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Stress analysis and applicability analysis of elliptical head

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Abstract: In view of the phenomenon that there is no uniform theoretical formula for the connection area between the elliptical head and the cylinder, the author takes the standard elliptical head as the research object. Firstly, the theoretical stress calculation formula of the elliptical head and the discontinuous area of the cylinder is derived according to the deformation continuity equation. Secondly, the experimental stress is measured by means of the internal pressure thin-walled vessel stress measuring apparatus. The theoretical stress and experimental stress in discontinuous region are analyzed and compared to verify the accuracy and applicability of the formula for calculating the theoretical stress of the elliptical head and the cylinder discontinuity region. The results show that the theoretical stress calculation formula of discontinuous region of elliptical head is obtained according to the equation of deformation continuity, edge force and edge moment, internal force and internal moment; The internal pressure load is kept unchanged, and for the theoretical longitudinal stress, the constant stress is greater than 0, which is the tensile stress, and decreases gradually from the vertex to the equator; For the theoretical circumferential stress, the change trend is more complex, which can be divided into three stages, and there is pressure stress. At the vertex, the magnitude of the meridional stress and the circumferential stress is approximately equal; The change of the change from point 8 to point 10 is affected by discontinuous stress, and the change trend is abrupt; The theoretical stress and experimental stress in discontinuous region of elliptical head are analyzed and compared, and the accuracy and applicability of the formula are verified. The results are of great significance for the stress measurement of internal pressure vessels.

Keywords: Cylinder; Elliptical head; Theoretical stress; Experimental stress

INTRODUCTION

With the continuous progress of science and technology and economy, chemical products play a more and more important role in people's daily life^[1]. The chemical industry is generally accompanied by high temperature, high pressure, high corrosion and other environmental characteristics, so the pressure vessel came into being^[2]. Pressure vessel, as the name suggests, is a vessel with internal pressure^[3]. According to the size of internal pressure and external pressure,

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it can be divided into internal pressure vessel and external pressure vessel [4]. In the chemical industry, the most used is internal pressure vessel. Internal pressure vessels can be divided into reaction vessels, heat exchange vessels, separation vessels and storage and transportation vessels according to their functions and purposes in the chemical industry. Different pressure vessels have different uses, but their structural composition is roughly the same, which is composed of cylinder, head, opening nozzle, support, sealing device and safety accessories, At the same time, as the main pressure components, the mechanical performance of cylinder and head affects the normal use of pressure vessels [5]. The loads of pressure vessel include internal pressure, external pressure, wind load, earthquake load, snow load, etc., but the effect of other loads can be ignored compared with internal pressure load. Under the internal pressure load, if the cylinder or head is studied separately, it only bears the membrane stress. Under the membrane stress, the corresponding membrane deformation occurs, that is, the radial displacement of the parallel circle and the meridian angle. Generally, the radial displacement and the meridian angle of the cylinder and head are different^[6]. This research method is simple and reliable, However, it is not considered that the cylinder and the head are welded together, that is, in the connection area of the cylinder and the head, the radial displacement of the parallel circle and the meridian angle are the same^[7]. Therefore, there will be a displacement constraint in the area near the connection between the cylinder and the head, which makes the radial displacement of the parallel circle and the meridian angle of the cylinder and head near the connection area equal, and the corresponding edge force Q_0 and edge moment M_0 will be generated^[8]. At this time, the pressure vessel bears the combined action of internal pressure, edge force Q_0 and edge moment M_0 , which causes the stress increase in the connection area between the cylinder and the head, It is called discontinuous effect, that is, the total stress near the connection of cylinder and head increases, which is prone to strength failure. Therefore, it has high economic and social benefits to analyze the stress near the connection of cylinder and head.

