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Legendre-Galerkin Method, Dynamical System, and Optimal Control for Solving Covid-19 Model

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Abstract: The advancement of numerical demonstrating of irresistible illnesses is a key examination territory in different fields including the nature and the study of disease transmission. One point of these models is to comprehend the elements of conduct in irresistible infections. For the new strain of Covid (Coronavirus), there is no immunization to secure individuals and to forestall its spread up until now. All things being equal, control procedures related to medical services, for example, social separating, isolation, travel limitations, can be adjusted to control the pandemic of Coronavirus. This article reveals insights into the dynamical practices of nonlinear Coronavirus models dependent on strategy: the Legendre-Galerkin strategy. We summon a novel sign stream chart that is utilized to depict the Coronavirus model. Based on the Legendre-Galerkin method, the covid-19 model. Mathematica, as one of the world's leading computational software, was employed for the implementation of solutions. The proposed numerical techniques provide are excellent. Through our numerical investigations, it is uncovered that social removing between possibly tainted people who are conveying the infection and solid people can diminish or intrude on the spread of the infection. The mathematical reenactment results are insensible concurrence with the investigation forecasts. The free balance and dependability focus for the Coronavirus model is researched. Likewise, the presence of a consistently steady arrangement is demonstrate.

Keywords: Covid-19 model; optimal control; existence of uniformly stable; signal stream chart; Legendre-Galerkin technique.

1 Introduction

Many researchers used Legendre polynomials in different methods to construct various mathematical models. These methods can solve different models see ([52]-[60]). The quantity of passages all throughout the planet has expanded significantly because of the spread of the new infection known as Covid (Coronavirus). The quick heightening of cases in practically all nations has made a genuine test for the whole world particularly when the World Wellbeing Association proclaimed that this infection has gotten a worldwide pandemic since its flare-up has spread quickly from China to the remainder of the world. Most nations all throughout the planet have executed recommended systems dependent on confining the development or venturing out to limit the spread of the conceivably lethal infection among countries. Regardless of the adverse consequence on accomplishing financial development, limited development is viewed as quite possibly the best approach to diminish the infection transmission in the worldwide local area. With the approach of the most recent fourteen days, the quantity of cases on the planet has filled dramatically in many influenced territories and come to more than 29 million all through the world. Subsequently, the spread of (Coronavirus) is broadly perceived as being quite possibly the main flare-up over the most recent forty years. At this stage, there is no antibody against the new Coronavirus and most people do not have any resistance that can protect them against diseases. This is the reason it is vital to address the current test of Coronavirus to forestall disease and to make a move to contain any additionally

spread of the infection. In view of the reports of specialists' clinical experts and devoted personnel, the infection is basically spread through drops. Along these lines, the scientists want to add to advance the possibility of social separating between conceivably contaminated people and solid people to diminish or decrease the ejection of Coronavirus in every one of the populaces. The test of Coronavirus is presently managing specialists from medication and atomic science to apply math toward numerical displaying that can assume a huge part in anticipating, evaluating, and controlling the likely episodes. Over the most recent couple of years, various numerical models have been created to give smart subtleties into numerous issues of interest including the transmission and control of irresistible illnesses. The on-going pandemic of Coronavirus, viral pneumonia break out in late 2019 in Wuhan, China, which has its spread worldwide across 210, countries is named as "SARS-CoV-2." It is seething around the world with a tremendous cost as far as human, monetary, and social effect. Inside a limited ability to focus, this brings a caution up in each country everywhere on the world like a pandemic sickness, which encourages every country to estimate the beneficiary preparatory activities to control and contain the wild spread of the infection as the seriousness of the illness will hurt human existence severely. Since the novel Covid is new to the world to figure some effect of the pandemic circumstance and to fabricate an alleviation plan, the similitude impacts of Serious Intense Respiratory Disorder (SARS) and Center East Respiratory Condition scourges in 2003 and 2009 were utilized for study and investigation. From the investigation of the underlying spread of Coronavirus, a considerable lot of the numerical models were utilized into the demonstration from benefactors across the world to decide the seriousness of the gravity of the pandemic. At whatever point an infectious sickness expands its feeder, it follows certain examples of spread, which broadly assists us with distinguishing and screening the elements of the illness flare-ups. The strategy we used to appraise the spread of the sickness is a factor that drives us to finish the actions to dispose of irresistible infections. The episode of the infection inside the country or state for time is normally nonlinear, which moves us to plan the framework where we can contemplate those dynamic nonlinear wonders. By this framework, we can be ready to characterize the transmission of such a virus infection, which assists us with interpreting the medicinal measures to stop or contain the spread of infectious illness.

