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Trajectory tracking of a mobile robot using adaptive sliding mode control

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Abstract— In this paper, a control system for a mobile four-wheeled robot is designed, whose task is to create stability and achieve proper performance in the execution of commands. Due to the nonlinear and time-varying dynamics, structural and parametric uncertainties of this robot, various control approaches are used in order to achieve stability, proper performance and minimize the effect of uncertainties and modeling errors, etc. The purpose of the control here is to follow a predetermined trajectory by adjusting linear and angular velocities in the presence of external disturbances and parametric uncertainty. In previous articles, the upper band of uncertainties has been assumed known. In this paper, and given that in practice, in many cases it is not possible to know the extent of uncertainties and disturbances in robotic systems, we have assumed that this upper band is unknown. Therefore, the sliding mode control law designed in the paper has been generalized and proved its stability so that by adding an adaptive part to the controller and converting it into a robust-adaptive sliding mode control, the upper band uncertainties are estimated online using these adaptive laws. The results of this typology are expressed in a separate theorem and proved to be stable. The results of simulation with MATLAB software show that the proposed controller ensures optimal performance under external disturbances and parametric uncertainty with less fluctuations.

Keywords— Mobile robot, Trajectory tracking, adaptive sliding mode control, Parametric uncertainty and disturbances

I. INTRODUCTION

In recent years, topics related to robotics have become one of the expanding fields of research. Among these, intelligent robots have gained a lot of popularity. However, the control and navigation of these devices is very difficult and safe and secure trajectory planning is one of the basic needs of these systems. A mobile robot should follow desired trajectories at a given time and may optimize criteria such as speed, security, environmental awareness, and so on along the trajectory. At each step, the robot should determine its position relative to the targets and define the appropriate control strategy to achieve the targets. Many works have been done on trajectory tracking of a mobile robot. In [1], state dependent Riccati equation (SDRE) method was used to solve the problem of autonomous navigation of the mobile robot in an environment where the obstacles are static and known. In [2], a SDRE based controller was employed to trajectory tracking of a robotic arm. In [3], modified artificial potential field algorithm was presented which allows the robot reach the target in an optimal trajectory without collision with obstacles and getting stuck in the local minimum. In [4], the trajectory tracking problem of nonholonomic wheeled mobile robots with bounded external disturbances and parameter uncertainties was addressed. Firstly, a kinematic controller is designed to generate the virtual velocity based on the kinematic error. Then, the sliding mode control with a modified reaching law is adopted to ensure the actual velocity can coverage to the virtual velocity in finite time based on the dynamic model. In [5], a neural approximation based model predictive control approach was proposed for tracking control of a nonholonomic wheel-legged robot in complex environments. In [6], an adaptive terminal sliding mode control scheme was presented for robust trajectory tracking control of an omnidirectional mobile robot. An adaptive online estimation law was designed in to overcome the total uncertainty. In [7], the problem of trajectory tracking for a non-holonomic mobile robot with non-random and random disturbances was investigated. A nonlinear disturbance observer with a developed Kalman filter was designed to observe the robot's speed and non-random disturbance. An error feedback controller and a kinematic controller have been proposed to achieve disturbance compensation and position tracking. The mean square exponential

constraint was presented for the robot estimation error. In [8], a mathematical model of a mobile robot with a magnetic gear used for mountaineering at certain industrial sites was developed and to achieve precise control of the robot's motion, an intelligent discrete algorithm based on dual dynamic programming was presented. In [9], the trajectory tracking model of an independent omni-directional mobile robot with a different friction model for each wheel and engine dynamics was obtained. In [10], asymptotic tracking control for non-holonomic wheeled mobile robot system under simultaneous actuator saturation and external perturbations was presented. A dynamic system was introduced to deal with actuator saturation, and radial basis function neural networks were used to approximate the unknown dynamics of the closed-loop system and an adaptive sliding mode feedback was used to compensate for estimation error as well as external disturbances. Much research has been done in the field of trajectory tracking using adaptive control method. For example, in [11], a Taylor series constructor approach was used for two-loop adaptive control of non-holonomic mobile robots. In [12], a real-time adaptive control framework was designed to achieve tracking of mobile robots. Based on the fuzzy neural network, an efficient control framework was provided with a combination of neural network controller and compensation controller. In [13], an adaptive neural network control scheme for a mobile wheeled robot with speed limits and non-holonomic constraints was presented. To overcome the robot's uncertainty, adaptive neural networks was used to approximate the robot's unknown dynamics and Lyapunov's function was used to guarantee speed limits. In [14], adaptive trajectory tracking of a mobile wheeled robot was performed based on constant time convergence with non-calibrated camera parameters. Initially, the rules for controlling the tracking of an adaptive trajectory with a fixed time were formulated. Then, using the terminal photosliding mode technology and adaptive control methods, the fixed trajectory tracking of the mobile robot with non-calibrated camera parameters was solved. Finally, the third-order sliding mode level was designed to solve a third-order chain system that converges to zero at a constant time. In the reviewed papers, the upper band of uncertainty is assumed to be known, but in this paper, we assume it to be unknown, and with the design of a new robust adaptive sliding mode controller, we estimate this upper band of uncertainty online using adaptive laws. From the simulation results, we see that we have reached the desired goal. The rest of paper is organized as follows. In section II, the mathematical model of four-wheeled robot is presented. The second-order sliding mode controller is developed in section III. The control designs of the Super Twisting and Adaptive Super Twisting are proposed in section IV. section V presents experimental real-time results and finally, the paper is concluded by some conclusions in section VI.