Elliptical head is composed of half ellipsoid and straight side section. Because of its small surface depth, continuous and smooth curvature radius and other advantages, it has great advantages in terms of manufacturing and stress, and is most widely used in modern chemical industry. Scholars at home and abroad have done a lot of research on the internal stress distribution of elliptical head, that is, to compare the membrane stress with the finite element analysis results or the experimental results, but this comparison is not accurate in a strict sense^[9-12]. On the one hand, the membrane stress only considers the influence of internal pressure, and ignores the influence of transverse force, bending moment and other bending forces; On the other hand, the elliptical head can not exist in isolation, it is an integral structure with the cylinder, so the cylinder and the elliptical head should be considered together, that is, in addition to the influence of internal pressure, the influence of edge force Q_0 and edge moment M_0 caused by discontinuity effect on the internal stress of the elliptical head should also be examined; Moreover, there is no unified theoretical calculation formula for the connection area between elliptical head and cylinder. Therefore, the author takes the standard elliptical head as the research object. Firstly, according to the deformation continuity equation, the theoretical stress calculation formula of the discontinuous area of the elliptical head and the cylinder is derived. Secondly, the experimental stress is measured with the help of the internal pressure thin-walled vessel stress measurement experimental device, and the theoretical stress and experimental stress of the discontinuous area are compared and analyzed, The accuracy and applicability of the theoretical stress calculation

formula for the discontinuous area of the elliptical head and cylinder are verified. The conclusions have certain guiding significance for the stress measurement of internal pressure vessels.

1 Calculation formula of theoretical stress in discontinuous area of elliptical head and cylinder

Firstly, the edge force Q_0 and the edge moment M_0 are calculated according to the deformation continuity equation, and then the internal force and the internal moment are calculated according to the edge force Q_0 and the edge moment M_0 . Finally, the theoretical stress calculation formula of the discontinuous area of the elliptical head can be obtained by substituting the internal pressure, the internal force and the internal moment into the calculation formula of the membrane stress and the bending stress. The specific analysis process is as follows:

1.1 Solution of edge force Q_0 and edge moment M_0 in discontinuous region of elliptical head

Fig.1 Spherical head with internal pressure

The connection structure between elliptical head and cylinder is shown in Figure 1. According to the literature report^[13], For the elliptic body of revolution 1, the second radius of curvature on the boundary is:

$$R_{20} = R, \quad P^* = N_\varphi \cos \varphi_0 = 0,$$

The radial displacement of parallel circle caused by edge force is as follows:

$$\Delta_1^{Q_0} = \frac{2\sqrt{3(1-\mu^2)}}{Et_1} \sqrt{\frac{R}{t_1}} RQ_0$$

The angle of meridian caused by edge force is as follows:

$$\theta_1^{Q_0} = -\frac{2\sqrt{3(1-\mu^2)}}{Et_1^2} RQ_0$$

The radial displacement of parallel circle caused by edge force distance is as follows:

$$\Delta_1^{M_0} = \frac{2\sqrt{3(1-\mu^2)}}{Et_1^2} RM_0$$

The angle of longitude caused by edge moment is:

$$\theta_1^{M_0} = -\frac{4[3(1-\mu^2)]^{\frac{3}{4}}}{Et_1^2} \sqrt{\frac{R}{t_1}} M_0$$

R_{20} is the second radius of curvature on the boundary, μ is Poisson's ratio, e is modulus of elasticity, t is thickness, ω is the meridional angle starting from the boundary, φ_0 is the meridional angle at the boundary. Where p^* is the meridional membrane internal force N at the edge connection φ . The component, direction and magnitude in the direction parallel to the circle are known, $P^*=N_\varphi \cos \varphi_0$. The edge deformation is included in the film deformation and should be removed.

For a body of revolution with a common tangent at the connecting edge: $\varphi_0=90^\circ$, $P^*=N_\varphi \cos \varphi_0=0$, which can be ignored;

For a revolution body with no common tangent at the connecting edge: $\varphi_0 \neq 90^\circ$, $P^*=N_\varphi \cos \varphi_0 \neq 0$ should be considered.

