimen; then, the fluid in the upstream reservoir flows through the rock specimen to the downstream reservoir, at which time the pressure in the upstream reservoir gradually decreases and the pressure in the downstream reservoir gradually increases; the pressure difference between the upper and lower reservoirs decays over time and gradually approaches equilibrium. The permeability of the rock specimens is calculated by fitting the pressure difference decay curves of the upper and lower reservoirs.

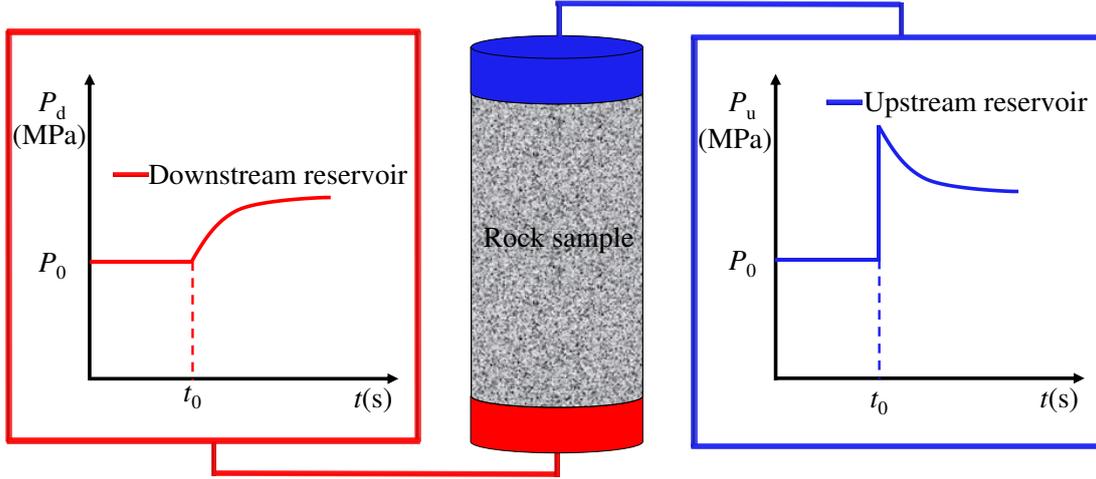


Fig. 2 Schematic diagram of the transient pulse test principle (Escoffier et al. 2005)

Brace et al. (1968) concluded that the pressure difference decay over time can be considered as a negative exponential function, i.e.,

$$P(t) = P_u(t) - P_d(t) = P_0 \cdot \exp\{-\alpha t\} \quad (1)$$

$$\alpha = \frac{KA}{\mu\beta_\omega L} \left(\frac{1}{V_u} + \frac{1}{V_d} \right) \quad (2)$$

where P_0 is the transient pressure pulse (Pa); $P(t)$ is the pressure drop (Pa); K is permeability (m^2); V_u and V_d represent the volumes of the upstream and downstream reservoirs (m^3), respectively; L is the height of the rock specimens (m) and A is the cross-sectional area (m^2); t is the duration of the permeability measurement (s); β_ω and μ are the compression coefficient (Pa^{-1}) and viscosity ($Pa \cdot s$) of water, respectively. As a result, the widely-used permeability formula can be represented as

$$K = \frac{\mu\beta_\omega L}{At} \ln \left(\frac{P_0}{P(t)} \right) \quad (3)$$

In Eq. (2), it can be seen that the parameter α is obtained by a best-fit analysis of the test pressure difference data, which is the most important step in calculating the permeability. Taking the logarithm of both sides of Eq. (1), and defining $R = \ln(P_0/P(t))$, Eq. (1) can be rewritten as

$$R = \alpha t \quad (4)$$

Due to the linear relationship between the logarithm of the pressure difference with time, Brace et al. (1968) determined the value of the attenuation coefficient α by plotting the pressure difference decay on semi-logarithmic paper against time, as shown in Fig. 3 (P_1 is the pore pressure of the upstream reservoir at time t_0 ; P_f is the final pressure).

