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Relativity and View Effects

Bernard Fischli (ab.fischli@sunrise.ch)
Institution Not Indicated

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Relativity and View Effects

Bernard A. Fischli

Summary

This paper describes a new interpretation of relativity.

The concept of rest frames clarifies the initial assumptions used in General Relativity and underlines the necessity to review whatever impacts relativity. Relativistic effects modifying paths, to be called view effects, are examined in addition to interactions. The Ehrenfest paradox is solved. View effects specific to each point of view are the solution. A curved space cannot be compatible with the paradox. Both the line of sight and the path of the object are contracted. The calculations are based on path speeds at path points independently of speed orientation and direction. Differences in clock time rates are examined. Special Relativity is mainly reduced to the examination of Lorentz boosts. The concept of a seen speed is introduced. Examples of view effects are discussed. The paths of stars seen from the earth are not compatible with the interpretation of Special Relativity.

The calculation of the deflection of light by the sun including relativistic view effects explains in detail why the deflection angle must be almost double the value obtained with Newton's laws. As already noted by Einstein the relativistic contribution to the deflection can only take place if the speed of light varies. This is the case due to gravitation as well as to relativistic view effects.

The compatibility of General Relativity with the new interpretation is discussed. The main argument for this compatibility is due to the use in General Relativity of a Pseudo-Riemannian geometry describing intrinsic views that are compatible with view effects. Several differences subsist.

Relativistic energy is examined. The relationship of Special Relativity between speed and energy is confirmed for electromagnetism where path speeds are limited to the speed of light. Such speed limits do not apply to gravitation where total energy is composed of rest mass energy plus a kinetic energy as defined in classical mechanics. Relativistic view effects modify paths without using energy above what is already included in the formula of the interaction. Dark energy should be recalculated using the energy described by the new interpretation.

The respective roles of relativity, gravitation and quantum mechanics are discussed. Quantum effects absorb a share of the energy provided by electromagnetism. Gravitation is independent of quantum effects.

Inertial behavior is due to exchanges of kinetic energy. Inertial mass results being the rest mass, but this does not introduce gravitation to General Relativity.

Relativistic view effects are specific to electromagnetism. When gravitation acts on objects that transfer their path information by photons, the paths of these photons will be impacted by view effects modifying that information.

1.On inertial reference frames

Galileo Galilei introduced the notion of inertial reference frames, to be called inertial frames, where the laws of motion would be valid in a Euclidian space. Newton proposed an absolute inertial frame in uniform motion relative to the stars. Einstein discarded the notion of an absolute inertial frame and developed Special Relativity that describes space transformations keeping the speed of light

constant. Inertial frames, as defined in Newton's first law, are reference frames where objects either remain at rest or move at constant velocity unless acted upon by a force.

This paper is focused on relativity which can be explained by starting from the classical description of interactions and then adding the impact of relativity matching the empirical evidence. Interactions valid in a flat space such as Newton's gravitation laws, to be called Newton's laws, will be further examined with an emphasis on gravitation. Electromagnetism will be analyzed when reviewing energy.

In this paper an observer is a person or an equipment registering positions and movements in space as recorded by instruments such as an eye or a sensor of a camera or of a telescope or of whatever can record pictures of the impact of light waves or gravitational waves or of whatever else. An observer always observes out of his own rest reference frame, to be called a rest frame. A rest frame {ct,x,y,z} can be interpreted to be an inertial frame relative to a chosen reference, in this case the observer. Thereby, the laws of motion described by Newton's laws remain valid in the rest frame of the observer. What could be the use of inertial frames not matching the rest frame of the observer? They would represent inertial frames from which no observations would be made.

The observer sees an object. The words seeing and viewing are here synonymous to recording. An object is the subject of the information recorded by an observer, for instance on a photon, a black hole neighborhood, a car, an electron, a point on a wall or whatever else.

The observer and the object are represented by their rest frames and the laws of motion describe the relative motions and positions of these rest frames as seen by the observer. Both the observer and the object are located at the origins of their rest frames.

In Special Relativity (1) the rest frame of the observer is $\{x^\alpha\} = \{ct, x, y, z\}$ and the rest frame of the object is $\{\boldsymbol{X}^\alpha\} = \{c\boldsymbol{\tau}, \boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}\}$

t is the time of the clock of the observer and $\boldsymbol{\tau}$ is the time of the clock of the object, c is the speed of light. The index α refers to the four variables.

In its rest frame the object always has the position $\{c\boldsymbol{\tau}, 0, 0, 0\}$

Then $$\frac{d^2\boldsymbol{X}^\alpha}{dp^2} = 0 \qquad [1]$$

where p is an affine parameter, for example the time $\boldsymbol{\tau}$.

Equation [1] is valid in space and time and can be rewritten as:

$$\frac{d^2\boldsymbol{X}^\alpha}{dp^2} = \frac{d}{dp}\left(\frac{\partial \boldsymbol{X}^\alpha}{\partial x^\mu}\frac{dx^\mu}{dp}\right) = 0$$

and results in the equation of General Relativity called the geodesic equation:

$$\frac{d^2x^\mu}{dp^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{dp}\frac{dx^\rho}{dp} = 0 \qquad [2]$$

with $$\Gamma^\mu_{\nu\rho} = \frac{\partial x^\mu}{\partial \boldsymbol{X}^\alpha}\frac{\partial^2 \boldsymbol{X}^\alpha}{\partial x^\nu \partial x^\rho}$$

$\Gamma^\mu_{\nu\rho}$ is called the Christoffel symbol.

Equation [1] is valid for all points on the path of the object and equation [2] is specific to the same path.

Equation [2] results exclusively from the rest frame concept. Contrary to General Relativity (1) it does not depend on the equivalence principle, and it is not limited to gravitation. At this stage of the theory there is no information on the sources impacting space and paths. The sources could be interactions and view effects. Interactions act using forces and by exchanges of energy. View effects modify the paths without using forces or exchanges of energy not yet included in interactions. In what follows, excepted electromagnetism, the interactions shall be as described in classical mechanics including Newton's laws.

A careful choice of the observer may simplify the calculations of interactions. When considering the deflection of light by the sun it is convenient to locate a virtual observer in the center of the sun and the observations shall be transmitted to the final observer usually located on earth.

2.On the importance of the movements of the observer

The importance of the movements of the observer relative to the observed object is explained by simple examples.