II. METHODS

A. model formulation

In this section, the kinematic and dynamic model of a four-wheeled robot and its control objectives are presented.

B. Modeling a four-wheeled robot

The four-wheeled robot model is developed with the following assumptions:

- Vehicle speed less than 10.4 km / h.
- The longitudinal slip of the wheel is ignored.
- The rigid vehicle moves on a horizontal plane.
- The lateral forces of the tire use its vertical load.

The kinematic model is expressed by the space vector $q = [x \ y \ \theta]^T$, x and y provide the center point position of vehicle corresponding to the earth frame and θ is the orientation of the robot. The dynamics of the state vector q is given as follows

$$\dot{q} = \begin{bmatrix} \cos(\theta) & -d \sin(\theta) \\ \sin(\theta) & d \cos(\theta) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (1)$$

Where v is the linear speed, ω is the angular speed and d is the distance from the point of instantaneous center of rotation and robot center of gravity. Dynamic model is presented by space vector $\eta = [v \ \omega]^T$ and dynamic equations is given as follows

$$\dot{\eta} = \begin{bmatrix} \frac{c_3}{c_1} \omega^2 - \frac{c_4}{c_1} v \\ -\frac{c_5}{c_2} v \omega - \frac{c_6}{c_2} \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{c_1} & 0 \\ 0 & \frac{1}{c_2} \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \quad (2)$$

Where $c_1, \dots, c_6$ are constant parameters and are given as some function of some physical parameters of the robot, such as the mass, moment inertia, motor parameters, etc. v_r and ω_r are the reference linear and angular velocities and they are the system control inputs. The two state vectors q and η are considered as measured variables [16].

C. Model uncertainty

System parameters cannot be specified precisely, especially when they depend on hardware and low speed control loops. All system parameters are considered as uncertainties. These parameters are as follows:

$$\begin{aligned} c_1 &= c_{01} + \delta c_1, c_2 = c_{02} + \delta c_2 \\ c_3 &= c_{03} + \delta c_3, c_4 = c_{04} + \delta c_4 \\ c_5 &= c_{05} + \delta c_5, c_6 = c_{06} + \delta c_6 \end{aligned} \quad (3)$$

Where c_{0i} is the nominal value of corresponding parameter, δc_i is parametric uncertainty such that $|\delta c_i| \leq \delta c_{0i} < |c_{0i}|$, where δc_{0i} is a known positive bound. Each parameter can be varied $\pm 20\%$ in relation to the nominal value.

D. Control objective

The aim of control is to force robot to track time varying desired trajectory as

$$q_r = [x_r, y_r]^T \quad (4)$$

Where x_r and y_r are the desired position and orientation in the earth frame. The vector q_r is given as function which depends explicitly on time. Moreover, the q_r can be generated by a reference kinematic model using a virtual model. The first and second derivatives of q_r are bounded due to robot limitation, i.e. maximum linear and angular velocities and maximum linear and angular accelerations. This objective can be achieved by providing the corresponded linear and angular reference velocities to the robot.

E. Second order sliding mode controller

The sliding control method, on the one hand, deviates from the control approach of switching, with the difference that the frequency of switching is generally unlimited, and on the other hand, it is close to the high-gain control method, with the property that instead of applying a signal with high gain, it applies a limited signal in a very short time, which somehow evokes the same unlimited signal in mind, thus, like high gain controllers, it increases the control system's robustness to uncertainty and disturbance. The mechanism of these controllers is based on imposing a constraint using a strong input. This means that we consider the level s for the system; If the system trajectory is placed in this manner, the constraint $s = 0$ will be fulfilled, otherwise the control system will force the system trajectory on the mentioned surface by applying the appropriate partial input. The mentioned surface is called the sliding surface and the movement of the trajectory on the mentioned surface and the fulfillment of the constraint $s = 0$ is called the sliding mode.

