A Dexterity Comparison for 6 DOF Hybrid Robots

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A Dexterity Comparison for 6 DOF Hybrid Robots

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Abstract. In this paper, new hybrid robots are suggested which divided the task into a position and orientation tasks. The position mechanism controls the position whereas the orientation one manipulates the orientation of the end effector. These robots consist of a translational parallel manipulator and a rotational serial or parallel mechanism. The 3UPU or Tricept parallel manipulator and a three-axis gimbaled system or parallel shoulder manipulator are chosen for translational and rotational movements, respectively. The main goal of this paper is analyzing the development and combination of serial and parallel manipulators in order to increase their features. According to this purpose, serial and parallel mechanisms with three DOF are combined in a way to encompass six DOF space. It is shown hybrid mechanisms with less coupling between their subsystems are capable of increasing robot characteristics.

Keywords: hybrid; robot; dexterity; optimization; stiffness

1. Introduction

Usually, industrial robots are made in accordance with serial manipulator architecture. Their main advantage is their large workspace with respect to their own volume and occupied floor space, easy forward kinematics and have a wide applicability in industry. Their main disadvantages are the low stiffness inherent to an open kinematic structure, errors are accumulated and amplified from link to link the fact that they have low effective load because of carrying the large weight of manipulators. Parallel manipulators have designed in a way that they have high structural stiffness. Their major drawback is their limited workspace (Campos et al., 2008; Yeshmukhametov et al., 2017). The hybrid inherit higher rigidity and higher loading capacity from serial and larger workspace and higher dexterity from parallel manipulators. The publication (Carbone & Ceccarelli, 2004) proposed a general method for analyzing stiffness of parallel-serial manipulators. This method is applied by Yang et al. (2008) to design and analyze a modular hybrid parallel-serial manipulator for robotised deburring of large jet engine components which consists of a 3-DOF (degree-of-freedom) planar parallel platform and a 3-DOF serial robotic arm. Yun and Li (2010) presents the design and modeling of a hybrid 6-DOF 8-PSS/SPS compliant dual redundant parallel robot with wide-range flexure hinges with high accurate positioning or rough positioning as well as a 6-DOF active vibration isolation and excitation to the payload placed on the moving platform. They also improved the structure and control algorithm optimization for a dual redundant parallel mechanism in order to achieve the feature of larger workspace, higher motion precision and better dynamic characteristics.

Most of the design methods of hybrid robots are based on the decomposition of translatory and rotary motions. Zeng et al. (2011) proposed a 4-DOF hybrid manipulator that allows for the realization of two translatory and two rotary output motions. Rahmani et al. (2014) composed two modules which consist of elementary manipulators with the parallel structure of Stewart Platform and shows that the proposed robots performed with high accuracy. A new reconfigurable parallel

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robotic manipulator is studied in the study of Coppola et al. (2014) and Unique characteristics are revealed and studied as a case study. It also solved a multi-objective optimization problem to carry out weighted stiffness, dexterity and workspace volume as the performance indices. Tian et al. (2016) presented a hybrid robot which includes two Stewart mechanisms in serial form, known as 2-(6UPS). It also addressed forward kinematic solution of 2-(6UPS) in order to develop a manipulator with additional constraints. The motion of spatial mechanisms with coupling chains (Tian et al., 2016) is two rotations and two translations (2R2T). Rastegarpanah et al. (2018) proposed a hybrid robot for ankle rehabilitation, which takes advantage of two important characteristics of parallel robots: stiffness and workspace. Rastegarpanah et al. (2018) also determined an optimum path based on maximum stiffness in the workspace of a 9 DoF hybrid parallel mechanism, which consists 9 parallel linear actuators and 2 serial moving platforms.