Observer one and observer two are standing together. It starts raining and there is no wind. Both see the object, a raindrop, falling on a straight-line path pointing to the center of the earth due to gravitation. Observer one has an umbrella and stays in place. Observer two has no umbrella and decides to walk home at constant speed. He notices that his front side gets more wet than the back and understands that this is a consequence of his speed relative to the raindrops which now follow a straight-line path not pointing to the center of the earth. Observer one still sees the raindrops following straight-line paths pointing to the center of the earth.

Observer two gets increasingly wet. He decides to run home. During the acceleration phase his speed relative to the raindrops progressively increases and so does the slope of his seen raindrop paths. Therefore, observer two sees the raindrops following a curved path. Observer one still sees a straight-line path.

Observer three is the raindrop travelling next to the observed raindrop. They have a relative speed of zero. Observer three sees the other raindrop not moving at all.

The movements of an observer relative to an object can have a strong impact on the perceived path of the object and the laws of perceived motion of the object must incorporate that impact to predict the values recorded by an observer. A camera always takes pictures out of its rest frame.

3.On the Ehrenfest paradox

The Ehrenfest paradox is about a disc rotating at constant angular velocity ω and whose circumference is subject to a relativistic contraction by a reciprocal Lorentz factor $1/\gamma$

$$\frac{1}{\gamma}=\sqrt{1-\frac{(\omega r_d)^2}{c^2}} \qquad [3]$$

when observed from the center of the disc where r_d is the uncontracted radius of the disc and c is the speed of light supposed to be constant in inertial frames. In what follows the paradox is a thought experiment. The disc rotates in a room in the presence of furniture to which the reciprocal Lorentz contraction applies as well.

A fixed observer does not rotate with the disc and a rotating observer rotates with the disc. Both are simultaneously located in the center of the disc. The fixed observer observes a contraction of the circumference of the disc but no contraction of the furniture whose speed relative to the fixed observer is zero. Simultaneously the rotating observer observes no contraction of the disc but a contraction of the furniture whose relative speed depends on the angular velocity of the rotating observer. No curved space can explain that. View effects can explain that.

The rotating observer sees a contracted table in the room. The fixed observer sees an uncontracted table. The rotation of an observer cannot contract a table, but view effects can contract the view of a table.

When the rotating observer sees different points on the table the local contraction specific to a point will depend on the speed of that point relative to the rotating observer and that speed depends on the relative distance to the point and on the angular velocity. Therefore, relativistic view contributions must be calculated for each point of view excepted for simple cases such as all the points on the circumference of the disc which are all subject to the same contraction.

A point on the radius r_d of the disc is shared with the circumference of a circle with a smaller radius r_c centered on the disc. As a curved space is excluded the circumference of that circle is equal to 2π times the radius r_c as observed from the center of the circle. Owing to the circular symmetry of that case a circle remains a circle after relativistic contractions and the proportionality of 2π between the circumference and the radius remains valid.

The fixed observer sees each point on the radius r_d moving at a different speed depending on its distance from the center of the disc. Using that speed, the contraction factor applicable at a point may be as calculated with equation [3]. A point close to the center would have a low path speed and contraction. A point at a greater distance to the center would have a higher path speed and a stronger contraction. An average of the contractions applying to the various points on the radius r_d will always be smaller than the contraction applying to the circumference of the circle with radius r_d which would not respect the proportionality factor of 2π.

A possible solution consists in introducing a factor specific to each circle and representing only the local contribution valid at the point common with the radius of the disc and applicable to the calculation of the overall contraction factor of a circle with radius r_d. The local contribution factor may be called the contribution factor of the circle with radius r_c.

The solution of the Ehrenfest paradox should respect the following conditions:

a) It should be compatible with view effects.
b) The formula of the Lorentz factor introduces a relativistic contraction dependent on the speed of light. This contraction is due to a view effect tied to that factor. Therefore, the solution should use somehow the Lorentz factor.
c) The path speed of the object impacts the relativistic view effects. A higher speed results in a higher contraction.
d) The solution must respect the proportionality factor of 2π between the circumference and the radius of any circular path contracted by a relativistic view effect.

The reciprocal Lorentz factor is proposed as a contribution factor. It could not be a contraction factor in this case as that would not respect condition d).

The overall contraction factor λ applicable to both the radius and the circumference of the disc, respecting thereby condition d), is obtained by integrating the contribution factors over r_d :

$$\int_0^{r_d} \sqrt{1-\frac{(r_c\omega)^2}{c^2}}dr_c$$

r_d is the uncontracted radius of the disc.

r_c is the uncontracted radius of a circle centered on the disc.

ω is the constant angular velocity of the disc.

v_c is the uncontracted path speed of a point on the circle with radius r_c .

v is the uncontracted path speed of a point on the circumference of the disc.

We have: $\omega = \frac{v}{r_d}$ and $v_c = r_c\omega = v\frac{r_c}{r_d}$ and $\beta = \frac{v}{c}$

$$\int_0^{r_d} \sqrt{1-\frac{(\beta r_c)^2}{r_d^2}}dr_c$$

The equation is solved by substituting r_c with $x = \frac{r_c}{r_d}\beta$

and by substituting x with u and x=sin(u)
and using Euler's formula.

The contracted value of the disc radius is:

$$r_d\left(\frac{1}{\beta}\left(\frac{\arcsin(\beta)}{2}+\frac{\sin(2\arcsin(\beta))}{4}\right)\right) = \lambda r_d \qquad [4]$$

with $0 \le \beta \le 1$ that is with a speed v of no more than the speed of light.

Path speed	*Overall contraction factor*
Speed of light C	$\frac{\pi}{4} = 0.7854$
$\frac{\pi}{12}\mathrm{C} = 0.2618\,\mathrm{C} = 78'485'665$ meters per second	0.988468

The impact of the overall contraction factor is negligible up to speeds very close to the speed of light.

The prevailing interpretation of Special Relativity supposes no contraction of the radius of the disc. Why would the contraction be limited to the circumference of a circle? Point specific view effects solve the paradox.

What could be retained from Special Relativity? A reciprocal Lorentz factor as equation [3] is used for the determination of contribution factors as well as for the calculation of a contracted length in a Lorentz boost which is a Lorentz transformation excluding rotations. In what follows the contribution of Special Relativity will be mainly based on the examination of Lorentz boosts which describe the impact of the constraints of the speed of light on a straight-line path with constant speed v .