Δ^P 、 θ^P is the radial displacement of parallel circle and the meridian angle (film deformation) under the action of internal pressure P ;

Δ^{Q_0} 、 θ^{Q_0} represents the radial displacement of parallel circle and the angle of meridian (bending deformation) under the action of edge force Q_0 ;

Δ^{M_0} 、 θ^{M_0} represents the radial displacement of the parallel circle and the meridian angle (bending deformation) generated by the edge moment M_0 ;

For cylindrical shell 2, the second radius of curvature on the boundary is:

$$R_{20} = R, \quad \varphi_0 = 90^\circ, \quad P^* = N_\varphi \cos \varphi_0 = 0$$

The radial displacement of parallel circle caused by edge force is as follows:

$$\Delta_2^{Q_0} = -\frac{2^4 \sqrt{3(1-\mu^2)}}{Et_2} \sqrt{\frac{R}{t_2}} R Q_0$$

The angle of meridian caused by edge force is as follows:

$$\theta_2^{Q_0} = -\frac{2\sqrt{3(1-\mu^2)}}{Et_2^2} R Q_0$$

The radial displacement of parallel circle caused by edge moment is as follows:

$$\Delta_2^{M_0} = \frac{2\sqrt{3(1-\mu^2)}}{Et_2^2} R M_0$$

The angle of longitude caused by edge moment is:

$$\theta_2^{M_0} = \frac{4[3(1-\mu^2)]^{3/4}}{Et_2^2} \sqrt{\frac{R}{t_2}} M_0$$

For revolution 1, the film deformation is as follows:

$$\Delta_1^P = \frac{P\alpha^2}{2Et_1} \left(2 - \frac{a^2}{b^2} - \mu\right)$$

$$\theta_1^P = 0$$

For revolution 2, the film deformation is as follows:

$$\Delta_2^P = \frac{PR^2}{2Et_2} (2 - \mu)$$

$$\theta_2^P = 0$$

$$\sum \Delta_1 = \Delta_1^P + \Delta_1^{Q_0} + \Delta_1^{M_0}$$

$$\sum \Delta_2 = \Delta_2^P + \Delta_2^{Q_0} + \Delta_2^{M_0}$$

$$\sum \theta_1 = \theta_1^P + \theta_1^{Q_0} + \theta_1^{M_0}$$

$$\sum \theta_2 = \theta_2^P + \theta_2^{Q_0} + \theta_2^{M_0}$$

$$\sum \Delta_1 = \sum \Delta_2$$

$$\sum \theta_1 = \sum \theta_2$$

$$t_1 = t_2,$$

$$\frac{pa^2}{2Et} \left(2 - \frac{a^2}{b^2} - \mu\right) + \frac{2^4 \sqrt{3(1-\mu^2)}}{Et} \sqrt{\frac{R}{t}} RQ_0 + \frac{2\sqrt{3(1-\mu^2)}}{Et^2} RM_0$$

$$= \frac{PR^2}{2Et} (2 - \mu) - \frac{2^4 \sqrt{3(1-\mu^2)}}{Et} \sqrt{\frac{R}{t}} RQ_0 + \frac{2\sqrt{3(1-\mu^2)}}{Et^2} RM_0$$

$$a = R, \quad Q_0 = \frac{P\sqrt{Rt}}{8^4 \sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2$$

$$\begin{aligned} & -\frac{2\sqrt{3(1-\mu^2)}}{Et^2} RQ_0 - \frac{4[3(1-\mu^2)]^{3/4}}{Et^2} \sqrt{\frac{R}{t}} M_0 \\ & = -\frac{2\sqrt{3(1-\mu^2)}}{Et^2} RQ_0 + \frac{4[3(1-\mu^2)]^{3/4}}{Et^2} \sqrt{\frac{R}{t}} M_0 \end{aligned}$$

$$Q_0 = \frac{P\sqrt{Rt}}{8^4 \sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2, \quad M_0 = 0$$

