# Spatial self-similar transformation and novel line rogue waves of the Fokas system

Zhang Jie-Fang [a,b] and Jin Mei-Zhen [c]

a) Institute of Intelligent Media Technology, Communication University of Zhejiang, Hangzhou 310018, Zhejiang, P.R. China;

b) Zhejiang Provincial Key Laboratory of Film and Television Media Technology, Communication University of Zhejiang, Hangzhou 310018, Zhejiang, P.R. China;

c) Network and Data Center, Communication University of Zhejiang, Hangzhou 310018, Zhejiang, P.R. China.

**Abstract:** Fokas system is a natural and simple extension of the (2+1)-dimensional Schrödinger equation which can be used to describe optical pulse propagation in the nonlocal optical fibers. We first propose a two-dimensional spatial self-similar transformation of the Fokas system which is translated into the (1+1)-dimensional nonlinear Schrödinger equation. And then construct its abundant line rogue waves excitation. It is found that the line rogue waves induced by the Akhmediev breathers and Kuznetsov-Ma solitons also have the short life characteristics which possessed for the line rogue waves induced by the Peregrine solitons and other higher-order rogue waves and multi-rogue waves of the (1+1) dimensional standard NLS equations. This is completely different from the evolution characteristics of line soliton induced by bright solitons and the multi-solitons, which keeping their shape and amplitude unchanged. The diagram shows the evolution characteristics of the resulting all kinds of line rogue waves. The new excitation mechanism of line rogue waves for the Fokas system revealed contributes to the new understanding of the localized coherent structure of high-dimensional nonlinear wave models.

**Keywords:** line rogue wave, Fokas system, Spatial self-similar transformation, Peregrine solitons, Akhmediev breathers, Kuznetsov-Ma solitons

**PACS:** 42.65.Tg;05.45.Yv; 02.30.Ik; 04.20.Jb

**Email** zhangjief@cuz.edu.cn

**Funding** This work is by the National Natural Science Foundation of China under Grant No. 61877053.

**Compliance with ethical standards**

**Conflict of interest** Authors declare that they have no conflict of interest.

**Ethical approval** Authors declare that they comply with ethical standards.

**PS** All data generated or analysed during this study are included in this published article.

# 1. Introduction

Rogue waves are a typical natural phenomenon that can occur in a variety of different environments. Research fields included in fluid mechanics[1-3], nonlinear optical systems[4-5], plasmas[6-7], Bose-Einstein condensates[8-9], turbulence[10], microwaves[11], super-fluids[12], atmosphere[13], communications[14], capillary systems[15], financial systems[16], particulate matter[17], and magnetic materials[18]. Rogue waves are named after the local big sea waves against the rough ocean background. They are characterized by large amplitude, high steepness, no warning, short life span, etc. They are highly energy-concentrated catastrophic waves, which have been reported by a large number of field measurements [19-20], because of seriously threatening the navigation safety of ships and the operation safety of Marine structures.

In (1+1) dimensional nonlinear Schrödinger (NLS) equation, Peregrine first discovered a novel soliton with spatiotemporal localization, which was called Peregrine soliton(PS) [21]. Therefore, PS as the prototype of the rogue wave has become a consensus, and (1+1) dimensional NLSE is naturally taken as an ideal model to excite the rogue wave. Akhmediev et al. pointed out that the rogue wave of (1+1) dimensional NLS equation is a kind of nonsingular rational form solution, which is the limit case of Akhmediev breathers (AB) [22] and Kuznetsov-Ma solitons (MS)[23-24], and found higher order rogue wave solutions and multiple rogue wave solutions of (1+1) dimensional NLS equation[25-31].

The novel characteristics, unique physical mechanism and valuable application background of rogue waves have always been a focus of the academic interest and research. Although rogue waves are dangerous and unwelcome in the open ocean, they are useful in optics for opening the way for powerful ultrashort pulses of peak energy concentration. In the past decades, the research on rogue waves has not only abundant theoretical results [32-47], but also abundant experimental verification [48-54]. It is worth pointing out that the understanding and characterization of the two-dimensional space rogue waves seem to be much less than that of the one-dimensional space rogue waves, so the search for the two-dimensional space nonlinear evolution model of the rogue wave excitation is still a focus of academic research.

In this paper, we focus on the Fokas system [55]

$$iu_t + u_{xx} + uV = 0, \qquad V_y = |u|_x^2, \tag{1}$$

which can be rewritten in the following form

$$iu_t + u_{xx} + u\int_{-\infty}^{\infty} |u|_x^2 \, dy = 0. \tag{2}$$

Eq. (2) is the simplest (2+1)-dimensional extension of the nonlinear Schrödinger (NLS) equation. Its soliton, lump, dromion, line rogue waves, higher-order Breather and Hybrid Solutions have been obtained by references[56−62] by using different method.

This paper is organized as follows: In Sect. 2, a two-dimensional spatial self-similar transformation is proposed. Five types of line-rogue waves are obtained and discussed at Sect. 3. Our results are summarized in Sect. 4.