Covids are encompassed, single-abandoned, positive-sense RNA infections having a place with the group of Coronaviridae [1]. They cause commonly gentle respiratory contaminations, despite the fact that they are at times deadly. Since their disclosure and first portrayal in 1965 [2], three significant, huge scope episodes have happened, brought about by arising, exceptionally pathogenic Covids, to be specific, the "Extreme Intense Respiratory Disorder" (SARS) flare-up in 2003 in territory China [3], the "Center East Respiratory Condition" (MERS) flare-up in 2012 in Saudi Arabia [4, 5] and the MERS flare-up in 2015 in South Korea [6, 7]. These flare-ups have brought about more than 8000 and 2200 confirmed SARS and MERS cases, separately [8]. As of late, a fourth Covid flare-up has happened in Wuhan, the capital city of the Hubei region and the seventh biggest city of Individuals' Republic of China [9, 10]. Since 31 December 2019, when the Wuhan Metropolitan Wellbeing Commission announced 27 instances of viral pneumonia, including 7 basically sick cases, the pneumonia flare-up has gotten impressive worldwide consideration. A tale of Covid was identified as the causative specialist by the Chinese experts on 7 January 2020, and on 10 January 2020, the World Wellbeing Association (WHO) assigned the novel Covid as 2019-nCoV. Around the same time, the WHO delivered a wide scope of interval direction for all nations on how they can get ready for adapting to this crisis, including how to screen for possibly contaminated individuals, gather and test tests, oversee patients, control and alleviate the weight produced by the disease in wellbeing habitats, keep up the correct medication supplies and viably speak with the lay public in regards to the new infection [11, 12]. By the morning of 23 January 2020, more than 571 confirmed cases with 17 passings had been accounted for in different pieces of terrain China, and in different nations including South Korea, Japan, Thailand, Singapore, the Philippines, Mexico and the US of America. Starting on 6 February 2020 (02:45 GMT), 28,276 cases, of which 3863 were in basic condition, and 565 passings had been accounted for. The transmission potential, frequently estimated as far as the essential proliferation number, the episode busy time and worth and term under current and developing intercession measures, stays indistinct, and warrants further examination. On 20 January 2020,

the Chinese government updated the law arrangements concerning irresistible illnesses to add the 2019-nCoV as a class B specialist (a microbe that can cause a plague flare-up). Around the same time, general wellbeing authorities reported a further correction to group the novel infection as a class A specialist (a microorganism that can cause a scourge in a brief timeframe). Some non-drug mediations (NPIs), including serious contact following followed by isolation of people possibly presented to the sickness, and segregation of tainted, indicative people, were carried out, yet their adequacy during the beginning phase was sketchy. Measuring the adequacy of these intercessions is of vital significance for Wuhan just as for different urban communities in their readiness and quick reaction to the importation of tainted cases. With the appearance of the Spring Celebration, huge efforts are required to activate a huge section of the populace, by which the novel Covid might be extensively reseeded. Outrageous, exceptional measures have been taken. For instance, on 23 January 2020, the Chinese specialists presented travel limitations influencing five urban communities (Wuhan, Huanggang, Ezhou, Chibi and Zhijiang), successfully closing down the development of in excess of 40 million individuals. Be that as it may, how these costly and asset concentrated measures can add to the counteraction and control of the contamination in these urban areas and different pieces of the country, and how long these movement limitations ought to be kept up, stay to be resolved. With regards to a novel Covid affecting a guileless populace, assessment of the essential propagation number is significant for deciding the potential and seriousness of a flare-up, and giving basic data to plan and carry out infection episode reactions as far as the identification of the most proper, proof based intercessions, alleviation measures and the assurance of the force of such projects to accomplish the maximal security of the populace with the negligible interference of social-financial exercises [8]. As perceived by the WHO [13], numerical models, particularly those which are convenient, assume a vital part in advising proof based choices by wellbeing choice and strategy producers. Supposedly, a couple of numerical models have so far been openly delivered, including a Bats-Hosts-Repository Individuals transmission network model and a returning voyager study intended to register belittled Covid cases [14, 15]. No investigation has zeroed in on the functional ramifications of general wellbeing mediations and measures.

Avoidance of Coronavirus, other than the significance of forcing general wellbeing and contamination control measures to forestall or diminish the transmission of SARS-CoV-2, the way to containing this worldwide pandemic is by inoculation to forestall SARS-CoV-2 disease in networks across the world. Exceptional endeavors in worldwide examination during this pandemic have brought about the advancement of novel immunizations against SARS-CoV-2 at a phenomenal speed to contain this viral ailment that has crushed networks worldwide and has had a descending spiraling impact on the worldwide economy [16, 17].

Inoculation triggers the insusceptible framework prompting the creation of killing antibodies against SARS-CoV-2. Consequences of a progressing global, fake treatment controlled, eyewitness, dazed, urgent adequacy preliminary announced that people 16 years old or more established getting two-portion routine the preliminary immunization BNT162b2 (mRNA-based, BioNTech/Pfizer) when given 21 days separated presented 95% insurance against Coronavirus with a wellbeing profile like other viral vaccines [18, 19]. Results from another multicenter, Stage 3, randomized, spectator dazed, fake treatment controlled preliminary exhibited that people who were randomized to get two dosages of mRNA-1273 (mRNA based, Moderna) antibody given 28 days separated showed 94.1% viability at forestalling Coronavirus sickness, and no security concerns were noted other than transient neighborhood and fundamental reactions. In light of the aftereffects of these antibody adequacy preliminaries, the FDA gave two EUAs, one on December 11, 2020, allowing the utilization of the BNT162b2 immunization, and another on December 18, 2020, conceding the utilization of the mRNA-1273 antibody for the avoidance of Coronavirus. A third immunization, Ad26.COV2.S for the avoidance of Coronavirus got EUA by the FDA on February 27, 2021, in light of a multicenter, fake treatment control, stage preliminary showed that a solitary portion of Ad26.COV2.S antibody presented 73% viability in the US in forestalling Coronavirus (information not yet distributed) [20-22], [23-25], [26-28].