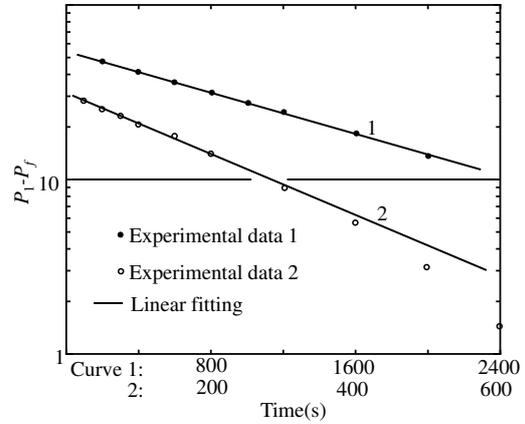


Fig. 3 Sample pressure difference decay curves by Brace et al. (1968) (semi-logarithmic graph)

However, many studies have shown that the decay characteristics of the transient pulse test pressure difference do not always satisfy the negative exponential decay, and that the fluid flow in rocks shows non-Darcy flow characteristics (Chen et al. 2014; Zhao et al. 2017; Zhou et al. 2020), i.e., the logarithm of the

pressure difference is not always linear with time. The typical pressure difference curve of a coal body affected by mining disturbance is obtained through the experiments in Section 3 of this paper (data from M4 specimen), as shown in Fig. 4. It can be seen that the logarithm of the pressure difference is not linear with time. In other words, when using the transient pulse method, the pressure difference decay characteristics of coal bodies affected by mining disturbances often deviate from the negative exponential function. To improve the accuracy of the traditional transient pulse test, some scholars have proposed a new fitting approach to describe the pressure difference decay. Zhao et al. (2017) performed seepage tests on limestone specimens and obtained the pressure difference decay over time with the negative exponential fitting curves, as shown in Fig. 5. It can be seen that the real data represented by the pressure difference test curve decays more slowly than the negative exponential model. Zhao et al. (2017) proposed a polynomial function fitting to describe the characteristics of the pressure difference decay accurately and obtained better results; however, there are more parameters leading to the calculation being too complicated. Yang et al. (2019) introduced the Mittag-Leffler function and considered the pressure difference curves as Mittag-Leffler decay; the permeability was also calculated more accurately by fitting the pressure difference test curves. In this paper, the exponential fitting formula proposed by Brace et al. (1968) is modified by introducing fractional calculus theory, and a new fractional derivative-based permeability formula is obtained to accurately describe the pressure difference decay characteristics of a coal body affected by mining disturbance to determine the permeability of the coal.

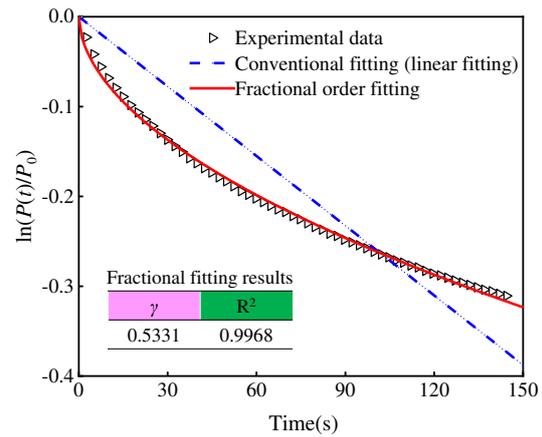


Fig. 4 Typical pressure difference decay curve of a mining-affected coal body based on the transient pulse test

2.2 Fractional derivative-based permeability estimation formula and preliminary validation considering mining-induced stresses

A new fitting method is proposed by introducing fractional calculus theory based on the exponential fitting proposed by Brace et al. (1968) to better fit the pressure difference decay curve considering mining-induced stresses to calculate the permeability of the specimen more accurately. First, Eq. (4) is rewritten into the differential form:

$$\frac{dR}{dt} = \alpha \quad (5)$$

Considering the memory effectiveness induced by solid-liquid interaction and time-dependent deformation of rocks (Yang et al. 2019), and referring to the fractional-order modeling ideas of non-Darcy flow (Zhou et al. 2018b, 2019, 2021; Yang et al. 2019; Wang et al. 2017). Eq. (5) can be further described in a fractional differential form:

$$\frac{d^\gamma R}{dt^\gamma} = \alpha \quad (6)$$

where d is the differential operator, γ is the fractional order.

For the solution of Eq. (6), the Caputo

fractional derivative is introduced. $f(t)$ is the function defined on the interval $(0, a)$, and its Caputo time fractional derivative is defined as (Kilbas et al. 2006)

$$\frac{d^\gamma f(t)}{dt^\gamma} = I^{1-\gamma} f'(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t (t-s)^{-\gamma} f'(s) ds \quad (7)$$

where $0 < \gamma \leq 1$, I^γ is the Riemann-Liouville fractional integral operator given by:

$$I^\gamma f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-s)^{\gamma-1} f(s) ds \quad (8)$$

where $\Gamma(\cdot)$ is the Gamma function. Considering the initial condition $R(0)=0$ and applying the Riemann-Liouville fractional integral in Eq. (8) to Eq. (6), we obtain

$$R = \alpha \frac{t^\gamma}{\Gamma(1+\gamma)} \quad (9a)$$

$$P(t) = P_0 \cdot \exp\left\{\frac{-\alpha t^\gamma}{\Gamma(1+\gamma)}\right\} \quad (9b)$$

