Improving Hybrid Models For Precipitation Forecasting By Combining Nonlinear Machine Learning Methods

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Improving hybrid models for precipitation forecasting by combining nonlinear machine learning methods

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Abstract

Precipitation forecast, especially on monthly and annual scales, is a key for optimal water resources management and planning, especially in semiarid climates with scarce water. The traditional hybrid models, in which two statistical models are used to separate and simulate linear and nonlinear components of precipitation time series, are still unable to provide accurate precipitation forecasts. This research aims to improve hybrid forecast models by combining one linear model and three nonlinear models with two preprocessing configurations: 1) using residuals of a linear model, representing the nonlinear component with different time steps and 2) using original time series of observations with different time steps, linear model simulations and residuals. Gene Expression Programming (GEP), Support Vector Regression (SVR) and Group Method of Data Handling (GMDH) models were used individually as in the traditional hybrid models and combinedly as in the proposed hybrid models in this study. The performance of the hybrid models was improved by different methods such as inverse variance (Iv) as an error-based method, least square regression, genetic algorithm and SVR. Two weather stations of Tabriz (annual) and Rasht (monthly) in Iran were selected to test the developed models. The results showed that Theil's coefficient, UII, decreased in configuration one for the Tabriz station by 9% and 15% for SVR and GMDH relative to GEP, suggesting that these two models performed better than GEP in the precipitation forecast. The error criteria used in developing the proposed hybrid models with all forecast combination methods better represent observations than the hybrid model. MSE decreased by 67% and Nash Sutcliffe increased by 5% in the Rasht station in configuration two when we combined the three models using GA to obtain the improved hybrid model relative to the hybrid model combined with SVR. Generally, the hybrid models when SVR, the error based methods and GA were incorporated showed better performance than traditional hybrid models. The developed models have implications for modeling highly nonlinear systems using full advantages of machine learning methods.

Keywords: Improved hybrid model, Precipitation, Forecast combination, Support vector regression

1. Introduction

Accurate precipitation forecast remains challenging despite enormous attempts to improve weather and climate models (Adhikari and Agrawal 2014), which is partly due to its complex nature. Stochastic models were extensively used for precipitation forecasting. The models such as autoregressive integrated moving average (ARIMA), seasonal ARIMA (SARIMA), vector autoregression (VAR) and generalized autoregressive conditional heteroskedasticity (GARCH) are among the common stochastic models (Wang et al., 2013; Papalaskaris et al., 2016; Murthy et al., 2018; Bouznad et al., 2020; Nugroho et al., 2014; Mehdizadeh et al., 2017). Despite the popularity of these models, their applications in representing a nonlinear characteristic of precipitation were not successful over the past. It is expected that the linear models not to be able to capture complex phenomena (Chen and Wang 2007). Alternatively, some machine learning techniques such as support vector regression (SVR) were used to solve nonlinear problems such as precipitation forecasting (Hamidi et al., 2015; Shenify et al., 2016), streamflow forecasting (Rasouli et al., 2012; 2020), freshwater-saltwater interface fluctuations prediction (Yoon et al., 2017) and groundwater level estimation (Moravej et al., 2020). The machine learning techniques also require a rigorous search algorithm to obtain optimal hyperparameters (Zeynoddin et al., 2018). According to the strengths of both linear and nonlinear models, it is reasonable to use a combination of models for forecasting problems. In this regard, hybrid models were introduced in the literature to assess the forecast accuracy compared to individual models (Chen and Wang 2007). A hybrid model, combining linear and nonlinear components of a complex process such as precipitation, can perform better than each of the composing models. In hybrid systems, nonlinear components can be obtained from the difference of observed and linear components, so called error series or residual time series (Chen et al., 2020). A combination of SARIMA and support vector machines (SVM) decreased mean absolute percentage error (MAPE) by 43% relative to the case when only SARIMA was used and by 32% relative to the case when only SVM was used in forecasting the production values of machinery industry (Chen and Wang, 2007). Another example of hybrid models was ARIMA and artificial neural network (ANN) models for particulate matter forecasting in urban areas (Diaz-Robles et al., 2008). A combination of SARIMA and ANN models was used to forecast monthly inflow, and the combined model had a high coefficient of determination relative to these two composing models (Moeeni et al., 2017). Another hybrid model was developed based on ARIMA coupled with ANN using GA to forecast the production value of