Recent researches refers to the efficient control techniques in order to improve the performances of the hybrid robots. Zhao et al. (2019) presents a novel 3 DOF serial-parallel hybrid leg which has improved the performances and reduced the manufacturing cost of the legs for quadruped robots. He et al. (2019) propose a control scheme for a hybrid manipulator which is used for the capturing mission in space. Liu and Yao (2019) proposed a serial-parallel hybrid worm-like robot based on two 3-RPS PMs with expandable platforms. Linearized error model of a 6-DOF polishing hybrid robot is formulated by Huang et al. (2019). The authors of the publication (Huang et al., 2019) considered all possible geometric source errors at link level. Kinematics and dynamics model of a 6-DOF serial-parallel hybrid humanoid arm is considered by Y. Li et al. (2020). Y. Li et al. (2020) has solved an optimal objective function to minimize the components coupling characteristics. He et al. (2020) presented a redundant hybrid finger mechanism actuated by flexible actuators and considered dexterity, velocity, and stiffness. J. Li et al. (2020) analyzed the correlation laws between optimization parameters and objectives of a 2UPR-RPS-RR hybrid-structure robot.

In this paper, typical 3DOF serial and parallel mechanisms are studied in order to develop 6DOF hybrid manipulator. The 3UPU, Tricept, Gimbaled and Shoulder manipulators are reviewed respectively. Finally, we consider different arrangement of these mechanisms and analyze the features of their characteristics. Some quantitative metrics are applied to compare between proposed arrangements.

2. Optimization criteria

It is necessary to have some quantitative metrics to compare hybrid robots. Dexterity is the most important kinematic metric of the parallel and serial manipulators. The dexterity of a robotic manipulator can be described as its ability to perform small displacements in arbitrary directions as easily as possible in its workspace. It is based on the condition number (κ) of the Jacobian matrix. The condition number of a full rank matrix J can be defined as:

$$\kappa = \sigma_{\max} / \sigma_{\min} \quad (1)$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the biggest and smallest singular values of the Jacobian matrix. The following objective function ($D.I$) to be minimized was proposed by Stoughton and Arai (1993).

$$D.I = \int_{V^*} \kappa \tilde{J} dV / V^* \quad (2)$$

where $\tilde{J}$ is the scaled Jacobian, and V is a the overall workspace volume. V^* is a centralized subregion of the overall workspace volume. In order to analyze the velocity and position of the hybrid robot, it is necessary first to treat them separately and thereafter link the individual steps into an integral procedure for the combined hybrid system. In this section, the structural characteristic of 3UPU, Tricept, Gimbaled and Shoulder manipulators are investigated. Then, we consider different arrangement of these mechanisms.

4. Translational Manipulator

This section aims to consider two translational parallel manipulators.

4.1 3UPU manipulator

The architecture of a 3UPU parallel manipulator is shown in Fig. 1 (a). Joshi and Tsai (2003) solved inverse and forward kinematics of a 3UPU and also explained the conditions which keep the moving platform from changing its orientation. The Jacobian of a 3UPU parallel manipulator is calculated as following:

$$J_{upu} v_M = \dot{q}_{3UPU}, J_{upu} = [s_1, s_2, s_3] \quad (3)$$

where the velocity vectors of the end effector and actuator joint rates, respectively, are denoted by v_M and $\dot{q}_i$. s_i is a unit vector pointing along the i th limb.

(a) A spatial 3 UPU parallel manipulator

(b) A tricept manipulator

Fig. 1 Translational Manipulators

4.2 Tricept manipulator

The architecture of the Tricept manipulator (Joshi & Tsai, 2003) is shown in Fig. 1 (b). As shown in Fig. 2, the fixed frame $\{G\}$ is attached to the fixed base with the x-axis pointing from V to g_1 and the coordinate frame $\{H\}$ is attached to the moving platform with the U-axis pointing from T to h_1 . The position vectors of points g_i and h_i with respect to coordinate frames $\{G\}$ and $\{H\}$, respectively, are denoted by ${}^G g_i$ and ${}^H h_i$. For the Tricept manipulator, a position vector, $T = VT$, is used to define the position of the moving platform.

$$\vec{T} = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T \quad (4)$$

A rotation matrix is defined as following.

$$C_H^G = \begin{bmatrix} s\varphi_2 & 0 & -c\varphi_2 \\ c\varphi_1 c\varphi_2 & s\varphi_1 & c\varphi_1 s\varphi_2 \\ c\varphi_2 s\varphi_1 & -c\varphi_1 & s\varphi_1 s\varphi_2 \end{bmatrix} \quad (5)$$

where φ_1 and φ_2 denote two successive rotations of the moving frame and are calculated as follows.