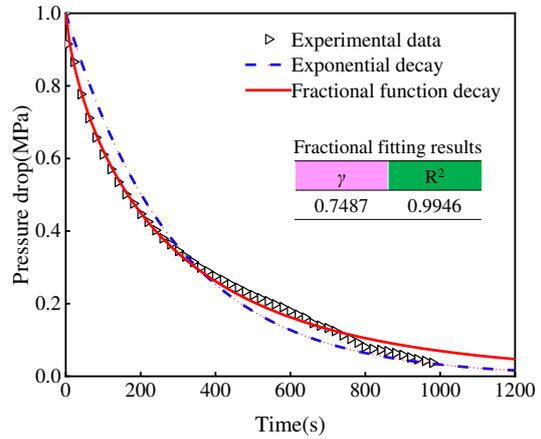
Therefore, Eq. (9) is the fractional derivative formula to obtain the pressure difference decay, and combining Eq. (9b) with Eq. (2), the fractional derivative-based permeability estimation formula with memory considering mining-induced stresses is obtained as

$$K_\gamma = \frac{\mu\beta_\omega L\Gamma(1+\gamma)}{At^\gamma \left(\frac{1}{V_u} + \frac{1}{V_d}\right)} \ln\left(\frac{P_0}{P(t)}\right) \quad (10)$$

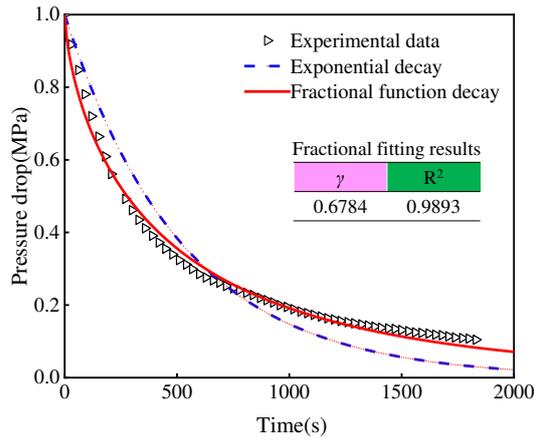
We can see that the fractional derivative-based permeability estimation formula with memory considering mining-induced stresses in Eq. (10) is degraded to the conventional transient method permeability formula in Eq. (3) when $\gamma = 1$, indicating that the permeability K calculated based on Darcy's law is a special case of the permeability with memory K_γ , i.e., $K = K_\gamma$ with γ

$= 1$.

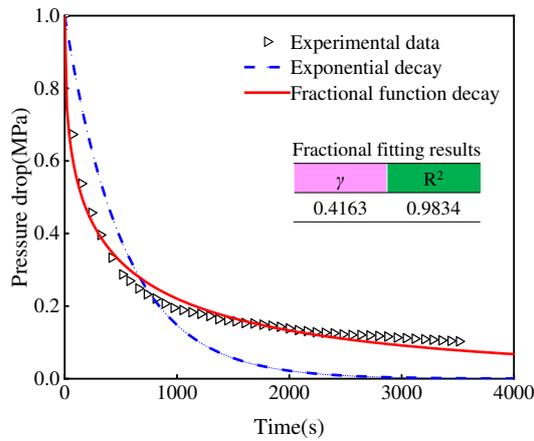
Furthermore, to perform a preliminary verification of Eq. (9), the use of the fractional pressure difference decay formula in Eq. (9) to fit the pressure difference is plotted in Fig. 5. It can be observed that the fractional function decay agrees much better with the experimental data than the exponential decay. It should be pointed out that although the influence of mining disturbance is not considered in the limestone sample SR3 shown in Fig. 5, the flow behavior presented still deviates from Darcy flow, refers to non-Darcy flow (Zhao et al. 2017). Therefore, the permeability of rocks determined by the exponential decay model corresponding to Darcy's law can only be regarded as an inaccurate reference permeability. The permeability of rocks can be calculated more precisely using Eq. (9). Fig. 6 shows the evolution characteristics of the reference permeability K measured using exponential decay and the permeability with memory K_γ measured through fractional function decay for the limestone specimen SR3 at different confining pressures.



