New Extended Topsis Method Based On The Entropy Measure And Picture Fuzzy Rough Set Information And Their Application In Decision Support System

Muhammad Ali Khan (✉ ali_khan@awkum.edu.pk)

AWKUM: Abdul Wali Khan University Mardan

Saleem Abdullah

Abdul Wali Khan University Mardan

Abbas Qadir

Abdul Wali Khan University Mardan

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NEW EXTENDED TOPSIS METHOD BASED ON THE ENTROPY MEASURE AND PICTURE FUZZY ROUGH SET INFORMATION AND THEIR APPLICATION IN DECISION SUPPORT SYSTEM

MUHAMMAD ALI KHAN, SALEEM ABDULLAH, AND ABBSS QADIR

ABSTRACT. In this article, we shall introduce a novel technique for order preference by similarity to ideal solution (TOPSIS)-based methodology to resolve multicriteria group decision-making problems within picture fuzzy environment, where the weights information of both the decision makers (DMs) and criteria are completely unknown. First, we briefly review the definition of picture fuzzy sets (PFS), score function and accuracy function of PFRSs and their basic operational laws. In addition, defined the generalized distance measure for PFRSs based on picture fuzzy rough entropy measure to compute the unknown weights information. Secondly, the picture fuzzy information-based decision-making technique for multiple attribute group decision making (MAGDM) is established and all computing steps are simply depicted. In our presented model, it's more accuracy and effective for considering the conflicting attributes. Finally, an illustrative example with robot selection is provided to demonstrate the effectiveness of the proposed picture fuzzy decision support approaches, together with comparison results discussion, proving that its results are feasible and credible.

1. INTRODUCTION

Multi-attribute group decision making (MAGDM) has made a significant contribution to decision support system since its beginnings [1, 2, 3, 4, 5, 6]. Robot selection for manufacturing industries is a multi-functional group decision-making problem that is frequently addressed by employing unprogrammed decision-making methodologies and including the company's massive contract. Various decision makers/analysis, such like development, research, engineering, and economics, are covered in a decision group. In reality, a single decision maker's interests may not be the same. When using the group decision maker (GDM) approach, the priority degree of each decision maker can have a large impact on the final result. The increased involvement of multi-functional teams in robot selection and estimating has a significant impact on the buying firm's efficiency. The representation of attribute value is a major issue in decision methods. Crisp numbers cause a problem in decision-making. Because in some circumstances, proving an attribute with crisp set can be challenging. As a result, decision-makers have the ability to make decisions at a good extent. The fuzzy set theory has been utilised to tackle collective

Key words and phrases. Picture fuzzy rough sets, Picture fuzzy rough entropy measure, Extended TOPSIS method, Group Decision Making Problems.

decision-making challenges in a number of aspects, including management, engineering, and social sciences. Fuzzy set theory's application to decision-making has a substantial impact.

Due to uncertainties, there are numerous issues that occur in decision-making. For this, In 1965, Zadeh was the first who gave the idea of fuzzy set (FS) [7] which consist only the membership grade function based on $[0,1]$. The Zadeh theory of fuzzy sets has a lot of interesting aspects. In the area of the fuzzy set theory, the challenge of making decisions that classify the elements in the given universe into more than one relevant place has been examined. According to Atanassov, there are numerous flaws with FS. He saw that the idea of negative membership grade might be there as well, which is an important issue to consider when assembling the completely suggested structure and impacts of the difficulties. As an alternative for proper values, the intuitionistic fuzzy (IF) set appropriately introduces this type of grade. The element of Atanassov's IF set [8] are provided in ordered pair that consists of positive and negative membership grade characteristics that follow constraint that the sum of both given functions lie between $[0,1]$.

After more outcomes Pawlak [9] is credited with being the first to explore the prevalent idea of rough sets (RS) theories. The classical set theory, that deals with inexact and inaccurate details, is generalised by this theory. Rough set analysis has advanced significantly in recent years, both in terms of practical applications and theoretical understanding. The idea of rough sets has been expanded in different ways by many researchers. By utilizing fuzzy relations instead of crisp binary relations, Dubois and Prade [10] invented the idea of fuzzy rough collection. Cornelis et al. [11] established a combined analysis of IF rough set applying the hybrid notion of IFS and rough set as link between all these two theories (IFRS). By using IFR approximation operators (AOs), Zhou and Wu [12] established a constructive and axiomatic analysis. By introducing the idea of crisp and fuzzy approximation space Zhou and Wu [13] created the idea of IFRS and rough IFS and presenting their constraining and axiomatic analysis in depth. The IF relation was established by Bustince and Burillo [14]. Zhang et al. [15] utilized general IF relations to examine the general structure of IFRS based on the principle of two universes. Yun and Lee [16] used topology to establish some properties of intuitionistic fuzzy rough approximation operator (IFRAO) based on intuitionistic fuzzy relation. Many number of IFRS extensions are examined See [17, 18, 19, 20] for more information. In addition, Mehmood et al. [21] invented the interaction between the rough and IFS and also discussed its various aggregation operators.

After more outcomes BC Cuong developed the picture fuzzy set (PFS) [22] by adding the neutral membership grade satisfying the condition that the sum of the positive, neutral and negative membership grades lie between $[0,1]$, which is the generalization of FSs, and IFSs. PFSs are clearly better suited to dealing with fuzziness and ambiguity than IFSs. Under the picture fuzzy environment, H. Garg [23] introduced the picture fuzzy weighted averaging operator, picture fuzzy ordered weighted and hybrid averaging operator. The correlation coefficient of PFS was given by P. Singh [24] in 2015. Wei [25, 26] developed a decision-making methodology based on the fuzzy weighted cross-entropy of picture, which is used to distinguish the alternative. Wang, X. Zhou, H. Tu, and S. Tao (2017) researched multiple attribute choice problems based on the picture fuzzy setting, as well as constructed and described various picture fuzzy geometric operators. due to lack of understanding

about the area of the problem and time constraints, DM sometimes uses picture fuzzy information, and knowledge about its weight is uncertain.

Numerous decision-making models have been presented in the research over the decades, with the methodology for order preference by similarity to ideal solution (TOPSIS) being one of the most often utilised and useful. The TOPSIS method was suggested by Hwang and Yoon [27] to handle with multi-attribute DMs. In DMs, the best alternative is the one with the lowest distance from the positive ideal solution (PIS) and the greatest distance from the negative ideal solution (NIS). In the first part [28] Chen demonstrated how to solve DMs using TOPSIS in the FS context. In recent years, many people have become interested in TOPSIS and have applied it to real-world DMs using various extended structures of FS [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41] in the field of decision sciences. In the first part, Chen demonstrated how to solve DMs using TOPSIS in the FS context. In recent years, many people have become interested in TOPSIS and have applied it to real-world DMs using various extended structures of FS in the domains of decision sciences [41, 42, 43]. It's also worth noting that the present TOPSIS procedure [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41] have the issue of requiring either DMs weight [37] or criteria weights [33, 40] or both [29, 31, 40, 41, 42, 43] to solve DMs. [44, 45] in which criteria weights are available, some authors assigned uncertain weight information regarding DMs. In MAGDM difficulties, some method utilizes uncertain weight information for criterion with known weight knowledge for DMs. According to researchers, there appears to be no strategy in the field for handling MAGDM problems using PFR data where the information regarding DM weights and criteria is completely unknown.

In this research, a novel modified TOPSIS-based approach is suggested to deal with the situation of unknown weight data for both DMs and criteria weights, as well as to handle the MAGDM issue once all the weights have been computed. It is necessary to choose the group decision/ideal opinion that is closest to each DMs decision matrix when solving the DMs. Ideal opinion is determined using the PFR average method in the proposed process. To evaluate the differences between two PFRs, a generalised distance measure is constructed. To find out the criteria weights using PFR fact in the given picture fuzzy rough, a generalised distance measure-based entropy measure is presented. In this research, TOPSIS is used to solve MAGDM problem. The main thought for calculating criteria weights using the entropy measure is that the lower the entropy measure of a criterion among alternatives, the higher the weight should be placed on that criterion, and instead, the lower weight should be placed on that criterion. A newly entropy measure for PFRs is described as keeping in mind the priority of its membership grades to better evaluate the fuzziness of PFRs and used to determine the criteria weights in addressing MAGDM problems with entirely unknown weight information using the PFRs entropy weight.

(i) Decision-making is provided by PFRNs, which means that DMs have more ability to share information about the alternative that match the criteria, i.e., DMs can easily apply their inaccurate judgement in the MAGDM process.

(ii) All DM weights and criteria are hidden behind the judgement values produced by DMs for alternatives that satisfying the criteria. As a result, all uncertain weights are generated and evaluated during the procedure using all of the decision-making.

(iii) In order to solve MSGDM challenges with fully unknown weight information, a novel TOPSIS-based methodology is presented to calculate the weights of DMs

(iv) The entropy weight approach is used to calculate the criteria weight in MAGDM situations with entirely uncertain weight information, using a newly established entropy measure for PFRSs.

(v) TOPSIS is further performed for each DM separately after analysing the weights of criteria, in order to avoid the loss of aggregate information.

(vi) Since aggregation is conducted in the final phase utilising determined weight of the DMs, there is no risk of losing collective information during the process.

The following is a summary of the paper's structure. The second section goes through some PFS and rough set concepts. The innovative concept of picture fuzzy rough entropy measure was introduced in section third. The modified TOPSIS method was established in Section forth to solve the uncertainties in MAGDM situations. In section five, an illustrated example of the planned MAGDM technique for robot selection for manufacturing units is shown, along with a comparison to existing decision-making approaches. In Section seven, the paper's study is concluded.

2. PRELIMINARIES

Here we sort out the essential knowledge about, fuzzy set, intuitionistic FS, picture FS and rough set.

Definition 1. [7] Assume that $\tilde{U}$ be a fixed set. A fuzzy set (FS) $\mathfrak{S}$ in the universe $\tilde{U}$ is a set having the form;

$$\mathfrak{S} = \{ \langle \tilde{h}, \rho_{\mathfrak{S}}(\tilde{h}) \rangle \mid \tilde{h} \in \tilde{U} \},$$

where the value $\rho_{\mathfrak{S}}(\tilde{h}) \in [0, 1]$ is called membership grade of $\tilde{h}$.

Definition 2. [8] Assume that $\tilde{U}$ be a fixed set. An intuitionistic FS $\mathfrak{S}$ in the universe $\tilde{U}$ is a set having the form;

$$\mathfrak{S} = \{ \langle \tilde{h}, \rho_{\mathfrak{S}}(\tilde{h}), \tilde{n}_{\mathfrak{S}}(\tilde{h}) \rangle \mid \tilde{h} \in \tilde{U} \},$$

where the values $\rho_{\mathfrak{S}}(\tilde{h}) \in [0, 1]$, $\tilde{n}_{\mathfrak{S}}(\tilde{h}) \in [0, 1]$ are called positive and negative membership grades of $\tilde{h}$ and $\rho_{\mathfrak{S}}(\tilde{h}) + \tilde{n}_{\mathfrak{S}}(\tilde{h}) \leq 1$, $\forall \tilde{h} \in \tilde{U}$. for each $\tilde{h} \in \tilde{U}$. The degree of hesitancy of IFS is determined by $v_{\mathfrak{S}}(\tilde{h}) = 1 - (\rho_{\mathfrak{S}}(\tilde{h}) + \tilde{n}_{\mathfrak{S}}(\tilde{h}))$.

