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Research Article

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Posted Date: August 30th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-775341/v1>

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Version of Record: A version of this preprint was published at Optical and Quantum Electronics on January 5th, 2022. See the published version at <https://doi.org/10.1007/s11082-021-03476-9>.

Intrinsic signal processed non-linearity tolerant novel 2-tier star constellation

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Abstract: In coherent optical systems, optical fiber non-linearity is a consistent limiting factor towards the effective signal-to-noise ratio though being mitigated by various digital signal processing based approaches. In this paper, an intrinsic method of signal processing based on the shape of the input constellation is employed to yield a novel non-linearity tolerant geometric constellation. 16-QAM back-to-back coherent system is optimized for minimum value of non-linear interference, and a novel 2-tier star constellation using sequential quadratic programming algorithm is proposed. The values of second and fourth order moments of input obtained for the optimized 2-tier star constellation are 1.19 and 1.70 respectively, resulting in an overall reduction of non-linear interference.

Keywords: *Constellation shaping, Geometric shaping, Sequential quadratic programming, Non-linearity mitigation, Non-linear interference noise, 16-Quadrature amplitude modulation.*

1. Introduction

The advent of high-speed digital signal processing has paved the way for coherent optical communication implementing spectral efficient and multi-level modulation formats, thereby increasing the channel capacity. The ever-growing traffic demands are being taken care of by optimal exploitation of the installed optical communication systems [1]. To keep up with the demand, various methods have been employed from time to time to exploit the system capacity to

Shannon limit, which include advanced modulation formats [2-4], different multiplexing techniques [5-6], coding and shaping techniques. However, optical fiber non-linearity has been a constant bottleneck to achieve the desired data rate over long-haul communication [7]. Digital signal processing (DSP) techniques that are robust against fiber non-linearities and offer sensitivity and spectral efficiency improvements have been widely used for the non-linearity mitigation. To undo with the complexity of the system, non-linearity tolerant systems based on one of the constellation shaping (CS) methods, geometric shaping are being consistently explored [8-10]. The systems employing geometric shaping are intrinsically tolerant to optical fiber non-linearity. 16-QAM employing geometric shaping is found a viable solution to achieve a single-carrier 400G long-haul optical transmission [11]. Further, to approach the Shannon limit in 16-QAM, coding and CS techniques are adopted, with later gaining more importance lately because coding technique is believed to have been put to an optimal use [12].

2. Geometric Shaping

Geometric shaping (GS) is based on uniform probability distribution of non-equidistant constellation points. It restricts high-energy symbols in the constellation, thereby lowering the peak-to-average power ratio which in turn reduces the non-linear effects [13]. The shaping is yielded by optimizing different criterion which include minimizing Euclidean distance, maximizing mutual information (MI) [14], maximizing generalized mutual information (GMI) [15], maximizing figure of merit (FOM) [16] and minimizing mean square error [17] under a given signal-to-noise ratio (SNR). These different optimization criteria with a constraint of uniform probability distribution to achieve an objective function of improved system capacity or tolerance to fiber non-linearity lead to various geometric constellations.

The basic GS includes square, rectangular and star/circular constellations as shown in Fig. 1.

a) Square (16-QAM)

b) Rectangular (32-QAM)

c) Star (16-QAM)

Fig. 1. Different geometric constellation shapes

The overall non-linear interference noise (NLIN) variance depending on the modulation format, and the symbols transmitted over the two channels being independent, uniformly and isotropically distributed, is given as [18]:

$$\sigma_{NLIN}^2 = P^3 \chi_1 + P^3 \chi_2 \left(\frac{\langle |b|^4 \rangle}{\langle |b|^2 \rangle^2} - 2 \right) \quad (1)$$