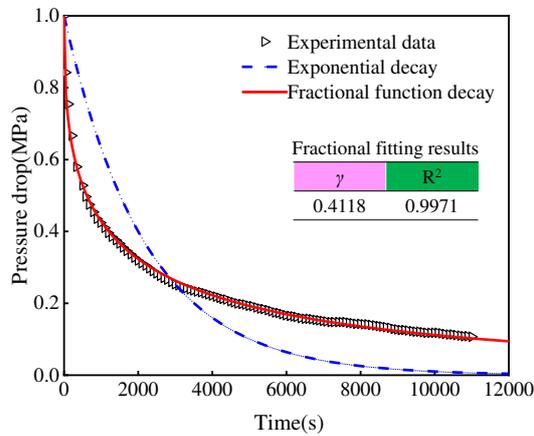
(a) 15MPa



(b) 20MPa



(c) 25MPa



(d) 30MPa

Fig. 5 Pressure difference curves of the limestone specimen SR3 at different confining pressures (Zhao et al. 2017)

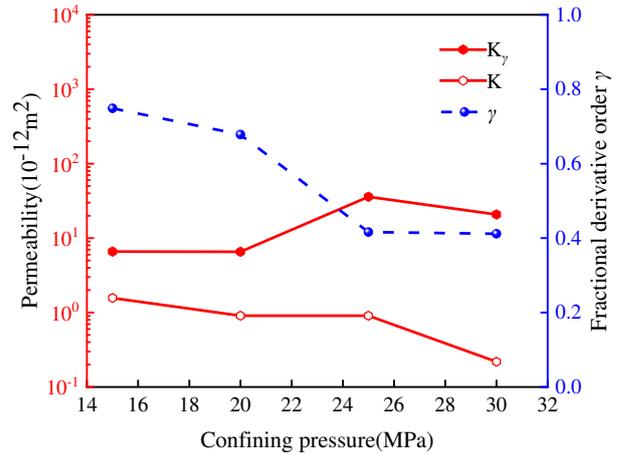


Fig. 6 Confining pressure versus permeability curve for the limestone specimen SR3

3. Experimental validation of the fractional derivative-based permeability estimation formula

3.1 Test method

To further verify the applicability of the new fractional derivative-based permeability estimation formula considering mining-induced stresses, a permeability test of coal specimens under triaxial unloading conditions based on the transient pulse method was conducted. Taking into consideration the real stress environment of a coal mass in front of the working face under mining disturbance (Fig. 1), we designed an appropriate permeability test for the coal specimens. The test had the following steps: (1) The axial and confining pressures increased to a hydrostatic pressure of 10 MPa at a loading rate of 10KN/min, respectively. (2) The axial pressure was loaded in steps of 20% of the peak strength of the specimen to peak stress, while the confining pressure was unloaded gradually by 1 MPa each time; the permeability was measured once after each reduction of the confining pressure, and a total of 4 times before the peak. (3) when the axial pressure reached the peak stress, the permeability was measured once, and subsequently, the unloading of the axial pressure was prepared, the way of unloading was changed to displacement control, and the unloading rate was set to 0.06 mm/min. (4) The axial pressure

was unloaded gradually with a 20% stress gradient of the peak strength and the confining pressure continued to be unloaded at a gradient of 1 MPa; the permeability was measured once after each unloading, and a total of 4 times after the peak was measured.

The loading and unloading stress paths are shown in Fig. 7.

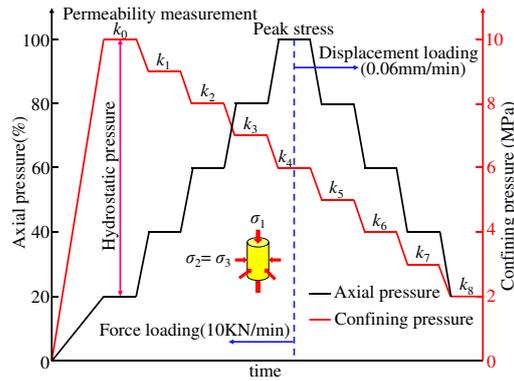


Fig. 7 Test method for measuring coal permeability under mining disturbance

3.2 Specimen preparation and test equipment

The coal specimens used for the permeability testing were taken from the No. 8 coal 28802 infill mining face of the Dongqu Mine in Taiyuan, Shanxi Province, China. The coal here is buried at a depth of about 400 m and the in-situ stress is about 10 MPa. The coal bodies drilled from the working face were processed, polished, and made into 50×100 mm

standard specimens, as shown in Fig. 8. Four specimens with complete appearance were selected for the test and numbered M1-M4. Then, the coal specimens were placed in a vacuum saturator for 3 days to fully saturate the specimens with water. Afterward, the saturated specimens were circumferentially sealed in thermo-shrinking plastic membranes, placed on the base pedestal of the triaxial cell, and the tests were performed according to the test method mentioned above.