$$\varphi_2 = \cos^{-1}\left(-\frac{t_x}{d_p}\right), \varphi_1 = A \tan^{-1}\left(\frac{t_z}{d_p s \varphi_2}, \frac{t_y}{d_p s \varphi_2}\right) \quad (6)$$

The Jacobian matrix of a 3-DOF position mechanism relates the linear velocity of the moving platform, $\dot{x}_T$, to the vector of actuated joint rates, $\dot{q}_{tri} = [\dot{q}_1, \dot{q}_2, \dot{q}_3]^T$, by $\dot{q}_{tri} = J_{q-tri} \dot{x}_{tri}$.

$$J_{q-tri} = \begin{bmatrix} s_1 & s_2 & s_3 \\ h_1 \times s_1 & h_2 \times s_2 & h_3 \times s_3 \end{bmatrix} \quad (7)$$

J_{q-tri} is a 3x6 matrix called the Jacobian of the actuated legs. Joshi and Tsai (2003) presents an equation which calculates ω_T as follows.

$$\omega_T = 1/d_3 \begin{bmatrix} 0 & -s\varphi_1/s\varphi_2 & c\varphi_1/c\varphi_2 \\ s\varphi_1 s\varphi_2 & s\varphi_1 c\varphi_1 c\varphi_2 & s\varphi_1^2 c\varphi_2 \\ -c\varphi_1 s\varphi_2 & -c\varphi_1^2 c\varphi_2 & -s\varphi_1 c\varphi_1 c\varphi_2 \end{bmatrix} = J_c v_T \quad (8)$$

Complementing Eq. (8) with the identity transformation $v_p = I v_p$, yields:

$$\dot{q} = J_{q-tri} \begin{bmatrix} v_T \\ \omega_T \end{bmatrix} = J_{q-tri} \begin{bmatrix} I_{3 \times 3} \\ J_c \end{bmatrix} v_T \quad (9)$$

Consequently

$$J_{Tricept} = J_{q-tri} J_{p-tri} \Rightarrow \dot{q} = J_{Tricept} v_p \quad (10)$$

where

$$J_{p-tri} = [I_{3 \times 3} \ J_c^T]^T \quad (11)$$

5. Rotational Manipulator

This section aims to consider a serial and a parallel rotational manipulator.

4.1 Gimbaled mechanism

The three axes gimbal configuration is used in many systems and can be regarded as the archetype for other configurations, such as a roll-pitch-yaw, mirror stabilization or tracking mechanisms. The dynamics and kinematics of the two axes gimbals configuration is analyzed by Ekstrand (2001). In order to achieve the Jacobian of the 3 gimbaled mechanism, four reference frames are introduced. A moving platform of the 3-UPU mechanism $\{M\}$, a frame $\{Y\}$ fixed to the $\overrightarrow{MP}$ limb, a frame $\{P\}$ fixed to the pitch gimbal, and a frame $\{R\}$ fixed to the roll gimbal. The angles of rotation α_1 , α_2 , and α_3 are defined in the following way.

1. The frame $\{M\}$ is carried into coincidence with the $\{Y\}$ frame by a positive angle of

rotation α_3 about the z -axis.

2. The frame $\{Y\}$ is carried into coincidence with the $\{P\}$ frame by a positive angle of rotation α_2 about the y -axis.

3. The frame $\{P\}$ is carried into coincidence with the $\{R\}$ frame by a positive angle of rotation α_1 about the x -axis.