The straight-line going from the observer to the object shall be called a ''line of sight''. A line of sight can represent an accelerated path as is the case for the radius of the Ehrenfest paradox.

A path subject to view effects can be of any origin such as interactions or movements of the observer. Any contracted infinitesimal path segment keeps the path orientation and direction unchanged. Both the infinitesimal uncontracted and contracted path segments are confined within the same angle of view of the uncontracted segment seen by the observer. The smaller contracted segments are parallel shifted to fit the angle as required to join the neighboring contracted infinitesimal segments. This parallel shift towards the observer keeps the angles of two triangles, each composed of the two sides of the angle of view plus of one of the path segments, unchanged. Therefore, the contraction factor of the path segment applies to the other two triangle sides. For infinitesimal path segments both such sides almost merge with the line of sight to which they apply that factor as a contribution factor specific to the contracted path segment. This is done independently of the respective side lengths and therefore of path orientation and direction.

The speed on a parallel shifted infinitesimal path segment fitting the angle of view is calculated from the uncontracted path speed v linearly reduced to match the position of the related point on the line of sight. This possible as a speed is determined by the segment length while the same time interval value, measured by the clock of the observer, applies to any such segments. That speed is used to calculate the corresponding contribution factor as done for the Ehrenfest Paradox disc and the overall contraction factor λ of each line of sight can be determined with formula [4] independently of speed orientation and direction. A line of sight contracted as per formula [4] ends at the position of a contracted path point and the seen path is built by all contracted points.

Special Relativity supposes a contraction valid only in the direction of the path speed. This implies a contraction factor based on velocity instead of speed. The new interpretation calculates the contractions of lines of sight using speeds independently of their orientation and direction.

A high path speed induces a high overall contraction of the line of sight. This is how the deflection of light by the sun is impacted by relativistic view effects. We record the paths of the past including the impact of relativistic view effects.

4.On time

A local time difference is a measure of the size of a local change. In classical mechanics the local change is the infinitesimal segment of the path of an object corresponding to the infinitesimal time difference. The path of the object and the times seen and measured in the rest frame of the observer are described by the laws of interactions applying in inertial frames.

In Special Relativity a similar relationship applies to a Lorentz boost:

$$d\tau^2 = -\frac{1}{c^2} ds^2 = dt^2 - \frac{1}{c^2} dx^2$$

τ is the time of the clock moving with the object.
t is the time of the clock moving with the observer.
x is the space coordinate of the rest frame of the observer applying to the path of the object.
ds^2 is an interval between two events. An event in a four-dimensional reference frame {ct,x,y,z} determines a unique position in space and time.

A relative movement between clocks impacts the difference in clock rates:

$$d\tau^2 - dt^2 = -\frac{1}{c^2} dx^2$$

The following relationships apply to a Lorentz boost where we have $v = dx/dt$

$$d\tau^2 = dt^2 - \frac{1}{c^2}dx^2 = dt^2\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{\gamma^2}dt^2 \qquad [5]$$

v is the constant relative speed between the clock of the observer and the clock of the object as calculated in the rest frame of the observer before relativistic contractions.

$\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ is the Lorentz factor.

Equation [5] introduces a difference in clock rates due to a constant speed v between two clocks with times t and τ. This effect is called velocity time dilation. An observer's clock time will always be dilated respective to the clock time of the object as the constant speed v results from previous accelerations of unknown origin implicitly supposed to locally impact the clock of the object by increasing its clock rate relative to the observer's clock.

Clock rates also depend on gravitational time dilation. A gravitational potential impacts the frequency of electromagnetic waves and thereby the rates of atomic clocks as well as the energy of the photons. The wave frequency including the local impact of gravitation is registered in the rest frame of the clock as described in the section ''On empirical evidence''.

The clock with the lowest gravitational potential always has the slowest rate. This differentiation is possible as the information on the origin and the value of the local impact on a clock is available.

When measuring time differences between two clocks the clock rate differences will be due to the velocity and gravitational time dilations encountered during the respective trips of the clocks. A corresponding difference in clock times depends on the time spent with each value of a difference in clock rate as done in 1971 by Haefele and Keating (2) who recorded a time difference on two flights around the world and their measures matched predictions with an accuracy of about 10%. The satellites of the Global Positioning System GPS are the continuous empirical evidence that different clock rates impact the performance of the system.

The formula $1/\gamma$ of the contribution factors applying to the calculation of lines of sight and therefore to view effects is also used for velocity time dilation. We have from [5]:

$$d\tau = \frac{1}{\gamma}dt$$

The values of both the uncontracted and the contracted infinitesimal segment lengths dL and dL_c can be calculated in the rest frame of the observer from the uncontracted path speed v by using the time intervals dt and $d\tau$ necessary to travel on each segment from an endpoint to the other one:

$dt \cdot v = dL$ and $d\tau \cdot v = dL_c$ therefore: $dL_c = \dfrac{1}{\gamma}dL$

Velocity time dilations and contracted segment lengths have the same contraction factor values. This is a consequence of the fact that both concern the path of the object. Their contraction factors represent the impact of the constraints of the speed of light at the same path location independently of view effects. All this supposes that Lorentz transformations apply.

A time difference results from the local impacts of the clock paths on each clock. The overall contraction factor λ of a line of sight is calculated from speeds specific to each point on the line of sight and does not apply to time differences as clock times are independent of lines of sight.

Gravitational and velocity time dilations contribute to determine the path of a satellite whose clock time is delivered to the observer.

5.On Lorentz boosts

Relativistic view effects impact the perception of the path of the object. Their initial conditions are specific to each path point and originate from the laws of interactions combined with the contributions from the relative movements between the observer and the object.

In a Lorentz boost the object advances at constant speed on a straight-line. When the observer is not located on that path, the relativistic view effects result in a parallel shift towards the observer. As per equation [4] the constant path speed applies the same overall contraction factor λ to all lines of sight and produces the shift. The infinitesimal segment lengths dL are all contracted by the same factor λ as well:

$$dL_c = \lambda dL$$

with dL_c and dL as previously defined.

An observer registers the movements of the object represented by a succession of path points whose lines of sight are contracted by relativistic view effects. The information received by the observer is mostly limited to the observation of the contracted path of the object. He often does not know what the object's clock indicates and must use his own clock to calculate a ''seen speed'' v_s of the object. The ''seen speed'' is the speed along the contracted path as recorded by the observer.