1.2 Solution of theoretical stress in discontinuous region of elliptical head

According to the literature report^[14-15], The internal force under the action of edge force Q_0 and edge moment M_0 , the internal moment is:

$$\begin{aligned} N_{\omega}^{(Q_0+M_0)} &= N_{\omega}^{Q_0} = \tan \omega e^{-k_1 \omega} Q_0 (\cos k_1 \omega - \sin k_1 \omega) \frac{R}{R_2} \\ &= \tan \omega e^{-k_1 \omega} \frac{P\sqrt{Rt}}{8^4 \sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2 (\cos k_1 \omega - \sin k_1 \omega) \frac{R}{R_2} \end{aligned}$$

$$\begin{aligned}
N_{\theta}^{(Q_0+M_0)} &= N_{\theta}^{Q_0} = 2e^{-k_1\omega} Q_0 \frac{\sqrt[4]{3(1-\mu^2)}}{\sqrt{kt}} R \cos k_1\omega \\
&= \frac{1}{4} P \operatorname{Re}^{-k_1\omega} \cos k_1\omega \left(\frac{a}{b}\right)^2 \sqrt{\frac{R}{R_2}} \\
M_{\omega}^{(Q_0+M_0)} &= M_{\omega}^{Q_0} = \frac{e^{-k_1\omega} Q_0 R}{\sqrt[4]{3(1-\mu^2)}} \sqrt{\frac{t}{R_2}} \sin k_1\omega \\
&= \frac{PRte^{-k_1\omega} \sin k_1\omega}{8\sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2 \sqrt{\frac{R}{R_2}}
\end{aligned}$$

In the elliptical head, the radius calculation formula is as follows:

$$\begin{aligned}
R_1 &= \frac{(a^4 y^2 + b^4 x^2)^{3/2}}{a^4 b^4} \\
R_2 &= \frac{(a^4 y^2 + b^4 x^2)^{1/2}}{b^2}
\end{aligned}$$

The internal torque is:

$$M_{\theta}^{(Q_0+M_0)} = M_{\theta}^{Q_0} = \mu M_{\omega}^{Q_0}$$

Total stress in edge area:

$$\begin{aligned}
\sum \sigma_{\omega} &= \sigma_{\omega}^P + \frac{N_{\omega}(Q_0 + M_0)}{t} \pm \frac{6M_{\omega}(Q_0 + M_0)}{t^2} \\
&= \frac{PR_2}{2t} + \frac{P \tan \omega e^{-k_1\omega} \sqrt{Rt} (\cos k_1\omega - \sin k_1\omega) \left(\frac{a}{b}\right)^2 R}{8t^4 \sqrt{3(1-\mu^2)} R_2} \pm \frac{3P \operatorname{Re}^{-k_1\omega} \sin k_1\omega}{4t \sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2 \sqrt{\frac{R}{R_2}} \\
\sum \sigma_{\theta} &= \sigma_{\theta}^P + \frac{N_{\theta}(Q_0 + M_0)}{t} \pm \frac{6M_{\theta}(Q_0 + M_0)}{t^2} \\
&= \frac{PR_2}{2t} \left(2 - \frac{R_2}{R_1}\right) + \frac{P \operatorname{Re}^{-k_1\omega} \cos k_1\omega}{4t} \left(\frac{a}{b}\right)^2 \sqrt{\frac{R}{R_2}} \pm \frac{6PRt \mu e^{-k_1\omega} \sin k_1\omega}{8t^2 \sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2 \sqrt{\frac{R}{R_2}}
\end{aligned}$$

For cylinders,

$$Q_0 = \frac{P\sqrt{Rt}}{8^4 \sqrt{3(1-\mu^2)}} \left(\frac{a}{b}\right)^2, \quad M_0 = 0, \quad k = \sqrt[4]{\frac{3(1-\mu^2)}{R^2 t^2}}$$