## **2.** Two-dimensional spatial self-similar transformation for the Fokas system

In order to construct the analytical two-dimensional localized coherent solutions of Eq. (1), we introduce the two-dimensional self-similar transformation in the form [63]

$$u(x,y,t) = \rho_1(t)\phi(\xi,\tau)\exp\left(i\varphi(x,y,t)\right), \quad v(x,y,t) = \rho_2(t)\psi(\xi,\tau), \tag{3}$$

to Eq. (1), where $\rho(t), \phi(\xi,t), \varphi(x,y,t)$ and $\xi = \xi(x,y,t), \tau = \tau(t)$ are respectively the undetermined function of the specified variable. We can get

$$\begin{aligned} &i\rho_1\tau_t\phi_\tau + \rho_1\xi_x^2\phi_{\xi\xi} + \rho_1\rho_2\phi\psi + i\rho_1\left(\xi_t + 2\varphi_x\xi_x\right)\phi_\xi \\ &+\rho_1\xi_{xx}\phi_\xi - \rho_1\left(\varphi_t + \varphi_x^2\right)\phi + i(\rho_{1t} + \rho_1\varphi_{xx})\phi + \rho_1\xi_{xx}\phi_\xi = 0, \end{aligned} \tag{4}$$

$$\rho_2\psi_\xi\xi_y = \rho_1^2 |\phi|_\xi \, \xi_x, \tag{5}$$

If we require that the unknown functions $\xi(x,t), \tau(t), \varphi(x,y,t)$ and $\rho(t)$ satisfy the following constraints

$$\xi_{xx} = 0, \tag{6}$$

$$\tau_t = 2\xi_x^2, \tag{7}$$

$$\xi_t + 2\varphi_x\xi_x = 0, \tag{8}$$

$$\varphi_t + \varphi_x^2 = 0, \tag{9}$$

$$\rho_{1t} + \rho_1 \varphi_{xx} = 0, \tag{10}$$

$$\rho_1^2 \xi_x = \rho_2 \xi_y = \xi_y \tau_t. \tag{11}$$

And yields from Eq. (5)

$$\psi = |\phi|^2 + C. \tag{12}$$

Then Eq. (1) reduces to a generalized (1+1)-dimensional NLS equation in the form

$$i\phi_t + \frac{1}{2}\phi_{\xi\xi} + \phi(|\phi|_\xi^2 + C) = 0. \tag{13}$$

It can be inferred from Eq. (6)

$$\xi(x, y, t) = \kappa(y, t)x + \iota(y, t). \tag{14}$$

From Eq. (8), We can assume that

$$\varphi(x, y, t) = -\frac{\alpha(y, t)}{2}x^2 - \beta(y, t)x - \delta(y, t). \tag{15}$$

where $\alpha(y,t), \beta(y,t), \delta(y,t)$ are three functions of the specified variables $(y,t)$ to be determined. Substituting Eq. (16) into Eq. (9) to find out

$$\alpha(y,t) = \frac{\alpha_0(y)}{1-\alpha_0(y)D(t)}, \quad \beta(y,t) = \frac{\beta_0(y)}{1-\alpha_0(y)D(t)}, \quad \delta(y,t) = \frac{\beta_0^2(y)D(t)}{2[1-\alpha_0(y)D(t)]} + \sigma(y), \tag{16}$$

where $D(t) = \int_0^t 2dt$ , $\alpha_0(y), \beta_0(y), \sigma(y)$ are three undetermined functions of the spatial variables $y$ respectively.

By combining Eqs. (14) and (16), it is obtained from Eq. (8)

$$\kappa(y,t) = \frac{\kappa_0}{1-\alpha_0 D(t)}, \quad \iota(y,t) = \frac{\kappa_0 \beta_0 D(t)}{1-\alpha_0 D(t)} + \lambda y, \tag{17}$$

$$\alpha(y,t) = \frac{\alpha_0}{1-\alpha_0 D(t)}, \quad \beta(y,t) = \frac{\beta_0}{1-\alpha_0 D(t)}, \quad \delta(y,t) = \frac{\beta_0^2 D(t)}{2[1-\alpha_0 D(t)]} + \sigma(y), \tag{18}$$

where $\alpha_0, \beta_0, \kappa_0, \lambda, \tau_0, \varphi_0$ are the five free real parameters, which describe the orientation and the of the triplet on the $(\xi, \tau)$ -plane. From Eqs. (10) and (11) and after sorting out, we get

$$\xi\left(x,y,t\right)=\frac{\kappa_0}{1-\alpha_0 D(t)}x+\lambda y+\frac{\kappa_0\beta_0 D(t)}{1-\alpha_0 D(t)},\qquad \tau(t)=\frac{\kappa_0^2 D(t)}{1-\alpha_0 D(t)}+\tau_0, \tag{19}$$

$$\varphi\left(x,y,t\right)=-\frac{\alpha_0}{2\left[1-\alpha_0 D(t)\right]}x^2-\frac{\beta_0}{1-\alpha_0 D(t)}x-\frac{\beta_0{}^2 D(t)}{2\left[1-\alpha_0 D(t)\right]}-\sigma(y), \tag{20}$$

$$\rho_1(t)=\frac{\rho_0}{\left[1-\alpha_0 D(t)\right]^{1/2}}=\frac{\sqrt{2\kappa_0\lambda}}{\left[1-\alpha_0 D(t)\right]^{1/2}},\quad \rho_2(t)=\frac{\kappa_0^2}{\left[1-\alpha_0 D(t)\right]^2}, \tag{21}$$

where $\sigma(y)$ is an arbitrary function of the spatial variable $y$. It indicates that the chirp of the system for the spatial variable $x$ is quadratic, while the chirp for the spatial variable $y$ can be of higher-order form. It's also worth pointing out that this phenomenon is first revealed not only for autonomous but also non-autonomous nonlinear systems.