The process of guiding the system to the sliding surface using the sliding mode controller has two main steps:

- Select the slip level
- Appropriate tangential input that can converge trajectory on the surface [16]

In general, the design of the sliding surface reduces the order of the closed-loop system and provides a robust way in the movement of the system towards the equilibrium point. It is appropriate to choose a control law that moves the system to this level and ensures that it stays on it.

In general, the design of the slip surface reduces the order of the closed system and provides a solid bed in the movement of the system towards the equilibrium point. It is appropriate to choose a control policy that moves the system to this level and ensures that it stays and stays on it. In fact, it can be said that the biggest problem of the sliding model method is how to define the switching level but in general, we can say that the level of switching must be exactly equal to the error. This error is actually the error that we want to reduce to zero in our system. Despite the proper design of the sliding controller, it may be observed during the practical implementation of the controller that there is an irregular oscillation at a very high frequency at the output of the system, which is called chattering.

F. Super Twisting algorithm

In this subsection the conventional ST algorithm control design is presented. Consider a single-input single-output nonlinear system [16]

$$\begin{aligned}\dot{x} &= f(x) + g(x)u, \\ y &= s(x).\end{aligned}\tag{5}$$

Where $x \in \chi \subset \mathbb{R}^n$ is the state variable and $u \in U \subset \mathbb{R}$ is the control input. f and g are smooth functions. $s(x)$ is called the sliding variable. The control objective of such a system is to force the sliding variable to zero. Assuming that the relative degree of the system is equal to 1 for $s(x)$. As SOSMC is applied, so the system bounds can be found in the second derivative of the sliding variable. The first and second derivative of $s(x)$ are given as follows

$$\begin{aligned}\dot{s} &= \frac{\partial}{\partial t}s(t, x) + \frac{\partial}{\partial x}[s(t, x)][f(x) + g(x)u], \\ \ddot{s} &= a(t, x, u) + b(t, x, u)\dot{u}(t).\end{aligned}\tag{6}$$

The function $a(x, t)$ and $b(x, t)$ are bounded for $x \in \chi$.

$$0 < K_m \leq b(t, x, u) \leq K_M, |a(t, x, u)| \leq C\tag{7}$$

The ST algorithm is defined by the following control law

$$u = -\alpha |s|^{\frac{1}{2}} \text{sign}(s) - \beta \int \text{sign}(s) dt,\tag{8}$$

Where α and β are positive bounded constants. The controller gains α and β should be over-estimated respecting to the system bounds with uncertainty in order to get a robust controller, as given as follows

$$\begin{aligned}\beta &> \frac{C}{K_m}, \\ \alpha^2 &\geq \frac{4CK_M(\beta + C)}{K_m^3(\beta - C)}\end{aligned}\tag{9}$$

G. Adaptive super Twisting algorithm

The main features of sliding mode control are closed-loop system robustness and limited time convergence, which can be achieved using a discontinuous function with high control gain. Awareness of the uncertainty bounds must be in order to design a controller, which is a difficult task from a practical point of view. This is compensated by increasing and overestimating the controller gains. The high gains in turn, increase chattering. Thus, an Adaptive ST (AST) algorithm is used to resolve the over-estimation condition of the controller gains. The controller gains are dynamically adapted to the parameter variations caused by system parametric uncertainty and external disturbance, which results in chattering reduction in steady state. The finite time convergence of the closed-loop system is derived using the Lyapunov function technique has been formally proved. The main difference of this algorithm comparing to the ST, is the controller force the sliding variable to a predefined neighborhood of the sliding surface. The controller gains α and β are dynamics and are expressed as function of the sliding variable and its neighborhood as follows

$$\begin{aligned}\dot{\alpha} &= \begin{cases} \vartheta \text{sign}(|s| - \mu), & \text{if } \alpha > \alpha_m, \\ \kappa & , \quad \text{if } \alpha \leq \alpha_m, \end{cases} \\ \beta &= \varepsilon \alpha,\end{aligned}\tag{10}$$

where $\vartheta, \mu, \alpha_m, \kappa$ and ε are positive constants. μ presents the accepted neighborhood of the sliding variable. The gains increase with a dynamic step equal to ϑ until μ is reached. Then, the dynamic gains decrease until the gain α is lower or equal to α_m [16].

H. Exact differentiator

The controller requires the knowledge of the first and second time derivatives of the set-point. Therefore, real time robust exact differentiator has been used

$$\begin{aligned} z_0 &= -\varepsilon_2 \sqrt{L} \sqrt{|z_0 - \rho|} \text{sign}(z_0 - \rho) + z_1, \\ z_1 &= -\varepsilon_1 L \text{sign}(z_1 - z_0), \end{aligned} \quad (11)$$

Where z_0 and z_1 are the real time estimates of ρ and $\dot{\rho}$, respectively. The parameters of the differentiator ε_1 and ε_2 are to be chosen in advance [17]. L is the only differentiator parameter to be tuned, and it has to satisfy the only condition $|\dot{\rho}| \leq \rho$ [16].