The internal force and internal moment acted by edge force Q_0 and edge moment M_0 are as follows:

$$N_x^{Q_0+M_0} = N_x^{Q_0} = 0$$

$$N_{\theta}^{(Q_0+M_0)} = N_{\theta}^{Q_0} = -2kRe^{-kx} [kM_0(\sin kx - \cos kx) + Q_0 \cos kx] = -2kRe^{-kx} Q_0 \cos kx$$

$$M_x^{(Q_0+M_0)} = \frac{e^{-kx}}{2k} [2kM_0(\cos kx + \sin kx) - 2Q_0 \sin kx] = \frac{-e^{-kx}}{k} Q_0 \sin kx$$

$$M_\theta^{(Q_0+M_0)} = \mu M_x^{(Q_0+M_0)}$$

The total stress in the edge zone is as follows:

$$\begin{aligned} \sum \sigma_x &= \sigma_x^P + \frac{N_x(Q_0 + M_0)}{t} \pm \frac{6M_x(Q_0 + M_0)}{t^2} \\ \sum \sigma_\theta &= \sigma_\theta^P + \frac{N_\theta(Q_0 + M_0)}{t} \pm \frac{6M_\theta(Q_0 + M_0)}{t^2} \end{aligned}$$

2 Solution of edge force Q_0 and edge moment M_0 in discontinuous region of elliptical head

In order to verify the correctness of the theoretical stress in the discontinuous area of the elliptical head, the experimental stress of the elliptical head under different pressures was tested with the help of the membrane stress test device of the internal pressure vessel. The accuracy of the theoretical stress and the experimental stress in the discontinuous area of the elliptical head was verified by comparing the errors of the theoretical stress and the experimental stress, so as to provide some reference for the stress distribution of the elliptical head.

2.1 Measurement of experimental stress in discontinuous area of elliptical head

The inner radius R of the elliptical head is 200 mm and the thickness t is 4 mm, $\alpha = 45^\circ$, Poisson's ratio $\mu = 0.3$, $\varphi_0 = 45^\circ$, 0Cr18Ni9, 206GPa. The experimental device is shown in Fig. 2. First, add water to the pressure pump, and through shaking the pressure pump up and down, the pipeline is filled with pressure, and the pressure is transmitted to the horizontal internal pressure experimental vessel, so as to produce stress and strain. The strain is measured by strain gauge. A total of ten strain gauge test points are set. The distance between each test point and the top of elliptical head is shown in Table 1. The test points 1 to 9 are located on the elliptical head, and the test point 10 is located on the cylinder. The distribution of test points is shown in Figure 3.

Fig.2 Experimental device for stress measurement of thin walled vessel with internal pressure

Fig.3 Distribution of test points

Tab. 1 Point distance from the vertex of ellipse head (mm)

Measuring point	1	2	3	4	5	6	7	8	9	10
distance	0	40	80	120	150	170	190	210	230	250

The geometric size of the oval head is kept unchanged, and the internal pressure P is 0.1MPa, 0.2MPa, 0.3MPa, 0.4MPa and 0.5MPa. By changing the internal pressure P , the influence of internal pressure on the longitudinal stress experimental value and circumferential stress experimental value of each test point is examined as shown in Figure 4 and figure 5. It can be seen that for the longitudinal stress, the internal pressure P is kept unchanged, and the value is constant more than 0. For tensile stress, the measuring point 1 is at the vertex, the measuring point 10 is at the equator, from the measuring point 1 to the measuring point 10, the longitudinal stress is decreasing gradually. At the measuring point 1, the meridional stress is the largest, and the longitudinal stress at the measuring point 10 is half of the stress value at the measuring point 1, and the decrease from the measuring point 8 to the measuring point 10 decreases, This is mainly because these measuring points are located in discontinuous region, which not only bear the membrane stress, but also bear the influence of discontinuous stress; For circumferential stress, the change trend is more complex, which can be divided into three stages. The first stage is from measuring point 1 to measuring point 6, and the circumferential stress is greater than 0, which is tensile stress; In the second stage, the circumferential stress is less than 0 from the measuring point 7 to the measuring point 9, which is the compressive stress; In the third stage, the circumferential stress is greater than 0 at the measuring point 10, which is the tensile stress. From the measuring point 7 to the measuring point 10, the stress decreases first and then increases. The reason is the same as the longitudinal stress, and is affected by discontinuous stress. At the measuring point 1, the longitudinal stress and circumferential stress are approximately the same. At the measuring point 6, the circumferential stress changes from tensile stress to compressive stress. This is because for the standard oval head, $a/b=2^{0.5}$, the circumferential stress will appear.