Therefore, the present study was undertaken to fill in this gap of knowledge.

2 Optimal Control for Covid-19 Modeling

Consider the state presented (1.2), in $\mathbb{R}^8$, with control functions admissible ([12, 13]):

$$\Omega = \left\{ (u_1(\cdot), u_2(\cdot)) \in \left(L^\infty(0, T_f) \right)^2 \mid 0 \leq u_1(\cdot), u_2(\cdot) \leq 1, \quad \forall t \in [0, T_f] \right\},$$

where T_f is time final, $u_1(\cdot)$ and $u_2(\cdot)$ are functions controls.

Defined the functional objective is (quadratic is the control variable)

$$J_1(\zeta_1, \zeta_2) = \int_0^{T_f} [B_1 S_q + B_2 I + B_3 u_1^2 + B_4 u_2^2] dt, \quad (2-1)$$

Minimizes function objective at it ([28-31], [32-34]):

$$J_2(\zeta_1, \zeta_2) = \int_0^{T_f} \eta(S, E, I, A, S_q, E_q, H, R, \zeta_1, \zeta_2, t) dt, \quad (2-2)$$

$$\text{where } \eta(S, E, I, A, S_q, E_q, H, R, \zeta_1, \zeta_2, t) = [B_1 S_q + B_2 I + B_3 u_1^2 + B_4 u_2^2]$$

subjected to the constraint

$$DS = b_1, DE = b_2, DI = b_3, DA = b_4, DS_q = b_5, DE_q = b_6, DH = b_7, DR = b_8. \quad (2-3)$$

The following initial conditions are satisfied:

$$S = S_0, E = E_0, I = I_0, A = A_0, S_q = S_{q0}, H(0) = H_0, R(0) = R_0. \quad (2-4)$$

OCP is defined, consider the a modified objective (cost) function:

$$\bar{\phi} = \int_0^{T_f} \left[\chi - \sum_{j=1}^8 \lambda_j w_j \right] dt, \quad (2-5)$$

where the objective function of Hamiltonian (2-5) and control smoking (1.2) are as follows:

$$\chi = \eta + \sum_{j=1}^8 \lambda_j w_j, \quad (2-6)$$

$$\begin{aligned} \chi = & B_1 S_q + B_2 I + B_3 u_1^2 + B_4 u_2^2 + B_5 RC + \lambda_1 \left(-u_1 (\beta C - C_q (1 - \beta)) S(I + \theta A) + \lambda u_2 S_q \right) \\ & + \lambda_2 \left(\beta u_1 (1 - q) S(I + \theta A) - \sigma E \right) + \lambda_3 \left(\sigma p E - (\delta I + \alpha + \gamma_I) I \right) + \lambda_4 \left(\sigma (1 - \rho) E - \gamma_A A \right) \\ & + \lambda_5 \left(u_1 (1 - \beta) C_q S(I + \theta A) - \lambda u_2 S_q \right) + \lambda_6 \left(u_1 \beta C_q S(I + \theta A) - \delta_q E_q \right) + \lambda_7 \left(\delta I + \delta_q E_q - (\alpha + \gamma_H) H \right) \\ & + \lambda_8 \left(\gamma_I I + \gamma_A A + \gamma_H H \right). \end{aligned} \quad (2-7)$$

Applying Pontryagin's maximum principal, from (2-5) and (2-7), conditions, necessary and sufficient for OPC is

$$D\lambda_1 = \frac{\partial \chi}{\partial S}, \quad D\lambda_2 = \frac{\partial \chi}{\partial E}, \quad D\lambda_3 = \frac{\partial \chi}{\partial I}, \quad D\lambda_4 = \frac{\partial \chi}{\partial A}, \quad D\lambda_5 = \frac{\partial \chi}{\partial S_q},$$

$$D\lambda_6 = \frac{\partial \chi}{\partial E_q}, \quad D\lambda_7 = \frac{\partial \chi}{\partial H}, \quad D\lambda_8 = \frac{\partial \chi}{\partial R}. \quad (2-8)$$

$$0 = \frac{\partial \chi}{\partial \xi_1}, \quad 0 = \frac{\partial \chi}{\partial \xi_2}. \quad (2-9)$$

$$DS = \frac{\partial \chi}{\partial \lambda_1}, \quad DE = \frac{\partial \chi}{\partial \lambda_2}, \quad DI = \frac{\partial \chi}{\partial \lambda_3}, \quad DA = \frac{\partial \chi}{\partial \lambda_4}, \quad DS_q = \frac{\partial \chi}{\partial \lambda_5},$$

$$DE_q = \frac{\partial \chi}{\partial \lambda_6}, \quad DH = \frac{\partial \chi}{\partial \lambda_7}, \quad DR = \frac{\partial \chi}{\partial \lambda_8} \quad (2-10)$$

$$\lambda_j, (T_j)=0, j=1,...,8. \quad (2-11)$$

where λ_j are multipliers Lagrange. Eqs. (2-9)-(2-10) exemplify the conditions necessary of Hamiltonian for the OPC.