Let $\mathfrak{S}_1 = \{ \rho_{\mathfrak{S}_1}, \tilde{n}_{\mathfrak{S}_1} \}$ and $\mathfrak{S}_2 = \{ \rho_{\mathfrak{S}_2}, \tilde{n}_{\mathfrak{S}_2} \} \in IFN(\tilde{U})$ with $\Phi > 0$. Then, the operational rules are as follows:

- (1) $\mathfrak{S}_1 \oplus \mathfrak{S}_2 = \{ \rho_{\mathfrak{S}_1} + \rho_{\mathfrak{S}_2} - \rho_{\mathfrak{S}_1} \rho_{\mathfrak{S}_2}, \tilde{n}_{\mathfrak{S}_1} \tilde{n}_{\mathfrak{S}_2} \};$
- (2) $\mathfrak{S}_1 \otimes \mathfrak{S}_2 = \{ \rho_{\mathfrak{S}_1} \rho_{\mathfrak{S}_2}, \tilde{n}_{\mathfrak{S}_1} + \tilde{n}_{\mathfrak{S}_2} - \tilde{n}_{\mathfrak{S}_1} \tilde{n}_{\mathfrak{S}_2} \};$
- (3) $\mathfrak{S}_1^{\beta} = \{ (\rho_{\mathfrak{S}_1})^{\beta}, 1 - (1 - \tilde{n}_{\mathfrak{S}_1})^{\beta} \};$
- (4) $\beta \cdot \mathfrak{S}_1 = \{ 1 - (1 - \rho_{\mathfrak{S}_1})^{\beta}, (\tilde{n}_{\mathfrak{S}_1})^{\beta} \};$

Definition 3. [22] Assume that $\tilde{U}$ be a fixed set. An picture FS $\mathfrak{S}$ in the universe $\tilde{U}$ is a set having the form;

$$\mathfrak{S} = \{ \langle \tilde{h}, \rho_{\mathfrak{S}}(\tilde{h}), \neg_{\mathfrak{S}}(\tilde{h}), \tilde{n}_{\mathfrak{S}}(\tilde{h}) \rangle \mid \tilde{h} \in \tilde{U} \},$$

where the values $\rho_{\mathfrak{S}}(\tilde{h}) \in [0, 1]$, $\neg_{\mathfrak{S}}(\tilde{h}) \in [0, 1]$ and $\tilde{n}_{\mathfrak{S}}(\tilde{h}) \in [0, 1]$ are called positive, neutral and negative membership grades of $\tilde{h}$ and $0 \leq \rho_{\mathfrak{S}}(\tilde{h}) + \neg_{\mathfrak{S}}(\tilde{h}) + \tilde{n}_{\mathfrak{S}}(\tilde{h}) \leq 1$, $\forall \tilde{h} \in \tilde{U}$. for each $\tilde{h} \in \tilde{U}$. Shortly the picture fuzzy set is represented by $\mathfrak{S} = (\rho_{\mathfrak{S}}, \neg_{\mathfrak{S}}, \tilde{n}_{\mathfrak{S}})$.

Let $\mathfrak{S}_1 = \langle \rho_{\mathfrak{S}_1}, \neg_{\mathfrak{S}_1}, \tilde{n}_{\mathfrak{S}_1} \rangle$ and $\mathfrak{S}_2 = \langle \rho_{\mathfrak{S}_2}, \neg_{\mathfrak{S}_2}, \tilde{n}_{\mathfrak{S}_2} \rangle \in PFN(\check{U})$ with $\beta > 0$. Then, the operational rules are as follows:

- (1) $\mathfrak{S}_1 \oplus \mathfrak{S}_2 = \{ \rho_{\mathfrak{S}_1} + \rho_{\mathfrak{S}_2} - \rho_{\mathfrak{S}_1} \cdot \rho_{\mathfrak{S}_2}, \neg_{\mathfrak{S}_1} \cdot \neg_{\mathfrak{S}_2}, \tilde{n}_{\mathfrak{S}_1} \cdot \tilde{n}_{\mathfrak{S}_2} \};$
- (2) $\mathfrak{S}_1 \otimes \mathfrak{S}_2 = \{ \rho_{\mathfrak{S}_1} \cdot \rho_{\mathfrak{S}_2}, \neg_{\mathfrak{S}_1} \cdot \neg_{\mathfrak{S}_2}, \tilde{n}_{\mathfrak{S}_1} + \tilde{n}_{\mathfrak{S}_2} - \tilde{n}_{\mathfrak{S}_1} \cdot \tilde{n}_{\mathfrak{S}_2} \};$
- (3) $\mathfrak{S}_1^\beta = \{ (\rho_{\mathfrak{S}_1})^\beta, (\neg_{\mathfrak{S}_1})^\beta, 1 - (1 - \tilde{n}_{\mathfrak{S}_1})^\beta \};$
- (4) $\beta \cdot \mathfrak{S}_1 = \{ 1 - (1 - \rho_{\mathfrak{S}_1})^\beta, (\neg_{\mathfrak{S}_1})^\beta, (\tilde{n}_{\mathfrak{S}_1})^\beta \};$

Definition 4. [12] Assume that $\check{U}$ be a fixed set, and $\mathcal{L} \in \check{U} \times \check{U}$ be a crisp relation. Then

- (1) if $(b, b) \in \mathcal{L}, \forall b \in \mathcal{L}$, then $\mathcal{L}$ is reflexive.
- (2) if $b, h \in \check{U}, (b, h) \in \mathcal{L}$ then $(h, b) \in \mathcal{L}$; then $\mathcal{L}$ is symmetric.
- (3) if $b, h, d \in \check{U}, (b, h) \in \mathcal{L}$ and $(h, d) \in \mathcal{L}$ then $(b, d) \in \mathcal{L}$. then $\mathcal{L}$ is transitive.

Definition 5. [12] Assume that $\check{U}$ be a fixed set and $\mathcal{L} \in \check{U} \times \check{U}$ be an arbitrary relation on set $\check{U}$. Then defined a set valued mapping $\mathcal{L}^*: \check{U} \rightarrow P(\check{U})$ as:

$$\mathcal{L}^*(b) = \{ h \in \check{U} : (b, h) \in \mathcal{L} \}, \text{ for } b \in \check{U}$$

where successor neighborhood is denoted by $\mathcal{L}^*(h)$ of an object p^* w.r.t. $\mathcal{L}$. Then the pair $(\check{U}, \mathcal{L})$ as known as crisp approximation space. The lower and upper approximation for any $\wp \subseteq \check{U}$ w.r.t. approximation space denoted by $(\check{U}, \mathcal{L})$, are defined as follows:

$$\underline{\mathcal{L}}(\wp) = \{ b \in \check{U} : \mathcal{L}^*(b) \subseteq \wp \},$$

$$\overline{\mathcal{L}}(\wp) = \{ b \in \check{U} : \mathcal{L}^*(b) \cap \wp \neq \emptyset \}.$$

Where the rough set is the combination of lower and upper approximation which are represented by $(\underline{\mathcal{L}}(\wp), \overline{\mathcal{L}}(\wp))$.

Definition 6. Assume that $\check{U}$ be a fixed set and $\mathcal{L}$ is any subset of a fixed set $\check{U}$. Then, $\mathcal{L} \in PFRS(\check{U} \times \check{U})$ is said to be an picture fuzzy PF relation. The pair $(\check{U}, \mathcal{L})$ is said to an picture fuzzy (PF) approximation space (PFAS). The upper and lower approximations of $\wp$ in term of PFAS $(\check{U}, \mathcal{L})$, for any $\wp \subseteq PFS(\check{U})$ are two picture fuzzy sets $\overline{\mathcal{L}}(\wp)$ and $\underline{\mathcal{L}}(\wp)$, which are defined as follows:

$$\underline{\mathcal{L}}(\wp) = \left\{ \left\langle b, \bar{u}_{\underline{\mathcal{L}}(\wp)}(b), \eta_{\underline{\mathcal{L}}(\wp)}(b), v_{\underline{\mathcal{L}}(\wp)}(b) \right\rangle \mid b \in \check{U} \right\}$$

$$\overline{\mathcal{L}}(\wp) = \left\{ \left\langle b, \bar{u}_{\overline{\mathcal{L}}(\wp)}(b), \eta_{\overline{\mathcal{L}}(\wp)}(b), v_{\overline{\mathcal{L}}(\wp)}(b) \right\rangle \mid b \in \check{U} \right\}$$

where

$$\bar{u}_{\underline{\mathcal{L}}(\wp)}(b) = \bigwedge_{h \in \check{U}} \{ \bar{u}_{\mathcal{L}}(b, h) \wedge \bar{u}_{(\wp)}(h) \}$$

$$\eta_{\underline{\mathcal{L}}(\wp)}(b) = \bigvee_{h \in \check{U}} \{ \eta_{\mathcal{L}}(b, h) \vee \eta_{(\wp)}(h) \}$$

and

$$v_{\underline{\mathcal{L}}(\wp)}(b) = \bigvee_{h \in \check{U}} \{ v_{\mathcal{L}}(b, h) \vee v_{(\wp)}(h) \}$$

$$\bar{u}_{\overline{\mathcal{L}}(\wp)}(b) = \bigvee_{h \in \check{U}} \{ \bar{u}_{\mathcal{L}}(b, h) \vee \bar{u}_{(\wp)}(h) \}$$

$$\eta_{\overline{\mathcal{L}}(\wp)}(b) = \bigwedge_{h \in \check{U}} \{ \eta_{\mathcal{L}}(b, h) \wedge \eta_{(\wp)}(h) \}$$

and

$$v_{\bar{\mathcal{L}}(\wp)}(b) = \bigwedge_{\hbar \in \check{U}} \{v_{\mathcal{L}}(b, \hbar) \wedge v_{(\wp)}(\hbar)\}$$

with condition $0 \leq \bar{u}_{\mathcal{L}(\wp)}(b) + \eta_{\mathcal{L}(\wp)}(b) + v_{\mathcal{L}(\wp)}(b) \leq 1$ also $0 \leq \bar{u}_{\bar{\mathcal{L}}(\wp)}(b) + \eta_{\bar{\mathcal{L}}(\wp)}(b) + v_{\bar{\mathcal{L}}(\wp)}(b) \leq 1$ Where $\bar{\mathcal{L}}(\wp)$ and $\mathcal{L}(\wp)$ is PFSs, so $\bar{\mathcal{L}}(\wp), \mathcal{L}(\wp) : PFS(\check{U}) \rightarrow PFS(\check{U})$ is upper and lower approximation operators. Then the pair

$$\begin{aligned} \mathcal{L}(\wp) &= (\underline{\mathcal{L}}(\wp), \bar{\mathcal{L}}(\wp)) \\ &= \left\{ \left\langle b, \begin{pmatrix} \bar{u}_{\mathcal{L}(\wp)}(b), \eta_{\mathcal{L}(\wp)}(b), v_{\mathcal{L}(\wp)}(b) \\ \bar{u}_{\bar{\mathcal{L}}(\wp)}(b), \eta_{\bar{\mathcal{L}}(\wp)}(b), v_{\bar{\mathcal{L}}(\wp)}(b) \end{pmatrix} : b \in \check{U} \right\} \right\} \end{aligned}$$

are said to be PF rough set. Shortly it is represented by $\mathcal{L}(\wp) = ((\underline{\rho}, \underline{\mathfrak{T}}, \underline{\tilde{n}}), (\bar{\rho}, \bar{\mathfrak{T}}, \bar{\tilde{n}}))$. The degree of hesitancy of PFRS is determined by $v_{\mathcal{L}(\wp)} = 1 - (\underline{\mathcal{L}}(\wp) + \bar{\mathcal{L}}(\wp))$ such that $v_{\mathcal{L}(\wp)} = 1 - (\underline{\rho} + \underline{\mathfrak{T}} + \underline{\tilde{n}})$ and $v_{\bar{\mathcal{L}}(\wp)} = 1 - (\bar{\rho} + \bar{\mathfrak{T}} + \bar{\tilde{n}})$.

Definition 7. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \bar{\mathcal{L}}(\wp_g)) = ((\underline{\rho}_g, \underline{\mathfrak{T}}_g, \underline{\tilde{n}}_g), (\bar{\rho}_g, \bar{\mathfrak{T}}_g, \bar{\tilde{n}}_g)) \in PFRS(\check{U})$ ($g \in \mathbb{N}$). The score ($\check{s}$) and accuracy (Ac) functions are defined as follows;

- (1) $\check{s}(\mathcal{L}(\wp_g)) = \frac{1}{6}(3 + \underline{\rho}_g + \bar{\rho}_g + \underline{\mathfrak{T}}_g - \bar{\mathfrak{T}}_g - \underline{\tilde{n}}_g - \bar{\tilde{n}}_g)$;
- (2) $Ac(\mathcal{L}(\wp_g)) = \frac{1}{6}(3 + \underline{\rho}_g + \bar{\rho}_g + \underline{\mathfrak{T}}_g + \bar{\mathfrak{T}}_g + \underline{\tilde{n}}_g + \bar{\tilde{n}}_g)$

Definition 8. Suppose $(\check{U}, \mathcal{L})$ be PF approximation space. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \bar{\mathcal{L}}(\wp_g)) \in PFRS(\check{U})$ ($g \in \mathbb{N}$). Then, weighted averaging aggregation operator can be defined as in the following;

$$\begin{aligned} PFRWA(\mathcal{L}(\wp_1), \mathcal{L}(\wp_1), \dots, \mathcal{L}(\wp_n)) &= \left(\sum_{g=1}^n \beta_g \underline{\mathcal{L}}(\wp_g), \sum_{g=1}^n \beta_g \bar{\mathcal{L}}(\wp_g) \right) \\ &= \left\{ \begin{pmatrix} 1 - \prod_{g=1}^n (1 - \rho_g)^{\beta_g}, \\ \prod_{g=1}^n (\mathfrak{T}_g)^{\beta_g}, \prod_{g=1}^n (\tilde{n}_g)^{\beta_g} \\ 1 - \prod_{g=1}^n (1 - \bar{\rho}_g)^{\beta_g}, \\ \prod_{g=1}^n (\bar{\mathfrak{T}}_g)^{\beta_g}, \prod_{g=1}^n (\bar{\tilde{n}}_g)^{\beta_g} \end{pmatrix} \right\} \end{aligned}$$

where β_g ($g = 1, 2, \dots, n$) is weight information of $(\mathcal{L}(\wp_1), \mathcal{L}(\wp_2), \dots, \mathcal{L}(\wp_n))$ i.e. $\beta_g \geq 0$; $\sum_{g=1}^n \beta_g = 1$.