If $\alpha_0=0$, we can obtain an special two-dimensional spatial self-similar transformations

$$\xi(x,y,t)=\kappa_0(x+\gamma y+2\beta_0 t),\quad \tau(t)=2\kappa_0^2 t+\tau_0, \tag{22}$$

$$\rho_1(t)=\rho_0=\sqrt{2\kappa_0\lambda},\quad \varphi\left(x,y,t\right)=-\beta_0 x-\sigma(y)-\beta_0^2 t, \tag{23}$$

where $\lambda=\kappa_0\gamma$. It indicates that the chirp of the system for the spatial variable $x$ vanish, but the chirp for the spatial variable $y$ still keep and can be of higher-order form. It's also worth pointing out that this phenomenon is also revealed for the first time.

The literature [64] has shown that Eq. (13) is still integrable. And it can transform into the well-known standard (1+1)-dimensional NLS equation. Without loss of generality, by setting $C=0$, Eq. (13) becomes as

$$i\phi_t+\frac{1}{2}\phi_{\xi\xi}+\phi\left|\phi\right|_{\xi}^2=0. \tag{24}$$

The reason why we transform Eq. (1) to Eq. (24) through the two-dimensional spatial self-similarity transformation is that many analytical and numerical methods are available for Eq. (14), and whose all kinds of meaningful solutions have been presented such as traveling wave solution, bright soliton solution, dark soliton solution, breathing soliton solution, Peregrine soliton(first-order rogue wave)solution, second-order rogue

wave solution and high-order rogue wave solution and rogue wave clusters[29,30,55].

For convenience, the two-dimensional spatial self-similarity transformations (3) with Eqs.(19)-(21) is called as the spatial self-similar transformations later.

## 3. Explicit construction of line rogue waves in the Fokas system

Since the traceable Eq. (24) has abundant coherent structures, such as soliton Peregrine soliton, the Akhmediev breathers (AB) and Kuznetsov-Ma solitons (KMS) and rogue waves etc., the corresponding results of the Fokas system (1) can be recovered by the two-dimensional spatial self-similarity transformation (3) with Eqs. (19)-(21). Here the focus is on the two-dimensional line rogue waves, which play an important and interesting role in the study.

With the help of the known rogue waves of the (1+1)-dimensional NLS equation, we discuss the plane line rogue wave of the Fokas system in detail and display the dynamics characteristics by graphs.

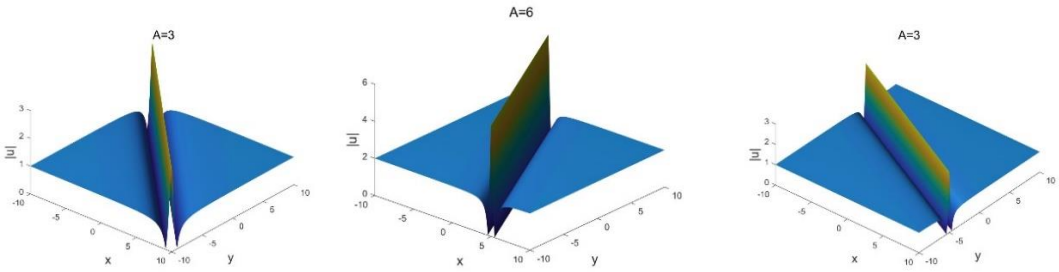
### 3. 1 First-order line rogue waves induced by the PS of Eq. (24)

By virtue of the self-similar transformations (3) with Eqs. (20)-(22) and the second-order rogue wave solutions of (1+1)-dimensional NLS equation (24)[29], we obtain the first-order line rogue wave solutions induced by the PS of Eq. (24) of the Fokas system (1) as follows

$$u_{1RW}(x,y,z)=\rho_1(t)\left(1-\frac{4+4i\xi}{1+4\xi^2+4\tau^2}\right)\exp\left\{i\left[\varphi(x,y,t)+\tau(t)\right]\right\}, \qquad (25)$$

where $\rho_1(t),\xi(x,y,t),\varphi(x,y,t),\tau(t)$ are presented by Eqs. (19)-(21).

Solution (25) involves four free parameters $\kappa_0,\lambda,\alpha_0,\beta_0,\tau_0$ to control the different types of rogue wave propagations. Figs.1 displays the first-order line rogue wave excitation of the Fokas system (1) on the $(x,y)$-plane for fixing $\alpha_0=\beta_0=1,\tau_0=0$ and selecting different value of $\kappa_0$, $\lambda$, respectively.




（a） (b) (c)

FIG.1. (Color online) The plots of the first-order line rogue wave (25) with Eqs. (19)-(21) on the $(x, y)$-plane when $t = 0$. The other free parameters are chosen as $\alpha_0 = \beta_0 = 1, \tau_0 = 0$ except $\kappa_0$, $\lambda$ are selected as the different value. (a) $\kappa_0 = 1, \lambda = 1$, (b) $\kappa_0 = 2, \lambda = 1$, (c) $\kappa_0 = 1, \lambda = 2$.