According to the Fig. 2 (a), we have the following transformations:

$$C_Y^M = \begin{bmatrix} C\alpha_3 & -S\alpha_3 & 0 \\ S\alpha_3 & C\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, C_P^Y = \begin{bmatrix} C\alpha_2 & 0 & S\alpha_2 \\ 0 & 1 & 0 \\ -S\alpha_2 & 0 & C\alpha_2 \end{bmatrix}, C_R^P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha_1 & S\alpha_1 \\ 0 & -S\alpha_1 & C\alpha_1 \end{bmatrix} \quad (12)$$

C_Y^M is the transformation from $\{Y\}$ to $\{M\}$, Similarly, C_P^Y is the transformation from $\{P\}$ to $\{Y\}$, and C_R^P is the transformation from $\{R\}$ to $\{P\}$. For the angular velocities of frames $\{Y\}$, $\{P\}$ and $\{R\}$, respectively, we introduce:

$$\begin{bmatrix} p_Y \\ q_Y \\ r_Y \end{bmatrix}; \begin{bmatrix} p_P \\ q_P \\ r_P \end{bmatrix}; \begin{bmatrix} p_R \\ q_R \\ r_R \end{bmatrix} \quad (13)$$

where p, q and r are the angular velocities components in each frame. The Jacobian of the rotational mechanism is calculated by mapping the angular velocities of the pitch gimbal into the fixed frame. The actuator joint rates of a rotating body as the set of $Z-Y-X$ Euler angles are:

$$\dot{\theta} = [\dot{\alpha}_1 \ \dot{\alpha}_2 \ \dot{\alpha}_3]^T \quad (14)$$

Determine the $J_{gimbaled}$ matrix which relates Euler angles to the angular velocities:

$$J_{P-sho} = \begin{bmatrix} J_{c-sho} \\ I_{3 \times 3} \end{bmatrix}, J_{c-sho} = \begin{bmatrix} 0 & c\gamma_2 c\gamma_1 l_p & c\gamma_2 s\gamma_1 l_p \\ -c\gamma_2 c\gamma_1 l_p & 0 & s\gamma_2 l_p \\ -c\gamma_2 s\gamma_1 l_p & -s\gamma_2 l_p & 0 \end{bmatrix} \quad (15)$$

$$\Omega = J_{gimbaled}(\alpha_1, \alpha_2, \alpha_3) \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \end{bmatrix}, J_{gimbaled}(\alpha_1, \alpha_2, \alpha_3) = \begin{bmatrix} C\alpha_3 C\alpha_2 & -S\alpha_3 & 0 \\ S\alpha_3 C\alpha_2 & C\alpha_3 & 0 \\ -S\alpha_2 & 0 & 1 \end{bmatrix} \quad (16)$$

(a) Gimbaled system

(b) Shoulder manipulator

Fig. 2 Rotational Manipulators

5.2 Shoulder mechanism

Sadjadian and Taghirad (2006) has solved the forward and inverse kinematic of the shoulder mechanism which is shown in the Fig. 2 (b). The parameters can be defined as:

$$\begin{aligned} l_b &= \|\overline{CE_i}\|, & l_p &= \|\overline{CN}\|, \\ l_d &= \|\overline{NN_i}\|_v, & l_k &= \|\overline{CN_i}\|_w \end{aligned} \quad (17)$$

where C and E_i are the center of the reference frame and end points of the actuators, respectively. N_i denotes the moving end points of the actuators, and the position of the moving platform center N is defined by:

$$N = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T \quad (18)$$

The Jacobian of the actuated and passive limbs of the shoulder manipulator can be derived by using the technique in [17]. The result is given below:

$$\begin{aligned} J_{q-sho} &= \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ C_E^F P_1 & C_E^F P & C_E^F P_1 & C_E^F P \end{bmatrix}, \\ {}^E N &= l_p \begin{bmatrix} s\gamma_2 & -c\gamma_2 s\gamma_1 & c\gamma_2 c\gamma_1 \end{bmatrix}^T \end{aligned} \quad (19)$$

where C_E^F is the transformation from $\{E\}$ to $\{F\}$, And γ_1, γ_2 and γ_3 are pitch, roll and yaw angles defined for a moving object in space.

$$C_E^F = \begin{bmatrix} c\gamma_3 c\gamma_2 & -s\gamma_3 c\gamma_2 & s\gamma_2 \\ s\gamma_3 c\gamma_1 + c\gamma_3 s\gamma_2 s\gamma_1 & c\gamma_3 c\gamma_1 - s\gamma_3 s\gamma_2 s\gamma_1 & -c\gamma_2 s\gamma_1 \\ s\gamma_3 s\gamma_1 - c\gamma_3 s\gamma_2 c\gamma_1 & c\gamma_3 s\gamma_1 + s\gamma_3 s\gamma_2 c\gamma_1 & c\gamma_2 c\gamma_1 \end{bmatrix} \quad (20)$$