3. DEVELOPMENT OF THE PFR ENTROPY MEASURE METHODOLOGY

In order to calculate the differences between the two PFRVs, this segment developed generalized and weighted generalized distance measures incorporating PFR information based on the distance model [46, 47]. In order to measure the fuzziness of PFRVs, we propose entropy measures for PFRS based on the developed distance operators.

3.1. PFR Measures.

Definition 9. Suppose $(\check{U}, \mathcal{L})$ be PF approximation space. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \bar{\mathcal{L}}(\wp_g))$, $K(\wp_g) = (\underline{K}(\wp_g), \bar{K}(\wp_g)) \in PFRS(\check{U})$ ($g \in \mathbb{N}$). Then, generalized distance measure (GDM) is described for any $\beta > 0$ ($\in \mathbb{R}$) as

$$d_G(\mathcal{L}, K) = \left(\frac{\frac{1}{2n} \sum_{g=1}^n \left(\left| \underline{\rho}_{\mathcal{L}_g} - \underline{\rho}_{K_g} \right|^\beta + \left| \underline{\imath}_{\mathcal{L}_g} - \underline{\imath}_{K_g} \right|^\beta + \left| \underline{\tilde{n}}_{\mathcal{L}_g} - \underline{\tilde{n}}_{K_g} \right|^\beta \right) + \frac{\frac{1}{2n} \sum_{g=1}^n \left(\left| \overline{\rho}_{\mathcal{L}_g} - \overline{\rho}_{K_g} \right|^\beta + \left| \overline{\imath}_{\mathcal{L}_g} - \overline{\imath}_{K_g} \right|^\beta + \left| \overline{\tilde{n}}_{\mathcal{L}_g} - \overline{\tilde{n}}_{K_g} \right|^\beta \right)}{\frac{1}{\beta}} \right)^{\frac{1}{\beta}}$$

Definition 10. Suppose $(\check{U}, \mathcal{L})$ be PF approximation space. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \overline{\mathcal{L}}(\wp_g))$, $K(\wp_g) = (\underline{K}(\wp_g), \overline{K}(\wp_g)) \in PFRS(\check{U})$ ($g \in \mathbb{N}$). Then, generalized distance measure (GDM) is described for any $\beta > 0 (\in \mathbb{R})$ as

$$d_{WG}(\mathcal{L}, K) = \left(\frac{\frac{1}{2n} \sum_{g=1}^n \beta_g \left(\left| \left(\underline{\rho}_{\mathcal{L}_g} \right) - \left(\underline{\rho}_{K_g} \right) \right|^\beta + \left| \left(\underline{\imath}_{\mathcal{L}_g} \right) - \left(\underline{\imath}_{K_g} \right) \right|^\beta + \left| \left(\underline{\tilde{n}}_{\mathcal{L}_g} \right) - \left(\underline{\tilde{n}}_{K_g} \right) \right|^\beta \right) + \frac{\frac{1}{2n} \sum_{g=1}^n \beta_g \left(\left| \left(\overline{\rho}_{\mathcal{L}_g} \right) - \left(\overline{\rho}_{K_g} \right) \right|^\beta + \left| \left(\overline{\imath}_{\mathcal{L}_g} \right) - \left(\overline{\imath}_{K_g} \right) \right|^\beta + \left| \left(\overline{\tilde{n}}_{\mathcal{L}_g} \right) - \left(\overline{\tilde{n}}_{K_g} \right) \right|^\beta \right)}{\frac{1}{\beta}} \right)^{\frac{1}{\beta}}$$

in which β_g ($g = 1, 2, \dots, n$) the weights satisfying condition $\beta_g \geq 0$ and $\sum_{g=1}^n \beta_g = 1$.

Remark 1. (1) The above mentioned distance measure are said to be Hamming distance, if $\beta = 1$.

(2) The above mentioned distance measure are said to be Euclidean distance, if $\beta = 2$.

Definition 11. Suppose $(\check{U}, \mathcal{L})$ be PF approximation space. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \overline{\mathcal{L}}(\wp_g)) \in PFRS(\check{U})$ ($g = \{1, 2\} \in \mathbb{N}$). Then,

the GDM defined in 9 reduced as follows, $\beta > 0 (\in \mathbb{R})$.

$$d_G(\mathcal{L}(\wp_1), \mathcal{L}(\wp_2)) = \left(\frac{\frac{1}{2} \left(\left| \left(\underline{\rho}_{\wp_1} \right) - \left(\underline{\rho}_{\wp_2} \right) \right|^\beta + \left| \left(\underline{\imath}_{\wp_1} \right) - \left(\underline{\imath}_{\wp_2} \right) \right|^\beta + \left| \left(\underline{\tilde{n}}_{\wp_1} \right) - \left(\underline{\tilde{n}}_{\wp_2} \right) \right|^\beta \right) + \frac{\frac{1}{2} \left(\left| \left(\overline{\rho}_{\wp_1} \right) - \left(\overline{\rho}_{\wp_2} \right) \right|^\beta + \left| \left(\overline{\imath}_{\wp_1} \right) - \left(\overline{\imath}_{\wp_2} \right) \right|^\beta + \left| \left(\overline{\tilde{n}}_{\wp_1} \right) - \left(\overline{\tilde{n}}_{\wp_2} \right) \right|^\beta \right)}{\frac{1}{\beta}} \right)^{\frac{1}{\beta}}$$

For $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \overline{\mathcal{L}}(\wp_g)) \in PFRS(\check{U})$ ($g = \{1, 2\} \in \mathbb{N}$), the GDM satisfied the following properties

- (1) $0 \leq d^g(\mathcal{L}(\wp_1), \mathcal{L}(\wp_2)) \leq 1$,
- (2) $d^g(\mathcal{L}(\wp_1), \mathcal{L}(\wp_2)) = 1$, iff $\mathcal{L}(\wp_1) = \mathcal{L}(\wp_2)$,
- (3) $d(\mathcal{L}(\wp_1), \mathcal{L}(\wp_2)) = d(\mathcal{L}(\wp_2), \mathcal{L}(\wp_1))$.

3.2. PFR Entropy Measure. In this part we invente a novel entropy measure for PFRSs, by using the notion of Guo and Song [46].

Definition 12. Suppose $(\check{U}, \mathcal{L})$ be PF approximation space. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \overline{\mathcal{L}}(\wp_g)) \in PFRS(\check{U})$ ($g \in \mathbb{N}$). Then, PFR entropy measure is described as

$$(3.1) \quad E(\mathcal{L}(\wp_g)) = \frac{1}{n} \sum_{g=1}^n \left[\{1 - d((\mathcal{L}(\wp_g), (\wp_g)^c))\} \frac{1 + (v_{\mathcal{L}(\wp_g)})}{2} \right].$$

where $v_{\mathcal{L}(\wp_g)}$ represented the indeterminacy of $\mathcal{L}(\wp_g)$

For $(\tilde{U}, \mathcal{L})$ be PF approximation space. Suppose $\mathcal{L}(\wp_g) = (\underline{\mathcal{L}}(\wp_g), \overline{\mathcal{L}}(\wp_g))$, $K(\wp_g) = (\underline{K}(\wp_g), \overline{K}(\wp_g)) \in PFRS(\tilde{U})$ ($g \in \mathbb{N}$). Then, PFR entropy measure holds the following propoerties:

- (1) $E(\mathcal{L}(\wp_g)) = 0$ iff $\mathcal{L}(\wp_g)$ is the crisp set,
- (2) $E(\mathcal{L}(\wp_g)) \leq E(K(\tilde{U}_g))$ if $\mathcal{L}(\wp_g) \leq K(\wp_g)$, that is $\underline{\mathcal{L}}(\wp_g) \leq \underline{K}(\wp_g)$ and $\overline{\mathcal{L}}(\wp_g) \leq \overline{K}(\wp_g)$,
- (3) $E(\mathcal{L}(\wp_g)) \leq E(\mathcal{L}(\wp_g)^c)$.

4. PFR IMPROVED TOPSIS

4.1. Picture fuzzy Rough MAGDM problem. We've created a methodology for dealing with ambiguity in decision-making (DM) when facing with picture fuzzy rough input. Let us a decision matrix represented by $D_{n \times m}$ consist n number of alternatives denoted by $\{S_1, S_2, S_3, \dots, S_n\}$ with m number of criteria/attributes $\{f_1, f_2, f_3, \dots, f_m\}$. And $W = \{\rho_1, \rho_2, \rho_3, \dots, \rho_m\}$ be the unknown weights of criteria satisfying the condition $\sum_{g=1}^m \rho_g = 1$ and $\rho_g \in [0, 1]$. The expert evaluation matrix is described as:

$$M = [\mathcal{L}(\wp_{ij}^{(k)})]_{\tilde{n} \times m}$$

$$= \begin{bmatrix} (\underline{\mathcal{L}}(\wp_{11}), \overline{\mathcal{L}}(\wp_{11})) & (\underline{\mathcal{L}}(\wp_{12}), \overline{\mathcal{L}}(\wp_{12})) & \cdots & (\underline{\mathcal{L}}(\wp_{1m}), \overline{\mathcal{L}}(\wp_{1m})) \\ (\underline{\mathcal{L}}(\wp_{21}), \overline{\mathcal{L}}(\wp_{21})) & (\underline{\mathcal{L}}(\wp_{22}), \overline{\mathcal{L}}(\wp_{22})) & \cdots & (\underline{\mathcal{L}}(\wp_{2m}), \overline{\mathcal{L}}(\wp_{2m})) \\ (\underline{\mathcal{L}}(\wp_{31}), \overline{\mathcal{L}}(\wp_{31})) & (\underline{\mathcal{L}}(\wp_{32}), \overline{\mathcal{L}}(\wp_{32})) & \cdots & (\underline{\mathcal{L}}(\wp_{3m}), \overline{\mathcal{L}}(\wp_{3m})) \\ \vdots & \vdots & \ddots & \vdots \\ (\underline{\mathcal{L}}(\wp_{n1}), \overline{\mathcal{L}}(\wp_{n1})) & (\underline{\mathcal{L}}(\wp_{n2}), \overline{\mathcal{L}}(\wp_{n2})) & \cdots & (\underline{\mathcal{L}}(\wp_{nm}), \overline{\mathcal{L}}(\wp_{nm})) \end{bmatrix}$$

where $\underline{\mathcal{L}}(\wp_{ij}) = \left\{ \left\langle b, \bar{u}_{\underline{\mathcal{L}}(\wp_{ij})}(b), \eta_{\underline{\mathcal{L}}(\wp_{ij})}(b), v_{\underline{\mathcal{L}}(\wp_{ij})}(b) \right\rangle \mid b \in \tilde{U} \right\}$ and

$\overline{\mathcal{L}}(\wp_{ij}) = \left\{ \left\langle b, \bar{u}_{\overline{\mathcal{L}}(\wp_{ij})}(b), \eta_{\overline{\mathcal{L}}(\wp_{ij})}(b), v_{\overline{\mathcal{L}}(\wp_{ij})}(b) \right\rangle \mid b \in \tilde{U} \right\}$ such that $0 \leq \bar{u}_{\underline{\mathcal{L}}(\wp_{ij})}(b) + \eta_{\underline{\mathcal{L}}(\wp_{ij})}(b) + v_{\underline{\mathcal{L}}(\wp_{ij})}(b) \leq 1$ and $0 \leq \bar{u}_{\overline{\mathcal{L}}(\wp_{ij})}(b) + \eta_{\overline{\mathcal{L}}(\wp_{ij})}(b) + v_{\overline{\mathcal{L}}(\wp_{ij})}(b) \leq 1$ are the PF rough values.

NOTE: All data about the weights of DMs and criteria is compeletly hidden in the decision-making setting.

4.2. PF TOPSIS method. The method is divided into five parts. The procedure contains five crucial parts. In the first section, a TOPSIS-based method for computing DM weights is proposed. In the first part, TOPSIS-based technique for compute the weights of DMs is proposed. The proposed entropy calculation is used to determine the weights of the criteria in the second section. The second part is about calculate the weights of the criteria using the proposed entropy measure. With PIM and NIM, the final aspect is a rating system based on degree of similarity to the ideal matrix. The last section is ranking approach using PIM and NIM tha is founded on the degree of similarity to the ideal matrix.