We construct the theorem at similar ([12], [13], [30, 31]):

Theorem 1.

If u_1 and u_2 are controls optimal with state corresponding variables, consequently they work out adjoint variables λ_j^* , $j=1,...,8$, accepts:

(i) Co-state equation

$$D\lambda_1 = \lambda_1^* \left(-u_1(\beta C + C_q(1-\beta))S(I + \theta A) \right) + \lambda_2^* \left(\beta u_1(1-q)S(I + \theta A) \right) + \lambda_5^* \left(u_1(1-\beta)C_q S(I + \theta A) \right) \\ + \lambda_6^* \left(u_1 \beta C_q S(I + \theta A) \right),$$

$$D\lambda_2 = \lambda_2^* (-\delta) + \lambda_3^* (\delta \rho) + \lambda_4^* (\delta(1-\rho)),$$

$$D\lambda_3 = B_2 + \lambda_1^* \left(-u_1(\beta C + C_q(1-\beta))S \right) + \lambda_2^* \left(\beta u_1(1-q)S \right) + \lambda_3^* \left(-(\delta_I + \alpha + \gamma_I) \right) \\ + \lambda_5^* \left(u_1(1-\beta)C_q S \right) + \lambda_6^* \left(u_1 \beta C_q S \right) + \lambda_7^* (\delta_I) + \lambda_8^* (\gamma_I),$$

$$D\lambda_4 = \lambda_1^* \left(-u_1(\beta C + C_q(1-\beta))S\theta \right) + \lambda_2^* \left(\beta u_1(1-q)S\theta \right) + \lambda_4^* (-\gamma_A) \\ + \lambda_5^* \left(u_1(1-\beta)C_q S\theta \right) + \lambda_6^* \left(u_1 \beta C_q S\theta \right) + \lambda_8^* (\gamma_A),$$

$$\begin{aligned}
D\lambda_5 &= B_1 + \lambda_1^* (-u_2 \lambda) + \lambda_2^* (-\lambda u_2), \\
D\lambda_6 &= \lambda_7^* (\delta_q) + \lambda_6^* (-\delta_q), \\
D\lambda_7 &= \lambda_7^* (-(\alpha + \gamma_H)) + \lambda_8^* (\gamma_H), \\
D\lambda_8 &= B_5.
\end{aligned} \tag{2-12}$$

(ii) With condition transversality:

$$\lambda_j^*(T_f) = 0. \tag{2-13}$$

(iii) Optimality conditions

$$\chi = \min_{0 \leq u_V^*, u_R^* \leq 1} \chi, \tag{2-14}$$

So, the functions control u_1^*, u_2^* .

For a lot of optimal control and model differentials, equations for solving models see ([1], [29]).

3 Existence of Uniformly Stable Solution

This subsection explores the existence of uniformly stable solution. Let us define

$$\begin{aligned}
g_1 &= -u_1(\beta C - C_q(1 - \beta))S(I + \theta A) + \lambda u_2 S_q, \\
g_2 &= \beta u_1(1 - q)S(I + \theta A) - \sigma E, \\
g_3 &= \sigma \rho E - (\delta_I + \alpha + \gamma_I)I, \\
g_4 &= \sigma(1 - \rho)E - \gamma_A A, \\
g_5 &= u_1(1 - \beta)C_q S(I + \theta A) - \lambda u_2 S_q \\
g_6 &= u_1 \beta C_q S(I + \theta A) - \delta_q E_q \\
g_7 &= \delta_I I + \delta_q E_q - (\alpha + \gamma_H)H \\
g_8 &= \gamma_I I + \gamma_A A + \gamma_H H.
\end{aligned} \tag{3.1}$$

Let $\Delta = \{S, E, I, A, S_q, E_q, H, R \in \mathfrak{R} : |S, E, I, A, S_q, E_q, H, R| \leq c, \quad t \in [0, T]\}$.

This means such all of the 8 functions $\frac{\partial g}{\partial S}, \frac{\partial g}{\partial E}, \frac{\partial g}{\partial I}, \frac{\partial g}{\partial A}, \frac{\partial g}{\partial S_q}, \frac{\partial g}{\partial E_q}, \frac{\partial g}{\partial H}$, and $\frac{\partial g}{\partial R}$

coincide with the Lipschitz case with regard to the 8 arguments, thereafter all of the 8 functions have absolutely continuous with regard to the 8 arguments ([12], [13]). For extra of fractional models ([32-34], [35-37], [38-40], [41-43], [44-46], [47-50]).

4 Invariance and Symmetry of the Proposed Model.

For system (1.1), the transformation

$(S, E, I, A, S_q, E_q, H, R) \rightarrow (S, -E, -I, -A, -S_q, -E_q, -H, -R)$ implies that the system is invariant.