Finally, the Jacobian of the shoulder manipulator was derived.

$$\begin{aligned} J_{p-sho} &= \begin{bmatrix} J_{c-sho} \\ I_{3 \times 3} \end{bmatrix}, J_{c-sho} = \begin{bmatrix} 0 & c\gamma_2 c\gamma_1 l_p & c\gamma_2 s\gamma_1 l_p \\ -c\gamma_2 c\gamma_1 l_p & 0 & s\gamma_2 l_p \\ -c\gamma_2 s\gamma_1 l_p & -s\gamma_2 l_p & 0 \end{bmatrix} \\ J_{sho} &= J_{q-sho} J_{p-sho} \end{aligned} \quad (21)$$

6. Optimization

The objective of optimization is to determine the values of the manipulator design variables which derived from a minimum dexterity index. The objective and design variables were scaled so that the optimization was performed with respect to dimensionless parameters. The design variables with units of length were divided by the radius of the base to obtain nondimensional design variables. The design vector for the 3UPU manipulator is

$$s_{d-3UPU} = [r_M / r_B, l_{\min} / r_B, \alpha_1, \alpha_2] \quad (22)$$

The design vectors of the Tricept and Shoulder manipulators, respectively, are

$$s_{d-Tri} = [r_m / r_b, l_{d \min} / r_b, \alpha_1, \alpha_2], s_{d-Sho} = [l_p / l_b, l_d / l_b, l_k / l_b, \alpha] \quad (23)$$

The results of the optimization criteria are shown in tables [1]. Be in accordance with the main purpose of this paper, we study the various arrangements of combined manipulators. In this case, some of the typical rotational and translational manipulators were studied. The serial Gimbaled mechanism and the parallel shoulder manipulator are examples of rotational mechanism; furthermore, the 3UPU and Tricept manipulators are examples of parallel translational mechanisms. The first developed hybrid manipulator which is studied contains the 3UPU and Gimbaled manipulators.

6.1 3UPU-Gimbaled manipulator

This arrangement is similar to two separate robots; because, there is no interaction between these mechanisms. The linear velocity of the center of the frame $\{M\}$ is equal to the linear velocities of the center of the roll gimbal. Indeed, the terms of velocity such as Coriolis effect does not appear in the linear velocity of the end-effector as compared to the fixed frame. Thus, the Jacobian matrix of the Hybrid robot can be derived as follows:

$$\begin{cases} J_{upu} v_M = \dot{d}_i \\ \omega = E \dot{\theta} \end{cases} \Rightarrow \begin{bmatrix} v_M \\ \omega \end{bmatrix} = \begin{bmatrix} J_{upu}^{-1} & 0 \\ 0 & E \end{bmatrix} \begin{bmatrix} \dot{d}_i \\ \dot{\theta} \end{bmatrix} \quad (24)$$

Eq. 24 illustrates that the Jacobian matrix is diagonal. Consequently, the rotational and translational movements are decoupled from each other.

Fig. 3 3UPU-Gimbaled hybrid robot

6.2 Tricept-Gimbaled manipulator

It is clear that translational movement of the Tricept manipulator makes the end effector to rotate. The angular velocity of the hybrid robot consists of an angular velocity of the Tricept mechanism and the Gimbaled mechanism.