where P is the average input power, b represents a single-polarization constellation point of the interfering channel, and χ_1 and χ_2 represent second-order noise (SON) and fourth-order noise (FON) coefficients respectively. It signifies the overall NLIN as a summation of modulation-independent and dependent noises, coined as SON and FON respectively. The fourth moment $\langle |b|^4 \rangle$ marks the dependence on modulation format, which in turn depends on constellation point arrangement in a spatial domain, in accordance to the equation as:

$$\hat{\mu}_k = \frac{E[|X - E[X]|^k]}{(E[|X - E[X]|^2])^{\frac{k}{2}}} \quad (2)$$

where $\hat{\mu}_k$ represents the standardized moment of input constellation X. The values obtained for the standardized moments, $\hat{\mu}_4$ and $\hat{\mu}_6$, second and fourth-order noise coefficients χ_1 and χ_2 over the different modulation and shaping formats in a non-linear optical channel are given in Table 1.

TABLE I. STANDARD MOMENTS AND HIGHER-ORDER NOISE COEFFICIENTS

Modulation	$\hat{\mu}_4$	$\hat{\mu}_6$	χ_1	χ_2
Square 16-QAM	1.32	1.96	7.85×10^{-7}	4.79×10^{-7}
Rectangular 32-QAM	1.368	2.214	2.14×10^{-6}	1.02×10^{-6}
Star/Circular 16-QAM	1.282	1.824	7.36×10^{-7}	4.11×10^{-7}

The values of the standardized moments, $\hat{\mu}_4$ and $\hat{\mu}_6$ along with the higher-order noise coefficients, χ_1 and χ_2 provide an insight of the variation of overall NLIN in an optical fiber channel with respect to the spatial arrangement of constellation points, which in turn leads to the incorporation of GS to minimize the effect. The dependence of NLIN on the spatial arrangement of the constellation points is exploited to obtain an optimized novel 2-Tier star constellation minimizing the values obtained for the standardized moments and the higher-order noise coefficients.

3. Proposed 2-Tier Star Constellation

A novel 2-Tier star constellation is proposed to inhibit the effect of non-linearity in long-haul optical communication systems obtained after optimizing the higher order moments of the input. This GS technique is seen as an effective non-linearity mitigation method as its NLIN performance is better when drawn parallels with the fundamental constellation shapes like square, rectangular or star. An efficient yet simple robust optimization method, sequential quadratic programming (SQP) approach is applied to yield the minimum values of second and fourth order moments of input [19]. As the baseline optimization technique, a classic SQP approach is applied to obtain an optimized 2-Tier star constellation. Within each step of the SQP, standardized moments for the

constellation are calculated within or on the boundary of the interval of a single constellation point that lead to its optimal or sub-optimal value. After that, the obtained value will replace the original nominal parameter value from the input star constellation. The process continues for the entire range of constellation points to yield an optimized 2-Tier star constellation. The entire process to generate the required optimized constellation with minimum values obtained for the objective function $\hat{\mu}_k$ is shown in algorithm below, where X represents an array of constellation points, a_k represents k^{th} constellation point and d_k represents the minimum step size.

3.1 SQP-based optimized 2-Tier star constellation algorithm

1: *Input*: $M=16$, number of constellation points.
2: *Initialize*: Generate star-QAM, an arbitrary constellation to be worked upon.
3: Calculate moments μ_k of the star constellation using eqn. (2)
5: Iteration $k=0$, find $\{\mu_k\}$, under constraints $\mathbb{R} \rightarrow -4 \leq \mathbb{R}^n \leq 4$ and constellations points being complex conjugates.

6: If $\mu_k > \epsilon$, find a_k such that $X_k + a_k d_k$ yields:

$$\mu_{k+1} < \mu_k$$

7: Continue till $\mu_k \leq \epsilon$.

8: Repeat step-3 to step-7 for entire input constellation.

9. Replace all input constellation points with the local minimum of an objective function.

10. Place all the constellation points in the 2-D space.

The algorithm is defined for pre-defined constraints over a 2-D space to obtain optimal or sub-optimal values of an objective function, optimizing an input star constellation along the dimensions of phase and amplitude to result into a 2-Tier star constellation.