the mechanical industry (Liang 2009). GA was used to optimize the ANN parameters, such as number of hidden neurons. The improvement of traditional hybrid model, especially in nonlinear part of time series, can significantly affect the accuracy of forecasts. Some decomposed time series from observed time series are often highly nonlinear and the use of a single nonlinear model for accurate forecasting of those time series is challenging. Also, forecasting residuals with large errors make the entire forecast series problematic (Mo et al., 2018). Therefore, there is a need to combine the forecast of nonlinear time series and avoid the large errors in some of the subseries by giving less weights to them. For example, the group method of data handling (GMDH) neural network was used to combine the nonlinear models (e.g., SVR, back-propagation (BP) neural network and genetic programming (GP)). The performance of the more recently improved hybrid models was better than individual models such as SARIMA or previously used hybrid models such as SARIMA-SVR and SARIMA-BP (Mo et al., 2018). Another approach to couple nonlinear time series is to use a weighted combination of the forecasts. In this method, the combination is based on assigning proper weights to the single models to better extract the right information from the time series with the right individual models (Song and Fu 2020). One of the important challenges related to forecasting combination is the precise selection of weights (Wang et al., 2019), which affects the accuracy of the forecasts. The simple average (Timmermann 2006), the error-based methods (Adhikari and Agrawal 2014, Song and Fu 2020), the least square regression method (Frietas and Rodrigues 2006), and the differential weighting methods (Winkler and Makridakis 1983a; Chen et al., 2004) are a few examples of weighted combination methods. The main purpose of this research is to improve a hybrid model in the field of precipitation forecasting with emphasis on the nonlinear part of time series (in two time steps: annual and monthly). ARIMA and SARIMA were used for modeling the linear part of the time series. Individual models such as SVR, GMDH and GEP were applied to model nonlinear patterns of time series. To better capture the complexity of nonlinear patterns of precipitation, instead of one single model, the results of three single models were combined. The error-based methods, the least square regression method, optimization with GA, artificial intelligence techniques such as SVR and ANN were used to combine the three individual models.

2. Materials and Methods

2.1. Case Study

To evaluate the performance of the proposed improved hybrid model, precipitation data were used in the Rasht and Tabriz weather stations in the Gilan and East Azerbaijan provinces in Iran with humid and semiarid climates. The mean annual precipitation is 279 mm in the Tabriz station and 1278 mm in the Rasht station. The coefficient of variation for precipitation is 0.25 and 0.2 in Tabriz and Rasht, respectively. Based on the climograph of these stations, monthly precipitation is more variable in the Rasht station than Tabriz station (Figures 1b and d). The climate of Rasht and Tabriz regions is very humid and semi-arid based on De Martonne climate classification (De Martonne, 1925), wet and cold dry based on Emberger climate classification (Emberger, 1952).

Figure 1: (a) Position and (b) climograph of Tabriz weather station in East Azerbaijan province and (c) position and (d) climograph of Rasht weather station in Gilan province in Iran over the 1992-2019 period.

2.2. Traditional hybrid models based on two linear and nonlinear models

One of the hypotheses related to the time series structure is the separation of time series into two linear and nonlinear components, as in Eq. 1 (Chen and Wang, 2007). The performance of the hybrid model depends on the estimated residuals, which are obtained from the difference between observations and linear components estimated by the linear model (Eq. 2). The nonlinear

component, therefore, is estimated with different models and configurations. Finally, the forecasted time series are obtained by summing estimated linear and nonlinear components (Zhang 2003).

$$y_t = L_t + N_t \quad (1)$$

$$e_t = y_t - \hat{L}_t \quad (2)$$

where L_t is the linear component, N_t denotes the nonlinear component, e_t is the error term or the residual component. The linear part of the time series can be modeled with ARIMA or SARIMA, while the modeling of the nonlinear component can be done with machine learning techniques such as SVR or ANN (Figure 2).

2.2.1 Modeling the linear component of time series

The forecasting processes of stochastic models such as ARMA, ARIMA, and SARIMA are based on the Box-Jenkins methodology (1976). The SARIMA model is an extension of the ARIMA model to represent seasonal variations. ARIMA and SARIMA models can be expressed as in Eqs. 3 and 4.

$$a_p(B)(1-B)^d Y_t = \theta_0 + e_q(B)r_t \quad (3)$$

$$a_p(B)A_p(B^s)(1-B)^d(1-B^s)^D Y_t = \theta_0 + e_q(B)E_Q(B^s)r_t \quad (4)$$

where $a_p(B)$, $e_q(B)$ and $A_p(B^s)$ and $E_Q(B^s)$ are polynomial with orders p , q and P , Q for autoregressive and moving average terms, non-seasonal and seasonal terms, respectively. $(1-B)$ and $(1-B^s)$ are the regular and seasonal differencing operators, respectively. d is the number of differencing operations, D is the number of seasonal differencing. Y_t is the observed value at time t , θ_0 is a fixed term, and r_t is a random error (Nwokike et al., 2020).

2.2.2 Individual models for modeling nonlinear components of time series

Support Vector Machine

SVM is a machine learning methodology (Vapnik, 1995), which is widely used in classification problems, regression estimation, pattern recognition and probability density function estimation. The general form of SVM for regressions is expressed as:

$$f(x) = \omega \times \varphi(x) + b \quad (5)$$

where $f(x)$ is the fitted function, ω is the weight of the input vector, $\varphi(x)$ is the map or kernel function to transform x_i to a high-dimensional feature space, and b is an adjustable factor. The penalty parameter in SVM can control the tolerance of the systematic outliers (Liu et al., 2014). One step in training a SVR model is to select the best kernel functions. Usually, radial basis and sigmoid functions show better performance than linear kernels (Xu et al., 2019; Zhu et al., 2018).

Figure 2: The structure of hybrid model with linear and nonlinear components and improved hybrid model with combined weighted forecasts instead of using one single model in nonlinear component.