Then if $(S, E, I, A, S_q, E_q, H, R)$ is a solution of model (1.1), then $(S, -E, -I, -A, -S_q, -E_q, -H, -R)$ is a solution of the same model too.

4.1 Dissipation of Proposed Model (1.1).

The divergence of proposed paradigm (1.1) is

$$\begin{aligned}\nabla \cdot Z &= \frac{\partial \dot{S}}{\partial S} + \frac{\partial \dot{E}}{\partial E} + \frac{\partial \dot{I}}{\partial I} + \frac{\partial \dot{A}}{\partial A} + \frac{\partial \dot{S}_q}{\partial S_q} + \frac{\partial \dot{E}_q}{\partial E_q} + \frac{\partial \dot{H}}{\partial H} + \frac{\partial \dot{R}}{\partial R} \\ &= -(\beta C - C_q(1-\beta))S(I + \theta A) - \sigma - (\delta_I + \alpha + \gamma_I) - \gamma_A - \lambda - S_q - (\alpha + \gamma_H).\end{aligned}$$

Then proposed model paradigm (1.1) is dissipative such as

$$\nabla \cdot Z = -(\beta C - C_q(1-\beta))S(I + \theta A) - \sigma - (\delta_I + \alpha + \gamma_I) - \gamma_A - \lambda - S_q - (\alpha + \gamma_H) < 0.$$

5 Solution Procedure of Legendre-Galerkin Method

5.1 Legendre Function Preliminaries

Orthogonal systems play a vital role in performing mathematical analysis. This can be due to functions belonging to very general classes can be expanded in series of orthogonal functions e.g., Fourier-Bessel series, Fourier series, etc. Orthogonal polynomials, $L_n(x)$; ($n = 0, 1, 2, \dots$) of degree n , represent an important class of orthogonal systems where many of the special functions encountered in the applications, such as Jacobi, Legendre, Laguerre, Hermite, and Chebyshev polynomials, are part of that class [52].

$$P_n(x) = 2^n \sum_{k=0}^n \binom{n}{k} \left(\frac{n+k-1}{2}\right) x^k, \quad (5-0)$$

with the orthogonality on the interval $[-1, 1]$

$$\int_{-1}^1 P_n(x) P_m(x) dx = \begin{cases} \frac{2}{2n+1}, & \text{if } m = n, \\ 0, & \text{if } m \neq n. \end{cases} \quad (5-1)$$

Lemma 1.1 [53] For arbitrary constants q, k, m , and j are positive integer

$$P_k^{(q)}(x) = \sum_{i=0}^{k-q} C_q(k, i) P_i(x), \quad (k+1) \text{ is even} \quad (5-2)$$

$$x^m P_j(x) = \sum_{n=0}^{2m} \Theta_{m,n}(j) P_{j+m-n}(x), \quad P_{-r}(x) = 0 \text{ for } r \geq 1 \quad (5-3)$$

where

$$C_q(k, i) = \frac{2^{q-1} (2i+1) \Gamma\left(\frac{q+k-i}{2}\right) \Gamma\left(\frac{q+k-i+1}{2}\right)}{\Gamma(q) \Gamma\left(\frac{2-q+k-i}{2}\right) \Gamma\left(\frac{3-q+k-i}{2}\right)}, \quad (5-4)$$

$$\begin{aligned} \Theta_{m,n}(j) &= \frac{(-1)^n 2^{j+m-n} m! (2j+2m-2n+1) \Gamma(j+1)}{\Gamma(j+m-n+1)} \\ &\times \sum_{k=\max(0, j-n)}^{j+m-n} \frac{\Gamma(j+k+1)}{2^k (n+k-j)! \Gamma(3j+2m-2n-k+2)} \\ &\times \sum_{\ell=0}^{j-k} \frac{(-1)^\ell \Gamma(2j+m-n-k-\ell+1) \Gamma(j+m+\ell-n+1)}{\ell! (j-k-\ell)! \Gamma(j-\ell+1) \Gamma(k+\ell+1)} \\ &\times {}_2F_1(j-k-n, j+m+\ell; 3j+2m-2n-k+2; 2), \end{aligned} \quad (5-5)$$

and

$${}_2F_1(N, a; 2a; 2) = \begin{cases} 0, & \text{for } N \text{ odd,} \\ \frac{\left(\frac{1}{2}\right)_N}{\left(a + \frac{1}{2}\right)_N}, & \text{for } N \text{ even.} \end{cases}$$

Theorem 1.1 Let k, l, q and m be arbitrary constants such that k, l, q and m are positive integer, then

$$\int_{-1}^1 P_k^{(q)}(x) P_l(x) dx = \begin{cases} 2 \sum_{i=0}^{k-q} \frac{C_q(k, i)}{2i+1}, & \text{if } l = i, \quad (k+1) \text{ is even,} \\ 0, & \text{if } l \neq i. \end{cases} \quad (5-6)$$

$$\int_{-1}^1 x^m P_k(x) P_l(x) dx = \begin{cases} 2 \sum_{n=0}^{2m} \frac{\Theta_{m,n}(j)}{2(j+m-n)+1}, & \text{if } l = j+m-n, \\ 0, & \text{if } l \neq j+m-n. \end{cases} \quad (5-7)$$

where $C_q(k, i)$ and $\Theta_{m,n}(j)$ are as defined in Eq.(5-4) and Eq.(5-5).