The procedure of SQP to obtain an optimized 2-Tier star constellation is summarized in Fig. 2. The entire structure of the SQP goes through a number of processes to yield the minimum value of an objective function.

Fig. 2. Flowchart of SQP to obtain optimized 2-Tier star constellation

It is evident from Fig. 2 that the entire structure of the proposed 2-Tier star constellation is employing a classic SQP with a few additional blocks/processes to minimize the objective function and satisfying the pre-defined constraints.

3.2. Optimized 2-Tier star constellation and decision region

The modification over the star constellation resulting in an optimized 2-Tier star constellation is shown in Fig. 3 (a). The constellation can be visualized as an amalgam of 2-level star and 4-layer circular constellation, with constellation points localized on 4 concentric circles and 16 possible phasors.

a) 2-Tier star constellation

b) Decision region

Fig. 3. Optimized 2-Tier star constellation for 16-QAM and its decision region

The reception of a constellation point (a symbol) at the receiver depends upon the decision region which is pre-defined for every constellation point in a constellation. The decision regions are not straight-forward for 2-Tier star constellation, but based on the decision boundaries of a basic star constellation, consisting of inclined regions with all possible phase angles. The decision making for 2-Tier star constellation requires the entire constellation divided into 8 sub-regions with phase angle $(2l + 1) \frac{\pi}{8}$, with each sub-region further divided almost at the mid-point to obtain triangular regions for inner and quadrilateral regions for outer constellation points as shown in Fig 3 (b). For inner constellation points, if the amplitude of lower and upper threshold rings is r_i and r_0

respectively and at time instance t and $t + 1$, the received symbol amplitude are r_t and r_{t+1} respectively, then the bit is set to 1, if it meets the condition

$$\frac{r_{t+1}}{r_t} > \frac{r_i + r_0}{2}$$

$$\frac{r_{t+1}}{r_t} < \frac{2}{r_i + r_0}$$

(3)

otherwise set to 0. The same decision making is applied for outer constellation points as well resulting in a 2-Tier system. Similarly, to retrieve the phases, if θ_t and θ_{t+1} are the phase angles of the sub-sequent received symbols, then the demodulated angle will be [20]

$$\theta_{dem} = (\theta_{t+1} - \theta_t) \bmod 2\pi$$

The decoded angle is quantized to the defined angle set of the constellation.

4. Set-up and Modeling

The experimental setup for 16-QAM back-to-back coherent system is simulated in Optisystem-16 software. The setup consists of a transmitter which includes a QAM sequence generator to generate multi-dimensional QAM symbols from the input binary sequence. QAM sequence generator decides about the GS and accordingly it is set as regular, circular/star or 2-Tier star constellation. The QAM sequence of the particular GS obtained from the generator is modulated using Mach-Zender modulator (MZM) and M-ary raised cosine pulse generator. The optical power of the signal is set as 0 dB and transmitted over non-linear optical fiber channel. The transmission over the link is done in accordance with the Manakov equations and split-step fourier method (SSFM). Using an optical amplifier, erbium doped fiber amplifier (EDFA), the signal is amplified in each span of the fiber link, before it is fed to the coherent receiver where continuous wave (CW) laser acts as a

local oscillator. Using coherent detection, the transmitted symbol of the particular QAM format and GS is decided in accordance to pre-defined decision thresholds. The performance of the setup is evaluated in terms of bit error rate (BER) and symbol error rate (SER), and non-linearity tolerance is calculated as NLIN variance. The block diagram of the setup employing GS is shown in Fig 4.