The seen speed v_s, valid for a Lorentz boost, is constant:

$$v_s = \frac{dL_c}{dt} = \lambda \frac{dL}{dt} = \lambda v$$

v is the constant speed of an object calculated in the in the rest frame of the observer with $v = dL/dt$ for a Lorentz boost. This formula is equivalent to $v = dx/dt$ as both relate to uncontracted values valid in the rest frame of the observer.

t is the time of the clock of the observer.

A seen speed v_s is always smaller than a positive real speed v as λ is then smaller than one.

In Special Relativity the three space components v_f^α of the four-velocity are defined as the division of the infinitesimal not contracted segments dx^α, not seen by the observer, by the infinitesimal clock time $d\tau$, not delivered by stars. For a Lorentz boost the following formula applies:

$$v_f = \frac{dx}{d\tau} = \frac{dx}{dt/\gamma} = \frac{dx}{dt}\gamma = v\gamma$$

τ is the time of the clock of the object

What should be interpreted as the real speed is the speed v as previously specified with, for a Lorentz boost:

$$v = v_f \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{\frac{1}{\frac{1}{v_f^2} + \frac{1}{c^2}}} \qquad [6]$$

and $$v_f = \sqrt{\frac{1}{\frac{1}{v^2} - \frac{1}{c^2}}} \qquad [7]$$

with the following values:

v	v_f
0.0995 C	0.1 C
0.4472 C	0.5 C
0.7071 C	C
0.8944 C	2 C
0.99995 C	10 C
C	∞

That the space component of the four-velocity may have an infinite value when the path speed is the speed of light requires further examination detailed in the section ''On energy and speed''.

Relativistic view effects are specific to observations. When swapping the roles of the observer and the object a length contraction is always seen by the one chosen as observer.

6.Examples of relativistic view effects

As calculated for the Ehrenfest paradox and due to view effects, the fixed observer located in the disc center sees a disc size smaller than the actual size.

Particle accelerators such as the CERN and Fermilab have paths contracted as described by Lorentz boosts. Particles rotating clockwise collide with particles rotating anticlockwise when the requested speed is reached. The particles are the objects, and their paths can be locally approximated by straight-lines. The observers are the particle detectors placed where the particles collide.

Relativistic view effects depend on the speed v of an object. That speed results from interactions and from the movements of the observer relative to the object. These movements contribute to the view effects by impacting the path speed used to calculate the contribution factors. An example is the rotation of the observer or of the disc as described for the Ehrenfest paradox. Observations made on earth include view effects due to the earth's rotation. The nearest star is Proxima Centauri at 4.244 light years and the contribution of the earth's rotation to the path speed is of 9740 times the speed of light whenever the observer does not compensate that rotation.

Stars are observed on earth to move on a circular path due to the rotation. This cannot be explained by Special Relativity that assumes an object's path contracted to a point in the center of the rest frame of the observer when the path speed reaches the speed of light. But such a path is compatible with a contraction of the line of sight by a factor of $\pi/4$. Contribution factors valid at points on the

line of sight with speeds above the speed of light should have a value of zero and would result in a smaller overall contraction factor.

Information delivered by light will not reach the observer whenever the speed relative to the object will be higher than the speed of light as described in the section ''On empirical evidence''. This does not apply to the path of stars seen from earth where the path speed is perpendicular to the line of sight. View effects do not impact angular motion. Furthermore, the telescopes for professional use are mounted since decades in a configuration with computerized tracking to compensate for rotations of any origin.

7.On the deflection of light by the sun

Bending of light by the sun has been an iconic case establishing Einstein as the foremost scientist in the new relativistic world of the early 20th century. The first calculations of the deflection angle date back to the 18th century and were based on Newton's laws with gravitational forces bending the path of the light particle called photon. Travelling on a straight-line path, the photon is attracted to the sun, but its high speed provides an escape after an exceedingly small deflection. A simple calculation is detailed in mathpages (3).

Expressed in a reference frame with the sun located in the center and the photon moving on an almost straight-line parallel to the x-axis with $y = r_0$ at $x = 0$, r_0 being the radius of the sun, the path deviation of the photon is, when supposing a uniform rate of acceleration a in the direction of the y axis:

$$y(t) = y_0 + v_0 t + \frac{1}{2} a t^2$$

t is the time

y_0 is the initial position

v_0 is the initial speed in the direction of the y axis

For an almost constant speed of the photon the time can be approximated by

$$t \approx \frac{x}{c}$$

c is the speed of light in vacuum with a value of 299 792 458 meters per second

We have:

$$\frac{dy}{dx} = \frac{v_0}{c} + \frac{a}{c^2} x = \tan(\theta)$$

θ is the angular coordinate with, for very small angles: $\tan(\theta) \approx \theta$

The gravitational acceleration calculated with Newton's law is:

$$a = \frac{MG}{r^2} \frac{r_0}{r}$$

where M is the mass of the sun, r is the distance from the center of the sun to the photon and G is the gravitational constant. The term $\frac{r_0}{r}$ is added to obtain the acceleration transverse to the path.

Then:

$$\frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{a}{c^2} = \frac{MGr_0}{c^2 r^3}$$

With $r^2 = r_0^2 + x^2$ the deflection angle of the photon becomes exactly:

$$\int_{-\infty}^{+\infty} \frac{MGr_0}{c^2\left(r_0^2 + x^2\right)^{1.5}} dx = \frac{2MG}{c^2 r_0} = 0.8754 \text{ seconds}$$

mathpages (3) goes on to a more rigorous approach based on a hyperbolic path with varying photon speeds. Compared to the previous simple calculation this adds terms of a Taylor series with corrections of about $2*10^{-6}$ seconds, a negligible amount.

As seen by the virtual observer located in the center of the sun, the contributions of the relativistic view effects to the deflection of light by the sun must match the non-relativistic path calculated with Newton's gravitation law and in particular the speed valid at each path point.

Given its sufficient accuracy, the same straight-line approach described above can be used to determine the view effects. A straight-line with path point speeds equal to those calculated for gravitation and seen by the same observer will be bent by view effects to the same deflection angle and the same acceleration required to reach the same path speeds as with gravitation. The total seen deflection angle will therefore be double the angle calculated from Newton's laws up to about $4*10^{-6}$ seconds due to the approximations used in the calculations. This result is compatible with the empirical evidence described in (4).