Proof. By using Lemma 5.1 beside Eq. (5-1), Theorem 1.1 is proved.

5.2 Legendre-Galerkin Method for Nonlinear COVID-19 Model

The approximate solutions are given by

$$S(t) = S_0 + \frac{1}{\gamma_A(\alpha+\gamma_i+\delta_i)} e^{-t(\alpha+\lambda+\gamma_i+\delta_i)} \left(c(e^{t(\alpha+\lambda+\gamma_i+\delta_i)} - e^{t(\alpha+\lambda-\gamma_A+\gamma_i+\delta_i)}) (q(-1+\beta) - \right.$$

$$\beta)\theta A_0 S_0(\alpha + \gamma_i + \delta_i) - \gamma_A \left(c(e^{t\lambda} - e^{t(\alpha+\lambda+\gamma_i+\delta_i)})(q(-1 + \beta) - \beta)I_0 S_0 - e^{t(\alpha+\gamma_i+\delta_i)}(-1 + e^{t\lambda})S_{q0}(\alpha + \gamma_i + \delta_i) \right) + \dots \quad (5.8)$$

$$\begin{aligned} \mathbf{E}(t) = & e^{-t\sigma} \mathbf{E}_0 + \\ & + \frac{1}{(\sigma - \gamma_A)(\alpha - \sigma + \gamma_i + \delta_i)} c e^{-t(\alpha+\gamma_i+\delta_i)} (-1 + q) \beta S_0 \left(-(-1 + e^{t(\alpha-\sigma+\gamma_i+\delta_i)}) I_0(\sigma - \gamma_A) \right. \\ & \left. + (e^{t(\alpha-\sigma+\gamma_i+\delta_i)} - e^{t(\alpha-\gamma_A+\gamma_i+\delta_i)}) \theta A_0(\alpha - \sigma + \gamma_i + \delta_i) \right) + \dots \end{aligned} \quad (5.9)$$

$$I(t) = e^{t(-\alpha-\gamma_i-\delta_i)} I_0 + \frac{e^{t(-\alpha-\gamma_i-\delta_i)}(-1 + e^{t(\alpha-\sigma+\gamma_i+\delta_i)})\rho\sigma E_0}{\alpha - \sigma + \gamma_i + \delta_i} + \dots \quad (5.10)$$

$$A(t) = e^{-t\gamma_A} A_0 + \frac{e^{-t(\sigma-\gamma_A)-t\gamma_A}(-1 + e^{t(\sigma-\gamma_A)})(-1 + \rho)\sigma E_0}{-\sigma + \gamma_A} + \dots \quad (5.11)$$

$$\begin{aligned} \mathbf{S}_q(t) = & e^{-t\lambda} \mathbf{S}_{q0} + \\ & \frac{1}{(\lambda - \gamma_A)(\alpha - \lambda + \gamma_i + \delta_i)} c e^{-t(\alpha+\gamma_i+\delta_i)} q(-1 + \beta) S_0 \left(-(-1 + e^{t(\alpha-\lambda+\gamma_i+\delta_i)}) I_0(\lambda - \gamma_A) + \right. \\ & \left. (e^{t(\alpha-\lambda+\gamma_i+\delta_i)} - e^{t(\alpha-\gamma_A+\gamma_i+\delta_i)}) \theta A_0(\alpha - \lambda + \gamma_i + \delta_i) \right) + \dots \end{aligned} \quad (5.12)$$

$$\begin{aligned} \mathbf{E}_q(t) = & e^{-t\delta_q} \mathbf{E}_{q0} + \\ & \frac{1}{(\alpha + \gamma_i + \delta_i - \delta_q)(-\gamma_A + \delta_q)} \{ c e^{-t(\alpha+\gamma_i+\delta_i)} q \beta S_0 \left((e^{t(\alpha-\gamma_A+\gamma_i+\delta_i)} - e^{t(\alpha+\gamma_i+\delta_i-\delta_q)}) \theta A_0(\alpha \right. \\ & \left. + \gamma_i + \delta_i - \delta_q) + (-1 + e^{t(\alpha+\gamma_i+\delta_i-\delta_q)}) I_0(-\gamma_A + \delta_q) \right) + \dots \end{aligned} \quad (5.13)$$

$$\begin{aligned} H(t) = & e^{t(-\alpha-\gamma_H)} h_0 + \frac{e^{-t(\alpha+\gamma_H)}}{(\gamma_H - \gamma_i - \delta_i)(\alpha + \gamma_H - \delta_q)} \left((-1 + e^{t(\gamma_H - \gamma_i - \delta_i)}) i_0 \delta_i (\alpha + \gamma_H - \delta_q) + \right. \\ & \left. (-1 + e^{t(\alpha+\gamma_H-\delta_q)}) M_0(\gamma_H - \gamma_i - \delta_i) \delta_q \right) + \dots \end{aligned} \quad (5.14)$$