$$\begin{aligned} \omega_R = \omega_{RH} + \omega_H = C_H^G {}^H \omega_R + \omega_H = \dots \\ \begin{bmatrix} s\varphi_2 & 0 & -c\varphi_2 \\ c\varphi_1 c\varphi_2 & s\varphi_1 & c\varphi_1 s\varphi_2 \\ c\varphi_2 s\varphi_1 & -c\varphi_1 & s\varphi_1 s\varphi_2 \end{bmatrix} \times \begin{bmatrix} C\alpha_3 C\alpha_2 & -S\alpha_3 & 0 \\ S\alpha_3 C\alpha_2 & C\alpha_3 & 0 \\ -S\alpha_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \end{bmatrix} + \dots \\ \frac{1}{d_3} \begin{bmatrix} 0 & -s_1 / s_2 & c_1 / c_2 \\ s_1 s_2 & s_1 c_1 c_2 & s_1^2 c_2 \\ -c_1 s_2 & -c_1^2 c_2 & -s_1 c_1 c_2 \end{bmatrix} v_T = C_H^G E(\alpha_1, \alpha_2, \alpha_3) \dot{\alpha} + J_{IT} v_T \end{aligned} \quad (25)$$

where ω_{RH} and ω_H denotes the angular velocities of the roll gimbal (with respect to the $\{H\}$ frame) and $\{H\}$ frame (with respect to the fixed frame) respectively. If the variable l_g of the

Gimbaled system is equal to zero, the linear velocity of the end effector will be produced by the Tricept manipulator as following:

$$\begin{aligned} v_T &= J_{Tri}^{-1} \dot{q}_{Tri} \xrightarrow{l_k=0} \dot{x}_T = v_R, \omega_R = C_H^G E(\alpha_1, \alpha_2, \alpha_3) \dot{\alpha} + J_{IT} J_{Tri}^{-1} \dot{q} \\ \begin{bmatrix} v_R \\ \omega_R \end{bmatrix} &= \begin{bmatrix} J_{Tri}^{-1} & 0 \\ J_{IT} J_{Tri}^{-1} & C_H^G E(\alpha_1, \alpha_2, \alpha_3) \end{bmatrix} \begin{bmatrix} \dot{q}_{Tri} \\ \dot{\alpha} \end{bmatrix} \end{aligned} \quad (26)$$

To be considered that there is one way interaction between subsystems of this type of hybrid manipulator. The Tricept affect on the angular movement of the end effector; on the other hand, the Gimbaled has no effect on the translational movement. The dexterity index of the Tricept-Gimbaled manipulator is shown in Table 1.

Fig. 4 Tricept-GIMBALED hybrid manipulator

6.3 Tricept-Shoulder manipulator

Based on this fact that both of the Tricept and shoulder manipulators make the end effector have translational and rotational motion, analysis of this robot is complex. The mapping between actuator's rate velocities of the shoulder mechanism and angular velocity of the end effector platform, with respect to $\{H\}$, is illustrated in equation (27).

$$\dot{q}_{sh} = J_{Sho} \omega_{TF}, \dot{q}_{Tri} = J_{Tri} v_T \quad (27)$$

The end effector position vector is given by:

$$\begin{aligned} {}^H x_{CN} &= l_p [s\gamma_2 \quad -c\gamma_2 s\gamma_1 \quad c\gamma_2 c\gamma_1]^T, \\ v_N &= v_T + C_E^{G \ E} v_N + \omega \times C_E^{G \ E} x_N, \{H\} = \{E\}, C_H^G = C_E^G \end{aligned} \quad (28)$$

Thus

$$\begin{aligned} \omega_F &= \omega_T + C_H^{G \ H} \omega_F = \frac{1}{d_3} \begin{bmatrix} 0 & -s\phi_1 / s\phi_2 & c\phi_1 / c\phi_2 \\ s\phi_1 s\phi_2 & s\phi_1 c\phi_1 c\phi_2 & s\phi_1^2 c\phi_2 \\ -c\phi_1 s\phi_2 & -c\phi_1^2 c\phi_2 & -s\phi_1 c\phi_1 c\phi_2 \end{bmatrix} J_{Tri}^{-1} \dot{q}_{Tri} + \dots \\ &\dots + C_H^{G \ H} J_{Sho}^H \dot{q}_{Sho} = J_{21} \dot{q}_{Tri} + J_{22} \dot{q}_{Sho} \end{aligned} \quad (29)$$