The acceleration of the photon induces changes in the contribution factors. As a contraction of the length of a straight-line path cannot bend a path by itself the bending is due to the contraction of the lines of sight. As the speed of light never reaches infinity an accelerated object advancing on a straight line is observed as advancing on a bent line due to a view effect.

Starting from the path due to an interaction as seen in the rest frame of the observer, the deflection angle can also be obtained by selecting enough path points, calculating the contraction factor λ of each line of sight using equation [4] with the path speed v valid at the considered point and interpolating between the contracted points. Such a matching of the gravitational acceleration due to Newton's law is equivalent to the approach described above. This method applies to any path with speeds of no more than the speed of light. For path speeds higher than light, points on the lines of sight moving faster than light should have contribution factors of zero value.

The additional deflection due to view effects is a consequence of the fact that such relativistic effects come on top of the initial conditions resulting from gravitation and that they can be calculated, up to negligible terms for this special case, as an additional bending of an accelerated straight-line path. This has the added advantage to explain the almost doubling of the deflection angle.

A more detailed description results from a decomposition of the hyperbolic path of the photon.

The impact parameter b determines the shortest distance from the center of the sun, where the virtual observer is located, to each of the asymptotes of the hyperbola, with:

$$b = -a\sqrt{\left(e^2 - 1\right)}$$

a is the semi-major axis that can be approximated up to a factor of 10^{-4} by $a = -\frac{\mu}{c^2}$

where c is the speed of light and μ is the standard gravitational parameter with, for the sun:

$$\mu = 1.3271244 * 10^{20} \frac{m^3}{\sec^2}$$

e is the eccentricity with, as calculated in (5):

$e \cong \frac{c^2 r_0}{GM} = 4.711 * 10^5$ with c, G, M and r_0 as previously defined

The impact parameter b has a calculated value of $6.951 * 10^8$ meters for both asymptotes, to be compared with a sun radius of $6.957 * 10^8$ meters.

The distance of closest approach is the periapsis distance r_p with:

$r_p = a(1-e) = 6.95638 * 10^8$ meters

As the radius of the sun, the impact parameter and the periapsis distance have almost the same value, the hyperbolic path of the photon can be described by the incoming asymptote up to its nearest point to the sun, to be called the impact point, then by a circular path ending at the impact point of the outgoing asymptote and finally by the outgoing asymptote. This is only possible as the hyperbolic path in an almost straight-line.

In gravitation the photon is accelerated in the straight-line path of the incoming asymptote and decelerated in the straight-line path of the outgoing asymptote. These accelerated and then decelerated straight-lines produce the contribution of the view effects to the total deflection angle as described above.

In the circular path between the two impact points the force of gravitation is perpendicular to the path of the photon, keeping thereby the path speed constant. The circular path will be impacted by a constant contraction factor keeping after contraction the angle of deflection between the impact points as calculated from gravitation. The relativistic view effect for a circular path will only confirm the deflection value due to gravitation without adding any contribution.

The speed of light in vacuum has always the same value when measured in reference frames with constant speed relative to the observer. This does not apply to a photon which is accelerated due to gravitation as seen in the rest frame of the virtual observer.

Einstein wrote on page 89 of his 1920 book on relativity (6):

Quote
…according to the general theory of relativity, the law of the constancy of the velocity of light in vacuo, which constitutes one of the two fundamental assumptions in the special theory of relativity and to which we have already frequently referred, cannot claim any unlimited validity. A curvature of rays of light can only take place when the velocity of propagation of light varies with position.
Unquote

This is the case due to gravitation as well as to relativistic view effects since a non-accelerated straight-line path is parallel shifted without any bending of the path.

Variations in the speed of light are compatible with relativistic view effects as these can be calculated for any photon speed.

The orbital speed of the earth around the sun is if of about 29'780 m/s. It is too small to impact the deflection angle of the photon seen from earth. The final observer measures the difference between the angular position of the photon received directly from the source and the one after bending by the sun and view effects. The result is independent of the rotation of the earth and the deflection angle will be as seen by the virtual observer.

A photon has no rest mass. According to Newton's second law:

$$f = ma$$

with f being the force, m the mass and a the acceleration, a photon may be accelerated to infinity at the slightest impact of a force. However, this equation can only be fully interpreted if the considered force is detailed in the equation.

The acceleration due to gravitation, calculated with Newton's laws, valid if one mass is much larger than the other and when observed from the center of the larger mass M_0, is:

$$a = \frac{GM_0}{r^2}\vec{r_u}$$

a is the acceleration of the small mass
G is the gravitational constant
r is the distance between the centers of the two masses
$\vec{r_u}$ is the unit vector in the direction of the larger mass

This is the acceleration used at the beginning of this section. It does not depend on the mass of a photon deflected by the sun. It depends only on the mass of the sun and on the distance to the center of the sun.

A photon will never reach an infinite speed when deflected. On the path from the periapsis point to the final observer the photon is decelerated as it is attracted to the sun and will reach again the speed c valid in an inertial frame when the attraction to the sun becomes negligible.

8.On General Relativity

General relativity has the best matching of empirical evidence with values calculated for gravitation and is considered as a monument in physics. The possible drawback is its opacity which blurs the interpretation of the theory behind thick mathematical smoke. The curved space that justified the theory being invalidated; the question is raised whether the new interpretation could put the theory on more solid ground.

Einstein's famous equation of General Relativity is:

$$E^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu}$$

$E^{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu}$ is the stress-energy tensor.

The Einstein tensor expresses curvatures in a pseudo-Riemannian manifold and depends on derivatives of first and second order as is in general the case for the equations used in physics (7) for flat spaces.

Newton's gravitation laws are valid in a flat space and use forces. A Newtonian potential is a representation of the potential impact of forces. General Relativity supposes a space curved by a Newtonian potential resulting in geodesic paths. A space curvature is another way to represent the

impact of the same forces. The geodesic equation [2] originates from the rest frame concept and applies to both the flat Minkowski space of Special Relativity and the curved space of General Relativity. Therefore, it seems reasonable to assume that both approaches are valid and that the paths calculated in General Relativity could be equivalent to those calculated in a flat space.

General Relativity systematically uses a Newtonian gravitational potential for the approximations made in the calculations of weak and slowly variable fields (1). Why should Newton's laws curve the space? They could be considered to introduce gravitation in the flat space of the rest frame of the observer. The same comment applies to the Schwarzschild metric of objects whose mass distribution respects a spherical symmetry. The Schwarzschild metric is extensively used to describe the characteristics of black holes. Michell and Laplace had already proposed the existence of black holes in the late 18th century. The description of gravitational waves also makes extensive use of Newtonian notions (1). All this is interpreted here as Einstein's choice of Newton's laws for gravitation with an added theoretical layer to include relativistic view effects.