III. CONTROL DESIGN FOR FOUR-WHEELED ROBOT

A Multi-Input Multi-Output (MIMO) SOSMC is developed for trajectory tracking control of SSMR. Let us recall that our control objective is to force the robot to follow predefined coordinates x_r and y_r with respect to time variations. In this section, the control design of the conventional ST and the proposed AST are presented [16].

A. Control design using ST algorithm

Two sliding manifolds s_1 and s_2 have been chosen to force the coordinate x and y to their reference points x_r and y_r , respectively:

$$\begin{aligned} s_1 &= \lambda_1 (x - x_r) + \dot{x} - \dot{x}_r, \\ s_2 &= \lambda_2 (y - y_r) + \dot{y} - \dot{y}_r, \end{aligned} \quad (12)$$

Where λ_1 and λ_2 are positive constants.

The sliding variables have been designed so that the system has a relative degree equal to 1 with respect to the sliding variables. Therefore, the system control inputs $[v_r, \omega_r]^T$ will appear in the first time derivative of these sliding variables, given as follows,

$$\begin{aligned} \dot{s}_1 &= \lambda_1 (v \cos \theta - d \omega \sin \theta - \dot{x}_r) + \left(\frac{c_3}{c_1} \omega^2 - \frac{c_4}{c_1} v + \frac{v_r}{c_1} \right) \cos \theta - v \omega \sin \theta \\ &\quad - d \left(-\frac{c_5}{c_2} v \omega - \frac{c_6}{c_2} \omega + \frac{\omega_r}{c_2} \right) \sin \theta - d \omega^2 \cos \theta - \ddot{x}_r, \\ \dot{s}_2 &= \lambda_2 (v \sin \theta + d \omega \cos \theta - \dot{y}_r) + \left(\frac{c_3}{c_1} \omega^2 - \frac{c_4}{c_1} v + \frac{v_r}{c_1} \right) \sin \theta + v \omega \cos \theta \\ &\quad + d \left(-\frac{c_5}{c_2} v \omega - \frac{c_6}{c_2} \omega + \frac{\omega_r}{c_2} \right) \cos \theta - d \omega^2 \sin \theta - \ddot{y}_r, \end{aligned} \quad (13)$$

Where $\dot{y}_r, \ddot{x}_r, \dot{x}_r$ and $\ddot{y}_r$ are calculated using the real time exact differentiator given in Eq. (11).

The first time derivatives of the s_1 and s_2 can be written as follows,

$$\begin{aligned} \dot{s}_1 &= \phi_1 + \gamma_1 v_r, \\ \dot{s}_2 &= \phi_2 + \gamma_2 \omega_r, \end{aligned} \quad (14)$$

Where

$$\begin{aligned} \phi_1 &= \phi_{01} + \delta \phi_1, & \gamma_1 &= \gamma_{01} + \delta \gamma_1, \\ \phi_2 &= \phi_{02} + \delta \phi_2, & \gamma_2 &= \gamma_{02} + \delta \gamma_2, \end{aligned} \quad (15)$$

Where $\phi_{01}, \phi_{02}, \gamma_{01}$ and γ_{02} are calculated using the nominal parameters (c_{0i}) while the bounds of $\delta \phi_1, \delta \phi_2, \delta \gamma_1$ and $\delta \gamma_2$ are estimated using the parametric uncertainty limits (δc_i) which can be varied $\pm 20\%$.

A FBL method is applied in order to compensate the known nonlinear part based on the nominal parameters. The system control inputs are given as follows

$$\begin{aligned} \nu_r &= \gamma_{01}^{-1}[\nu_1 - \phi_{01}], \\ \omega_r &= \gamma_{02}^{-1}[\nu_2 - \phi_{02}]. \end{aligned} \quad (16)$$

Where ν_1 and ν_2 are the controller outputs of the ST algorithm. The ST controller outputs ν_1 and ν_2 can be written as follows:

$$\begin{aligned} \nu_1 &= -\alpha_1 |s_1|^{\frac{1}{2}} \text{sign}(s_1) - \int \beta_1 \text{sign}(s_1) dt, \\ \nu_2 &= -\alpha_2 |s_2|^{\frac{1}{2}} \text{sign}(s_2) - \int \beta_2 \text{sign}(s_2) dt. \end{aligned} \quad (17)$$

Applying (16) and (17) to (14), the dynamics of the sliding variable can be written as follows

$$\begin{aligned} \dot{s}_1 &= \hat{\phi}_1 + \hat{\gamma}_1 (-\alpha_1 |s_1|^{\frac{1}{2}} \text{sign}(s_1) - \int \beta_1 \text{sign}(s_1) dt) \\ \dot{s}_2 &= \hat{\phi}_2 + \hat{\gamma}_2 (-\alpha_2 |s_2|^{\frac{1}{2}} \text{sign}(s_2) - \int \beta_2 \text{sign}(s_2) dt). \end{aligned} \quad (18)$$