Fig 4. Block diagram of 16-QAM back-to-back coherent optical system implementing geometric shaping

BER test set calculates probability of BER in accordance to:

$$BER = \frac{1}{\log 2(M)} \cdot 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc} \left(\sqrt{\frac{3 \cdot \log 2(M) \cdot E_b}{2(M-1)N_0}} \right) - \left(1 - \frac{2}{\sqrt{M}} + \frac{1}{M}\right) \operatorname{erfc}^2 \left(\sqrt{\frac{3 \cdot \log 2(M) \cdot E_b}{2(M-1)N_0}} \right) \quad (5)$$

where E_b is energy of a bit and N_0 is the noise power.

The equation for probability of symbol being in error, given that the signal (symbol) to noise ratio is $\frac{E_s}{N_0}$ is given as:

$$SER = 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc} \left(\sqrt{\frac{3E_s}{2(M-1)N_0}} \right) - \left(1 - \frac{2}{\sqrt{M}} + \frac{1}{M}\right) \operatorname{erfc}^2 \left(\sqrt{\frac{3E_s}{2(M-1)N_0}} \right) \quad (6)$$

In GS, SER is upper bounded as

$$SER \leq \frac{1}{M} \sum_{i=0}^{M-1} \sum_{j \neq i} \frac{1}{2} \operatorname{erfc} \left(\frac{d_{ij}}{2\sqrt{N_0}} \right) \quad (7)$$

where d_{ij} is the Euclidean distance between any two constellation points c_i and c_j , $\operatorname{erfc}(x)$ is the complementary error function equal to $\frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) dt$ and $\frac{N_0}{2}$ is the variance in each dimension of the Gaussian noise. Eqn. (7) is an approximate upper bound, considered a true value only when N_0 is equal to zero [21].

The analysis of NLIN is taken up with at least two channels for inter-channel and a unitary channel for intra-channel non-linear impairments. In an optical fiber, electric field vector is given as [18]

$$u(0, t) = \sum_n \underline{a}_n g(0, t - nT) + e^{-i\Omega t} \sum_n \underline{b}_n g(0, t - nT) \quad (8)$$

where the symbols $\underline{a}_n$ and $\underline{b}_n$ represent the transmitted data in the channel of interest (COI) and adjacent interfering channel, respectively, in the n -th time-slot, having a symbol duration of T . $g(0, t)$ is the fundamental injected waveform, with energy normalized and orthogonal in time, i.e.

192 $\int g^*(0, t - nT)g(0, t - n'T) = \delta_{n,n'}$ [18]. The waveform of a single pulse reaching any arbitrary
 193 point z along the fiber is given as

$$194 \quad g(z, t) = \exp\left(-\frac{i}{2}\beta''z\frac{d^2}{dt^2}\right)g(0, t) \quad (9)$$

195 where β'' coefficient of fiber dispersion.

196 In order to detect the data symbols $\underline{a}_0$ under the influence of NLIN, a matched filter is used prior
 197 to coherent detection in order to isolate the COI and mitigate the effect of linear noise. The received
 198 signal is expressed as $\underline{r}_0 = \underline{a}_0 + \Delta\underline{a}_0$, where $\Delta\underline{a}_0$ represents the NLIN contribution. The NLIN
 199 contribution over the dual-polarization is extended as

$$200 \quad \Delta\underline{a}_0 = i\frac{8}{9}\gamma\sum_{l,k,m}S_{l,k,m}\underline{a}_k\underline{a}_m\underline{a}_l + i\frac{8}{9}\gamma\sum_{l,k,m}X_{l,k,m}(\underline{b}_k\underline{b}_m\mathbf{I} + \underline{b}_m\underline{b}_k)\underline{a}_l \quad (10)$$

201 where γ is the fiber non-linearity coefficient, $\mathbf{I}$ is an identity matrix of order 2×2 .