The following steps are implemented to solve the PFRS MAGDM problem using a TOPSIS-based procedure:

Step-1(a): Construct the experts evaluation matrices $(E)^k$.

$$\begin{bmatrix} (\underline{\mathcal{L}}(\wp_{11}^k), \overline{\mathcal{L}}(\wp_{11}^k)) & (\underline{\mathcal{L}}(\wp_{12}^k), \overline{\mathcal{L}}(\wp_{12}^k)) & \cdots & (\underline{\mathcal{L}}(\wp_{1m}^k), \overline{\mathcal{L}}(\wp_{1m}^k)) \\ (\underline{\mathcal{L}}(\wp_{21}^k), \overline{\mathcal{L}}(\wp_{21}^k)) & (\underline{\mathcal{L}}(\wp_{22}^k), \overline{\mathcal{L}}(\wp_{22}^k)) & \cdots & (\underline{\mathcal{L}}(\wp_{2m}^k), \overline{\mathcal{L}}(\wp_{2m}^k)) \\ (\underline{\mathcal{L}}(\wp_{31}^k), \overline{\mathcal{L}}(\wp_{31}^k)) & (\underline{\mathcal{L}}(\wp_{32}^k), \overline{\mathcal{L}}(\wp_{32}^k)) & \cdots & (\underline{\mathcal{L}}(\wp_{3m}^k), \overline{\mathcal{L}}(\wp_{3m}^k)) \\ \vdots & \vdots & \ddots & \vdots \\ (\underline{\mathcal{L}}(\wp_{n1}^k), \overline{\mathcal{L}}(\wp_{n1}^k)) & (\underline{\mathcal{L}}(\wp_{n2}^k), \overline{\mathcal{L}}(\wp_{n2}^k)) & \cdots & (\underline{\mathcal{L}}(\wp_{nm}^k), \overline{\mathcal{L}}(\wp_{nm}^k)) \end{bmatrix}$$

where k represents the number of expert.

Step-1(b): Evaluate normalized experts matrics $(\check{N})^k$, that is

$$(4.1) \quad (\check{N})^k = \begin{cases} \mathcal{L}(\wp_{ij}) = \left(\left(\rho_{ij}, \underline{\mathfrak{T}}_{ij}, \tilde{n}_{ij} \right), \left(\overline{\rho}_{ij}, \overline{\mathfrak{T}}_{ij}, \tilde{n}_{ij} \right) \right) & \text{if For benefit} \\ (\mathcal{L}(\wp_{ij}))^c = \left(\left(\tilde{n}_{ij}, \underline{\mathfrak{T}}_{ij}, \rho_{ij} \right), \left(\tilde{n}_{ij}, \overline{\mathfrak{T}}_{ij}, \rho_{ij} \right) \right) & \text{if For cost} \end{cases}$$

Step-2(a): The expert ideal matrix (EIM) is calculated using PFRWA operator, which is closer to each expert information.

$$EIM = \begin{pmatrix} EI_{11} & EI_{12} & \cdots & EI_{1n} \\ EI_{21} & EI_{22} & \cdots & EI_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ EI_{m1} & EI_{m1} & \cdots & EI_{mn} \end{pmatrix}$$

where

$$\begin{aligned} EI_{ij} &= \sum_{k=1}^e \frac{1}{e} \check{N}_{ij}^{(k)} \\ &= \left\{ \begin{pmatrix} 1 - \prod_{k=1}^e \left(1 - \left(\rho_{ij}^{(k)} \right)^{\frac{1}{e}} \right), \\ \prod_{k=1}^e \left(\underline{\mathfrak{T}}_{ij}^{(k)} \right)^{\frac{1}{e}}, \prod_{k=1}^e \left(\tilde{n}_{ij}^{(k)} \right)^{\frac{1}{e}} \end{pmatrix}, \begin{pmatrix} 1 - \prod_{k=1}^e \left(1 - \left(\overline{\rho}_{ij}^{(k)} \right)^{\frac{1}{e}} \right), \\ \prod_{k=1}^e \left(\overline{\mathfrak{T}}_{ij}^{(k)} \right)^{\frac{1}{e}}, \prod_{k=1}^e \left(\tilde{n}_{ij}^{(k)} \right)^{\frac{1}{e}} \end{pmatrix} \right\}. \end{aligned}$$

Step-2(b): Compute the expert right ideal matrix (ERIM) and expert left ideal matrix (ELIM) as follows:

$$ERIM = \begin{pmatrix} RIM_{11} & RIM_{12} & \cdots & RIM_{1n} \\ RIM_{21} & RIM_{22} & \cdots & RIM_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ RIM_{m1} & RIM_{m2} & \cdots & RIM_{mn} \end{pmatrix}$$

where

$$RIM_{ij} = \left\{ \left(\check{N}_{ij}^{(k)} \right) : \max_k \left[\check{c} \left(\check{N}_{ij}^{(k)} \right) \right] \right\},$$

and

$$ELIM = \begin{pmatrix} LIM_{11} & LIM_{12} & \cdots & LIM_{1h} \\ LIM_{21} & LIM_{22} & \cdots & LIM_{2h} \\ \vdots & \vdots & \ddots & \vdots \\ LIM_{m1} & LIM_{m1} & \cdots & LIM_{mn} \end{pmatrix}$$

where

$$LIM_{ij} = \left\{ \left(\check{N}_{ij}^{(k)} \right) : \min_k \left[\check{s}\check{c} \left(\check{N}_{ij}^{(k)} \right) \right] \right\}$$

Step-2(c): Using Definition 9 evaluate the distance of $N_{ij}^{(k)}$ to EIM , $ERIM$ and $ELIM$ as follows $DEIM$, $DERIM$ and $DELIM$ respectively.

$$DEIM_i^{(k)} = \left(\begin{aligned} & \frac{1}{2n} \sum_{j=1}^n \left(\left| \left(\rho_{\check{N}_{ij}^{(k)}} \right) - \left(\rho_{EI_{ij}} \right) \right|^\beta + \left| \left(\overline{\rho_{\check{N}_{ij}^{(k)}}} \right) - \left(\overline{\rho_{EI_{ij}}} \right) \right|^\beta \right) + \\ & \frac{1}{2n} \sum_{j=1}^n \left(\left| \left(\tilde{n}_{\check{N}_{ij}^{(k)}} \right) - \left(\tilde{n}_{EI_{ij}} \right) \right|^\beta + \left| \left(\overline{\tilde{n}_{\check{N}_{ij}^{(k)}}} \right) - \left(\overline{\tilde{n}_{EI_{ij}}} \right) \right|^\beta \right) \end{aligned} \right)^{\frac{1}{\beta}}$$

$$DERIM_i^{(k)} = \left(\begin{aligned} & \frac{1}{2n} \sum_{j=1}^n \left(\left| \left(\rho_{\check{N}_{ij}^{(k)}} \right) - \left(\rho_{RID_{ij}} \right) \right|^\beta + \left| \left(\overline{\rho_{\check{N}_{ij}^{(k)}}} \right) - \left(\overline{\rho_{RID_{ij}}} \right) \right|^\beta \right) + \\ & \frac{1}{2n} \sum_{j=1}^n \left(\left| \left(\tilde{n}_{\check{N}_{ij}^{(k)}} \right) - \left(\tilde{n}_{RID_{ij}} \right) \right|^\beta + \left| \left(\overline{\tilde{n}_{\check{N}_{ij}^{(k)}}} \right) - \left(\overline{\tilde{n}_{RID_{ij}}} \right) \right|^\beta \right) \end{aligned} \right)^{\frac{1}{\beta}}$$

$$DELIM_i^{(k)} = \left(\begin{aligned} & \frac{1}{2n} \sum_{j=1}^n \left(\left| \left(\rho_{\check{N}_{ij}^{(k)}} \right) - \left(\rho_{LID_{ij}} \right) \right|^\beta + \left| \left(\overline{\rho_{\check{N}_{ij}^{(k)}}} \right) - \left(\overline{\rho_{LID_{ij}}} \right) \right|^\beta \right) + \\ & \frac{1}{2n} \sum_{j=1}^n \left(\left| \left(\tilde{n}_{\check{N}_{ij}^{(k)}} \right) - \left(\tilde{n}_{LID_{ij}} \right) \right|^\beta + \left| \left(\overline{\tilde{n}_{\check{N}_{ij}^{(k)}}} \right) - \left(\overline{\tilde{n}_{LID_{ij}}} \right) \right|^\beta \right) \end{aligned} \right)^{\frac{1}{\beta}}$$

for $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, e$.

Step-2(d): Evaluate closeness indices (CIs) "Yue [44]" as follows:

$$CI^{(k)} = \frac{\sum_{i=1}^m DEIM_i^{(k)} + \sum_{i=1}^m DELIM_i^{(k)}}{\sum_{i=1}^m DEIM_i^{(k)} + \sum_{i=1}^m DERIM_i^{(k)} + \sum_{i=1}^m DELIM_i^{(k)}}$$

For $k = 1, 2, \dots, e$.

Step-2(e): Expert weight information is evaluated as follows:

$$\Upsilon^{(k)} = \frac{CI^{(k)}}{\sum_{k=1}^e CI^{(k)}}$$

Step-3(a): Evalute revised expert ideal matrix (R_vEIM) based on develop entropy measure as:

$$R_vEIM_{ij} = \sum_{k=1}^{\hat{j}} \Upsilon_{ij}^{(k)} \tilde{N}^{(k)}$$

$$= \left\{ \begin{array}{l} \left(\begin{array}{l} 1 - \prod_{k=1}^e \left(1 - \left(\underline{\rho}_{ij}^{(k)} \right) \right)^{\Upsilon^{(k)}}, \\ \prod_{k=1}^e \left(\underline{\tau}_{ij}^{(k)} \right)^{\Upsilon^{(k)}}, \prod_{k=1}^e \left(\underline{\tilde{n}}_{ij}^{(k)} \right)^{\Upsilon^{(k)}} \end{array} \right), \\ \left(\begin{array}{l} 1 - \prod_{k=1}^e \left(1 - \left(\overline{\rho}_{ij}^{(k)} \right) \right)^{\Upsilon^{(k)}}, \\ \prod_{k=1}^e \left(\overline{\tau}_{ij}^{(k)} \right)^{\Upsilon^{(k)}}, \prod_{k=1}^e \left(\underline{\tilde{n}}_{ij}^{(k)} \right)^{\Upsilon^{(k)}} \end{array} \right) \end{array} \right\}$$

Step-3(b): Equation 3.1, calculates the entropy measure for each criteria, as shown below:

$$EA_j = (E_j(\underline{\mathcal{L}}), E_j(\overline{\mathcal{L}})) = E(R_vEIM_{1j}, R_vEIM_{2j}, \dots, R_vEIM_{mj}), \quad j = 1, 2, \dots, n.$$

Step-3(c): The following is how attribute weights are determinted.

$$\beta \tilde{A}_j = \frac{1 - \left(\frac{E_j(\underline{\mathcal{L}}) + E_j(\overline{\mathcal{L}})}{2} \right)}{n - \sum_{j=1}^n \left(\frac{E_j(\underline{\mathcal{L}}) + E_j(\overline{\mathcal{L}})}{2} \right)}, \quad j = 1, 2, \dots, n.$$

Step-4(a): The following is how the weighted normalised experts matices are calculated using the attribute weight vector:

$$EM(\tilde{N})_{ij}^{(I)} = \sum_{k=1}^e \beta \tilde{A}_j \tilde{N}_{ij}^{(I)}$$

$$= \left(\begin{array}{l} \left(1 - \left(1 - \left(\underline{\rho}_{ij}^{(k)} \right) \right)^{\beta \tilde{A}_j} \right), \left(\underline{\tau}_{ij}^{(k)} \right)^{\beta \tilde{A}_j}, \left(\underline{\tilde{n}}_{ij}^{(k)} \right)^{\beta \tilde{A}_j} \\ \left(1 - \left(1 - \left(\overline{\rho}_{ij}^{(k)} \right) \right)^{\beta \tilde{A}_j} \right), \left(\overline{\tau}_{ij}^{(k)} \right)^{\beta \tilde{A}_j}, \left(\underline{\tilde{n}}_{ij}^{(k)} \right)^{\beta \tilde{A}_j} \end{array} \right)$$

for each $k = 1, 2, \dots, e$.