Group Method of Data Handling

GMDH, or polynomial neural network, is a self-organization technique, which is used for simulating complex systems (Jeddi and Sharifian, 2020; Mo et al., 2018). The model has a set of

neurons in which different pairs in each layer are connected through a polynomial and in each layer, new neurons are built up for the next layer. The complicated discrete form of the Volterra functional series (Volterra, 1959), such as in Eq. 6 were applied for general connection between the input and output variables which is also known as the Kolmogorov-Gabor (K-G) polynomial (Mo et al., 2018).

For modeling nonlinear input/output relationships, the Volterra series can be used (Marzocca et al., 2008).

$$y = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots \quad (6)$$

where y is output variables, x is input variables, a is a coefficient.

Gene Expression Programming

GEP, which was introduced as a new function model mining algorithm by Ferreira (2001), was based on Darwin's theory of evolution. GEP superiority to GA and GP is its convergence speed and capability in solving complex problems. Chromosomes and expression trees are the main components of GEP. The start of GEP is based on the random generation of chromosomes that form one or more genes in the name of sub-expression tree (sub-ET). The best solution is to link the sub-ETs through algebraic or Boolean (AND, OR, NOT) functions. Each sub-ET is a part of the solution with less complexity and provides information about the given process less than the corresponding expression tree. Finally, the power of GP blocks is applied with GEP to model nonlinear pattern of a complex system through a multi gene structure (Deng et al., 2018- Yousefi et al., 2017; DanandehMehr, 2018).

2.3. Improved hybrid models based on one linear model and a combination of nonlinear models

One approach to increase the efficiency of the traditional hybrid models in simulating the nonlinear component is to utilize a combination of individual models to forecast residuals instead of using a single model (Mo et al., 2018). In this study, GEP, SVR and GMDH were used to model the residuals with two configurations, composed of residual subseries with different antecedent time steps ($t-1$), ($t-2$), ($t-3$) or composed of linear component ($\hat{L}_t$), original observed precipitation time series with antecedent time steps (y_{t-1} or y_{t-12}) and residual subseries with different time steps, and

the forecasts of mentioned models were combined by different strategies such as the least square regression method to obtain a weighted combination of the three individual models. The steps of the improved hybrid model structure proposed in this study include (Figure 2):

1) Stochastic models (ARIMA for annual and SARIMA for monthly time series) are applied to model linear patterns of the precipitation time series ($\hat{L}_t$).

2) Calculating error or residual subseries (e_t) based on Eq. 2 using observed values (y_t) and estimated linear time series in step 1 ($\hat{L}_t$).

3) Determining two configurations to model residual subseries for annual (y_{an}) and monthly (y_{mon}) precipitation. In these configurations, ε_t denotes the random error.

- Configuration one based on residual subseries with different antecedent time steps ($t-1$), ($t-2$), ($t-3$):

$$e_t = f(e_{t-1}, e_{t-2}, e_{t-3}) + \varepsilon_t$$

- Configuration two based on linear component ($\hat{L}_t$), original observed time series with antecedent time steps (y_{t-1} or y_{t-12}) and residual subseries with different antecedent time intervals from step 1:

$$y_{t-an} = f(y_{t-1}, y_{t-2}, \hat{L}_t, e_{t-1}, e_t) + \varepsilon_t$$

$$y_{t-mon} = f(y_{t-1}, y_{t-12}, \hat{L}_t, e_{t-1}, e_t) + \varepsilon_t$$

4) Forecasting residual subseries with two configurations in step 3 from each of GEP, SVR and GMDH nonlinear models. In the configurations with residual subseries, e_t , as the output, it is necessary to sum up the estimated linear and nonlinear components.

5) Obtaining weighted forecasts $\hat{y}_t^{n, c}$: We combine the forecasts of the three nonlinear models with the most reasonable weights (Song and Fu 2020). The general structure of weighted forecasts is expressed in Eq. 7:

$$\hat{y}_t^{n, c} = \sum_{i=1}^m w_i \cdot \hat{y}_t^{n, i, c}$$

$$n = \begin{pmatrix} 1 & \text{Monthly} \\ 2 & \text{Annual} \end{pmatrix}, \quad i = \begin{pmatrix} 1 & \text{GEP} \\ 2 & \text{SVR} \\ 3 & \text{GMDH} \end{pmatrix}, \quad c = \begin{pmatrix} \text{Configuration 1} \\ \text{Configuration 2} \end{pmatrix} \quad (7)$$

223

224 where w_i is the weight coefficient of the i th individual model and m is the number of models
 225 ($\sum_{i=1}^m w_i = 1$).

226

227 The inverse variance method, Iv, as in Eq. 8, is one of the weighted combining methods that are
 228 based on the inversion of the forecast errors of the corresponding models. Another method is the
 229 mean square error inverse, MSEI, as in Eq. 9, which can be categorized as an error-based method
 230 (Adhikari and Agrawal, 2014; Song and Fu, 2020). The forecast errors (Eqs. 8 and 9) are calculated
 231 by obtaining the sum of squared error (SSE) in Eq. 8 and symmetric mean absolute percentage
 232 error (SMAPE) in Eq. 9, which are defined as in Eqs. 10-11.