202 The higher order noise coefficients $S_{l,k,m}$ and $X_{l,k,m}$ are given by

$$203 \quad S_{l,k,m} = \int_0^L dz f(z) \int_{-\infty}^{\infty} dt g^*(z, t)g(z, t - lT) \times g^*(z, t - kT)g(z, t - mT) \quad (11)$$

$$204 \quad X_{l,k,m} = \int_0^L dz f(z) \int_{-\infty}^{\infty} dt g^*(z, t)g(z, t - lT) \times g^*(z, t - kT - \beta''\Omega z)g(z, t - mT - \beta''\Omega z) \quad (12)$$

206 where L is the length of the optical fiber link and $f(z)$ is the function is its loss/gain profile,
 207 representing both intra-channel and inter-channel interference [18]. A non-linearity component,
 208 phase-noise exists within NLIN, which in turn depends on the modulation format of the co-
 209 propagating channels. The phase-noise is contributed by the diagonal elements of the 2×2 matrices
 210 $X_{0,k,m}(\underline{b}_k\underline{b}_m\mathbf{I} + \underline{b}_k\underline{b}_m)$ which are further divided into two subgroups. The first subgroup consists
 211 of the terms where $k = m$, whereas the second subgroup consists of the terms where $k \neq m$. The
 212 coefficients $X_{0,m,m}$ are real-valued, the diagonal contributions from the first group are at a complex

angle of $\pi/2$ from the detected symbol a_0 , constituting phase-noise. The same hold good for the terms where $k \neq m$, such that $X_{0,m,m} = X_{0,m,k}^*$ [18]. In a single-polarization transmission, NLIN coefficients χ_1 and χ_2 , from eqn. (1) are given by equations (13) and (14) multiplied by T^3 , whereas in case of dual polarizations the equations need to be multiplied by $\frac{8}{27}$ and $\frac{20}{81}$, respectively.

$$\chi_1 = \frac{32}{27} \gamma^2 \sum_{l,k,m} |X_{l,k,m}|^2 \quad (13)$$

$$\chi_2 = \frac{80}{81} \gamma^2 \sum_{l,m} |X_{l,m,m}|^2 \quad (14)$$

Other than higher order moments and noise coefficients, some real coefficients generally called as additional noise terms contribute to the variation of NLIN. Mathematically, the dependence of NLIN on these additional terms is given as [22]:

$$\sigma_{NLIN}^2 = P_{tx}^3 [\chi_0 + (\hat{\mu}_4 - 2) \cdot \chi_4 + (\hat{\mu}_4 - 2)^2 \cdot \acute{\chi}_4 + \hat{\mu}_6 \cdot \chi_6] \quad (15)$$

where $\chi_0, \chi_4, \acute{\chi}_4$ and χ_6 are real coefficients that contribute to the fiber non-linearities. Though, the value of these terms is very small yet considered during the non-linear analysis of fiber-optic systems. The standard moments $\hat{\mu}_4$ and $\hat{\mu}_6$, NLIN coefficients χ_1 and χ_2 together with the modulation-dependent coefficients $\chi_0, \chi_4, \acute{\chi}_4$ and χ_6 define the value of NLIN in a system. The variation of modulation-dependent coefficients $\chi_0, \chi_4, \acute{\chi}_4$ and χ_6 over different modulations formats is given in Table II.

TABLE II. PROBABILITY DISTRIBUTION AND NLI TERMS

Modulation	P_X	χ_0	χ_4	$\acute{\chi}_4$	χ_6
Square 16-QAM	Uniform	1.143×10^{-7}	1.117×10^{-7}	2.143×10^{-7}	1.171×10^{-7}
Rectangular 32-QAM	Uniform	1.137×10^{-7}	1.103×10^{-7}	2.117×10^{-7}	1.167×10^{-7}
Star/Circular 16-QAM	Uniform	1.136×10^{-7}	1.116×10^{-7}	2.114×10^{-7}	1.163×10^{-7}

The NLIN variance increases with the increase in the value of standard moments $\hat{\mu}_4$ and $\hat{\mu}_6$, NLIN coefficients χ_1 and χ_2 and modulation-dependent coefficients χ_0 , χ_4 , χ_4 and χ_6 . It is evident that the values obtained for the standard moments and modulation-dependent coefficients vary over different modulation schemes and geometric shapes which in turn lead to the variation in overall NLIN of the fiber-optic system.