The main argument for the equivalence of the two approaches results from the use of a pseudo-Riemannian geometry in General Relativity. That geometry is intrinsic. It describes what an observer located on any surface will see and measure in contrast to an extrinsic point of view where the observer is in a space of higher dimension. In other words, a pseudo-Riemannian geometry is specific to view effects of all sorts and could be compatible with relativistic view effects. The rest frame concept introduces intrinsic views as well. Moreover, the geometry applies the same gravitational potential formula independently on whether space is flat or curved.

General Relativity calculates the path of the object in the rest frame of the observer by using a Newtonian gravitational potential providing the initial conditions for the relativistic view effects. Intrinsic views of a pseudo-Riemannian geometry adapt the length and orientation of a line of sight to match a path point. The corresponding relativistic constraints result in a contraction of the line and modify the path seen by the observer.

Does General Relativity match the results of the new interpretation by applying relativistic contribution factors as done to lines of sight with the Ehrenfest paradox? This would explain why both give the same value for the deflection of light. However, the impact of rotations on relativistic views may not have always been considered with General Relativity but should be examined and clarified. Calculations relating to a virtual observer must be corrected if necessary to fit the real observer and General Relativity may include curved space solutions not compatible with a flat space.

The new interpretation retains that the laws of nature apply to the rest frame of the observer instead of the Minkowski space of Special Relativity. A closer examination is required to sort out where General Relativity and the new interpretation produce the same results regarding energy and where they do not.

9.On Energy and Speed

The relativistic energy-momentum equation of an object is:

$$E_t^2 = m_0^2 c^4 + p^2 c^2 \qquad [8]$$

E_t is the total relativistic energy

m_0 is the rest mass of the object

c is the speed of light

p is the magnitude of the three-dimensional relativistic momentum of the object with:

$$p^i = \frac{m_0 v^i}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = m_0 v^i \gamma$$

v^i is the space component i of the path speed v previously defined.

The term $m_0 c^2$ represents the energy content of the mass of the object at rest. The term $p^2 c^2$ represents the contribution of relativistic kinetic energy. The relativistic kinetic energy is defined relative to an observer. The rest mass energy does not depend on a path speed and is not impacted by relativistic view effects.

The Lorentz boost represented by the Lorentz factor γ introduces a limited speed of light impacting the calculation of energy and remaining unchanged when measured in reference frames with a constant speed relative to the observer.

Many experiments were conducted to confirm the relativistic relations using electromagnetic forces and measuring in particular the Lorentz factor γ

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

which applies to the calculation of relativistic contractions.

One of the most accurate measurements was conducted by Meyer et al (8). The expression

$$Y = \frac{m/m_0}{\sqrt{\left(1 - \frac{p^2}{m_0^2 c^2}\right)}}$$

originates from the Lorentz factor γ and has a calculated value of 1. The relativistic mass m is defined by: $m = \gamma m_0$ and p by: $p = m_0 v \gamma$

The experimental set-up consisted in deflecting relativistic electrons on a path where the electrons were first deflected by magnetic forces and then by electrostatic forces. The electrostatic forces were adjusted until the electrons could be detected by the sensor and the electrostatic deflector was then calibrated using protons.

The measured mean value was: $Y = 1.00037 \pm 0.00036$

Many other experimental approaches have been conducted. Their measures include relativistic effects related to electromagnetism whenever the corresponding forces have been used in the set-up. The important result is that the empirical evidence confirms the Lorentz factor. This has a consequence. The reciprocal Lorentz factors used in the section ''On the Ehrenfest paradox'' may concern view effects specific to electromagnetism.

Special Relativity defines energy using the following formula:

$$E_t = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \gamma m_0 c^2 \qquad [9]$$

Equations [8] and [9] are equivalent and each can be calculated from the other equation.

Equation [9] is more compact and therefore more difficult to understand. It is usually interpreted as meaning that the speed of an object is limited to the speed of light except for trivial cases where no energy is transferred and that the total energy increases towards infinity the closer the path speed gets to the speed of light. This traditional interpretation must be closely examined given the arguments detailed in this paper.

As described for the Ehrenfest paradox, a rotation of the disc produces a relativistic view effect with contraction values depending on the path speed v. The same view effects can be obtained with a fixed disc and a rotating observer. A rotating observer will observe stars moving with a path speed of many times the speed of light. How could the rotation of an observer be the source of an infinite amount of energy when the path speed reaches the speed of light? A rotation of an observer cannot be that. View effects due to the rotation of an observer require no energy and this was not considered by the theories of relativity so far.

Relativistic view effects participate in the transmission of information on the path of the object. This is done along lines of sight specific to path points. The calculation of such view effects uses Lorentz factors. A Lorentz factor introduces the dependency on the speed of light which remains unchanged when calculated in any reference frame with a constant speed relative to the observer. The supposition that the path speed v of any object should be limited to the speed of light results from an interpretation of equation [9] which must be subjected to a more detailed analysis.

The total relativistic energy E_t can be expanded in a Maclaurin series:

$$E_t = m_0 c^2 + \frac{1}{2} m_0 v^2 + m_0 c^2 \left\{ \frac{3}{8}\left(\frac{v}{c}\right)^4 + \frac{5}{16}\left(\frac{v}{c}\right)^6 + \cdots \right\}$$

The term $m_0 c^2$ is the rest mass energy and $\frac{1}{2} m_0 v^2$ is the kinetic energy as generally used in classical mechanics. The other terms in powers of $\frac{v}{c}$ starting from $\left(\frac{v}{c}\right)^4$ depend on the ratio of the path speed to the speed of light and must be examined.

Photons travel at the speed of light. As confirmed by experiments (9), electrons have an upper speed limit which is the speed of light. Given the evidence gathered in accelerators, this particularity can be extended to all particles impacted by electromagnetism, and these cannot exceed the prevailing speed limit whatever the energy input may be. Moreover, the relationship of Special Relativity between speed and kinetic energy was measured and confirmed (10) as well. Therefore, the energy equations [8] and [9], which depend on the Lorentz factor, apply to electromagnetic interactions.