Step-4(b): $PIM^{(k)}$ and $NIM^{(k)}$ for each EMs can be calculated by using weighted normalized experts matices $EM(\tilde{N})_{ij}^{(k)}$, which are defined as follows:

$$PIM^{(k)} = \left\{ \left(EM(\tilde{N})_{ij}^{(k)} \right) : \max_i \left[\check{s}\check{c} \left(EM(\tilde{N})_{ij}^{(k)} \right) \right] \right\}, \quad j = 1, 2, \dots, n$$

and

$$NIM^{(k)} = \left\{ \left(EM(\tilde{N})_{ij}^{(k)} \right) : \min_i \left[\check{s}\check{c} \left(EM(\tilde{N})_{ij}^{(k)} \right) \right] \right\}, \quad j = 1, 2, \dots, n$$

Step-4(c): $\hat{W}\check{G}DM$ are evaluated by using the definition 10 from $EM(\check{N})^{(k)}$ to $PIM^{(k)}$ and $PIM^{(k)}$ which is represented and defined as follows:

$$DIM_i^{+(k)} = \left(\begin{array}{c} \frac{1}{2n} \sum_{g=1}^n \Upsilon C_j \left(\begin{array}{c} \left| \left(\rho_{EM(\check{N})^{(k)}} \right) - \left(\rho_{PIM^{(k)}} \right) \right|^\beta + \left| \left(\tau_{EM(\check{N})^{(k)}} \right) - \left(\tau_{PIM^{(k)}} \right) \right|^\beta \\ + \left| \left(\tilde{n}_{EM(\check{N})^{(k)}} \right) - \left(\tilde{n}_{PIM^{(k)}} \right) \right|^\beta \end{array} \right) + \\ \frac{1}{2n} \sum_{g=1}^n \Upsilon C_j \left(\begin{array}{c} \left| \left(\overline{\rho}_{EM(\check{N})^{(k)}} \right) - \left(\overline{\rho}_{PIM^{(k)}} \right) \right|^\beta + \left| \left(\overline{\tau}_{EM(\check{N})^{(k)}} \right) - \left(\overline{\tau}_{PIM^{(k)}} \right) \right|^\beta \\ + \left| \left(\overline{\tilde{n}}_{EM(\check{N})^{(k)}} \right) - \left(\overline{\tilde{n}}_{PIM^{(k)}} \right) \right|^\beta \end{array} \right) \end{array} \right)^{\frac{1}{\beta}}$$

and

$$DIM_i^{- (k)} = \left(\begin{array}{c} \frac{1}{2n} \sum_{g=1}^n \Upsilon C_j \left(\begin{array}{c} \left| \left(\rho_{EM(\check{N})^{(k)}} \right) - \left(\rho_{NIM^{(k)}} \right) \right|^\beta + \left| \left(\tau_{EM(\check{N})^{(k)}} \right) - \left(\tau_{NIM^{(k)}} \right) \right|^\beta \\ + \left| \left(\tilde{n}_{EM(\check{N})^{(k)}} \right) - \left(\tilde{n}_{NIM^{(k)}} \right) \right|^\beta \end{array} \right) + \\ \frac{1}{2n} \sum_{g=1}^n \Upsilon C_j \left(\begin{array}{c} \left| \left(\overline{\rho}_{EM(\check{N})^{(k)}} \right) - \left(\overline{\rho}_{NIM^{(k)}} \right) \right|^\beta + \left| \left(\overline{\tau}_{EM(\check{N})^{(k)}} \right) - \left(\overline{\tau}_{NIM^{(k)}} \right) \right|^\beta \\ + \left| \left(\overline{\tilde{n}}_{EM(\check{N})^{(k)}} \right) - \left(\overline{\tilde{n}}_{NIM^{(k)}} \right) \right|^\beta \end{array} \right) \end{array} \right)^{\frac{1}{\beta}}$$

or each $i = 1, 2, \dots, m$.

Step-4(d): The following is how each EM's revised closeness indices (RCIs) are calculated:

$$\acute{R}\check{C}\check{I}_i^{(k)} = \frac{DIM_i^{- (k)}}{DIM_i^{+(k)} + DIM_i^{- (k)}}$$

Step-5: Through using EMs weights, compute the final revised closeness indices (FRCIs):

$$F\acute{R}\check{C}\check{I}_i = \sum_{k=1}^e \Upsilon^{(k)} \cdot \acute{R}\check{C}\check{I}_i^{(k)}$$

Rank the obtained FRCIs values in decreasing order; the alternative with the highest value is our best option.

5. THE PROPOSED IMPROVED TOPSIS METHOD'S NUMERICAL APPLICATION

The developed MAGDM method is inially demonstrated in this section with a numerical application involving robot selection. Then, in order to demonstrate the characteristics and advantages of the proposed technique, a comparison ia made between it and aother decision-making techniques that use picture fuzzy rough information.

5.1. Example. Now a days we are facing many problems like selection of a robot for a particular industrial application has

always been a crucial issue due to the market's availability of various types of industrial robots with varying capacities, functionality, facilities, and specifications. For this we will present MAGDM a practical example to find the best optimal solution for selecting the different types of industrial robots.

Let us suppose there are four differents types of industrial robots S_1, S_2, S_3 and

S_4 with different features and there are three professional experts $\check{D}_i (i = 1, 2, 3)$ having unknown weights vectors. The experts assessed these four robots concerning the five criteria $\{f_1, f_2, f_3, f_4, f_5\}$ with unknown weight vector which are given as under.

(1) Performance (Static and Dynamic)

A robot's performance characteristics are divided into two categories: static and dynamic. The values given under steady state conditions are called static characteristics, while dynamic characteristics refer to the robot's time-dependent behaviour.

(2) Instrumentation and Control Systems

These are the characteristics that are in charge of making important decisions based on input values from sensors and transducers, as well as monitoring and calculating the quantities of controllable parameter values.

(3) Operating Environment

Robotic systems must operate in difficult and unpredictable settings, so the ability to communicate with and cope with the environment, whether on land, underwater, in the air, underground, or in space, is a vital skill.

(4) Robotic Architecture

The geometry and movements needed to push around the robot's surroundings are important in the design and analysis of the robot.

(5) General and Physical

These characteristics are a mix of cost-effective and desirable technological features relevant to a robot's efficiency and quality that aren't needed to complete any task but are extremely useful during installation and/or operation.

The invited decision makers are divided into three expert panels

$$\text{Expert Information} = \{(E)^1, (E)^2, (E)^3\},$$

where each expert panel is required to provide unified evaluation results in the form of picture fuzzy rough values with unknown expert and criteria weight information.

The expert evaluation information in the form of picture fuzzy rough values is enclosed in Table-1-3:

Table-1(a): Expert Information $(E)^1$

	f_1	f_1	f_3
S_1	$\left(\begin{array}{c} (0.5, 0.2, 0.3) \\ (0.7, 0.1, 0.2) \end{array} \right)$	$\left(\begin{array}{c} (0.5, 0.1, 0.3) \\ (0.6, 0.3, 0.1) \end{array} \right)$	$\left(\begin{array}{c} (0.1, 0.1, 0.1) \\ (0.3, 0.2, 0.4) \end{array} \right)$
S_2	$\left(\begin{array}{c} (0.4, 0.1, 0.5) \\ (0.7, 0.2, 0.1) \end{array} \right)$	$\left(\begin{array}{c} (0.5, 0.2, 0.2) \\ (0.3, 0.2, 0.3) \end{array} \right)$	$\left(\begin{array}{c} (0.5, 0.2, 0.3) \\ (0.8, 0.1, 0.1) \end{array} \right)$
S_3	$\left(\begin{array}{c} (0.5, 0.3, 0.2) \\ (0.6, 0.3, 0.1) \end{array} \right)$	$\left(\begin{array}{c} (0.5, 0.4, 0.1) \\ (0.6, 0.3, 0.1) \end{array} \right)$	$\left(\begin{array}{c} (0.7, 0.2, 0.1) \\ (0.6, 0.2, 0.3) \end{array} \right)$
S_4	$\left(\begin{array}{c} (0.8, 0.1, 0.1) \\ (0.5, 0.4, 0.1) \end{array} \right)$	$\left(\begin{array}{c} (0.6, 0.3, 0.1) \\ (0.4, 0.3, 0.2) \end{array} \right)$	$\left(\begin{array}{c} (0.1, 0.1, 0.1) \\ (0.6, 0.3, 0.1) \end{array} \right)$

Table-1(b): Expert Information $(E)^1$

	f_4	f_5
S_1	$\begin{pmatrix} (0.5, 0.2, 0.1) \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2, 0.3) \\ (0.3, 0.1, 0.1) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.6, 0.3, 0.1) \\ (0.6, 0.2, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.3, 0.2, 0.4) \\ (0.5, 0.2, 0.1) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.2, 0.3, 0.4) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.3, 0.2) \\ (0.3, 0.2, 0.2) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.5, 0.1, 0.4) \\ (0.4, 0.5, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2, 0.2), \\ (0.5, 0.2, 0.1) \end{pmatrix}$

Table-2(a): Expert Information $(E)^2$

	f_1	f_1	f_3
S_1	$\begin{pmatrix} (0.7, 0.1, 0.2), \\ (0.3, 0.1, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.5, 0.2, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.4, 0.2, 0.1), \\ (0.8, 0.1, 0.1) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.5, 0.1, 0.3), \\ (0.5, 0.4, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.1, 0.3), \\ (0.4, 0.5, 0.1) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.2, 0.6, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.4, 0.3, 0.2), \\ (0.3, 0.6, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.3, 0.4, 0.2), \\ (0.7, 0.2, 0.1) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.4, 0.5, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.5, 0.3, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.2, 0.3, 0.3), \\ (0.7, 0.1, 0.1) \end{pmatrix}$

Table-2(b): Expert Information $(E)^2$

	f_4	f_5
S_1	$\begin{pmatrix} (0.3, 0.5, 0.2), \\ (0.6, 0.2, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.6, 0.2, 0.2) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.4, 0.5, 0.1), \\ (0.3, 0.5, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.1, 0.2), \\ (0.5, 0.3, 0.2) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.2, 0.6, 0.2), \\ (0.4, 0.2, 0.3) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.2, 0.3, 0.3) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.5, 0.1, 0.3) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.1, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$

Table-3(a): Expert Information $(E)^3$

	f_1	f_1	f_3
S_1	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.3, 0.2, 0.4) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.5, 0.4, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2, 0.3), \\ (0.5, 0.4, 0.1) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.4, 0.5, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.6, 0.1, 0.3), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.4, 0.3, 0.2), \\ (0.3, 0.5, 0.2) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.3, 0.4, 0.2), \\ (0.6, 0.2, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.5, 0.2, 0.3) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.2, 0.5, 0.2) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.4, 0.2, 0.3), \\ (0.6, 0.1, 0.3) \end{pmatrix}$	$\begin{pmatrix} (0.4, 0.3, 0.2), \\ (0.3, 0.4, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.3, 0.4, 0.1), \\ (0.4, 0.3, 0.1) \end{pmatrix}$

Table-3(b): Expert Information (E)³

	f_4	f_5
S_1	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.5, 0.3, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.1, 0.2), \\ (0.6, 0.2, 0.2) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.4, 0.3, 0.2), \\ (0.6, 0.2, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.5, 0.3, 0.2) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.4, 0.5, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.6, 0.2, 0.3) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.4, 0.1, 0.3), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.5, 0.3, 0.1) \end{pmatrix}$

Step-1(b): According to the experts, the all attributes f_1 , f_2 , f_3 , f_4 and f_5 are benefits type,

Step-2(a): The EIM is calculated in Table-4:

Table-4(a): Expert Ideal Matrix

	f_1	f_1	f_3
S_1	$\begin{pmatrix} (.688, .126, .182), \\ (.471, .126, .252) \end{pmatrix}$	$\begin{pmatrix} (.688, .126, .144), \\ (.535, .288, .100) \end{pmatrix}$	$\begin{pmatrix} (.353, .150, .114), \\ (.587, .200, .159) \end{pmatrix}$
S_2	$\begin{pmatrix} (.506, .144, .246), \\ (.551, .342, .100) \end{pmatrix}$	$\begin{pmatrix} (.608, .159, .182), \\ (.517, .262, .144) \end{pmatrix}$	$\begin{pmatrix} (.468, .182, .262), \\ (.561, .292, .126) \end{pmatrix}$
S_3	$\begin{pmatrix} (.480, .288, .200), \\ (.495, .330, .126) \end{pmatrix}$	$\begin{pmatrix} (.551, .288, .126), \\ (.480, .330, .114) \end{pmatrix}$	$\begin{pmatrix} (.527, .317, .126), \\ (.541, .271, .182) \end{pmatrix}$
S_4	$\begin{pmatrix} (.583, .215, .144), \\ (.568, .229, .144) \end{pmatrix}$	$\begin{pmatrix} (.541, .262, .159), \\ (.405, .330, .200) \end{pmatrix}$	$\begin{pmatrix} (.204, .229, .114), \\ (.583, .208, .100) \end{pmatrix}$