A multi-span 16-QAM back-to-back coherent system is simulated over a non-linear fiber employing GS with the main parameters listed in Table III.

TABLE III. NON-LINEAR SIMULATION SETUP PARAMETERS

Parameter	Value	
Symbol rate	28 GBaud	238
RRC roll-off factor	0.2	239
Fiber non-linear coefficient	1.3 W/km	240
Fiber dispersion	17 ps/nm/km	241
Fiber attenuation	0.2 dB/km	242
Span length	80 km	243
EDFA noise figure	5 dB	243

5. Results and Discussions

The proposed GS based 2-Tier star 16-QAM is simulated and analysed over a non-linear fiber channel, under the input parameters as given in Table III,. The analysis is based on the parameters like bit error rate (BER), symbol error rate (SER) and NLIN. BER and SER are symbolic to the overall performance of the system and NLIN gives a measure of non-linearity tolerance.

5.1 BER and SER analysis

In Fig. 5, the BER performance of 16-QAM undergoing optimized geometric constellation shaping, 2-Tier star is studied under the static spectral efficiency and filter roll-off conditions. The analysis is carried out over a non-linear fiber having the total propagation distance of 80 km. It is observed that at low input power, when there is minimal effect of non-linearity, regular (square) constellation shaping undergoes minimal BER with respect to star/circular constellation shaping, higher order modulation format, rectangular 32-QAM or proposed 2-Tier star constellation.

Fig. 5. BER vs. E_b/N_0 for different geometric shapes

Fig. 6. SER vs. E_s/N_0 for different geometric shapes

Beyond a particular threshold of signal-to-noise ratio, approximated at around 8 dB, the variation in BER with respect to different geometric shaping changes is found minimal for 2-Tier star constellation underpinning the geometric shaping as a non-linearity tolerant method for high power input. The minimum BER of 1.46×10^{-4} is obtained for 2-Tier star constellation with respect to 4.27×10^{-4} and 1.77×10^{-3} for star/circular and square constellation respectively for the maximum

signal-to-noise ratio of 12 dB. The variation is attributed to equation (5), resulting into waterfall curves, with 2-Tier star constellation standing out as best choice for higher power inputs or higher values of non-linear interference. Since star constellation gives better BER performance over other fundamental shapes at higher values of SNR, 2-Tier star constellation, an optimized star constellation a modification over it yields even better values.

In Fig. 6, the SER performance is found to be at an optimum level for 2-Tier star constellation, thereby encouraging geometric shaping as an important method of non-linearity tolerance in a defined range of SNR. SER is found to be minimum for square constellation bounded at 12 dB of SNR in accordance with equation (7), maintaining maximum possible minimum Euclidean distance for square constellation. However, the high data-rates for the same constellation at very high input power (or SNR) requires strict SER/BER requirements, undergoes cell-to-cell interference, thereby maintaining 2-Tier star constellation as an optimal solution at high input powers.

5.2 NLIN analysis

In order to attain high data rates, the system is subjected to high input powers resulting into severe NLIN. As observed 2-Tier star constellation yields minimum BER beyond a certain input power, NLIN of the different geometric constellation shapes is calculated, using the standardized moments for input, second and fourth-order noise and modulation-dependent coefficients.

The values of standard moments, $\hat{\mu}_4$ and $\hat{\mu}_6$, higher order noise coefficients, second-order, χ_1 and fourth-order, χ_2 over different GS are given in Table IV. The values of standard moments, $\hat{\mu}_4$ and $\hat{\mu}_6$ are obtained using an equation (2). Likewise, higher order noises coefficients, second-order χ_1 and fourth-order χ_2 are obtained using equation (11) and (12).