Fig.4 Influence of internal pressure on longitudinal stress experimental value of each test point

Fig.5 Influence of internal pressure on experimental value of circumferential stress at each test point

2.2 Calculation of theoretical stress in discontinuous region of elliptical head

According to the arc length formula, the meridional angles (radian system) of test points 1 to 9 starting from the boundary are 1.57, 1.83, 0.62, 0.60, 0.43, 0.33, 0.23, 0.11 and 0.06 respectively; The test point 10 is distributed on the cylinder. According to the conversion, the distance in the meridional direction from the boundary of the test point 10 is 10 mm. The distance between the measuring point and the edge of the elliptical head is shown in Table 2, so the stress enhancement coefficient of each measuring point is shown in Table 3.

Tab. 2 Distance between measuring point and edge of elliptical head (mm)

Measuring point	1	2	3	4	5	6	7	8	9
R_1	400	382	260	253	185	138	96	61	53
R_2	400	394	347	343	309	281	248	214	204

The stress enhancement coefficient of each measuring point calculated by the above formula is shown in Table 3:

Tab. 3 Stress enhancement coefficient of each measuring point									
Measuring point	1	2	3	4	5	6	7	8	9
K	5142	4875	3117	3015	2086	1491	968	577	491

By substituting the above data into the total stress calculation formula of elliptical head and cylinder edge area, the theoretical meridional stress and theoretical circumferential stress from measuring point 1 to measuring point 10 can be obtained, as shown in Fig. 6 and Fig. 7. It can be seen that the change law of stress is basically consistent with the experimental stress, keeping the internal pressure load unchanged, for the theoretical meridional stress, constant greater than 0, it is tensile stress, and gradually decreases from measuring point 1 (vertex) to measuring point 10 (equator); For the theoretical circumferential stress, the compressive stress appears. At the apex, the theoretical meridional stress is approximately equal to the theoretical circumferential stress.

Fig.6 Influence of internal pressure on theoretical value of longitudinal stress at each test point

Fig.7 Influence of internal pressure on theoretical value of circumferential stress at each test point

2.3 Analysis and comparison of two kinds of elliptical head

In order to further compare the accuracy of the two analysis methods for elliptical head, the internal pressure load of 0.1MPa is taken as an example, and the comparison diagram of theoretical stress and experimental stress is shown in Fig. 8 and Fig. 9. It can be seen that the variation law of the theoretical stress and the experimental stress of the elliptical head is basically the same, which further verifies the correctness and reliability of the theoretical method.

Fig.8 Longitudinal stress under two analysis methods

Fig.9 Circumferential stress under two analysis methods

3 conclusion

1) According to the deformation continuity equation, edge force Q_0 and edge moment M_0 , internal force and internal moment, the theoretical stress calculation formula of discontinuous area of elliptical head is obtained.

2) When the internal pressure load remains unchanged, the theoretical meridional stress is always greater than 0, which is tensile stress and decreases gradually from the apex to the equator; For the theoretical circumferential stress, the change trend is complex, which can be divided into three stages, and the compressive stress appears. At the apex, the meridional stress and circumferential stress are approximately equal; The change from point 8 to point 10 is affected by discontinuous stress, and the change trend is abrupt.

3) By comparing and analyzing the theoretical stress and experimental stress in the discontinuous area of elliptical head, the accuracy and applicability of the theoretical stress calculation formula in the discontinuous area of elliptical head and cylinder are further verified.

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