In case of using the ST algorithm, the controller gains $\alpha_1, \alpha_2, \beta_1$ and β_2 are estimated under the conditions (7) and (9). For the AST algorithm the controller gains will be adapted dynamically using Eq. (10) and they do not require estimation studies. $\hat{\phi}_1, \hat{\gamma}_1, \hat{\phi}_2, \hat{\gamma}_2$ are uncertainty functions and they are given as

$$\begin{aligned} \hat{\phi}_1 &= \delta\phi_1 - \phi_{01} \frac{\delta\gamma_1}{\gamma_{01}}, \hat{\gamma}_1 = 1 + \frac{\delta\gamma_1}{\gamma_{01}}, \\ \hat{\phi}_2 &= \delta\phi_2 - \phi_{02} \frac{\delta\gamma_2}{\gamma_{02}}, \hat{\gamma}_2 = 1 + \frac{\delta\gamma_2}{\gamma_{02}}. \end{aligned} \quad (19)$$

The controller gains of the ST algorithm have been tuned accordingly to the following values $\alpha_1 = \alpha_2 = 0.8$ and $\beta_1 = \beta_2 = 0.1$ with respect to condition (9).

B. Control design using AST algorithm

In this case study, the trajectory tracking controller is developed using the adaptive ST controller. The sliding variables of the adaptive controller are identical to those of the robust controller given in Eq. (12). The difference is that the controller gains need not be determined as they are dynamic. Two sliding manifolds s_3 and s_4 have been chosen:

$$\begin{aligned} s_3 &= \lambda_3 (x - x_r) + \dot{x} - \dot{x}_r \\ s_4 &= \lambda_4 (y - y_r) + \dot{y} - \dot{y}_r \end{aligned} \quad (20)$$

With λ_3 and λ_4 being positive constants. The same steps of the previous control design are followed, starting with derivative of the sliding variables and then applying a feedback linearization using nominal parameters to compensate for non-linearity. The first derivative of the sliding variables can be written as

$$\begin{aligned} \dot{s}_3 &= \hat{\phi}_1 + \hat{\gamma}_1 (-\alpha_3 |s_3|^{\frac{1}{2}} \text{sign}(s_3) - \int \beta_3 \text{sign}(s_3) dt) \\ \dot{s}_4 &= \hat{\phi}_2 + \hat{\gamma}_2 (-\alpha_4 |s_4|^{\frac{1}{2}} \text{sign}(s_4) - \int \beta_4 \text{sign}(s_4) dt). \end{aligned} \quad (21)$$

Where the adaptive controller gains α_3 and α_4 are given as follows,

$$\begin{aligned}\dot{\alpha}_3 &= \begin{cases} \mathcal{G}_1 \text{sign}(|s_3| - \mu_1), & \text{if } \alpha_3 > \alpha_{3m}, \\ \kappa_1, & \text{if } \alpha_3 \leq \alpha_{3m}, \end{cases} \\ \beta_3 &= \varepsilon_3 \alpha_3, \end{aligned} \quad (22)$$

$$\begin{aligned}\dot{\alpha}_4 &= \begin{cases} \mathcal{G}_2 \text{sign}(|s_4| - \mu_2), & \text{if } \alpha_4 > \alpha_{4m}, \\ \kappa_2, & \text{if } \alpha_4 \leq \alpha_{4m}, \end{cases} \\ \beta_4 &= \varepsilon_2 \alpha_4. \end{aligned} \quad (23)$$

IV. SYSTEM MODEL AND INNOVATION

The kinematic and dynamic model of the mobile robot is considered as Equations (1) and (2). Considering $q_r = [x_r, y_r]^T$ as a reference trajectory, the aim is to design the input speeds v_r and ω_r such that despite the uncertainty in the dynamic parameters (c_i) in model (1), the robot follow a predetermined optimal. In previous section, the high band of uncertainties was assumed to be known.

In practical applications, in many cases, it is not possible to know the extent of uncertainties and disturbances in robotic systems. In this paper we assume that the high band of uncertainties is unknown.

Therefore, the sliding mode control law designed here has been generalized so that by adding an adaptive part to the controller and converting it to a robust-adaptive sliding mode control, the high band of uncertainties be estimated using these online adaptive rules.

V. ALGORITHM DESIGN

A flowchart illustrating the steps of adaptive sliding mode controller design is given in Fig. 1.