TABLE IV. STANDARD MOMENTS AND HIGHER-ORDER NOISE COEFFICIENTS

Modulation	$\hat{\mu}_4$	$\hat{\mu}_6$	χ_1	χ_2
Square 16-QAM	1.32	1.96	7.85×10^{-7}	4.79×10^{-7}
Star/Circular 16-QAM	1.282	1.824	7.36×10^{-7}	4.11×10^{-7}
2-Tier Star 16-QAM	1.192	1.702	4.38×10^{-8}	2.59×10^{-8}

Modulation-dependent coefficients χ_0 , χ_4 , $\acute{\chi}_4$ and χ_6 for the different geometric constellation shapes of 16-QAM i.e. regular (square), circular and 2-Tier star are obtained using equation (11), (12) and (15). The values obtained for different geometric constellation shapes under uniform probability distribution are given in Table V.

TABLE V. PROBABILITY DISTRIBUTION AND NLI TERMS

Modulation	P_X	χ_0	χ_4	$\acute{\chi}_4$	χ_6
Square (16-QAM)	Uniform	1.143×10^{-7}	1.117×10^{-7}	2.143×10^{-7}	1.171×10^{-7}
Star/Circular (16-QAM)	Uniform	1.136×10^{-7}	1.116×10^{-7}	2.114×10^{-7}	1.163×10^{-7}
2-Tier Star (16-QAM)	Uniform	7.213×10^{-8}	6.166×10^{-8}	5.819×10^{-8}	3.224×10^{-8}

The SNR performance of an optical communication system with varying GS is given in Fig. 7 (a). It is observed from the figure that the SNR is influenced by both SON approximated as Gaussian and non-linear FON noise. 2-Tier star GS results in increased value of SNR over varying input power ranging from -5 dBm to 1 dBm. Based on the results obtained for higher order moments, higher order noise and modulation-dependent coefficients for different geometric shapes, NLIN is calculated according to equation (1) and its variation with respect to the range of input powers, from -8 dBm to 2 dBm is observed in Fig. 7 (b).

Fig. 7. a) SNR vs Optical input power b) NLIN vs input power for different geometric shapes

Since the overall distribution of SON is Gaussian and the value of FON obtained over low input powers is very small, the variation of NLIN is linear for low input powers, ranging from -8 dB to -2 dB which eventually becomes non-linear over the increased values of input power, attributed to higher values of FON. 2-Tier star constellation is found to be most tolerant to non-linearity, well in conformity with the values obtained for higher order moments and noise coefficients using eqn (2) and (15) respectively. The values of NLIN obtained for 2-Tier star constellation range from -62.6 dBm to -45 dBm. The overall gain, with respect to square and star constellation is found to be 1.12 and 0.89 dBm respectively.

Variation of higher order noises, SON and FON for different geometric shapes with respect to number of spans over a fixed optical link length is observed in Fig. 8. It can be observed from Fig. 8 (a) that SON behaves completely Gaussian, increases linearly with the increase in number of

spans. The variation goes well in accordance with eqn (1) with 2-Tier star constellation standing out as the best solution in comparison to square and star constellation. The minimum values of SON obtained over the number of spans ranging from 1 to 20 spans are -74 dB and -32 dB respectively. However, it is observed from Fig. 8 (b) that the behaviour of FON is completely non-Gaussian. FON is calculated using eqn (1) and (14).

Fig. 8. a) Second order noise (SON) vs Number of spans b) Fourth- order noise (FON) vs Number of spans

The difference in the values of FON over different GS is maximum for a single span which considerably reduces over the increasing values of spans, attributed to increased number of incomplete collisions of non-linear interacting pulses. The variation of 2-Tier star constellation stands out with minimal values of FON. In 2-Tier star constellation, the maximum gain of 1.12 dBm is obtained over the other constellation shapes. The variation of FON for all three geometric constellation shapes taper towards its fag end over the range of spans, attributed to the decreased number of incomplete collisions occurring over a large number of spans.