233

$$w_i = \frac{e_i^{-1}}{\sum_{i=1}^m e_i^{-1}} \quad (8)$$

$$w_i = \frac{e_i^{-\frac{1}{2}}}{\sum_{i=1}^m e_i^{-\frac{1}{2}}} \quad (9)$$

$$e_i = SSE = \sum_{t=1}^n (y_t - \hat{y}_t^{n,i,c})^2, \quad i = 1, 2, \dots, m \quad (10)$$

$$e_i = SMAPE = \frac{1}{N} \sum_{t=1}^n \frac{|y_t - \hat{y}_t^{n,i,c}|}{\frac{y_t + \hat{y}_t^{n,i,c}}{2}}, \quad i = 1, 2, \dots, m \quad (11)$$

238

239 The least-square regression (LSR) method was used in this study to minimize the sum of squared
 240 error for a linear combination of weighted forecasts. The matrix form of Eq. 7 can be written as in
 241 Eq. 12. Therefore, sum of squared error between observed and simulated ($y_i, \hat{y}_i$) values can be
 242 described with Eq. 13. At the end, the weights vector can be obtained by minimizing SSE as in Eq.
 243 14 (Frietas and Rodrigues, 2006; Adhikari and Agrawal, 2014).

$$\hat{Y} = UW \quad U = \begin{bmatrix} \hat{y}_1^1 & \hat{y}_1^2 & \dots & \hat{y}_1^m \\ \hat{y}_2^1 & \hat{y}_2^2 & \dots & \hat{y}_2^m \\ \vdots & \vdots & \ddots & \vdots \\ \hat{y}_n^1 & \hat{y}_n^2 & \dots & \hat{y}_n^m \end{bmatrix}, w = [w_1, w_2, \dots, w_m]^T \quad (12)$$

$$SSE = Y^T Y - 2W^T U^T Y + W^T U^T U W \quad (13)$$

$$W = (U^T U)^{-1} U^T Y \quad (14)$$

247

248 The GA optimization method was used to find the weights so that the error between observed and
 249 simulated precipitation was minimized. The objective function and constraints were expressed as
 250 below (Prudêncio and Ludermir, 2006).

251

Minimize:

$$error(\hat{y}_t^{n,c}) = \sum_{t=1}^n (y_t - \hat{y}_t) = \sum_{t=1}^n (y_t - \sum_{i=1}^m (w_i \cdot \hat{y}_t^{n,i,c})) \quad (15)$$

Subject to:

$$\sum_{i=1}^m w_i = 1 \quad \text{and} \quad w_i \geq 0 \quad (16)$$

254

255 Additionally, precipitation values forecasted by GEP, SVR and GMDH were used as inputs to the
 256 SVR model to improve the forecasts (Eq. 17) further. A sensitivity analysis was carried out based
 257 on the type of kernel function and penalty parameter.

258

$$\hat{y}_t^{n,c} = f_{SVR}^{(kernel-penalty)}(\hat{y}_t^{n,1,c}, \hat{y}_t^{n,2,c}, \dots, \hat{y}_t^{n,m,c}) \quad (17)$$

260

261 An ANN model was also used to improve the forecasts by GEP, SVR and GMDH models (Eq.
 262 18). A sensitivity analysis was carried out to obtain optimal number of neurons in the hidden layer
 263 and type of activation function.

$$\hat{y}_t^{n,c} = f_{ANN}^{(neuron,activation\ function)}(\hat{y}_t^{n,1,c}, \hat{y}_t^{n,2,c}, \dots, \hat{y}_t^{n,m,c}) \quad (18)$$

2.4. Performance evaluation of the improved hybrid models

In order to assess the performance of the model, evaluation metrics can be used which some of them are used in this study to compare the performance of hybrid and improved hybrid models. The used criteria or metrics are mean square error (RMSE), relative root mean square error (RRMSE), mean absolute error (MAE), mean square error (MSE), Theil's coefficient of UI, UII, residual predictive deviation (RPD), APB, modified index of agreement (d_m), adapted mean absolute percentages error (AMAPE), accuracy improved (AI) and geometric mean error ratio (GMER).

$$RMSE = \frac{1}{N} \sqrt{\sum_{i=1}^N (O_i - P_i)^2} \quad (19)$$

$$RRMSE = \frac{RMSE}{\bar{O}} \quad (20)$$

$$MSE = \frac{\sum_{i=1}^N (P_i - O_i)^2}{N} \quad (21)$$

$$UI = \frac{\left[\sum_{i=1}^N (P_i - O_i)^2 \right]^{0.5}}{\left[\sum_{i=1}^N (O_i)^2 \right]^{0.5} + \left[\sum_{i=1}^N (P_i)^2 \right]^{0.5}} \quad (22)$$

$$UII = \frac{\left[\sum_{i=1}^N (P_i - O_i)^2 \right]^{0.5}}{\left[\sum_{i=1}^N (O_i)^2 \right]^{0.5}} \quad (23)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N \left| \frac{P_i - O_i}{O_i} \right| \quad (24)$$

$$RPD = \frac{SD}{RMSE} \quad (25)$$

$$NSE = 1 - \frac{\sum_{i=1}^n (P_i - O_i)^2}{\sum_{i=1}^n (O_i - \bar{O})^2} \quad (26)$$