The equations (29) and (30) yields

$$\begin{aligned}
\omega_F &= \omega_T + C_H^{G^H} \omega_F = J_{21} \dot{q}_{Tri} + J_{22} \dot{q}_{Sho}, \\
\dot{x}_N &= J_{Tri}^{-1} \dot{q}_{Tri} + C_H^G \begin{bmatrix} 0 & c\gamma_2 c\gamma_1 l_p & c\gamma_2 s\gamma_1 l_p \\ -c\gamma_2 c\gamma_1 l_p & 0 & s\gamma_2 l_p \\ -c\gamma_2 s\gamma_1 l_p & -s\gamma_2 l_p & 0 \end{bmatrix} J_{Sho}^h \dot{q}_{Sho} + \dots \\
(J_{21} \dot{q}_{Tri} + J_{22} \dot{q}_{Sho}) \times C_H^{G^H} x_{CN} &= J_{11} \dot{q}_{Tri} + J_{12} \dot{q}_{Sho}
\end{aligned} \tag{30}$$

Thus,

$$\begin{bmatrix} \dot{x} \\ \omega \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_{Tri} \\ \dot{q}_{Sho} \end{bmatrix} \tag{31}$$

The design variables for the Tricept-Gimbaled hybrid robot is shown in table 1. The dexterity index of the Tricept-Shoulder hybrid robot is calculated in two cases. In the first case, the optimized design variables of the Tricept and shoulder manipulators are used; while, in the second case, the design variables are optimized by minimizing the dexterity index of the hybrid robot. The result shows that the interactions between Tricept and shoulder mechanism increase the dexterity index of the robot. It seems that the interactions make the robot less dexterous.

6. Stiffness analysis

When a manipulator performs a given task, the end effector exerts force and/or moment onto its environment. The reaction force and/or moment will cause the end-effector to be deflected away from its desired location. Thus, the stiffness of a manipulator has a direct impact on its position accuracy. Let $\tau = [\tau_1, \tau_2, \dots, \tau_n]$ be the vector of actuated joint torques or forces and $\Delta q = [\Delta q_1, \Delta q_2, \dots, \Delta q_n]$ be the corresponding vector of joint deflections. Then we can relate Δq to τ by $n \times n$ diagonal matrix, $X = \text{diag}[k_1, k_2, \dots, k_n]$, as follows:

$$\tau = X \Delta q \tag{32}$$

The end-effector force F is related to the end effector displacement, Δx , by the transpose of the stiffness matrix.

$$F = K \Delta x, \tag{33}$$

For serial and parallel manipulators, the stiffness matrix can be respectively obtained as follows.

$$K_{PAR} = J^T X J, K_{SER} = J^{-T} X J^{-1} \tag{34}$$

In this section, a formulation is proposed for a stiffness performance index by using the obtained stiffness matrix. A numerical investigation has been carried out on the effects of design parameters and fundamental results are discussed in the paper. The compliance displacement of the hybrid manipulators can be expressed as

$$\begin{aligned}
x &= \Delta x_{PAR} + \Delta x_{SER}, \\
\Delta x_{SER} &= K_{SER}^{-1} C_{ST}^T F, \Delta x_{PAR} = K_{SER}^{-1} C_{PT} F
\end{aligned} \tag{35}$$

The matrixes C_{ST} and C_{PT} are giving F_S and F_T as a function of F with respect to the parallel or serial frames. In fact, matrix C_{ST} maps the end-effector force F into the serial manipulator's base frame; likewise, matrix C_{PT} maps F into the serial manipulator's base

frame. Therefore, the stiffness matrix of the hybrid manipulator can be written as

$$K_{HYB}^{-1} = K_{SER}^{-1}C_{ST} + K_{PAR}^{-1}C_{PT} \quad (36)$$

Matrixes K_{SER}^{-1} , K_{PAR}^{-1} , C_{ST} and C_{PT} are calculated for hybrid manipulator. Equation (36) shows that the stiffness of a hybrid manipulator depends on it's position and direction. λ_i , eigenvalue of the stiffness matrix, represents the stiffness of the manipulator in the corresponding eigenvector direction (v_i). λ_{min} denotes the minimum eigenvalue while λ_{max} the maximum eigenvalue, then the minimum stiffness occurs in the v_{min} direction and maximum stiffness occurs in the v_{max} direction. For comparison purpose, the stiffness constant is taken to be 700 N/m for all linear or revolute actuators. The maximum and minimum stiffness values at each point within the workspace of the three manipulators have been computed. The maximum and the minimum stiffness mappings for the three manipulators at the elevation $z = 0.7$ are shown in Fig. 5 through 7.