Relativistic view effects apply to electromagnetism. They modify the path of a photon recorded by the observer. The deflection of light by the sun provides the empirical evidence matching this interpretation.

Lorentz factors are used in the calculation of relativistic view effects and apply as well to the energy formula of electromagnetism which includes all the energy specific to view effects.

No laboratory measures confirm the validity of Lorentz factors with gravitation, and this casts doubt on the pertaining energy formula.

Let us examine equation [9]. That equation:

$E_t = \dfrac{m_0 c^2}{\sqrt{1-\dfrac{v^2}{c^2}}} = \gamma m_0 c^2$ gives a complex number with an imaginary part if the fraction v/c is greater than one thereby limiting the speed v to no more than the speed of light. However, the terms of the Maclaurin series including the fraction v/c originate from the use of a Lorentz factor γ valid for the energy of electromagnetism where it represents the impact of the limitations in the speed of light. The first two terms of the Maclaurin series are:

$$m_0 c^2 + \frac{1}{2} m_0 v^2$$

$m_0 c^2$ is the rest mass energy and $1/2\, m_0 v^2$ is the kinetic energy specific to the observer and they should remain valid in gravitation. Do terms in v/c apply to gravitation? Or do they only concern electromagnetism?

In his 1905 paper (11) introducing Special Relativity, Einstein calculated the energy formula valid for electromagnetism starting from Maxwell's equations and then applying a Lorentz boost to the forces. Regarding his calculation of the relativistic mass, the English translation mentions on page 22:

Quote
We remark that these results as to the mass are also valid for ponderable material points, because a ponderable material point can be made into an electron (in our sense of the word) by the addition of an electric charge, no matter how small.
Unquote

He then used the same argument for ''kinetic energy'' being the work corresponding to the total energy E_t of equation [9] less the rest mass energy, but his argument is not a proof. Furthermore, why should an energy calculated starting from Maxwell's equations apply to gravitation? General Relativity calculates the supposed curved space starting from a Newtonian gravitational potential which is not compatible with Maxwell's equations.

Therefore, the energy formula of electromagnetism should not apply to gravitation. The empirical evidence must be further examined to settle this matter.

The relativistic energy-momentum equation [8] is calculated using the path speed v with no need to introduce additional variables such as the three space components of the four velocity v_f which describe a velocity with no meaning in physics.

Calculations are always done in the rest frame of the observer and an energy impacting the observer will be assigned to an impact on an object if that impact is equivalent to a difference in potential energy between the observer and the object. A rotation of an observer has no impact on energy, but it induces view effects specific to the observer.

10.On empirical evidence

Empirical evidence has traditionally been considered as the supreme test of any theory in physics but will not represent the fundamental laws of nature whenever view effects induce a deformation of reality. Relativistic view effects are new to physics and their impact must be analyzed.

Electromagnetic forces are negligeable in the universe scale as positive and negative charges cancel out over large distances where gravitation rules. The Planck Collaboration (12) found results that suggest our universe is spatially flat to a 1σ accuracy of 0.2%.

An upper speed limit remaining unchanged in vacuum when calculated in any reference frame with a constant speed relative to the observer and valid for electromagnetism implies that a photon cannot be accelerated above the speed of light when subjected to electromagnetic forces as otherwise no upper speed limit would be valid. These constraints are incorporated in the related energy formula [9] by applying a Lorentz boost calculated with a Lorentz factor γ.

Einstein wrote (6) that the law of the constancy of the velocity of light cannot claim any unlimited validity and that a curvature of light can only take place if the speed of light varies with position, in other words if a photon is accelerated or decelerated. This is done by Newton's gravitation laws which calculate the deflection of light by the sun matching the empirical evidence (4) and signifies that these laws are not concerned by the speed limit applicable to electromagnetism. Therefore, the terms including the fraction v/c in the Maclaurin series of the relativistic energy E_t do not apply to gravitation. The total energy E_g valid for gravitation is then given by:

$$E_g = m_0 c^2 + \frac{1}{2} m_0 v^2$$

This implies that the calculation of dark energy shall be reviewed by applying the correct energy formula for gravitation and by discarding view effects as they do not contribute to energy.

A photon is limited to the speed of light c except when accelerated by gravitation. However, on very short distances, electrostatic forces are about 10^{40} times stronger than gravitational forces whose impact on the photon speed is usually very limited unless acted upon by major objects of the universe and a photon will be decelerated back to speed c when leaving the gravitational potential of the object. Consequently, calculations of relativistic view effects should remain valid in most cases for observations of planets and stars excluding both black holes and the big bang which have strong impacts on photon speeds. Whenever light significantly exceeds the speed c, the calculation of the view effects may still be done with formula [4] using the light speed valid at the considered location and contribution factors of zero value at points on the line of sight with speeds faster than light.

Lorentz factors do not relate to gravitation where Galilean transformations apply. Electromagnetism relates to Lorentz factors and Lorentz transformations apply. Both types of transformations are valid in the flat rest frame of the observer. In that inertial frame a photon travels in vacuum at the speed of light c corrected due to the impact of gravitation which uses Galilean transformations. Relativistic view effects apply to electromagnetism and determine the overall contractions of lines of sight. These rules have been employed to calculate the deflection of light by the sun. Furthermore, gravitational time dilation is empirical evidence of the effects of gravitation on electromagnetic waves.

Relativistic view effects do not apply to gravitation, which is unrelated to Lorentz transformations. However, when gravitation acts on objects that transfer their path information by photons, the paths of these photons will be changed by view effects, modifying thereby the information.

When calculated in the rest frame of observer one emitting light signals, such signals will be limited to the prevailing speed of light and never reach observer two located on an object impacted by gravitation and resulting in a speed higher than light relative to observer one. Consequently, observer one and observer two will be mutually invisible. When information on the positions is limited to a transmission by light neither of these observers can record relative speeds higher than the prevailing speed of light.

Rotations impacting the observer increase the speed of objects relative to the observer. Such contributions to the speed do not represent real movements of an object and they must be sorted out from the speed v to get meaningful data on paths.

Speeds v corrected for rotations of the observer can still be higher than the speed of light and remain visible unless they are speeds relative to the observer.

The cosmic distance ladder must be reviewed to include the additional impacts due to view effects and to path speeds higher than the speed of light c whether they are speeds relative to the observer or not.