331 The variation of SON and FON with respect to number of spans is extended to the variation of
 332 NLIN for different geometric shapes under different fixed span lengths. The variation of NLIN
 333 with respect to number of spans is shown in Fig. 9.

Fig. 9. NLIN vs number of spans for different geometric constellation shapes,
 with fixed span length a) 25 km to b) 50 km c) 100 km to d) 200 km

With the minimum number of spans, NLIN behaves completely Gaussian, which is attributed to minimal collisions over the fiber length ranging from 25 km to 200 km. However, with the increase in number of spans, the increased values of non-linearity shift the overall behaviour of the system from being completely Gaussian, to non-linear attributed to increased value of FON and consequently NLIN.

2-Tier star constellation stands out with minimal values obtained for NLIN ranging from -38.4 dBm to -30.8 dBm over the fixed span length of 25 km and increasing number of spans which consequently increases with the increase in span length. The variation of NLIN for all three geometric constellation shapes is in coherence with FON. Fig 10. shows the variation of the mitigation gain of 2-Tier star constellation with respect to SNR of the optical signal.

Fig. 10. Non-linearity benefit of 2-Tier star constellation

The maximum value of the gain obtained is 1.12 dB when calculated for the highest OSNR value of 24 dB. The received OSNR in an optical fiber link is given as:

$$OSNR \approx 58 - 10\log_{10}(N_{EDFA}) + P_{Launch} - Loss - NF \quad (16)$$

where N_{EDFA} is the chain of EDFA optical amplifiers, P_{Launch} is input optical power and NF is the noise figure of an EDFA.

The simulation results based on autocorrelation function (ACF) obtained over one of the diagonal elements from eqn (10), and extended to almost the entire range of symbols, give an insight into the overall non-linearity tolerance. The behavior of ACF over different GS obtained from the higher order noise coefficients is shown in Fig. 11.

Fig. 11. a) Temporal autocorrelation function and b) Mitigation gain

The curves are obtained by computing the higher order noise coefficients $S_{l,k,m}$ and $X_{l,k,m}$ given in eqn. (11) and (12) over a non-linear optical fiber channel. It can be seen from the figure that the 2-Tier star constellation exhibits the longest temporal correlation whereas for square constellation,

the values obtained are comparatively shorter. The results signify that the long temporal correlation obtained for 2-Tier star constellation intrinsically offer better tolerance to non-linearity in comparison to square and star constellations NLIN mitigation gain for 2-Tier star constellation is calculated as the ratio of NLIN variance of 2-Tier star constellation to square and star constellation. It is evident from the figure that the non-linearity tolerant 2-Tier star constellation yields a gain of 1.12 and 0.89 over square and star constellations respectively.

6. Conclusion and Future Scope

In this paper, GS is employed over a long-haul communication system and the behaviour of the system is studied both analytically and experimentally. The numerical simulations are carried out for both Gaussian and non-Gaussian behaviour of the channel to include linear and non-linear impairments. It is found that Geometric constellation shaping has a strong bearing to BER and NLIN for a coherent optical communication system employing higher order modulation format. Square constellation yields minimum value of BER for a maximum bounded input power attributed to minimum Euclidean distance, but beyond the maximum bound of input power, the proposed novel 2-Tier star constellation provides an optimal solution. In long-haul optical communication system operated at high input powers, constellation shaping has proved to be an intrinsic solution to mitigate the non-linearity effect. 2-Tier GS, a modified star constellation has an improved non-linearity tolerance with minimal receiver sensitivity requirements. The values of SON and FON obtained are minimal for optimized 2-Tier star constellation resulting in lowering the values of NLIN. Therefore, the system is seen tolerant to non-linearity in comparison to the rest of geometric constellation shapes.

391 In future, alongside geometric shapes, other shaping techniques like probabilistic shaping and
392 hybrid shaping techniques can be exploited and synergistically used along digital signal processing
393 to increase the overall capacity of the system, with minimum values of BER and NLIN.

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