(a) Maximum stiffness

(b) Minimum stiffness

Fig. 5 Stiffness analysis of the optimized 3UPU-Gimbaled hybrid manipulator

(a) Maximum stiffness

(b) Minimum stiffness

Fig. 6 Stiffness analysis of the optimized Tricept-Gimbaled hybrid manipulator

(a) Maximum stiffness

(b) Minimum stiffness

Fig. 7 Stiffness analysis of the optimized Tricept-Shoulder hybrid manipulator

The maximum stiffness of the Tricept-Gimbale manipulator is higher than 3UPU-gimbale and Tricept-Shoulder manipulators, and the minimum stiffness of the 3UPU-Gimbale manipulator is higher than Tricept-Gimbale and Tricept-Shoulder manipulators. The stiffness matrix of the Tricept-Gimbale and 3UPU-Gimbale is better conditioned than the Tricept-Shoulder manipulator.

Table 1: Dexterity Index and optimized design variables

case	<i>D.I.</i>	design vectors
3UPU	2.2	[0.004 0.7226 120° 240°]
Tricept	2.195	[0.003 0.7084 120° 240°]
Gimbale	1.3357	*
Shoulder	4.77	[0.649 0.012 0.257 39.73°]
3UPU-GIMBALED	3.0677	[0.004 0.7226 120° 240° *]
Tricept-GIMBALED	6.889	[0.03 0.7084 120° 240° 0]
Tricept-shoulder (1)	24.8789	[0.03 0.7084 120° 240° 0.949 0.122 0.49 39.7°]
Tricept-shoulder (2)	7.07	[0.006 0.8984 142° 254° 0.258 0.03 0.58 51.7°]

7. Conclusions

A study of hybrid manipulator which are developed from separated serial and parallel, one with three 3 DOF translational movements and another with 3 DOF rotational movement, are presented. The closed-form inverse kinematics and the Jacobian matrices of these manipulators are derived. It is showed that the Jacobian matrix of the hybrid manipulator is a function of the Jacobian of the serial and parallel manipulators. Also, the optimized dimensions of different types of hybrid manipulators are obtained. According to the dexterity of optimized manipulators, it is concluded that less coupling between rotational and translational mechanisms increases dexterity. it is shown that the dexterity of the hybrid robot whose translational movement is fully decoupled from its rotational mechanism possess a higher global condition index as compared to the other optimized hybrid manipulators.

Declarations

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. I am enclosing herewith a manuscript entitled “A Dexterity Comparison for 6 DOF Hybrid Robots” for publication in Chinese Journal of Mechanical Engineering. The Corresponding author of this manuscript is MohammadAli Mohammadkhani and contribution of the authors as mentioned below with their responsibility in the research.

1. MohammadAli Mohammadkhani
2. Ahmad Haghighi

With the submission of this manuscript I would like to undertake that:

- All authors of this research paper have directly participated in the planning, execution, or analysis of this study;

- All authors of this paper have read and approved the final version submitted;
- The contents of this manuscript have not been copyrighted or published previously;
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Submitted manuscript is a Research Article.

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Figures

Figure 1

Translational Manipulators (a) A spatial 3 UPU parallel manipulator (b) A tricept manipulator

Figure 2

Rotational Manipulators (a) Gimbaled system (b) Shoulder manipulator

Figure 3

3UPU-Gimbaled hybrid robot

Figure 4

Tricept-GIMBALED hybrid manipulator

Figure 5

Stiffness analysis of the optimized 3UPU-Gimbaled hybrid manipulator (a) Maximum stiffness (b) Minimum stiffness

Figure 6

Stiffness analysis of the optimized Tricept-Gimbaled hybrid manipulator (a) Maximum stiffness (b) Minimum stiffness

Figure 7

Stiffness analysis of the optimized Tricept-Shoulder hybrid manipulator (a) Maximum stiffness (b) Minimum stiffness