Table-4(b): Expert Ideal Matrix

	f_4	f_5
S_1	$\begin{pmatrix} (.561, .311, .126), \\ (.568, .262, .126) \end{pmatrix}$	$\begin{pmatrix} (.643, .159, .182), \\ (.517, .159, .159) \end{pmatrix}$
S_2	$\begin{pmatrix} (.475, .356, .126), \\ (.517, .271, .159) \end{pmatrix}$	$\begin{pmatrix} (.601, .159, .200), \\ (.499, .300, .159) \end{pmatrix}$
S_3	$\begin{pmatrix} (.541, .288, .159), \\ (.339, .311, .229) \end{pmatrix}$	$\begin{pmatrix} (.631, .229, .126), \\ (.345, .229, .262) \end{pmatrix}$
S_4	$\begin{pmatrix} (.506, .144, .229), \\ (.506, .246, .144) \end{pmatrix}$	$\begin{pmatrix} (.535, .252, .159), \\ (.568, .262, .100) \end{pmatrix}$

Step-2(b): The ERIM and ELIM are calculated in Table-5 & 6:

Table-5(a): Expert Right Ideal Matrix

	f_1	f_1	f_3
S_1	$\begin{pmatrix} (0.5, 0.2, 0.3), \\ (0.7, 0.1, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.5, 0.2, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.5, 0.2, 0.1) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.4, 0.5, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.5, 0.3, 0.2), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.4, 0.5, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.4, 0.3, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.4, 0.3, 0.2) \end{pmatrix}$

Table-5(b): Expert Right Ideal Matrix

	f_4	f_5
S_1	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.6, 0.2, 0.2) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.6, 0.2, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.5, 0.3, 0.2) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.7, 0.2, 0.1), \\ (0.4, 0.5, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.5, 0.2, 0.3) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.5, 0.1, 0.3) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$

Table-6(a): Expert Left Ideal Matrix

	f_1	f_1	f_3
S_1	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.3, 0.2, 0.4) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.6, 0.3, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.3, 0.2, 0.4) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.5, 0.1, 0.3), \\ (0.5, 0.4, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2, 0.2), \\ (0.3, 0.3, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.1, 0.3), \\ (0.4, 0.5, 0.1) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.2, 0.6, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.4, 0.3, 0.2), \\ (0.3, 0.6, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.4, 0.1), \\ (0.2, 0.5, 0.2) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.4, 0.2, 0.3), \\ (0.6, 0.1, 0.3) \end{pmatrix}$	$\begin{pmatrix} (0.4, 0.3, 0.2), \\ (0.3, 0.4, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.3, 0.4, 0.1), \\ (0.4, 0.3, 0.1) \end{pmatrix}$

Table-6(b): Expert Left Ideal Matrix

	f_4	f_5
S_1	$\begin{pmatrix} (0.6, 0.3, 0.1), \\ (0.5, 0.3, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2, 0.3), \\ (0.3, 0.1, 0.1) \end{pmatrix}$
S_2	$\begin{pmatrix} (0.4, 0.5, 0.1), \\ (0.3, 0.5, 0.2) \end{pmatrix}$	$\begin{pmatrix} (0.3, 0.2, 0.4), \\ (0.5, 0.3, 0.1) \end{pmatrix}$
S_3	$\begin{pmatrix} (0.6, 0.2, 0.2), \\ (0.2, 0.3, 0.4) \end{pmatrix}$	$\begin{pmatrix} (0.8, 0.1, 0.1), \\ (0.2, 0.3, 0.3) \end{pmatrix}$
S_4	$\begin{pmatrix} (0.5, 0.1, 0.4), \\ (0.4, 0.5, 0.1) \end{pmatrix}$	$\begin{pmatrix} (0.5, 0.2, 0.2), \\ (0.5, 0.2, 0.1) \end{pmatrix}$

Step-2(c): Using Definition 9, the distance of $N_{ij}^{(k)}$ to EIM , $ERIM$ and $ELIM$ as follows in Table-7 ($DEIM$), Table-8 ($DERIM$) and Table-9 ($DELIM$) respectively.

Table-7: $DEIM$

	S_1	S_2	S_3	S_4
Expert-1 (E) ¹	0.35764	0.34466	0.22843	0.25898
Expert-2 (E) ²	0.25478	0.25485	0.42702	0.28381
Expert-3 (E) ³	0.23824	0.28378	0.35128	0.25709

Table-8: $DERIM$

	S_1	S_2	S_3	S_4
Expert-1 (E) ¹	0.524404	0.45825	0.29580	0.43301
Expert-2 (E) ²	0.38078	0.42426	0.62649	0.08660
Expert-3 (E) ³	0.1825	0.1425	0.1375	0.185

Table-9: *DELIM*

	S_1	S_2	S_3	S_4
Expert-1 (E) ¹	0.41833	0.42426	0.49749	0.38405
Expert-2 (E) ²	0.47434	0.32015	0.44158	0.47958
Expert-3 (E) ³	0.32787	0.46368	0.56568	0.18027

Step-2(d): The closeness indices is calculated as follows:

$$\begin{array}{ccc} CI^{(1)} & CI^{(2)} & CI^{(3)} \\ 0.742781 & 0.726000 & 0.659046 \end{array}$$

Step-2(e): Expert weight information is calculated as follows:

$$\begin{array}{ccc} \Upsilon^{(1)} & \Upsilon^{(2)} & \Upsilon^{(3)} \\ 0.349 & 0.341 & 0.310 \end{array}$$

Step-3(a): The revised expert ideal matrix is calculated in Table-10:

Table-10(a): Revised Expert Ideal Matrix (R_vEIM)

	f_1	f_1	f_3
S_1	$\left(\begin{array}{c} (.129, .724, .786), \\ (.213, .630, .724) \end{array} \right)$	$\left(\begin{array}{c} (.130, .628, .784), \\ (.168, .784, .628) \end{array} \right)$	$\left(\begin{array}{c} (.020, .629, .629), \\ (.069, .723, .831) \end{array} \right)$
S_2	$\left(\begin{array}{c} (.097, .630, .870), \\ (.213, .724, .630) \end{array} \right)$	$\left(\begin{array}{c} (.130, .722, .722), \\ (.069, .722, .784) \end{array} \right)$	$\left(\begin{array}{c} (.130, .723, .785), \\ (.276, .629, .629) \end{array} \right)$
S_3	$\left(\begin{array}{c} (.129, .786, .724), \\ (.167, .786, .630) \end{array} \right)$	$\left(\begin{array}{c} (.130, .831, .628), \\ (.168, .784, .628) \end{array} \right)$	$\left(\begin{array}{c} (.214, .723, .629), \\ (.168, .723, .785) \end{array} \right)$
S_4	$\left(\begin{array}{c} (.275, .630, .630), \\ (.129, .832, .630) \end{array} \right)$	$\left(\begin{array}{c} (.168, .784, .628), \\ (.098, .784, .772) \end{array} \right)$	$\left(\begin{array}{c} (.020, .629, .629), \\ (.168, .785, .629) \end{array} \right)$

Table-10(b): Revised Expert Ideal Matrix (R_vEIM)

	f_4	f_5
S_1	$\left(\begin{array}{c} (.229, .0705, .606), \\ (.180, .0770, .606) \end{array} \right)$	$\left(\begin{array}{c} (.117, .748, .805), \\ (.062, .660, .660) \end{array} \right)$
S_2	$\left(\begin{array}{c} (.180, .770, .606), \\ (.180, .705, .705) \end{array} \right)$	$\left(\begin{array}{c} (.062, .748, .847), \\ (.117, .805, .660) \end{array} \right)$
S_3	$\left(\begin{array}{c} (.180, .705, .705), \\ (.047, .770, .819) \end{array} \right)$	$\left(\begin{array}{c} (.117, .805, .748), \\ (.062, .748, .748) \end{array} \right)$
S_4	$\left(\begin{array}{c} (.139, .606, .819), \\ (.104, .860, .606) \end{array} \right)$	$\left(\begin{array}{c} (.117, .748, .748), \\ (.117, .748, .660) \end{array} \right)$

Step-3(b): Equation 3.1 is used to calculate the entropy measure to every attribute:

$\hat{E}\hat{A}_1$	$\hat{E}\hat{A}_2$	$\hat{E}\hat{A}_3$	$\hat{E}\hat{A}_4$	$\hat{E}\hat{A}_5$
0.379458	0.373381	0.371396	0.334755	0.444944

Step-3(c): The weights of the attributes are computed as follows:

$\beta\hat{A}_1$	$\beta\hat{A}_2$	$\beta\hat{A}_3$	$\beta\hat{A}_4$	$\beta\hat{A}_5$
0.2	0.202	0.201	0.217	0.18

Step-4(a): The weighted normalized experts matrices are computed in Table 11-13, as follows:

Table-11(a): Weighted Normalized Expert Matrix $(W_NEM)^1$

	f_1	f_2	f_3
S_1	$\begin{pmatrix} (.129, .724, .786), \\ (.213, .630, .724) \end{pmatrix}$	$\begin{pmatrix} (.130, .628, .784), \\ (.168, .784, .628) \end{pmatrix}$	$\begin{pmatrix} (.020, .629, .629), \\ (.069, .723, .831) \end{pmatrix}$
S_2	$\begin{pmatrix} (.097, .630, .870), \\ (.213, .724, .630) \end{pmatrix}$	$\begin{pmatrix} (.130, .722, .722), \\ (.069, .722, .784) \end{pmatrix}$	$\begin{pmatrix} (.130, .723, .785), \\ (.276, .629, .629) \end{pmatrix}$
S_3	$\begin{pmatrix} (.129, .786, .724), \\ (.167, .786, .630) \end{pmatrix}$	$\begin{pmatrix} (.130, .831, .628), \\ (.168, .784, .628) \end{pmatrix}$	$\begin{pmatrix} (.214, .723, .629), \\ (.168, .723, .785) \end{pmatrix}$
S_4	$\begin{pmatrix} (.275, .630, .630), \\ (.129, .832, .630) \end{pmatrix}$	$\begin{pmatrix} (.168, .784, .628), \\ (.098, .784, .722) \end{pmatrix}$	$\begin{pmatrix} (.020, .629, .629), \\ (.168, .785, .629) \end{pmatrix}$

Table-11(b): Weighted Normalized Expert Matrix $(W_NEM)^1$

	f_4	f_5
S_1	$\begin{pmatrix} (.229, .705, .606), \\ (.180, .770, .606) \end{pmatrix}$	$\begin{pmatrix} (.117, .748, .805), \\ (.062, .660, .660) \end{pmatrix}$
S_2	$\begin{pmatrix} (.180, .770, .606), \\ (.180, .705, .705) \end{pmatrix}$	$\begin{pmatrix} (.062, .748, .847), \\ (.117, .805, .660) \end{pmatrix}$
S_3	$\begin{pmatrix} (.180, .705, .705), \\ (.047, .770, .819) \end{pmatrix}$	$\begin{pmatrix} (.117, .805, .748), \\ (.062, .748, .748) \end{pmatrix}$
S_4	$\begin{pmatrix} (.139, .606, .819), \\ (.104, .860, .606) \end{pmatrix}$	$\begin{pmatrix} (.117, .748, .748), \\ (.117, .748, .660) \end{pmatrix}$

Table-12(a): Weighted Normalized Expert Matrix $(W_NEM)^2$

	f_1	f_2	f_3
S_1	$\begin{pmatrix} (.213, .630, .724), \\ (.068, .630, .724) \end{pmatrix}$	$\begin{pmatrix} (.277, .628, .628), \\ (.130, .722, .628) \end{pmatrix}$	$\begin{pmatrix} (.097, .723, .629), \\ (.069, .723, .831) \end{pmatrix}$
S_2	$\begin{pmatrix} (.129, .630, .786), \\ (.129, .832, .630) \end{pmatrix}$	$\begin{pmatrix} (.215, .722, .628), \\ (.168, .784, .628) \end{pmatrix}$	$\begin{pmatrix} (.130, .723, .785), \\ (.276, .629, .629) \end{pmatrix}$
S_3	$\begin{pmatrix} (.167, .724, .724), \\ (.043, .902, .630) \end{pmatrix}$	$\begin{pmatrix} (.098, .784, .722), \\ (.069, .901, .628) \end{pmatrix}$	$\begin{pmatrix} (.214, .723, .629), \\ (.168, .723, .785) \end{pmatrix}$
S_4	$\begin{pmatrix} (.097, .870, .630), \\ (.168, .786, .630) \end{pmatrix}$	$\begin{pmatrix} (.168, .722, .722), \\ (.130, .784, .722) \end{pmatrix}$	$\begin{pmatrix} (.043, .785, .785), \\ (.214, .629, .629) \end{pmatrix}$