Fig. 8 Standard specimens

The transient pulse tests were conducted using the MTS815 rock mechanics test system (Fig. 9). The maximum axial loading capacity of this system is 4600 KN, the maximum confining pressure and pore water pressure are 140 MPa, and the system is configured with a transient pulse apparatus for measuring the permeability of rocks.

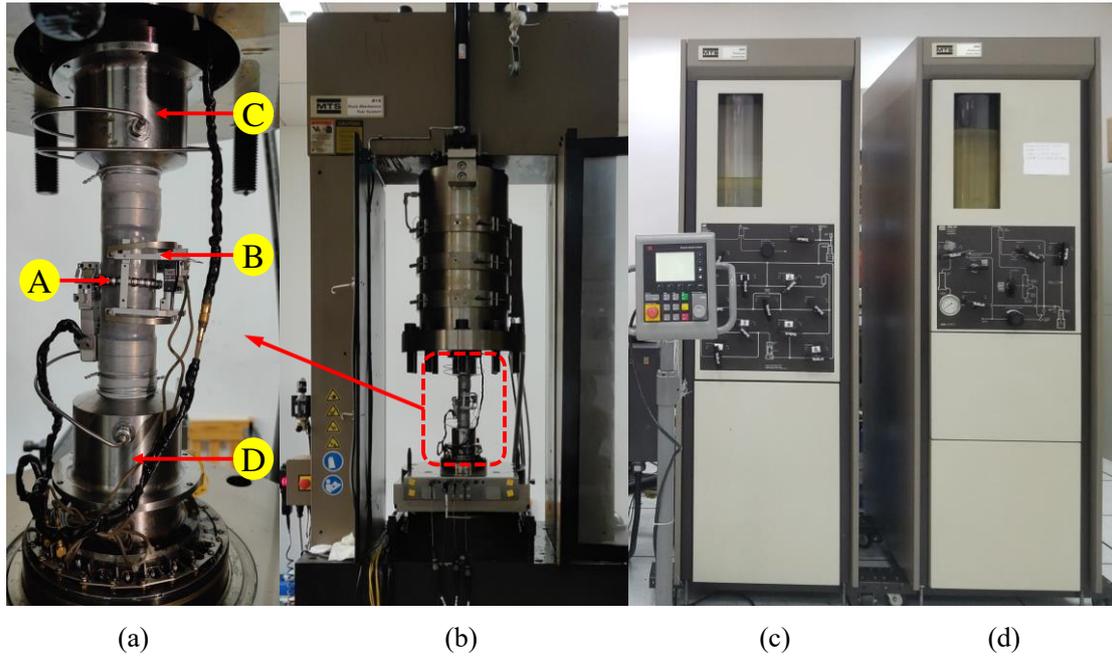


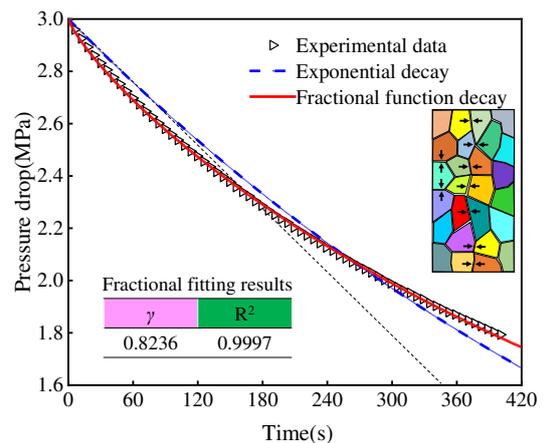
Fig. 9 MTS815 rock mechanics test system ((a) transient pulse apparatus (A: Lateral Strain Gauge, B: Axial Strain Gauge, C: Upstream Reservoir, D: Downstream Reservoir); (b) triaxial cell; (c) pore pressure intensifier; (d) confining pressure intensifier)