Fig. 1. The framework of the adaptive sliding mode controller

A. Redefinition of sliding manifolds

First, sliding manifolds are considered as (12). Then, to satisfy sliding mode condition, the access rule is selected as follows:

$$\dot{s}_i(t) = -rs_i - \rho \operatorname{sgn}(s_i) \quad (24)$$

Where ρ and r are non-negative constants and $\text{sgn}(\cdot)$ is sign function. By derivation from sliding manifolds (12) and substituting in system model (1) the following equations are obtained

$$\begin{aligned}\dot{s}_1 &= \lambda_1(v \cos \theta - d\omega \sin \theta - \dot{x}_r) + \left(\frac{c_3}{c_1}\omega^2 - \frac{c_4}{c_1}v + \frac{v_r}{c_1}\right)\cos \theta - v\omega \sin \theta \\ &\quad - d\left(-\frac{c_5}{c_2}v\omega - \frac{c_6}{c_2}\omega + \frac{\omega_r}{c_2}\right)\sin \theta - d\omega^2 \cos \theta - \ddot{x}_r, \\ \dot{s}_2 &= \lambda_2(v \sin \theta + d\omega \cos \theta - \dot{y}_r) + \left(\frac{c_3}{c_1}\omega^2 - \frac{c_4}{c_1}v + \frac{v_r}{c_1}\right)\sin \theta + v\omega \cos \theta \\ &\quad + d\left(-\frac{c_5}{c_2}v\omega - \frac{c_6}{c_2}\omega + \frac{\omega_r}{c_2}\right)\cos \theta - d\omega^2 \sin \theta - \ddot{y}_r,\end{aligned}\tag{25}$$

The equation (25) can be rewritten as follows

$$\begin{aligned}\dot{s}_1 &= \phi_1 + \gamma_1 v_r, \\ \dot{s}_2 &= \phi_2 + \gamma_2 \omega_r,\end{aligned}\tag{26}$$

In which nominal terms and terms containing parametric uncertainties can be separated as follows:

$$\begin{aligned}\phi_1 &= \phi_{01} + \delta\phi_1, & \gamma_1 &= \gamma_{01} + \delta\gamma_1, \\ \phi_2 &= \phi_{02} + \delta\phi_2, & \gamma_2 &= \gamma_{02} + \delta\gamma_2,\end{aligned}\tag{27}$$

By applying (27) in (26), the derivative of the sliding manifolds can be rewritten as follows by separating the terms with uncertainty:

$$\begin{aligned}\dot{s}_1 &= \phi_{01} + \delta\phi_1 + (\gamma_{01} + \delta\gamma_1)v_r = \phi_{01} + \gamma_{01}v_r + \alpha_1 \\ \dot{s}_2 &= \phi_{02} + \delta\phi_2 + (\gamma_{02} + \delta\gamma_2)\omega_r = \phi_{02} + \gamma_{02}\omega_r + \alpha_2\end{aligned}\tag{28}$$

Where $\alpha_1 = \delta\phi_1 + \delta\gamma_1 v_r$ and $\alpha_2 = \delta\phi_2 + \delta\gamma_2 \omega_r$ are terms with uncertainties.

Assumption 1. Let assume that the terms with uncertainties in (28) have known high band such that they be limited as follows

$$\begin{aligned}\|\alpha_1\| &\leq \beta_1 \\ \|\alpha_2\| &\leq \beta_2\end{aligned}\tag{29}$$

Where β_1 and β_2 are unknown.

The purpose is to design an adaptive sliding mode control law is to achieve the desired trajectory convergence for the robot without knowing the high band of uncertainties. In this case, the values β_1 and β_2 are estimated by an adaptive law.

B. Controller design

Assume $\hat{\beta}_1$ is an estimate of quantity β_1 and $\hat{\beta}_2$ is an estimate of quantity β_2 . This being the case, the adaptive sliding mode control law is considered as follows

$$\begin{aligned}v_r &= \gamma_{01}^{-1} \left\{ -\phi_{01} - rs_1 - \rho \hat{\beta}_1 \text{sgn}(s_1) \right\} \\ \omega_r &= \gamma_{02}^{-1} \left\{ -\phi_{02} - rs_2 - \rho \hat{\beta}_2 \text{sgn}(s_2) \right\}\end{aligned}\tag{30}$$

In which the law of adaptive updating of the parameters are considered as follows:

$$\begin{aligned}\dot{\hat{\beta}}_1(t) &= k_2 \rho s_1^T \text{sgn}(s_1) \\ \dot{\hat{\beta}}_2(t) &= k_2 \rho s_2^T \text{sgn}(s_2)\end{aligned}\tag{31}$$

Where $k_1, k_2 > 0$ are positive gains in adaptive terms.

Theorem 1. Consider the mobile robot with dynamic equations (1) and assumption 1. If constants r, k_1, k_2, ρ are selected such that $r > 0, k_1, k_2 > 0, \rho > 1$ and the sliding manifold is also defined as (12), then by applying the control input in (30) along with the adaptive rules in (31), the tracking error will be reached zero and the mobile robot will follow the desired trajectory.