Table-12(b): Weighted Normalized Expert Matrix $(W_NEM)^2$

	f_4	f_5
S_1	$\begin{pmatrix} (.074, .860, .705), \\ (.180, .705, .606) \end{pmatrix}$	$\begin{pmatrix} (.194, .748, .660), \\ (.152, .748, .748) \end{pmatrix}$
S_2	$\begin{pmatrix} (.104, .860, .606), \\ (.074, .860, .705) \end{pmatrix}$	$\begin{pmatrix} (.194, .660, .748), \\ (.117, .805, .748) \end{pmatrix}$
S_3	$\begin{pmatrix} (.047, .895, .705), \\ (.104, .705, .770) \end{pmatrix}$	$\begin{pmatrix} (.251, .660, .660), \\ (.039, .805, .805) \end{pmatrix}$
S_4	$\begin{pmatrix} (.180, .770, .606), \\ (.139, .606, .707) \end{pmatrix}$	$\begin{pmatrix} (.117, .847, .660), \\ (.152, .805, .660) \end{pmatrix}$

Table-13(a): Weighted Normalized Expert Matrix (W_NEM)³

	f_1	f_2	f_3
S_1	$\begin{pmatrix} (.275, .630, .630), \\ (.068, .724, .832) \end{pmatrix}$	$\begin{pmatrix} (.215, .722, .628), \\ (.130, .831, .628) \end{pmatrix}$	$\begin{pmatrix} (.130, .723, .785), \\ (.130, .831, .785) \end{pmatrix}$
S_2	$\begin{pmatrix} (.167, .786, .630), \\ (.097, .870, .630) \end{pmatrix}$	$\begin{pmatrix} (.168, .628, .784), \\ (.168, .784, .628) \end{pmatrix}$	$\begin{pmatrix} (.097, .785, .723), \\ (.069, .869, .723) \end{pmatrix}$
S_3	$\begin{pmatrix} (.068, .832, .724), \\ (.167, .724, .724) \end{pmatrix}$	$\begin{pmatrix} (.215, .722, .628), \\ (.130, .722, .784) \end{pmatrix}$	$\begin{pmatrix} (.130, .831, .629), \\ (.043, .869, .723) \end{pmatrix}$
S_4	$\begin{pmatrix} (.097, .724, .786), \\ (.167, .630, .786) \end{pmatrix}$	$\begin{pmatrix} (.098, .784, .722), \\ (.069, .831, .722) \end{pmatrix}$	$\begin{pmatrix} (.069, .831, .629), \\ (.097, .785, .629) \end{pmatrix}$

Table-13(b): Weighted Normalized Expert Matrix (W_NEM)³

	f_4	f_5
S_1	$\begin{pmatrix} (.180, .770, .606), \\ (.139, .770, .705) \end{pmatrix}$	$\begin{pmatrix} (.194, .660, .748), \\ (.152, .748, .748) \end{pmatrix}$
S_2	$\begin{pmatrix} (.104, .770, .705), \\ (.180, .705, .606) \end{pmatrix}$	$\begin{pmatrix} (.194, .748, .660), \\ (.117, .805, .748) \end{pmatrix}$
S_3	$\begin{pmatrix} (.229, .705, .606), \\ (.104, .860, .606) \end{pmatrix}$	$\begin{pmatrix} (.117, .847, .660), \\ (.117, .748, .805) \end{pmatrix}$
S_4	$\begin{pmatrix} (.104, .606, .707), \\ (.180, .770, .606) \end{pmatrix}$	$\begin{pmatrix} (.152, .748, .748), \\ (.748, .805, .660) \end{pmatrix}$

Step-4(b): The $PEM^{(k)}$ and $\check{N}EM^{(k)}$ for each EMs are computed as follows:

Table-14(a): Positive Expert Matrix for each EMs

	f_1	f_2	f_3
$PEM^{(1)}$	$\begin{pmatrix} (.129, .786, .724), \\ (.167, .786, .630) \end{pmatrix}$	$\begin{pmatrix} (.130, .831, .628), \\ (.168, .784, .628) \end{pmatrix}$	$\begin{pmatrix} (.130, .723, .875), \\ (.276, .629, .629) \end{pmatrix}$
$PEM^{(2)}$	$\begin{pmatrix} (.097, .870, .630), \\ (.167, .786, .630) \end{pmatrix}$	$\begin{pmatrix} (.215, .722, .628), \\ (.168, .784, .628) \end{pmatrix}$	$\begin{pmatrix} (.097, .723, .629), \\ (.276, .629, .629) \end{pmatrix}$
$PEM^{(3)}$	$\begin{pmatrix} (.167, .786, .630), \\ (.097, .870, .630) \end{pmatrix}$	$\begin{pmatrix} (.215, .722, .628), \\ (.130, .722, .628) \end{pmatrix}$	$\begin{pmatrix} (.069, .831, .629), \\ (.097, .785, .629) \end{pmatrix}$

Table-14(b): Positive Expert Matrix for each EMs

	f_4	f_5
$PEM^{(1)}$	$\begin{pmatrix} (.229, .705, .606), \\ (.180, .770, .606) \end{pmatrix}$	$\begin{pmatrix} (.117, .748, .748), \\ (.117, .748, .660) \end{pmatrix}$
$PEM^{(2)}$	$\begin{pmatrix} (.180, .770, .606), \\ (.139, .606, .770) \end{pmatrix}$	$\begin{pmatrix} (.117, .847, .660), \\ (.152, .805, .660) \end{pmatrix}$
$PEM^{(3)}$	$\begin{pmatrix} (.229, .705, .606), \\ (.104, .860, .606) \end{pmatrix}$	$\begin{pmatrix} (.117, .847, .660), \\ (.117, .748, .805) \end{pmatrix}$

Table-15(a): Negative Expert Matrix for each EM s

	f_1	f_2	f_3
$\tilde{N}EM^{(1)}$	$\begin{pmatrix} (.097, .630, .870), \\ (.213, .724, .630) \end{pmatrix}$	$\begin{pmatrix} (.130, .722, .722), \\ (.069, .722, .784) \end{pmatrix}$	$\begin{pmatrix} (.020, .629, .629), \\ (.069, .723, .831) \end{pmatrix}$
$\tilde{N}EM^{(2)}$	$\begin{pmatrix} (.129, .630, .786), \\ (.129, .832, .630) \end{pmatrix}$	$\begin{pmatrix} (.098, .784, .722), \\ (.069, .901, .628) \end{pmatrix}$	$\begin{pmatrix} (.130, .629, .785), \\ (.097, .869, .629) \end{pmatrix}$
$\tilde{N}EM^{(3)}$	$\begin{pmatrix} (.097, .724, .786), \\ (.167, .630, .786) \end{pmatrix}$	$\begin{pmatrix} (.098, .874, .722), \\ (.069, .831, .722) \end{pmatrix}$	$\begin{pmatrix} (.130, .831, .629), \\ (.043, .869, .723) \end{pmatrix}$

Table-15(b): Negative Expert Matrix for each EM s s

	f_4	f_5
$\tilde{N}EM^{(1)}$	$\begin{pmatrix} (.139, .676, .819), \\ (.104, .860, .606) \end{pmatrix}$	$\begin{pmatrix} (.062, .748, .847), \\ (.117, .805, .660) \end{pmatrix}$
$\tilde{N}EM^{(2)}$	$\begin{pmatrix} (.047, .895, .705), \\ (.104, .705, .770) \end{pmatrix}$	$\begin{pmatrix} (.194, .660, .748), \\ (.117, .805, .748) \end{pmatrix}$
$\tilde{N}EM^{(3)}$	$\begin{pmatrix} (.180, .770, .606), \\ (.139, .770, .705) \end{pmatrix}$	$\begin{pmatrix} (.194, .660, .748), \\ (.152, .748, .784) \end{pmatrix}$

Step-4(c): Distance are computed as follows:

Table-16: $DEIM$

	S_1	S_2	S_3	S_4
Expert-1 $\left(\check{D}\check{I}\check{S}_i^{+(1)}\right)$	0.1139	0.0861	0.0923	0.1013
Expert-2 $\left(\check{D}\check{I}\check{S}_i^{+(2)}\right)^2$	0.1180	0.1334	0.1869	0.1493
Expert-3 $\left(\check{D}\check{I}\check{S}_i^{+(3)}\right)^3$	0.1070	0.0878	0.0717	0.1186

and

Table-17: $DEIM$

	S_1	S_2	S_3	S_4
Expert-1 $\left(\check{D}\check{I}\check{S}_i^{-(1)}\right)$	0.0963	0.1099	0.1174	0.1013
Expert-2 $\left(\check{D}\check{I}\check{S}_i^{+(2)}\right)$	0.1391	0.0700	0.0874	0.1296
Expert-3 $\left(\check{D}\check{I}\check{S}_i^{-(3)}\right)$	0.0945	0.1091	0.0868	0.0766

Step-4(d): The revised closeness indices for each EM are determined as follows:

Table-18: $DEIM$

	S_1	S_2	S_3	S_4
Expert-1 $\left(\check{R}\check{C}\check{I}_i^{(1)}\right)$	0.5417	0.4395	0.4402	0.5196
Expert-2 $\left(\check{R}\check{C}\check{I}_i^{(2)}\right)$	0.4590	0.6560	0.6815	0.5353
Expert-3 $\left(\check{R}\check{C}\check{I}_i^{(3)}\right)$	0.5309	0.4460	0.4525	0.6075

Step-5: When using EMs weights, the final revised closeness indices (RCI) are calculated as follows:

Table-19:

Alternatives	S_1	S_2	S_3	S_4
$F\check{R}\check{C}\check{I}s$	0.5101	0.5253	0.5162	0.2677

Hence, $\hat{S}_2$ is our best altenative.

6. COMPARISON ANALYSIS

The benefits of the created methodology are demonstrated in this part by comparing the characteristics of the proposed upgraded TOPSIS method and the designed MAGDM approach. This comparision is made by analyzing the prpperties of several decision-making techniques found in the letrature, The EDAS approach for intuitionistic fuzzy rough aggregation operators is described in the technique [21]. Table-20-23 displays the evaluation EM information.

Table-20(a): Expert Information $(E)^1$

	f_1	f_2	f_3
S_1	$((0.6, 0.3), (0.8, 0.2))$	$((0.8, 0.1), (0.7, 0.2))$	$((0.9, 0.1), (0.4, 0.3))$
S_2	$((0.5, 0.2), (0.9, 0.1))$	$((0.6, 0.2), (0.4, 0.3))$	$((0.4, 0.1), (0.8, 0.1))$
S_3	$((0.6, 0.4), (0.5, 0.3))$	$((0.6, 0.3), (0.5, 0.2))$	$((0.6, 0.1), (0.4, 0.3))$
S_4	$((0.7, 0.1), (0.4, 0.2))$	$((0.4, 0.6), (0.5, 0.4))$	$((0.2, 0.1), (0.5, 0.2))$

Table-20(b): Expert Information $(E)^1$

	f_4	f_5
S_1	$((0.9, 0.1), (0.7, 0.2))$	$((0.5, 0.1), (0.4, 0.2))$
S_2	$((0.7, 0.2), (0.5, 0.1))$	$((0.4, 0.1), (0.6, 0.4))$
S_3	$((0.4, 0.5), (0.3, 0.6))$	$((0.6, 0.4), (0.4, 0.3))$
S_4	$((0.5, 0.4), (0.3, 0.5))$	$((0.5, 0.3), (0.4, 0.2))$

Table-21(a): Expert Information $(E)^2$

	f_1	f_2	f_3
S_1	$((0.6, 0.1), (0.3, 0.2))$	$((0.8, 0.2), (0.6, 0.3))$	$((0.6, 0.3), (0.7, 0.1))$
S_2	$((0.4, 0.1), (0.5, 0.3))$	$((0.9, 0.1), (0.4, 0.2))$	$((0.4, 0.5), (0.4, 0.1))$
S_3	$((0.7, 0.2), (0.2, 0.7))$	$((0.4, 0.2), (0.2, 0.6))$	$((0.5, 0.3), (0.5, 0.3))$
S_4	$((0.5, 0.5), (0.6, 0.2))$	$((0.6, 0.3), (0.4, 0.3))$	$((0.3, 0.1), (0.5, 0.4))$

Table-21(b): Expert Information $(E)^2$

	f_4	f_5
S_1	$((0.5, 0.1), (0.7, 0.3))$	$((0.9, 0.1), (0.8, 0.1))$
S_2	$((0.4, 0.3), (0.2, 0.7))$	$((0.8, 0.2), (0.4, 0.5))$
S_3	$((0.2, 0.7), (0.7, 0.2))$	$((0.5, 0.3), (0.3, 0.1))$
S_4	$((0.6, 0.2), (0.4, 0.5))$	$((0.6, 0.3), (0.2, 0.3))$

Table-22(a): Expert Information $(E)^3$

	f_1	f_2	f_3
S_1	$((0.7, 0.1), (0.2, 0.1))$	$((0.8, 0.2), (0.6, 0.3))$	$((0.7, 0.3), (0.6, 0.2))$
S_2	$((0.7, 0.2), (0.3, 0.4))$	$((0.5, 0.3), (0.4, 0.3))$	$((0.5, 0.1), (0.3, 0.6))$
S_3	$((0.4, 0.2), (0.5, 0.1))$	$((0.9, 0.1), (0.9, 0.1,))$	$((0.4, 0.3), (0.2, 0.6))$
S_4	$((0.5, 0.1), (0.7, 0.3))$	$((0.5, 0.2), (0.4, 0.6))$	$((0.3, 0.2), (0.1, 0.3))$

Table-22(b): Expert Information $(E)^3$

	f_4	f_5
S_1	$((0.5, 0.2), (0.4, 0.1))$	$((0.8, 0.1), (0.7, 0.2))$
S_2	$((0.7, 0.3), (0.5, 0.1))$	$((0.6, 0.3), (0.4, 0.1))$
S_3	$((0.8, 0.2), (0.3, 0.4))$	$((0.6, 0.4), (0.3, 0.2,))$
S_4	$((0.3, 0.1), (0.4, 0.2))$	$((0.4, 0.2), (0.5, 0.3))$

Expert weight information is calculated as follows:

$$\begin{array}{ccc} \Upsilon^{(1)} & \Upsilon^{(2)} & \Upsilon^{(3)} \\ 0.336 & 0.347 & 0.317 \end{array}$$

Attributes weights are computed as follows:

$\beta \tilde{A}_1$	$\beta \tilde{A}_2$	$\beta \tilde{A}_3$	$\beta \tilde{A}_4$	$\beta \tilde{A}_5$
0.201	0.192	0.207	0.198	0.202

With the help of EMs weights thr final closeness indices ($FRCIs$) are evaluated as follows:

Table-23:

Alternatives	S_1	S_2	S_3	S_4
$\check{FRCIs}$	0.5783	0.6681	0.5878	0.3935

As a result $\hat{S}_2$ is our best option.