3.3 Test results and analysis

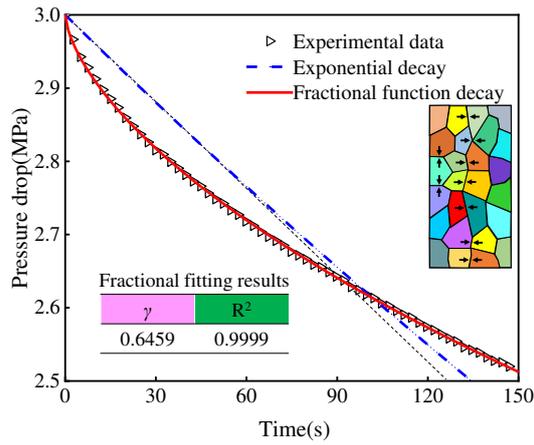
Due to the development of fractures in the coal specimens selected for the test, it is difficult to measure the permeability under post-peak unloading conditions. The post-peak permeability of the specimens in this test was measured only once before the specimens were destroyed completely and the MTS815 program was stopped. The stress of specimen M1 reached its peak when the fourth permeability measurement was taken, so only three measurements were taken before the peak. The M3 specimen was damaged before the permeability measurement was performed because its compressive strength was too low, and the permeability could not be measured. For the MTS815 setup in the present study, the dynamic viscosity of water μ is $1.01 \times 10^{-3} \text{Pa}\cdot\text{s}$, and the compression coefficient of water β_w is $5.56 \times 10^{-8} \text{Pa}^{-1}$. The reference volume of the upstream and downstream water reservoirs is $V_u = V_d = 3.32 \times 10^{-7} \text{m}^3$.

The experimental pressure difference data of the coal specimens were analyzed by fitting

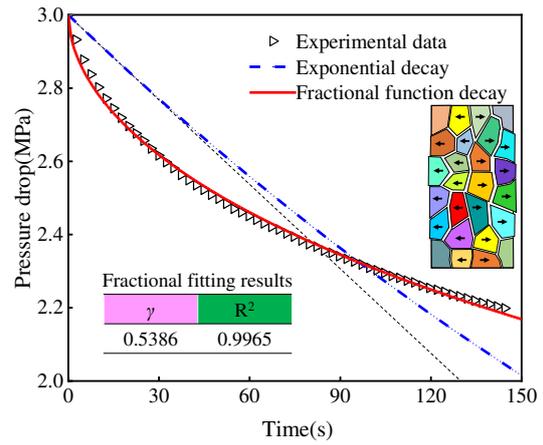
the negative exponential formula Eq. (1) and the fractional derivative formula Eq. (9) to verify the applicability of the proposed fractional derivative-based permeability estimation formula considering mining-induced stresses. Fig. 10 shows the pressure difference decay curves of the M4 coal specimen measured through the transient pulse method.