$$AMAPE = \frac{1}{n} \sum_{i=1}^n \left(\frac{|P_i - O_i|}{\frac{1}{n} \sum_{i=1}^n O_i} \right) \times 100 \quad (27)$$

$$APB = \frac{\sum_{i=1}^n |P_i - O_i|}{\sum_{i=1}^n O_i} \times 100 \quad (28)$$

$$d_m = 1 - \frac{\sum_{i=1}^n |P_i - O_i|}{\sum_{i=1}^n (|P_i - \bar{O}| + |O_i - \bar{O}|)} \quad (29)$$

$$AI = \frac{S - S_h}{S} \times 100 \quad (30)$$

$$GMER = \exp\left(\frac{1}{n} \sum_{i=1}^n \ln\left(\frac{P_i}{O_i}\right)\right) \quad (31)$$

where P_i is the forecasted value, O_i is the observed value, $\bar{O}$ is the mean of observed value, S is MAE of the hybrid model, S_h is MAE of the improved hybrid model. Minimum values of $RMSE$, $RRMSE$, MAE , MSE , APB and $AMAPE$ indicate the similarity between forecasted and observed values (Ruiz-Aguilar et al., 2014; del Carmen Bas et al., 2017; DanandehMehr, 2018). Theil's coefficient (Theil, 1966; 1961) were used for evaluating models forecasting accuracy (UI) and quality of forecasting (UII). $UI=0$ and $UII=0$ are indicative of perfect forecasts (Zeynoddin et al., 2018). AI greater than 0 indicates the superior performance of the hybrid model, whereas AI less or equal to 0 shows that the hybrid model does not outperform any individual model (Chen and Zhu 2013). The high NSE indicates the good performance of a model (DanandehMehr 2018) and its value is one for a perfect model. The RPD values are less than 1.4 and greater than two show poor and high performance, respectively. The range of d_m is between 0 and 1, where 1 is an indicative of high performance of the model (Ruiz-Aguilar et al., 2014; Duveiller et al., 2016). $GMER > 1$ indicates overestimation and $GMER < 1$ shows underestimation (Abdelbaki 2016).

301

302 3. Results

303 The observation period for precipitation data was 1964-2019 for annual and 2000-2019 for
304 monthly time series. The investigated dataset was split into three subsets, 80% for calibration, 10%
305 for validation, and 10% for verification. In order to check the sufficiency of data length for
306 modeling, the Hurst coefficient (Hurst et al., 1965) was determined 0.8 and 0.6 in Tabriz and Rasht,
307 respectively (when the Hurst coefficient is more than 0.5 data length is sufficient for modeling).

308

309

3.1. Modeling linear component of precipitation time series

ARIMA and SARIMA models were applied as linear models for annual and monthly time series, respectively. The presence of seasonality was checked and assured for SARIMA model application in precipitation time series for Rasht station using autocorrelation function and partial autocorrelation function. The autocorrelation function had intervals of 12, clearly proving that there is a seasonal cycle in the Rasht precipitation. Thus, seasonal differencing in SARIMA model is required. The results of preprocessing steps in precipitation time series analysis were shown in Table 1. The normality tests of Kolmogorov-Smirnov (Kolmogorov,1933) and Shapiro-Wilk tests (Shapiro and Wilk, 1965) confirmed the normality of the Tabriz precipitation time series. The significance level of the normality tests was less than the critical value for the Rasht precipitation time series. Therefore, a transformation of the time series was necessary. After transformation, the significance level of Kolmogorov-Smirnov and Shapiro-Wilk tests improved to reach 0.2 and 0.167, respectively. Furthermore, the Augmented Dickey-Fuller (ADF) test (Said and Dickey 1984) was used to check the stationarity of the time series. The statistics t_α was greater than the critical values (-3.6 and -2.57 for Tabriz and Rasht at 1% level), suggesting that both time series were non-stationary (Table 1). The Mann-Kendall test (seasonal Mann-Kendall test for Rasht time series) showed a significant decreasing trend in precipitation observed in both stations. The ADF test was checked once again after applying for differencing order on time series and the statistics t_α was recalculated and found to be less than the critical values and hence, the precipitation values to become stationarity. To find the orders of models, some range of parameters order is examined, ARIMA and SARIMA: $a \in \{0, 1, 2, 3, 4, 5, 6\}$, $e \in \{0, 1, 2, 3, 4, 5\}$ and $a, A, e, E \in \{0, 1, 2, 3\}$.

Out of many optional models, SARIMA(0,1,1) $\times$ (0,1,1)₁₂ and ARIMA(0,1,2) had the minimum values of SBC and BIC for Rasht and Tabriz precipitation (Table1). The SBC decreased from ARIMA(4,1,1) to ARIMA(0,1,2) by 6.6% and from SARIMA(1,1,0)(0,1,0)₁₂ to SARIMA(0,1,2)(0,1,1)₁₂ by 28.5%. The significance levels of the Shapiro-Wilk test for Rasht and Tabriz residuals were 0.8 and 0.64, which were indicative of normal residuals.