6.1. Results and Discussion. The details are given by the decision maker in the form of picture fuzzy rough sets. We utilised the picture improved TOPSIS plan to resolve the information in the comparison section, considering the neutral term to be zero. In terms of obtaining outcomes, S_2 is the best option, which is the same as the one stated in [21].

As a result, the suggested methodology seems to be more realistic, practicable, useful, and generalised for solving MAGDM problems with uncertain information between EMs and criteria.

7. CONCLUSION

PFRS are new and effective generalised procedures that have been chosen as operable opportunity to manage the uncertainties and ambiguity associated with MAGDM difficulties, and so DMs feel much more comfortable using PFRS information in their judgement than IFS, IFRS, and PFRS. In this work, an unique improved TOPSIS-based decision-making strategy is developed to address with MAGDM problems in a PFRS system with entirely unknown DMs and criteria weights. To construct the PFRS entropy weight structure for evaluating the criteria weights over PFRS data, a GDM-based unique PFRS entropy measure is provided. Aggregation is conducted in the last steps, utilising the determined DMs

weights to create a final ranking of alternatives, to prevent the failure of collective information during the process. Finally, numerical examples are illustrated to present the applicability and advantage of the introduced technique. Furthermore, the suggested method can be enhanced for future studies by adding other existing fuzzy sets and applying them to various MCGDM issues involving undetermined DM and criteria weights.

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Declaration

Conflicts of interest: The authors declare that they have no conflict of interest.

REFERENCES

- [1] Garg, H., 2016. A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making. *International Journal of Intelligent Systems*, 31(9), pp.886-920.
- [2] Ashraf, S., Mahmood, T., Abdullah, S. and Khan, Q., 2019. Different approaches to multi-criteria group decision making problems for picture fuzzy environment. *Bulletin of the Brazilian Mathematical Society, New Series*, 50(2), pp.373-397.
- [3] Rahman, K. and Abdullah, S., Some new generalized interval-valued Pythagorean fuzzy aggregation operators using einstein t-norm and t-conorm. *Journal of Intelligent & Fuzzy Systems*, (Preprint), pp.1-22.
- [4] Ashraf, S., Abdullah, S. and Smarandache, F., 2019. Logarithmic Hybrid Aggregation Operators Based on Single Valued Neutrosophic Sets and Their Applications in Decision Support Systems. *Symmetry*, 11(3), p.364.
- [5] Khan, M.J., Kumam, P., Ashraf, S. and Kumam, W., 2019. Generalized Picture Fuzzy Soft Sets and Their Application in Decision Support Systems. *Symmetry*, 11(3), p.415.
- [6] Ashraf, S., Abdullah, S., Mahmood, T. and Aslam, M., 2019. Cleaner Production Evaluation in Gold Mines Using Novel Distance Measure Method with Cubic Picture Fuzzy Numbers. *International Journal of Fuzzy Systems*, pp.1-14.
- [7] Zadeh, L.A., 1965. Fuzzy sets. *Information and control*, 8(3), pp.338-353.
- [8] Atanassov, K., 1986. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.*, 20, pp.87-96.
- [9] Pawlak, Z., 1982. Rough sets. *International journal of computer & information sciences*, 11(5), pp.341-356.
- [10] Dubois, D. and Prade, H., 1990. Rough fuzzy sets and fuzzy rough sets. *International Journal of General System*, 17(2-3), pp.191-209.
- [11] Cornelis, C., De Cock, M. and Kerre, E.E., 2003. Intuitionistic fuzzy rough sets: at the crossroads of imperfect knowledge. *Expert systems*, 20(5), pp.260-270.
- [12] Zhou, L. and Wu, W.Z., 2008. On generalized intuitionistic fuzzy rough approximation operators. *Information Sciences*, 178(11), pp.2448-2465.
- [13] Zhou, L. and Wu, W.Z., 2011. Characterization of rough set approximations in Atanassov intuitionistic fuzzy set theory. *Computers & Mathematics with Applications*, 62(1), pp.282-296.
- [14] Bustince, H. and Burillo, P., 1996. Structures on intuitionistic fuzzy relations. *Fuzzy sets and systems*, 78(3), pp.293-303.
- [15] Zhang, X., Zhou, B. and Li, P., 2012. A general frame for intuitionistic fuzzy rough sets. *Information Sciences*, 216, pp.34-49.
- [16] Yun, S.M. and Lee, S.J., 2015. Intuitionistic fuzzy rough approximation operators. *International Journal of Fuzzy Logic and Intelligent Systems*, 15(3), pp.208-215.
- [17] Chakrabarty, K., Gedeon, T. and Koczy, L., 1998. Intuitionistic fuzzy rough set. In *Proceedings of 4th joint conference on information sciences (JCIS)*, Durham, NC (pp. 211-214).
- [18] S. P. Jena, S. K. Ghosh, and B. K. Tripathy, "Intuitionistic fuzzy rough sets," *Notes IFS.*, vol. 8, no. 1, pp. 1-18, 2002.
- [19] Samanta, S.K. and Mondal, T.K., 2001. Intuitionistic fuzzy rough sets and rough intuitionistic fuzzy sets. *Journal of Fuzzy Mathematics*, 9(3), pp.561-582.
- [20] Zhou, L., Wu, W.Z. and Zhang, W.X., 2009. On characterization of intuitionistic fuzzy rough sets based on intuitionistic fuzzy implicators. *Information Sciences*, 179(7), pp.883-898.

- [21] Chinram, R., Hussain, A., Mahmood, T. and Ali, M.I., 2021. EDAS method for multi-criteria group decision making based on intuitionistic fuzzy rough aggregation operators. *IEEE Access*, 9, pp.10199-10216.
- [22] Cuong, B.C. and Kreinovich, V., 2013. Picture Fuzzy Sets-a new concept for computational intelligence problems. In *2013 Third World Congress on Information and Communication Technologies (WICT 2013)* (pp. 1-6). IEEE.
- [23] H. Garg, Some Picture Fuzzy Aggregation Operators and Their Applications to Multicriteria Decision-Making, *Arab J Sci Eng.* 42:(2017) 5275—5290. <https://doi.org/10.1007/s13369-017-2625-9>.
- [24] P. Singh, Correlation coefficients for picture fuzzy sets, *Journal of Intelligent & Fuzzy Systems* 28 (2015) 591–604.
- [25] G. Wei, Some Cosine Similarity Measures for Picture Fuzzy Sets and Their Applications to Strategic Decision Making, *Informatica*, 28(3) (2017) 547–564.
- [26] Qiyas, M., Khan, M.A., Khan, S. and Abdullah, S., 2020. Concept of Yager operators with the picture fuzzy set environment and its application to emergency program selection. *International Journal of Intelligent Computing and Cybernetics*.
- [27] Hwang, C.L. and Yoon, K., 1981. Methods for multiple attribute decision making. In *Multiple attribute decision making* (pp. 58-191). Springer, Berlin, Heidelberg.
- [28] Chen, C.T., 2000. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy sets and systems*, 114(1), pp.1-9.
- [29] Beg, I. and Rashid, T., 2013. TOPSIS for hesitant fuzzy linguistic term sets. *International Journal of Intelligent Systems*, 28(12), pp.1162-1171.
- [30] Biswas, A. and Kumar, S., 2018. An integrated TOPSIS approach to MADM with interval-valued intuitionistic fuzzy settings. In *Advanced Computational and Communication Paradigms* (pp. 533-543). Springer, Singapore.
- [31] Cables, E., García-Cascales, M.S. and Lamata, M.T., 2012. The LTOPSIS: An alternative to TOPSIS decision-making approach for linguistic variables. *Expert Systems with Applications*, 39(2), pp.2119-2126.
- [32] Chen, T.Y. and Tsao, C.Y., 2008. The interval-valued fuzzy TOPSIS method and experimental analysis. *Fuzzy sets and systems*, 159(11), pp.1410-1428.
- [33] Li, D.F., 2010. TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 18(2), pp.299-311.
- [34] Liang, D. and Xu, Z., 2017. The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets. *Applied Soft Computing*, 60, pp.167-179.
- [35] Mohagheghi, V., Mousavi, S.M. and Vahdani, B., 2017. Enhancing decision-making flexibility by introducing a new last aggregation evaluating approach based on multi-criteria group decision making and Pythagorean fuzzy sets. *Applied Soft Computing*, 61, pp.527-535.
- [36] Park, J.H., Park, I.Y., Kwun, Y.C. and Tan, X., 2011. Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment. *Applied Mathematical Modelling*, 35(5), pp.2544-2556.
- [37] Xu, Z. and Zhang, X., 2013. Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information. *Knowledge-Based Systems*, 52, pp.53-64.
- [38] Yang, Y., Ding, H., Chen, Z.S. and Li, Y.L., 2016. A note on extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, 31(1), pp.68-72.
- [39] Zeng, S., Chen, J. and Li, X., 2016. A hybrid method for Pythagorean fuzzy multiple-criteria decision making. *International Journal of Information Technology & Decision Making*, 15(02), pp.403-422.
- [40] Zhang, X. and Xu, Z., 2014. Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, 29(12), pp.1061-1078.
- [41] Boran, F.E., Genç, S., Kurt, M. and Akay, D., 2009. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. *Expert Systems with Applications*, 36(8), pp.11363-11368.
- [42] Shih, H.S., 2008. Incremental analysis for MCDM with an application to group TOPSIS. *European Journal of Operational Research*, 186(2), pp.720-734.

- [43] Torlak, G., Sevcli, M., Sanal, M. and Zaim, S., 2011. Analyzing business competition by using fuzzy TOPSIS method: An example of Turkish domestic airline industry. *Expert Systems with Applications*, 38(4), pp.3396-3406.
- [44] Yue, Z., 2011. An extended TOPSIS for determining weights of decision makers with interval numbers. *Knowledge-Based Systems*, 24(1), pp.146-153.
- [45] Yue, Z., 2013. An avoiding information loss approach to group decision making. *Applied Mathematical Modelling*, 37(1-2), pp.112-126.
- [46] Guo, K. and Song, Q., 2014. On the entropy for Atanassov's intuitionistic fuzzy sets: An interpretation from the perspective of amount of knowledge. *Applied Soft Computing*, 24, pp.328-340.
- [47] Biswas, A. and Sarkar, B., 2019. Pythagorean fuzzy TOPSIS for multicriteria group decision-making with unknown weight information through entropy measure. *International Journal of Intelligent Systems*, 34(6), pp.1108-1128.

DEPARTMENT OF MATHEMATICS, ABDUL WALI KHAN UNIVERSITY, MARDAN 23200, PAKISTAN
E-mail address: `ali_khan@awkum.edu.pk`

DEPARTMENT OF MATHEMATICS, ABDUL WALI KHAN UNIVERSITY, MARDAN 23200, PAKISTAN
E-mail address: `saleemabdullah@awkum.edu.pk`

DEPARTMENT OF MATHEMATICS, ABDUL WALI KHAN UNIVERSITY, MARDAN 23200, PAKISTAN
E-mail address: `abbasqadir@awkum.edu.pk`