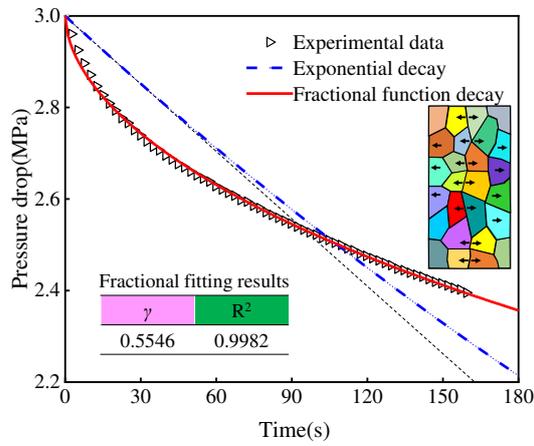
(a) M4- K_0



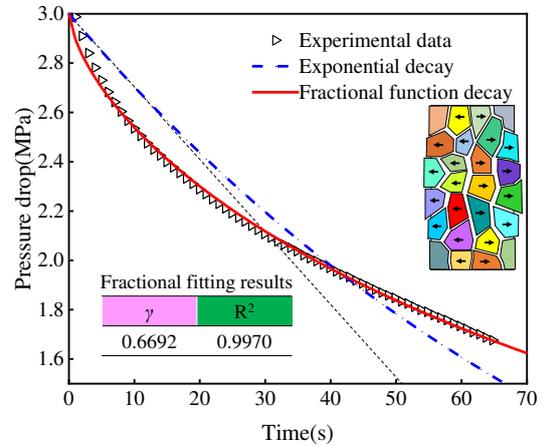
(b) M4-K₁



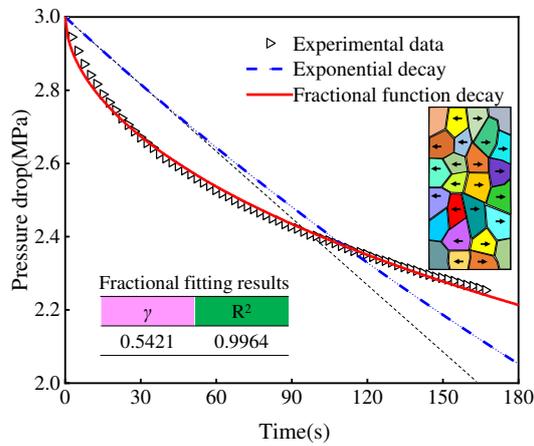
(e) M4-K₄



(c) M4-K₂



(f) M4-K₅



(d) M4-K₃

Fig. 10 Pressure difference curves of the M4 coal specimen (a) M4-K₀, (b) M4-K₁, (c) M4-K₂, (d) M4-K₃, (e) M4-K₄, (f) M4-K₅

It can be seen from Fig. 10 that the experimental data of the pressure difference of the coal specimens do not match with the exponential decay. The fractional function decay proposed in this paper describes the pressure difference decay more accurately and allows for more accurate measurement of the permeability of the coal specimens. Notably, the fractional function decay degenerates to an exponential decay conforming to Darcy flow when the order of the fractional derivative $\gamma=1$. In other words, the order of the fractional derivative γ can be used as an indicator to determine Darcy flow. The pressure difference curves of the other coal specimens were fitted and analyzed by the same method, and the results show that the new estimation formula for fractional derivative

permeability considering mining-induced stresses based on the transient pulse test can be used to determine the permeability of coal specimens accurately.

The curves of the axial strain versus the deviatoric stress and the permeability of the coal specimens are plotted in Fig. 11, where the permeability K and K_γ are calculated using Eq. (3) and Eq. (10), respectively; the order γ corresponding to the fractional derivative for the determination of permeability is also given, which can be used as an indicator for the determination of Darcy flow. In addition, it can be seen from both Fig. 6 and Fig. 11 that the reference permeability K calculated using Eq. (3) is low compared with the permeability with memory K_γ calculated using Eq. (10) under non-Darcy flow conditions. Meanwhile, the evolution of the permeability of the specimens based on Eq. (3) and Eq. (10) is basically consistent. Therefore, the fractional derivative-based permeability estimation formula considering mining-induced stresses proposed in this paper can more precisely determine the permeability evolution characteristics of mining-affected coal bodies and replace the traditional permeability formula completely.

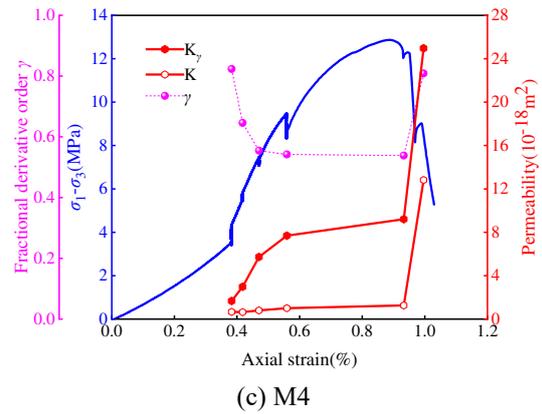
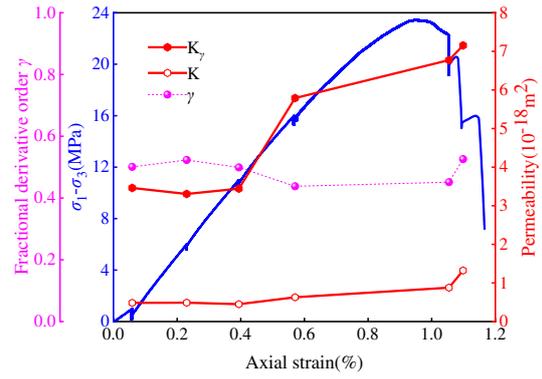
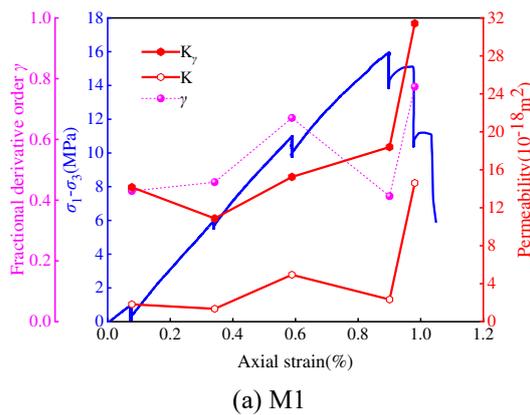