Table 1: The results of stochastic modeling using different Box-Jenkins steps for each precipitation time series

Station		Tabriz	Rasht
Normality Test	Kolmogorov-Smirnov	0.157	0.000
	Shapiro-Wilk	0.399	0.000
Stationarity test	ADF	-1.87	-0.34
Identification of model	Model	ARIMA(0,1,2)	SARIMA (0,1,1)×(0,1,1) ₁₂
	SBC	347.31	456.88
	BIC	8.47	2.55
	Parameters	e ₁ =0.64, e ₂ =0.5	e ₁ =0.96, E ₁₂ =0.9
Diagnostics	P-value	0.002, 0.017	0.0001, 0.0001
	P-value	0.497	0.6

3.2. Modeling nonlinear component of precipitation time series

In this study, different models (e.g., SVR, GEP, and GMDH) and two residual configurations were used to model the nonlinear component of the time series. A proper parameter estimation has an important role in the accuracy of precipitation forecasts. Therefore, a sensitivity analysis was conducted to identify the optimal parameters and kernel function types (e.g., Figure 3). The parameters of SVR, GEP and GMDH were summarized in Table 2, which were found from the sensitivity analysis. Some of GEP model parameters include addition (+), mutation rate (0.06), one-point recombination rate (0.2), two-point recombination rate (0.3), gene recombination rate (0.2) and IS transposition rate (0.2).

Figure 3: Sensitivity analysis of support vector regression model for two time series related to kernel function and penalty parameter

Table 2: Estimated parameters for each of the individual models of Gene Expression Programming (GEP), Support Vector Regression (SVR) and Group Method of Data Handling (GMDH)

model	Parameters	Tabriz		Rasht	
		Configuration			
		First	Second	First	Second
GEP	Number of generations	1000	100	100	80
	Chromosome length	50	50	50	50
	Head size	8	7	7	7
	Number of genes	4	3	3	3
SVR	Kernel function	radial basis function	sigmoid function	linear function	radial basis function
	Penalty parameter	0.5	1	0.5	2
GMDH	Maximum number of neurons in a layer	5	10	10	10
	Maximum number of layers	5	7	5	5
	Selection pressure	0.6	0.5	0.6	0.1

RMSE decreased by 15% when we switched the kernel function from sigmoid to radial basis function with $C = 0.5$ and by 8.8% when we changed the C parameter of the radial basis function from 2 to 0.5 in the Tabriz station and for the configuration one. Another comparison was

conducted to show the performance of single individual models with two residual configurations (Figure 4).

Figure 4: Comparison of the performance of two configurations for the nonlinear component of time series with single individual models and error criteria of a) Theil's coefficient of UI and UII and b) RMSE and MAE. T1, T2, R1 and R2 denote Tabriz and Rasht station for the two configurations (Table 2).

As shown in Figure 4, the error criteria decreased from configuration one to two. For instance, Theil's coefficient of UII decreased by 56% for GEP and 62% for GMDH from configuration one to two in Rasht. By considering all models and all error criteria the average percent decreasing in Tabriz was greater than Rasht (46%). The error criteria increased by using GEP model. For instance, MAE decreased by 5% when GEP was replaced by SVR and by 24% when GEP was replaced by GMDH in configuration 1 in Tabriz. In Rasht, UI decreased by 42% when GEP was replaced by SVR and by 25% when GEP was replaced by GMDH in configuration two. In general, the performance of GMDH and SVR for Tabriz and Rasht precipitation forecasting was reasonably high, and the difference between the error values of SVR and GMDH for Rasht time series in configuration one is low.

3.3. Improved hybrid models

The main advantage of the improved hybrid models to traditional hybrid models is to combine forecasts from multiple models instead of using one single model in nonlinear part of time series. The performance of improved hybrid models with different combinations and weighting methods was compared against the hybrid model, composed of one linear and only one nonlinear model (Table 3 and Figure 5).

Table 3: Comparison of the hybrid and improved hybrid models. Three Gene Expression Programming (GEP), Support Vector Regression (SVR) and Group Method of Data Handling (GMDH) were combined using four error based (Iv+SSE, MSEI+SSE, Iv+SMAPE, MSEI+SMAPE) and four machine learning based methods (SVR, GA, LSR and ANN) for configuration two.