### 3.2 Second-order line rogue waves induced by the second-order rouge wave of Eq. (24)

By virtue of the spatial self-similar transformations (3) with Eqs. (20)-(22) and the second-order rogue wave solutions of (1+1)-dimensional NLS equation (24)[30], we obtain the second-order line rogue wave solutions induced by the second-order rogue wave of Eq. (24) of the Fokas system (1) as follows

$$u_{2RW}(x, y, t) = \rho_1(t)\left(1 + \frac{A_2 + iB_2}{C_2}\right)\exp\left\{i\left[\varphi(x, y, t) + \tau(t)\right]\right\}, \tag{26}$$

where

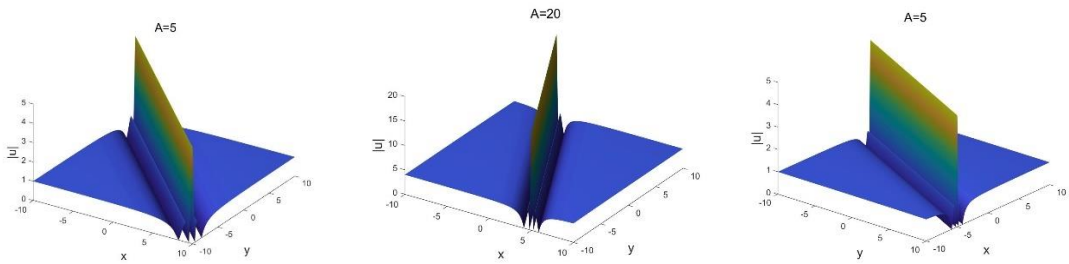
$$A_2 = -\frac{3}{8} + 3\xi^2 + 2\xi^4 + 9\tau^2 + 12\xi^2\tau^2 + 10\tau^4, \tag{27}$$

$$B_2 = \left(-\frac{15}{4} - 6\xi^2 + 4\xi^4 + 2\zeta^2 + 8\xi^2\zeta^2 + 4\zeta^4\right)\zeta, \tag{28}$$

$$C_2 = \frac{3}{32} + \frac{9}{8}\xi^2 + \frac{\xi^4}{2} + \frac{2\xi^6}{3} + \frac{33}{8}\zeta^2 - 3\xi^2\zeta^2 + 2\xi^4\zeta^2 + \frac{9}{2}\zeta^4 + 2\xi^2\zeta^4 + \frac{2}{3}\zeta^6, \tag{29}$$

and $\rho_1(t), \xi(x, y, t), \varphi(x, y, t), \tau(t)$ are presented by Eqs. (19)-(21).

Solution (26) involves five free parameters $\kappa_0, \lambda, \alpha_0, \beta_0, \tau_0$ to control the different types of rogue wave propagations. Figs.2 exhibits the second-order line rogue wave excitation of the Fokas system (1) on the $(x, y)$-plane for selecting fixed value $\alpha_0 = \beta_0 = 1, \tau_0 = 0$ and different value of $\kappa_0$, $\lambda$, respectively.




(a) (b) (c)

FIG.2 (Color online) The plots of the second-order line rogue wave (26) with Eqs. (19)-(21) and Eqs.(27)-(29) on the $(x, y)$-plane when $t = 0$. (a) $\kappa_0 = 1, \lambda = 1,$ (b) $\kappa_0 = 2, \lambda = 1,$ (c) $\kappa_0 = 1, \lambda = 2.$ The other free parameters are chosen as $\alpha_0 = \beta_0 = 1, \tau_0 = 0$ except $\kappa_0, \lambda$ are selected as the different value.

### 3.3 Line rogue waves induced by the rogue wave clusters of Eq. (24)

As we know the rogue wave clusters are some special high-order rogue wave solutions for (1+1)-dimensional NLS equation. The central idea of these solutions is that the high-order rogue wave solutions can be decomposed into a series of lower-order rouge wave solutions. And a variety of structures are formed by these lower-order rouge waves interrelated and interact with each other on the $(x,t)$-plane. There are some well-known rogue wave clusters, just like the triangular rogue wave cascades and the circular rogue wave clusters. And in addition, there are many articles on the rogue wave clusters. In the next discussion, we adopt the simplest clusters, which is called rouge wave triplets. Rouge wave triplets is a special second-order rogue wave solution. It describes that three Peregrine soliton interact with each other's on the $(x,t)$-plane. By virtue of the self-similar transformations (3) with Eqs. (20)-(22) and the rogue wave clusters of (1+1)-dimensional NLS equation (24)[30], we obtain the line rogue wave solutions induced by the rogue wave clusters of Eq. (24) of the Fokas system (1) as follows

$$u_{RWC}(x,y,t) = \rho(t)\left(1 + \frac{G_2 + iH_2}{D_2}\right)\exp\left\{i\left[\varphi(x,y,t) + \tau(t)\right]\right\}, \tag{30}$$

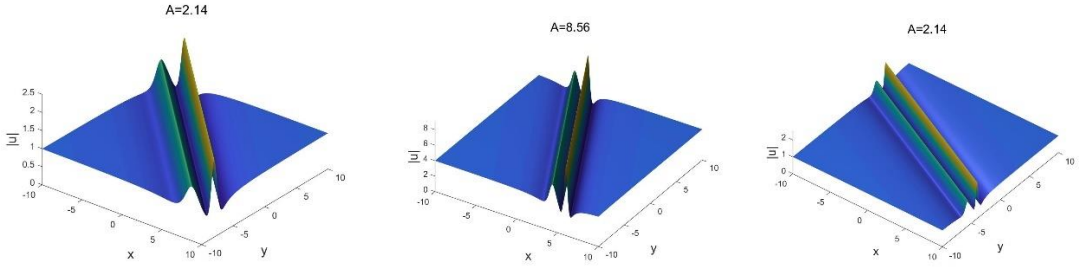
where

$$G_2 = 12\left[3 - 16\xi^4 - 24\xi^2(4\tau^2 + 1) - 4\mu\xi - 80\tau^4 - 72\tau^2 + 4\delta\tau\right], \tag{31}$$