Prof. By substituting the control law (30) in (28) we have

$$\begin{aligned}\dot{s}_1 &= -rs_1 - \rho\hat{\beta}_1 \operatorname{sgn}(s_1) + \alpha_1 \\ \dot{s}_2 &= -rs_2 - \rho\hat{\beta}_2 \operatorname{sgn}(s_2) + \alpha_2\end{aligned}\quad (32)$$

Now we choose the Lyapunov function as follows

$$V = \frac{1}{2}s_1(t)^T s_1(t) + \frac{1}{2}s_2(t)^T s_2(t) + \frac{1}{2k_1}\tilde{\beta}_1(t)^2 + \frac{1}{2k_2}\tilde{\beta}_2(t)^2 \quad (33)$$

Where $\tilde{\beta}_1(t) = \hat{\beta}_1(t) - \beta_1$ and $\tilde{\beta}_2(t) = \hat{\beta}_2(t) - \beta_2$

Then by deriving from V we have:

$$\dot{V} = s_1(t)^T \dot{s}_1(t) + s_2(t)^T \dot{s}_2(t) + \frac{1}{k_1}\tilde{\beta}_1(t)\dot{\tilde{\beta}}_1 + \frac{1}{k_2}\tilde{\beta}_2(t)\dot{\tilde{\beta}}_2 \quad (34)$$

Then by substituting (32) in (34) we have

$$\begin{aligned}\dot{V} &= s_1(t)^T \{-rs_1 - \rho\hat{\beta}_1 \operatorname{sgn}(s_1) + \alpha_1\} \\ &+ s_2(t)^T \{-rs_2 - \rho\hat{\beta}_2 \operatorname{sgn}(s_2) + \alpha_2\} + \frac{1}{k_1}\tilde{\beta}_1(t)\dot{\tilde{\beta}}_1 + \frac{1}{k_2}\tilde{\beta}_2(t)\dot{\tilde{\beta}}_2 \\ &= -r\|s_1(t)\|^2 - \rho\hat{\beta}_1\|s_1(t)\| + s_1(t)^T \alpha_1 - r\|s_2(t)\|^2 \\ &- \rho\hat{\beta}_2\|s_2(t)\| + s_2(t)^T \alpha_2 + \frac{1}{k_1}(\hat{\beta}_1(t) - \beta_1)\dot{\hat{\beta}}_1 + \frac{1}{k_2}(\hat{\beta}_2(t) \\ &- \beta_2)\dot{\hat{\beta}}_2\end{aligned}\quad (35)$$

Now by substituting proposed adaptive rules (31) in (35) we have:

$$\begin{aligned}\dot{V} &= -r\|s_1(t)\|^2 - \rho\hat{\beta}_1\|s_1(t)\| + s_1(t)^T \alpha_1 - r\|s_2(t)\|^2 - \rho\hat{\beta}_2\|s_2(t)\| \\ &+ s_2(t)^T \alpha_2 + \rho(\hat{\beta}_1(t) - \beta_1)\|s_1(t)\| + \rho(\hat{\beta}_2(t) - \beta_2)\|s_2(t)\| \\ &= -r\|s_1(t)\|^2 + s_1(t)^T \alpha_1 - r\|s_2(t)\|^2 + s_2(t)^T \alpha_2 \\ &- \rho\beta_1\|s_1(t)\| - \rho\beta_2\|s_2(t)\|\end{aligned}\quad (36)$$

Now, according to assumption 1, $s_1(t)^T \alpha_1 \leq \beta_1 s_1(t)$ and $s_2(t)^T \alpha_2 \leq \beta_2 s_2(t)$, the upper band of the derivative of Lyapunov function can be written as follows:

$$\begin{aligned}\dot{V} &\leq -r\|s_1(t)\|^2 + \beta_1\|s_1(t)\| - r\|s_2(t)\|^2 + \beta_2\|s_2(t)\| - \rho\beta_1\|s_1(t)\| - \rho\beta_2\|s_2(t)\| \\ &= -r\left(\|s_1(t)\|^2 + \|s_2(t)\|^2\right) + (1 - \rho)(\beta_1\|s_1(t)\| + \beta_2\|s_2(t)\|)\end{aligned}\quad (37)$$

Thus, with the conditions $\rho > 1$ and $r > 0$ since Lyapunov function is definite positive and its time derivative is negative, it can be concluded that according to Lyapunov stability theorem and in accordance with the previous theorem, the error values converge to zero. Therefore, when the sliding manifolds are zero, the error values in the tracking of the desired trajectory in the plane are zero.

VI. RESULTS AND DISCUSSION

In this section, simulation results are given to show the efficiency of proposed method.