$$\begin{aligned}
R(t) = R_0 + \frac{1}{(\alpha + \gamma_H)(\alpha + \gamma_i + \delta_i)} e^{-t(2\alpha + \gamma_A + \gamma_H + \gamma_i + \delta_i)} & \left(e^{t(\alpha + \gamma_A + \gamma_H)} (-1 + e^{t(\alpha + \gamma_i + \delta_i)}) i_0(\alpha \right. \\
& + \gamma_H) \gamma_i + e^{t(\alpha + \gamma_A + \gamma_i + \delta_i)} (-1 + e^{t(\alpha + \gamma_H)}) h_0 \gamma_H (\alpha + \gamma_i + \delta_i) \\
& \left. + e^{t(2\alpha + \gamma_H + \gamma_i + \delta_i)} (-1 + e^{t\gamma_A}) A_0 (\alpha + \gamma_H) (\alpha + \gamma_i + \delta_i) \right).
\end{aligned} \tag{5.15}$$

From the initial values in Wuhan, China, the parameters in the approximate solutions (5.8)-(5.15) are given by as $c = 14.781$, $\beta = 2.1011$; $q = 1.8887 \times 10^{-8}$; $w = 0.13266$; $\lambda = 1/14$; $\rho = 0.86834$; $\delta_I = 0.3266$; $\delta_q = 0.1259$; $\gamma_I = 0.33029$; $\gamma_A = 0.13978$; $\gamma_H = 0.11624$; $\alpha = 1.7826 \times 10^{-5}$, $S_0 = 11081000$, $E_0 = 105.1$; $I_0 = 27.679$; $\theta = 0.5$; $A_0 = 53.839$; $\sigma = 1/7$; $S_{q0} = 739$; $E_{q0} = 1.1642$; $h_0 = 1$; $\delta_i = 0.3266$.

The approximate solutions (5.8)-(5.15) are given by:

$$\begin{aligned}
S(t) = 11081000 - 8.077550840783672 \\
\times 10^{10} e^{-0.8681163974285714t} (-0.17951293599126503 e^{0.2112085714285714t} \\
- 0.8204870731575477 e^{0.7283363974285714t} + 9.148812735028278 \times 10^{-9} e^{0.796687826t} \\
+ 1. e^{0.8681163974285714t}) + 2.943437315081433 \\
\times 10^{14} e^{-4.340581987142855t} (0.03223199288688491 e^{3.026766335142855t} \\
+ 0.2946409779302137 e^{3.543894161142855t} - 2.963183621861236 \\
\times 10^{-9} e^{3.612245589714284t} - 0.35915010013294 e^{3.683674161142855t} \\
+ 0.6733473336595697 e^{4.061021987142855t} - 9.937923116376702 \\
\times 10^{-9} e^{4.129373415714284t} - 0.0024212077611237555 e^{4.1977248442857125t} \\
- 1.6386489836866025 e^{4.200801987142855t} + 5.104696820586916 \\
\times 10^{-12} e^{4.269153415714284t} + 1. e^{4.340581987142855t})
\end{aligned} \tag{5.16}$$

$$\begin{aligned}
E(t) = 105.1 e^{-t/7} + 3.010570602728641 \times 10^{12} e^{-0.6569078259999999t} & (-0.006154958441363934 \\
& - 0.9938450415586361 e^{0.5140506831428571t} + 1. e^{0.517127826t}) - 2.190978407569755 \\
& \times 10^{16} e^{-3.272230558571426t} (0.00048584392087149024 e^{1.9584149065714267t} \\
& + 0.004823171677329365 e^{2.4755427325714265t} - 4.952172416946539 \\
& \times 10^{-11} e^{2.543894161142855t} - 0.00616582475255329 e^{2.615322732571426t} \\
& + 0.01849921393298708 e^{2.9926705585714264t} - 4.125496904340082 \\
& \times 10^{-10} e^{3.0610219871428552t} - 1.0176424043165633 e^{3.129373415714283t} \\
& + 1. e^{3.1324505585714264t} + 0.000004646764974935146 e^{3.129373415714283t} t)
\end{aligned} \tag{5.17}$$

$$\begin{aligned}
A(t) = & 53.839e^{-0.13978t} + 642.4078922934066e^{-0.2826371428571428t}(-1.e^{0.13978t} + 1.e^{0.14285714285714285t}) \\
& + 5.662453222217899 \\
& \times 10^{10}e^{-0.6599849688571426t}(0.011902199285953595e^{0.0030771428571427784t} \\
& + 322.9765687516541e^{0.5171278259999998t} - 322.98847095094004e^{0.5202049688571426t} \\
& + 1.e^{0.5202049688571426t}t)
\end{aligned} \tag{5.18}$$