$$H_2 = 24\left[\tau(15-16\xi^4+24\xi^2-4\mu\xi)-8(4\xi^2+1)\tau^3-16\tau^5\right] + 24\delta(2\tau^2-2\xi^2-0.5), \tag{32}$$

$$D_2 = 64\xi^6+48\xi^4(4\tau^2+1)+12\xi^2(3-4\tau^2)^2+64\tau^6+432\tau^4+396\tau^2+9 + \mu\left[\mu+4\xi(12\tau^2-4\xi^2+3\right]+\delta\left[\delta+4\tau(12\xi^2-4\tau^2-9)\right]. \tag{33}$$

and $\rho_1(t), \xi(x,y,t), \varphi(x,y,t), \tau(t)$ are presented by Eqs. (20)-(22) and $\mu, \delta$ are the two free real parameters which describe the orientation and the of the triplet on the $(\xi,\tau)$-plane.




(a) (b) (c)

FIG.3 (Color online) (Color online) The plots of the second-order line rogue wave (30) with Eqs. (19)-(21) and Eqs. (31-(33)) on the $(x,y)$-plane when $t=0$. (a) $\kappa_0=1, \lambda=1$, (b) $\kappa_0=2, \lambda=1$, (c) $\kappa_0=1, \lambda=2$. The other free parameters are chosen as $\alpha_0=\beta_0=1, \tau_0=0$, $\mu=\delta=50$ except $\kappa_0$, $\lambda$ are selected as the different value.

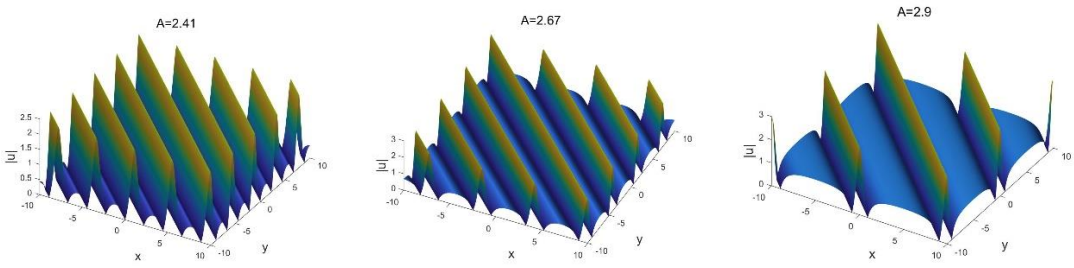
### 3.4 Periodic line rogue waves induced Akhmediev breathers of Eq. (24)

By virtue of the self-similar transformations (3) with Eqs. (20)-(22) and Akhmediev breather solutions [66] of (1+1)-dimensional NLS equation (24), we obtain the periodic line rogue wave induced by the Akhmediev breather of the (1+1)-dimensional NLS equation (24) of the Fokas system as follows

$$u_{KB}(x,y,t)=\rho(t)\left[\frac{(1-4a)\cosh(b\tau)+\sqrt{2a}\cos(\omega\xi)+ib\sinh(b\tau)}{\sqrt{2a}\cos(\omega\xi)-\cosh(b\tau)}\right]\exp\left\{i\left[\varphi(x,y,t)+\tau(t)\right]\right\}, \tag{29}$$

where $\rho_1(t), \xi(x,y,t), \varphi(x,y,t), \tau(t)$ are presented by Eqs. (19)-(21), $a$ ( $0\langle a\langle 0.5$ ) is a positive parameter, $b=\left[8a(1-2a)\right]^{1/2}, \omega=2(1-2a)^{1/2}$.

Fig. 4 shows plots of the periodic line rogue wave excitation on the $(x,y)$-plane with different propagation variables selecting different $\kappa_0, a$ when $t=0$.




(a) (b) (c)

FIG.4 (Color online) The plots of the periodic line rogue wave (29) on the $(x, y)$-plane when $t = 0$. The parameter $a$ is chosen as (a) $a = 0.25$, (b) $a = 0.35$, (c) $a = 0.45$. The other free parameters are selected as $\kappa_0 = \lambda = \alpha_0 = \beta_0 = 1, \tau_0 = 0$.

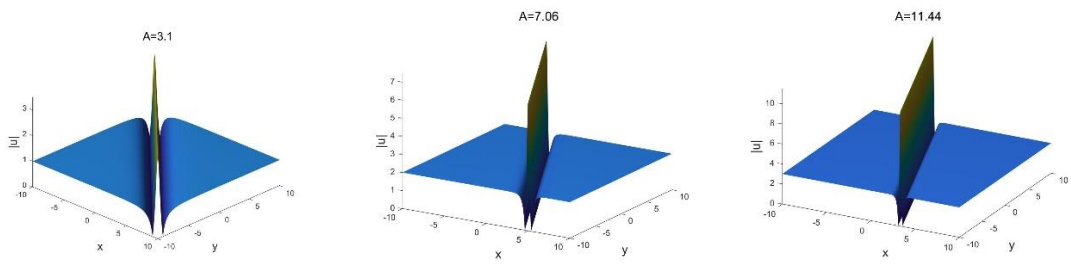
### 3.5 Line rogue waves induced Kuzenetov-Ma solitons of Eq. (24)