In the Adaptive Super Twisting algorithm, with the knowledge of the high band of uncertainties, the controller parameters are set as follows

$$\vartheta_1 = \vartheta_2 = 0.25, a_{3m} = a_{4m} = 0.001, \mu_1 = \mu_2 = 0.02, \kappa_1 = \kappa_2 = 0.1, \varepsilon_1 = \varepsilon_2 = 0.2$$

The trajectory tracking control is implemented on the embedded computer using Matlab/Simulink software on Windows operating system. The sample time fixed step of the controller program on Matlab is set 2 times slower than the robot communication rate, i.e. 10 ms. The robot linear and angular velocities are limited up-to 0.7 m/s and 360 deg/s, and the linear and angular accelerations are limited up-to 0.2 m/s² and 360 deg/s².

The requested trajectory is selected to have a circular shape as function of time (t), and it is given as follow

$$x_r = \cos(0.25t)$$

$$y_r = \sin(0.25t)$$

Noted, that the experiment of each controllers is performed for 75 s, which equivalent for 3 circles, i.e. 3 periods. The initial condition of the robot on the earth frame is ($x = 0, y = 0, \theta = 0$) in meter.

The proposed control scheme is performed using Adaptive Super Twisting algorithm based on the second-order sliding mode control. The controller parameters must be set such that the gains be stabilized quickly and without fluctuation, while ensuring the stability of the closed loop.

Figures 2, 3 and 4 show Robot trajectory and reference trajectory, robot trajectory and reference trajectory in the x direction and robot trajectory and reference trajectory in the y direction, respectively.

Fig. 2. Robot trajectory and reference trajectory

Fig. 3. Robot trajectory and reference trajectory in the x direction

Fig. 4. Robot trajectory and reference trajectory in the y direction

The proposed controller has been designed with the knowledge of the high band of uncertainties and by selecting the following control parameters:

$$\lambda_1 = 0.7, \lambda_2 = 1.1$$

$$r = 10, \rho = 8$$

$$k_1 = 2, k_2 = 3$$

In addition, random numbers have been used to indicate uncertainty. Simulink is the model used in MATLAB as shown in Fig. 5.

Fig. 5. Matlab/Simulink scheme

The simulation results with the initial conditions $q_0 = [2, 4, 30]^T$ are shown in Fig. 6- Fig. 10.

Fig. 6. Errors in x and y directions

Fig. 7. Control input

Fig. 8. Robot trajectory and reference trajectory

Fig. 9. Robot trajectory and reference trajectory in x and y directions

Fig. 10. Adaptive parameters

As can be seen in simulation results, by applying the adaptive sliding mode control law, firstly, the robot follows the desired trajectory well and the error values converge to zero, and secondly, the values of the adaptive parameters also are converged to limited values.

VII. CONCLUSION

In this paper, a robust controller is proposed for trajectory tracking of a mobile robot, which is based on the Adaptive Super Twisting algorithm. The AST controller adapts its gains dynamically to keep the control action just sufficient. Therefore its practical implementation does not need continuous approximation of the sign function for chattering reduction, and the robustness and finite time convergence properties are not compromised. Since in many practical applications, it is not possible to know the range of uncertainties and disturbances in robotic systems, in this paper, the high band of uncertainties is unknown. Therefore, the sliding mode control law designed in the paper has been generalized and proven stable such that by adding an adaptive part to the controller and converting it into a robust-adaptive sliding mode control, the high band of uncertainties be estimated using these online adaptive rules. The simulation results indicates the practicality of proposed method in trajectory tracking of mobile robots in the aforementioned conditions.

•Abbreviations

ASMC: Adaptive Sliding Mode Controller, ST algorithm: Super Twisting algorithm

DECLARATIONS

• ETHICS APPROVAL AND CONSENT TO PARTICIPATE

All the authors of this manuscript would like to declare that mutually agreed no conflict of interest. All the results of this paper with respect to the experiments on human subject.

• CONSENT FOR PUBLICATION

Not Applicable

• AVAILABILITY OF DATA AND MATERIAL

The data is available on request with prior concern to the first author of this paper.

• COMPETING INTERESTS

“The authors declare that they have no competing interests.”

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• AUTHORS' CONTRIBUTIONS

All the authors conceived the idea, developed the method, and conducted the experiment.

Prof. Seyyed Mohammad Hosseini Rostami – Formulation of methodology and experiments

Prof. Fatemeh jahangiri hosseinabadi - Data Analysis and Performance Analysis

Prof . Xiaofeng Yu– Overview of the proposed approach and decision analysis

Prof. Jingyu Zhang, Algorithm design

Prof. Heyuan Shi, data sources

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• AUTHORS' INFORMATION (OPTIONAL)

Nil

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