Fig. 11 Stress-strain curve for the permeability measurement of the coal specimens (a) M1, (b) M2, (c) M4

4 Discussion

To further understand the significance of the fractional derivative order γ mentioned above, $\alpha = 0.02$ is brought into Eq. (9) to obtain a set of pressure difference decay curves at different γ values. It is worth noting that the pressure difference curve at $\gamma = 1$ is an exponential decay curve agreeing with Darcy flow. Fig. 12 shows that the non-Darcy flow changes to Darcy flow as the fractional order increases. In fact, a lower fractional derivative order γ commonly represents the stronger solid-liquid interaction and the memory effect of non-Darcy flow (Yang et al. 2019). With the increase in the value of the order γ , the solid-liquid interaction and the memory effect of non-Darcy flow decrease gradually. Although the fractional order γ can be used as an indicator to determine Darcy and non-Darcy flow, it is not clear how the order γ

distinguishes between the high-velocity flow from the low-velocity flow in non-Darcy flow, and further research is needed.

In addition, although the coal specimens have large discreteness, it can still be seen from Fig. 11 that the permeability of the coal specimens shows a slightly decreasing trend in the pre-peak pressure-density phase; the permeability then gradually increases in the plastic phase and increases sharply in the post-peak phase. It is worth noting that the change in the fractional derivative order γ is complex, as shown in Fig. 11, which indicates that a large number of cracks are generated inside the specimens when the stress in the coal specimens changes, resulting in complex solid-liquid interactions within the specimens.

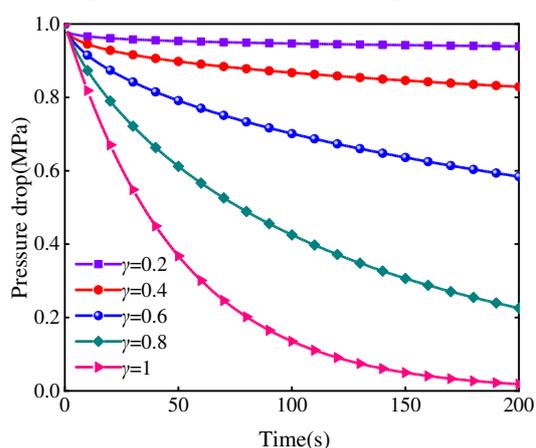


Fig. 12 The curves of the pressure difference decay at different γ

5 Conclusions

In this paper, based on the transient pulse test, a formula to estimate the permeability of coal subjected to mining-induced stresses is established and verified. The main conclusions are as follows:

(1) The conventional permeability formula based on the transient pulse test is modified by introducing fractional calculus theory to obtain a new fractional derivative-based permeability estimation formula considering mining-induced stresses. The results show that the new fractional derivative-based permeability estimation

formula can describe the pressure difference decay more accurately, and thus calculate the permeability of coal specimens more precisely than the conventional permeability formula. Furthermore, the permeability evolution of the coal specimens obtained from the two formulations is basically consistent; thus, the accuracy of the fractional derivative-based permeability estimation formula considering mining-induced stresses is further verified.

(2) The curves of the pressure difference decay at different γ reveals that non-Darcy flow shifts to Darcy flow as γ increases, and in particular, the fractional derivative-based permeability estimation formula with memory considering mining-induced stresses degenerates to the conventional permeability formula corresponding to Darcy flow when $\gamma = 1$.

(3) The permeability evolution of the coal specimens was analyzed. The permeability shows a slightly decreasing trend in the pre-peak pressure-density phase, following which the permeability gradually increases in the plastic phase, and then increases sharply in the post-peak phase. With the change in the stress in the coal specimen, a large number of cracks appear within the specimen, and the fractional derivative order γ shows a more complicated variation, which indicates that the solid-liquid interaction inside the specimen also becomes complicated.

Acknowledgements The present work is supported by the National Natural Science Foundation of China (51827901, 51674266), the Yueqi Outstanding Scholar Program of CUMTB (2017A03), and the Fundamental Research Funds for the Central Universities (2021YJSNY32). The financial supports are gratefully acknowledged.

Authors' contributions Hongwei Zhou: Put forward research innovation points. Senlin Xie: Analyzed the data and wrote the manuscript. Wenhao Jia: Performed the experiments. Jiawei

Zhao: Conceived and designed the experiments.
Shuai Yang: Modified article language error.
Qing Wei: Review & Editing. Xiangyu Wang:
Language polishing.

Data availability The data can be available from the authors upon request.

Declaration of interest statement

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant support for this work that could have influenced its outcome.

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