Similarly, by virtue of the self-similar transformations (3) with Eqs. (20)-(22) and the Kuzenetov-Ma[66] solitons of (1+1)-dimensional NLS equation (24), we obtain the line rogue wave of the Fokas system as follows

$$u_{KMS}(x,y,t)=\rho_1(t)\left[\frac{(1-4a)\cosh(B\tau)+\sqrt{2a}\cos(\Omega\xi)+iB\sinh(B\tau)}{\sqrt{2a}\cos(\Omega\xi)-\cosh(B\tau)}\right]\exp\left\{i\left[\varphi(x,y,t)+\tau(t)\right]\right\}, \quad (31)$$

where $\rho_1(t), \xi(x,y,t), \varphi(x,y,t), \tau(t)$ are presented by Eqs. (19)-(21), $a$ ($0.5\langle a \le 1$) is a positive parameter, $B = i\left[8a(2a-1)\right]^{1/2}, \Omega = 2i(2a-1)^{1/2}$.

Fig.5 shows plots of the line rogue wave excitation induced by the Kuzenetov-Ma solitons of the (1+1)-dimensional NLS equation (24) on the $(x, y)$-plane with different $\kappa_0, a$ when $t = 0$.




(a) (b) (c)

FIG.5 (Color online) The plots of the periodic line rogue wave (29) on the $(x, y)$-plane when $t = 0$. The parameter $a$ is chosen as (a) $\kappa_0 = 1, a = 0.55$, (b) $\kappa_0 = 2, a = 0.8$, (c) $\kappa_0 = 3, a = 0.99$.

The other parameters are selected as $\lambda = \alpha_0 = \beta_0 = 1, \tau_0 = 0.$

## 4. Conclusion

In this paper, the theory of self-similar transformation is further developed. The two-dimensional spatial self-similar transformation is constructed based on the Fokas system. And the novel line rogue wave excitation is studied in depth. It is found that the line rogue wave induced by the Akhmediev breathers (AB) and Kuznetsov-Ma solitons (KMS) also have the short life characteristics which possessed for the line rogue waves induced by the Peregrine solitons and other higher-order rogue waves and multi-rogue waves of the (1+1) dimensional standard NLS equations. This is completely different from the evolution characteristics of line soliton structures induced by bright solitons and multi-solitons, which keeping their shape and amplitude unchanged. The diagram shows the evolution characteristics of the resulting all kinds of spatial rogue waves.

The new excitation mechanism of line rogue waves revealed contributes to the new understanding of the coherent structure of high-dimensional nonlinear wave models. It should be pointed out that the method, which provides an enlightening way, can be used to research other generalized (2+1)-dimensional NLS models in the field of nonlinear science.

**References**

[1] OnoratoM., Osborne, A. R., Serio, M., Bertone, S.: Freak Waves in Random Oceanic Sea States. Phys. Rev. Lett. **86**, 5831(2001)

[2] Kharif, C., Pelinovsky, E., Slunyaev, A.: Rogue Waves in the Ocean. Springer-Verlag, Berlin(2009)

[3] Adcock, T. A. A., Taylor, P. H.: The physics of anomalous (‘rogue’) ocean waves. Rep. Prog. Phys.**77**, 105901(2014)

[4] Dudley, J. M., Genty, G., Eggleton, B. J.: Harnessing and control of optical rogue waves in supercontinuum generation. Opt. Express **16**, 3644–3651(2008)

[5] Kasparian, J., Bejot , P., Wolf, J. P., Dudley, J. M.: Optical rogue wave statistics in laser filamentation. Opt. Express **17**:12070–12075(2009)

[6] Shukla, P. K., Moslem, W. M.: Alfvénic rogue waves. Phys. Lett. A **376**(12-13), 1125–1128(2012)

[7] Tsai, Y. Y., Tsai, J. Y., Lin. I.: Generation of acoustic rogue waves in dusty plasmas through three-dimensional particle focusing by distorted waveforms. Nature Phys.

**12(6),** 573-577(2016)

[8] Bludov, Y. V., Konotop, V. V., Akhmediev, N.: Matter rogue waves. Phys. Rev. A **80,** 033610(2009)

[9] Vinayagam, P. S., Radha, R., Porsezian, K.: Taming rogue waves in vector Bose–Einstein condensates. Phys. Rev. E **88,** 042906(2013)

[10] Ginzburg, N. S., Rozental, R. M., Sergeev, A. S., Fedotov, A. E., Zotova, I. V., Tarakanov, V. P.: Generation of rogue waves in gyrotrons operating in the regime of developed turbulence. Phys. Rev. Lett. **119,** 034801(2017)

[11] Hohmann, R., Kuhl, U., Stockmann, H. J., Kaplan, L., Heller, E. J.: Freak waves in the linear regime: a microwave study. Phys. Rev. Lett. **104,** 093901(2010)

[12] Ganshin, A. N., Efimov, V. B., Kolmakov, G. V., Mezhov-Deglin, L. P., McClintock, P. V. E.: Observation of an inverse energy cascade in developed acoustic turbulence in superfluid helium. Phys. Rev. Lett. **101,** 065303(2008)

[13] Stenflo, L., Marklund, M.: Rogue waves in the atmosphere. J. Plasma Phys.**76,** 293–295(2010)

[14] Hammani, K., Kibler, B., Finot, C., Morin, P., Millot, G.: Peregrine soliton generation and breakup in standard telecommunications fiber. Opt. Lett. **36,** 112–114 (2011).