$$\begin{aligned}
S_q(t) = & 739e^{-t/14} \\
& - 1502.535510389303e^{-0.7252592545714286t}(-0.107173312618706e^{0.06835142857142862t} \\
& - 0.8928266873812939e^{0.5854792545714286t} + 1.e^{0.6538306831428572t}) \\
& + 7273848.199122141e^{-3.7691534157142805t}(0.013652061810143121e^{2.4553377637142813t} \\
& + 0.1296348087598582e^{2.972465589714281t} - 1.31589041905024 \times 10^{-9}e^{3.0408170182857095t} \\
& - 0.16139961665976396e^{3.1122455897142807t} + 0.3622514177293506e^{3.489593415714281t} \\
& - 6.014447218305942 \times 10^{-9}e^{3.55794484428571t} \\
& - 0.0019395261323268999e^{3.626296272857138t} - 1.3421991381769234e^{3.629373415714281t} \\
& + 1.e^{3.697724844285709t}).
\end{aligned} \tag{5.19}$$

$$\begin{aligned}
E_q(t) = & 1.1642e^{-0.1259t} + 12944.586834398624e^{-0.670787826t}(-0.02617301509751275e^{0.013880000000000003t} \\
& - 0.9738269849024873e^{0.531007826t} + 1.e^{0.544887826t}) - 8.536953685846727 \\
& \times 10^7e^{-3.387853415714281t}(0.0023214045747880292e^{2.0740377637142817t} \\
& + 0.022788251647984353e^{2.5911655897142816t} - 2.33288980952792 \\
& \times 10^{-10}e^{2.65951701828571t} - 0.028933059256547822e^{2.730945589714281t} \\
& + 0.07977525448342258e^{3.1082934157142814t} - 1.602246578630015 \\
& \times 10^{-9}e^{3.1766448442857103t} - 0.0013283000500193941e^{3.2449962728571387t} \\
& - 1.0746235495640923e^{3.2480734157142814t} + 1.e^{3.261953415714281t})
\end{aligned} \tag{5.20}$$

$$\begin{aligned}
H(t) = & 1.e^{-0.116257826t} \\
& + 31.921759750033623e^{-0.66655t}(-0.5237976480945624e^{0.009642174000000003t} \\
& - 0.4762023519054375e^{0.54065t} + 1.e^{0.550292174t}) \\
& + 101766.35565315833e^{-0.7166714908571429t}(0.0009258109972406025e^{0.059763664857142906t} \\
& - 0.0030600605340758737e^{0.5738143480000001t} + 0.6630007924132848e^{0.5768914908571429t} \\
& - 1.6608665428764495e^{0.5907714908571428t} + 1.e^{0.6004136648571429t})
\end{aligned} \tag{5.21}$$

$$\begin{aligned}
R(t) = & 2 + 68.75571072170078e^{-0.9129456520000001t}(-0.20241030028246748e^{0.256037826t} \\
& - 0.7830476833832982e^{0.7731656520000001t} - 0.014542016334234376e^{0.7966878260000001t} \\
& + 1.e^{0.9129456520000001t}) \\
& + 74.64711217749436e^{-1.181702794857143t}(0.21046681774653264e^{0.5247949688571429t} \\
& + 7.635017935997784e^{1.038845652t} - 8.605930940314238e^{1.041922794857143t} \\
& + 0.18801617828273953e^{1.0558027948571431t} - 0.42756999171281834e^{1.065444968857143t} \\
& + 1.e^{1.181702794857143t})
\end{aligned}
\tag{5.22}$$

$$\begin{aligned}
I(t) = & 27.679e^{-0.656907826t} - 2.070405815456379 \times 10^{25}e^{-0.656907826t}(-1. + 1.e^{1.110223024625156 \times 10^{-16}t} \\
& + 3.487368493187135 \times 10^{-14}e^{0.5140506831428572t} - 3.488086109177916 \\
& \times 10^{-14}e^{0.5171278260000001t}) \\
& + 25.3622945843254e^{-0.7997649688571429t}(-1.e^{0.1428571428571429t} + 1.e^{0.656907826t})
\end{aligned}
\tag{5.23}$$

Figure 2: the relation of variable I and t.

Figure 3: the relation of variable R and t.

Figure 4: the relation of variable IH and t.

Figure 5: the relation of variable Eq and t.

Figure 6: the relation of variable Sq and t.

Figure 7: the relation of variable A and t.

Figure 8: the relation of variable E and t.

Figure 9: the relation of variable S and t.

6 Results and discussion

The plotted curves reveal complete overlaps between the Legendre-Galerkin solution. Furthermore, the comparison charts demonstrate a very good agreement between the numerical results estimated via Legendre-Galerkin and the corresponding procedure introduced by [51]. These observations suggest that

the lateral deflection and bending moment profiles can be represented accurately by the proposed method. However, insignificant differences between the location of peak deflection and bending moment from proposed methods and the solution proposed by [51] can also be observed regardless of the type of head condition.

7 Conclusion

In the present study, the Legendre-Galerkin method has been introduced to obtain simplified numerical approaches for understanding the behavior of Covid-19. The free infection balance and soundness point for the Coronavirus model are addressed. The model is portrayed by a novel sign stream graph where the signal stream chart is a scheme agent that is applied to display the interrelation among the system states and becomes ours to utilize scheme-theoretic stuff to find novel brow of the system. Through our numerical investigations, the seriousness of the infection is explained, which shows more impact by expanding the contact number. The mathematical recreations show that the nearby association among helpless and irresistible people is a significant danger factor for spreading the infection while keeping up actual distance is fundamental to decrease the danger of spreading the infection.

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