Model type	Precipitation time series	Structure of hybrid model	RMSE	RRMSE	MSE	RPD	APB	dm	UII	AMPE	NSE
Hybrid	Tabriz	ARIMA+SVR	11.88	0.041	141.21	4.5	0.034	0.86	0.04	40.83	0.94
		ARIMA+GEP	17.59	0.061	309.6	3.05	0.052	0.77	0.06	40.85	0.87
		ARIMA+GMDH	9.87	0.034	97.49	5.45	0.028	0.89	0.034	40.83	0.95
Improved hybrid: Combination of nonlinear models	Tabriz	Iv+SSE	8.81	0.03	77.67	6.1	0.023	0.9	0.03	40.82	0.96
		MSEI+SSE	7.36	0.025	54.21	7.3	0.02	0.91	0.025	40.82	0.97
		Iv+SMAPE	7.73	0.027	59.9	6.95	0.023	0.9	0.026	40.82	0.97
		MSEI+ SMAPE	8.91	0.031	79.46	6.03	0.024	0.89	0.03	40.82	0.96
		SVR	5.57	0.019	31.03	9.6	0.017	0.93	0.019	40.81	0.98
		GA	9.39	0.032	88.31	5.7	0.025	0.89	0.032	40.82	0.96
		LSR	9.01	0.031	81.2	5.97	0.025	0.89	0.031	40.82	0.96
		ANN	7.64	0.026	58.43	7.04	0.02	0.91	0.026	40.82	0.97
Hybrid	Rasht	SARIMA+SVR	20.02	0.187	400.9	3.77	0.142	0.97	0.15	4.58	0.92
		SARIMA+GEP	30.51	0.28	931.3	2.47	0.19	0.94	0.23	4.64	0.82
		SARIMA+GMDH	24.22	0.22	586.7	3.11	0.18	0.96	0.18	4.63	0.89
Improved hybrid: Combination of nonlinear models	Rasht	Iv+SSE	15.92	0.149	253.4	4.74	0.11	0.98	0.12	4.56	0.95
		MSEI+SSE	17.2	0.16	296.17	4.39	0.12	0.98	0.13	4.56	0.94
		Iv+SMAPE	16.52	0.154	272.9	4.57	0.11	0.98	0.12	4.55	0.95
		MSEI+ SMAPE	18.08	0.169	326.9	4.17	0.13	0.98	0.13	4.57	0.94
		SVR	13.03	0.12	177.56	5.66	0.091	0.99	0.1	4.53	0.96
		GA	11.48	0.1	131.84	6.57	0.078	0.99	0.088	4.52	0.97
		LSR	14.37	0.13	206.55	5.26	0.11	0.98	0.11	4.55	0.96
		ANN	17.7	0.16	294.99	4.39	0.1	0.98	0.13	4.54	0.94

Figure 5: Performance of the hybrid models for configuration one and the Rasht and Tabriz stations

As illustrated in Figure 5, the error criteria degraded for the hybrid model using GEP as a single model (ARIMA+GEP in Table 3), relative to other single nonlinear hybrid models or combined nonlinear hybrid models. The forecast accuracy was improved when the nonlinear models were combined in the form of the hybrid model. For instance, in configuration two for Tabriz station, RMSE improved by 5 to 43% when the ARIMA+GMDH hybrid model was replaced by the proposed multi-nonlinear hybrid models such as Iv+SSE, MSEI+SSE, Iv+SMAPE, MSEI+SMAPE, SVR, GA, LSR and ANN (Table 3). The three improved hybrid models with the highest performance in different stations and configurations are those combined with: 1) Iv+SMAPE, GA

and SVR methods for Tabriz station and configuration one; 2) SVR, MSEI+SSE and ANN methods for Tabriz station and configuration two; 3) SVR, MSEI+SMAPE and GA for Rasht station and configuration one; and 4) GA, SVR and LSR methods for Rasht station and configuration two.

In general, the hybrid models combined with SVR, GA, and the error-based methods performed better than the other methods. The average RPD criteria were 0.42 and 4.33 in the traditional hybrid model combined with one linear and one single model for Tabriz series in configurations one and two. These values were 1.17 and 3.11 for the Rasht station. The average of RPD criteria in proposed hybrid models combined with one linear and multiple nonlinear methods were improved and reached 0.58 and 6.83 for Tabriz station and 1.26 and 4.9 for Rasht station in configuration one (Figure 5) and two (Table 3). The RPD values of Rasht and Tabriz precipitation time series in configuration two are in the high range in terms of model performance (Table 3).

For evaluating the performance of hybrid models combined with multiple nonlinear methods, an AI criterion (Eq. 30) was calculated for the three best models. This calculation was compared with the best model of the traditional hybrid model. AI of Tabriz precipitation in configuration one for the hybrid models combined with Iv+SMAPE, GA and SVR methods was 44%, 20% and 19.3%, respectively. AI in configuration two for the hybrid models combined with SVR, MSEI+SSE and ANN methods was 41%, 29% and 27%, respectively. AI of Rasht time series in configuration one for the hybrid model combined with SVR, MSEI+SMAPE and GA methods was 6.7%, 2.9% and 2.7% respectively. In configuration two, AI was 44%, 35% and 19%, respectively for the hybrid models combined with GA, SVR and LSR methods. Like other performance criteria, AI values for hybrid models combined with different methods were improved compared with other traditional hybrid models. Among the error-based combinations of three nonlinear methods, Iv+SMAPE and MSEI+SMAPE performed the best for Tabriz and Rasht stations for configuration one and MSEI+SSE and Iv+SSE for configuration two. The observed and forecasted precipitation series were shown in Figure 6 for the verification period and traditional and proposed hybrid models.