[15] Shats, M., Punzmann, H., Xia, H.: Capillary Rogue Waves. Phys. Rev. Lett. **104,** 104503(2010)

[16] Yan, Z. Y.: Vector financial rogue waves. Phys. Lett. **375(48)**, 4274-4279(2011)

[17] Han, J. F., Liang, T., Duan, W. S.: Possibility of the existence of the rogue wave and the super rogue wave in granular matter. Euro. Phys. J. E **42,** 5(2019)

[18] Copus, M. G., Camley, R. E.: Creation of magnetic rogue waves. Phys. Rev. B, **102,** 220410(R) (2020)

[19] Dysthe, K., Krogstad, H., Muller, P.: Oceanic rogue waves. Ann. Rev. Flu. Mech. **40,** 287-310(2008)

[20] Benetazzo, A., Ardhuin, F., Bergamasco, F., Cavaleri, L., Guimarães, P. V., Schwendeman, M., Sclavo, M., Thomson, J., Torsello, A.: On the shape and likelihood of oceanic rogue waves. Sci. Rep. **7,** 8276(2017).

[21] Peregrine, D. H.: Water-waves, nonlinear Schrödinger equations and their Solutions.

J. Aust. Math. Soc. B **25,** 16–43 (1983)

[22] Akhmediev N., Korneev, V. I.: Modulation instability and periodic solutions of the nonlinear Schrödinger equation. Theor. Math. Phys. **69,** 1089–1093(1986)

[23] Kuznetsov, E.: Solitons in a parametrically unstable plasma. Sov. Phys. Dokl. **22,** 507–508(1977)

[24] Ma, Y. C., Perturbed plane-wave solutions of the cubic Schrödinger equation. Stud. Appl. Math. **60,** 43–58(1979)

[25] Akhmediev, N., Ankiewicz, A., Soto-Crespo, J. M.: Rogue waves and rational solutions of the nonlinear Schrödinger equation. Phys. Rev. E **80,** 026601(2009)

[26] He, J. S., Zhang, H. R., Wang, L. H., Porsezian, K., Fokas, A. S.: Generating mechanism for higher-order rogue waves. Phys. Rev. E **87,** 052914(2013)

[27] Dubard, P., Matveev, V. B.: Multi-rogue waves solutions to the focusing NLS equation and the KP-I equation. Nat. Hazards Earth Syst. Sci. **11,** 667–672(2011)

[28] Ankiewicz, A., Clarkson, P. A., Akhmediev, N.: Rogue Waves, rational solutions, the patterns of their zeros and integral relations. J. Phys. A **43,** 122002(2010)

[29] Kedziora, D. J., Ankiewicz,A, Akhmediev N.:Triangular rogue wave cascades. Phys. Rev. E **86,** 056602(2012)

[30] Kedziora D J, Ankiewicz, A., Akhmediev, N.: Circular rogue wave clusters. Phys. Rev. E **84,** 056611(2011)

[31] Onorato, M., Residori, S., Bortolozzo, U., Montina, A., Arecchi, F.: Rogue waves and their generating mechanisms in different physical contexts. Phys. Rep. **528,** 47–89(2013)

[32] Soto-Crespo, J. M., Devine, N., Akhmediev, N.: Integrable turbulence and rogue waves: breathers or solitons ? . Phys. Rev. Lett. **116,** 103901(2016)

[33] Akhmediev, N., Dudley, J. M., Solli, D. R., Turitsyn, S. K.: Recent progress in investigating optical rogue waves. J. Opt. **15,** 060201(2013)

[34] Ohta, Y., Yang, J. K., General high-order rogue waves and their dynamics in the nonlinear Schrödinger equation. Proc. R. Soc. Lond. Ser. A **468,** 1716–1740(2012)

[35] Christian, K., Efim, P.: Physical mechanisms of the rogue wave phenomenon. Euro. J. Mech. / B Fluids **22(6),** 603-634(2003).

[36] Wang L H, He J S, Xu H, Wang J, Porsezian K, Generation of higher-order rogue

waves from multibreathers by double degeneracy in an optical fiber, Phys. Rev. E, 2017,95:042217.

[37] Dudley, J. M., Dias, F., Erkintalo, M., Genty, G.: Instabilities, breathers and rogue waves in optics. Nature Photo. **8,** 755–764(2014)

[38] Ankiewicz, A., Akhmediev, N. Rogue wave-type solutions of the mKdV equation and their relation to known NLSE rogue wave solutions. Nonlinear Dyn. **91,** 1931–1938 (2018).

[39] Yin, H.M., Tian, B., Zhang, C.R., Du, X.X., Zhao, X.C.: Optical breathers and rogue waves via the modulation instability for a higher-order generalized nonlinear Schrödinger equation in an optical fiber transmission system. Nonlinear Dyn. **97,** 843–852 (2019)

[40] Wang, YY., Dai, CQ., Zhou, GQ. et al. Rogue wave and combined breather with repeatedly excited behaviors in the dispersion/diffraction decreasing medium. Nonlinear Dyn. **87,** 67–73 (2017).

[41] Temgoua, D.D.E., Tchoula Tchokonte, M.B., Maaza, M. et al. Contrast of optical activity and rogue wave propagation in chiral materials. Nonlinear Dyn. **95,** 2691–2702 (2019).