The sun produces only a limited acceleration on a nearby object. A black hole has a stronger impact. The strongest impact results close to the big bang as soon as Newtonian gravitation applies and results in extremely high relative object speeds. Newtonian gravitation laws are valid on the photon scale and may help validate the inflation period. Invisible objects described above may correspond to the dark matter content. An important share of invisible matter may have been absorbed by black holes. The impact on cosmology of speeds higher than the speed of light of objects subject to extreme forces and amounts of energy, such as due to black holes and to the big bang, remains to be determined.

11.On Relativity and Quantum effects

Particle accelerators use electromagnetic forces. Accelerations to speeds close to the speed of light produce high energy collisions resulting in decays of highly energetic particles as described by the Standard Model. An ever more important share of the energy provided by the accelerators is absorbed by the particles the closer they get to the speed of light. The remaining energy is transformed in kinetic energy of the particles until, at the speed of light, the energy would be completely absorbed by the particles themselves. Particles absorb kinetic energy as well during collisions. No particle may travel faster than at the speed of light under these conditions. The energy for the collisions is supplied by electromagnetic forces and quantum electrodynamics is the link to the Standard Model.

The total relativistic energy E_t of electromagnetism is empirical evidence (10) and its Maclaurin series:

$$E_t = \gamma m_0 c^2 = m_0 c^2 + \frac{1}{2} m_0 v^2 + m_0 c^2 \left\{ \frac{3}{8}\left(\frac{v}{c}\right)^4 + \frac{5}{16}\left(\frac{v}{c}\right)^6 + \cdots \right\} \qquad [10]$$

describes the respective roles of rest mass energy, kinetic energy and energy absorbed by particles due to quantum effects. The energy supplied by these accelerators is transformed into other representations of energy each impacting the particle as recorded by the observer.

The constraints of the speed of light do not apply to gravitation and therefore no energy due to quantum effects is absorbed by the particles.

Gravitation accelerates photons and thereby impacts their speed and frequency. This results in kinetic energy relative to an observer located for example in the center of a mass as described for the deflection of light by the sun.

The total gravitational energy E_g of objects with a mass is given by:

$$E_g = m_0 c^2 + \frac{1}{2} m_0 v^2$$

12.On inertial mass

Interactions of any kind use exchanges of energy including kinetic energy. Exchanges of energy explain inertial behavior. An object can be accelerated when absorbing kinetic energy from whatever source and can be decelerated when transferring kinetic energy to other objects. An inertial reaction is proportional to the exchanged kinetic energy. Relativistic view effects introduce a distortion of reality and do not contribute to inertial behavior.

The work w is the energy transferred to or from an object using a force along a path and according to the work-energy principle it is equal to the change in the kinetic energy E_k with:

$$w = \int_C f \cdot ds = \Delta E_k$$

f is the force acting along a path C.

Then, using Newton's second law, we have:

$$w = \int_C m_i a \cdot ds = \Delta E_k \qquad [11]$$

a is the acceleration and m_i is the inertial mass from Newton's second law.

In gravitation and electromagnetism, kinetic energy depends on the rest mass m_0. The integration of the acceleration a solves equation [11]:

$$m_i \frac{1}{2}\left(v_2^2 - v_1^2\right) = m_0 \frac{1}{2}\left(v_2^2 - v_1^2\right)$$

v_1 is the speed at the beginning of the path and v_2 is the speed at the end of the path.

Therefore: $m_i = m_0$ applies to gravitation and electromagnetism.

The rest frame of the object makes certain that it is always possible to cancel the impact of all interactions at any location of the object independently on whether inertial masses exist or not.

13.On the respective impacts of relativity

In electromagnetism the speed limit of objects remains unchanged in any reference frame with a constant speed relative to the observer. Particles with an electric charge and photons cannot be accelerated beyond that limit in that case. The energy formula includes these constraints. Relativistic view effects are specific to electromagnetism.

Gravitation remains as described by Newton. It is valid in a flat space with no speed limit and even accelerates photons. When gravitation acts on objects that transfer their path information by photons, the paths of these photons will be impacted by view effects modifying the information.

The impact of gravitational forces on the speed of massive objects usually results in speeds smaller than the speed of light unless they are acted upon by major bodies of the universe which can generate relative speeds higher than light.

The Ehrenfest paradox postulates a disc rotating at constant speed and then adds view effects calculated using Lorentz transformations, thereby impacting the paths seen by observers located in the disc center.

References

1) Blanchet Luc, Introduction à la Relativité Générale, Institut d'Astrophysique de Paris, UMR 7095 du CNRS, Université Pierre et Marie Curie, 75014 Paris, France
2) Hafele J. C., Keating Richard E., Around-the-World Atomic Clocks, Predicted Relativistic Time Gains and Observed Relativistic Time Gains, Science, New Series, Vol. 177, No. 4044 (Jul. 14, 1972), pp. 166-170
3) mathpages, 6.3 Bending Light, pp. 1-5, https://www.mathpages.com/rr/s6-03/6-03.htm
4) Will Clifford M., The 1919 measurement of the deflection of light, Department of Physics, University of Florida, Gainsville FL 32611, USA
5) Soares Domingos S.L., Newtonian gravitational deflection of light revised, Physics Department, ICEX, UFMG, C.P. 702, 30.123-970, Belo Horizonte, Brazil.
6) Einstein Albert, Relativity, The Special and General Theory, Digital Reprint, Elegant Books, p. 89
7) Reference (1), p.50
8) Meyer V., Reichart W., Staub H.H., Experimentelle Untersuchung der Massen-Impulsrelation des Elektrons, Helvetia Physica Acta, Band 36 (1963), Heft VII, Persistenter Link: http://doi.org/10.5169/seals-113412
9) Guiragossián Z.G.T. et all, Relative Velocity Measurements of Electrons and Gamma Rays at 15 GeV, Physical Review Letters, 34, 335, Published 10 February 1975, DOI: https://doi.org/10.1103/PhysRevLett.34.335
10) Bertozzi William, Speed and Kinetic Energy of Relativistic Electrons, American Journal of Physics 32, 551 (1964), https://doi.org/10.1119/1.1970770
11) Einstein Albert, Zur Elektrodynamik bewegter Körper, Annalen der Physik, Volume 322, Issue 10 / p. 891-921, first published: June 30,1905, https://doi.org/10.1002/andp.19053221004, English translation: On the electrodynamics of moving bodies, p. 22
12) Planck Collaboration, Planck 2018 results. VI. Cosmological parameters, September 14, 2020, arXiv: 1807.06209v3, p. 42