[42] Mukam, S.P.T., Souleymanou, A., Kuetche, V.K., Bouetou, T.B.: Generalized Darboux transformation and parameterdependent rogue wave solutions to a nonlinear shrodinger system. Nonlinear Dyn. **93(2)**, 373–383 (2018)

[43] Yu, W., Liu, W., Triki, H., Zhou, Q., Biswas, A., Belic, M.R.: Control of dark and anti-dark solitons in the (2+ 1)-dimensional coupled nonlinear Schrodinger equations with perturbed dispersion and nonlinearity in a nonlinear optical system. Nonlinear Dyn. **97,** 471–483 (2019)

[44] Dai, CQ., Wang, YY. & Zhang, JF. Managements of scalar and vector rogue waves in a partially nonlocal nonlinear medium with linear and harmonic potentials. Nonlinear Dyn. **102,** 379–391 (2020).

[45] Ye, Y., Liu, J., Bu, L. et al. Rogue waves and modulation instability in an extended Manakov system. Nonlinear Dyn. **102**, 1801–1812 (2020)

[46] Mukam, S.P.T., Souleymanou, A., Kuetche, V.K. et al. Generalized Darboux transformation and parameter-dependent rogue wave solutions to a nonlinear

Schrödinger system. Nonlinear Dyn. **93,** 373–383 (2018)

[47] Zhang, R.F., Li, M.C., Yin, H.M.: Rogue wave solutions and the bright and dark solitons of the (3+1)-dimensional Jimbo-Miwa equation. Nonlinear Dyn. **103,** 1071–1079 (2021)

[48] Birkholz, S., Nibbering, E. T. J.: Spatiotemporal rogue events in optical multiple filamentation. Phys. Rev. Lett. **111,** 243903(2013)

[49] Solli, D. R. Ropers, C., Koonath, P., Jalali, B.: Optical rogue waves. Nature **450,** 1054-1057(2007）

[50] Solli, D. R., Ropers, C., Jalali, B.: Active control of rogue waves for stimulated supercontinuum generation. Phys. Rev. Lett. **101,** 233902(2008)

[51] Kibler, B., Fatome, J., Finot, C., Millot, G., Genty, G., Wetzel, B., Akhmediev, N., Dias, F., Dudley, J.M.: Observation of Kuznetsov-Ma soliton dynamics in optical fiber. Sci. Rep. **2,** 463(2012)

[52] Chabchoub, A., Hoffmannn N. P., Akhmediev, N.: Rogue wave observation in a water wave tank. Phys. Rev. Lett.**106,** 204502(2011)

[53] Bailung, H., Sharma, S. K., Nakamura, Y.: Observations of peregrine solitons in a multicomponent plasma with negative ions. Phys. Rev. Lett.**107**, 255005(2011)

[54] Xiong, H., Gan, J. H., Wu, Y.: Kuznetsov-Ma Soliton dynamics based on the mechanical effect of light .Phys. Rev. Lett.**119:**153901(2017)

[55] Fokas, A. S.: On the simplest integrable equation in 2+1. Inverse Problems 1994, **10,** L19-22(1994)

[56] Estévez, P. G.: Darboux transformation and solutions for an equation in 2+1 dimensions. J. Math. Phys. **4,** 1406-1419(1999)

[57] Radha, R., Lakshmanan, M.: Localized coherent structures and integrability in a generalized (2+1)-dimensional nonlinear Schrödinger equation. Chaos, Solitons and Fractals **8,** 17-25(1997)

[58] Radha, R., Lakshmanan, M,: Generalized dromions in the (2 + 1) dimensional long dispersive wave (2LDW) and scalar nonlinear Schrödinger (NLS) equations. Chaos, Solitons and Fractals **10,** 1821-1824(1999)

[59] Villarroel, J., Prada, J., Estévez, P. G.: Dynamics of lump solutions in a 2+1 NLS equation. Stud. Appl. Math. **122,** 95-410(2009)

[60] Rao, J., Mihalache, D., Chen, Y., He, J.: Lump-soliton solutions to the Fokas system. Phys. Lett. A **383,** 1138-1142(2019)

[61] Rao, J. G., Wang, L. H., Zhang, Y., He, J, S.: Rational Solutions for the Fokas System. Commun. Theor. Phys. **64,** 605-618 (2015)

[62] Chen, T. T., Hu, P. Y., He, J. S.: General Higher-Order Breather and Hybrid Solutions of the Fokas System. Commun. Theor. Phys. **71,** 496-508(2019)

[63] Fermann, M. E.: Self-Similar Propagation and Amplification of Parabolic Pulses in Optical Fibers. Phys. Rev. Lett. **84(26)**, 6010-6013(2000)

[64] Kundu, A.: Integrable nonautonomous nonlinear Schrödinger equations are equivalent to the standard autonomous equation. Phys. Rev. E **79,** 015601(R) (2009)

[65] Akhmediev, N., Ankiewicz, A., Taki, M.: Waves that appear from nowhere and disappear without a trace. Phys. Lett. A **373**, 675-(2009)

[66] Kimmoun, O., Hsu, H. C., Branger, H., Li, M. S., Chen, Y. Y., Kharif, C., Onorato, M., Kelleher, E.J. R., Kibler, B., Akhmediev, N., Chabchoub, A.: Modulation Instability and Phase-Shifted Fermi-Pasta-Ulam Recurrence. Sci. Rep. **6**, 8516 (2016)