Figure 6: Comparison of observed and forecasted precipitation by 11 hybrid models for configuration 2 for the (a) Tabriz and (b) Rasht weather stations in Iran in the verification period

The maximum and minimum annual precipitation were observed in 2018 and 2017 in Tabriz (Figure 6a), which were preserved with all improved hybrid models combined with multiple error based and other combination methods. The observed monthly average of precipitation was 107 mm in Rasht, which was close to the monthly average forecasted by the hybrid model combined with GA (110 mm). The mean annual precipitation observed was 1279 mm in Rasht and its associated forecasted value was 1317 mm according to the hybrid model combined with GA (Figure 6b). The minimum precipitation was observed in June 2019 in Rasht, and again it was preserved in all hybrid models, except for the one combined with the LSR method. The scatter plots of the two best improved hybrid models with the highest performance were illustrated in

Figure 7 and compared against observations and the conventional hybrid methods of ARIMA+GMDH and SARIMA+SVR.

Figure 7: A scatter plot of observations against forecasted precipitation values in the (a) Tabriz and (b) Rasht weather stations with traditional hybrid models (ARIMA+GMDH and SARIMA+SVR) and two weighted hybrid models combined using support vector regression (SVR) and genetic algorithm (GA).

The R-square of the lines fitted to ARIMA+GMDH as a hybrid model and SVR and MSEI+SSE as improved hybrid model outputs were respectively 0.96, 0.99 and 0.96 in Tabriz and the R-square of the lines fitted to SARIMA+SVR as a hybrid model, and SVR and GA as improved hybrid models were respectively 0.87, 0.99 and 0.95 in Rasht. The R-square of the fitted line was increased using improved hybrid models 3 to 14%.

Figure 8: Boxplot of observed and forecasted precipitation in the (a) Tabriz and (b) Rasht weather stations with conventional hybrid model (ARIMA+GMDH) and four weighted hybrid models combined using an error based MESI+SSE method, support vector regression (SVR), genetic algorithm (GA) and least square regression (LSR)

The forecasts of the hybrid (e.g., ARIMA+GMDH) and improved hybrid models (e.g., MESI+SSE, SVR, GA, and LSR) were compared in Figure 8. These plots showed that observed precipitation matches with forecasts by the improved hybrid models (Figure 8). In Tabriz, the variability of precipitation values forecasted by the hybrid model combined with SVR was similar to the spread of observations. From the point of lower and upper whisker view, the high similarity was observed between observed and the hybrid model combined with SVR. The median box-plot

of observed and proposed hybrid models had good match. The same were true for observations and forecasts by the hybrid model combined with GA for the Rasht time series (Figure 8).

4. Discussion

The purpose of this study was to improve the accuracy of the traditional hybrid model with one linear and one nonlinear in their structures. To achieve this, we 1) properly modeled the linear component of time series using ARIMA or SARIMA; 2) precisely selected the structures of the two configurations with antecedent residual subseries; and with antecedent observations and residual subseries and linear model simulations; 3) properly selected a model to represent residual subseries using an artificial intelligence technique (Liang, 2009; Zheng, 2003; Chen, et al., 2010; Moeeni et al., 2017); and 4) accurately estimated parameters of the selected model. The average RMSE, MAE, UI, and UII decreased from configuration one to two in GEP model for Tabriz and Rasht time series by 88% and 56%, respectively; In this study, the performance of SVR and GMDH was better than GEP model. The average RMSE, MAE, UI and UII decreased by 69%, 56% and 61% when we switched from configuration one to two in SVR, GEP, and GMDH models for Rasht. RMSE of SVR, GEP, and GMDH models forecasts was 12, 18 and 10 mm, respectively in configuration two for Tabriz. For estimation of monthly reference evapotranspiration in Iran, SVR had good performance than GEP (Ahmadi et al., 2021). A study investigates the performance of the multivariate regression spline, least-square support vector regression, GEP and ANN for estimation of monthly long-term rainfall and the best model was least square support vector regression (Mirabbasi et al., 2019). SVM model improved the forecasted error compared to ANN in the internal process of time variation analysis related to precipitation forecasting (Parviz, 2020). The RMSE of the GEP model forecasts decreased by 44% and 4% for configuration one when we decreased generation number from 1000 to 100 was and head size from 8 to 7.

Hybrid models increased forecasting efficiency by combining two types of models and capturing linear and nonlinear patterns of time series. For example, in configuration one, the RMSE, RRMSE and MSE decreased by 4%, 5%, and 8.5% when we replaced SARIMA with the improved hybrid model combined with SVR, by 30%, 25%, and 44% when we replaced ARIMA with the improved hybrid model combined with GMDH. Improvement of precipitation forecasting with the hybrid model based on temporal variations, relative to SARMIA model was observed in Rasht and Gorgan

stations in Iran with 48% and 24% improvements in RMSE scores (Parviz and Rasouli 2019). Furthermore, the hybrid model structure was more important in forecasting. For instance, two hybrid models based on temporal variations and decomposition of precipitation time series, which were compared with a hybrid model based on time series decomposition (linear and nonlinear components) had the minimum error (Parviz 2020).

The hybrid model performance in modeling complex phenomena has drawbacks as the nonlinearity of the decomposed time series may be high, and one model is insufficient to capture all variabilities (Mo et al., 2018). Furthermore, the error series may have high volatility and irregularity (Chen et al., 2020). Therefore, the improvement of hybrid models is necessary. The main direction of the improved hybrid model was to combine the forecast of single models with different methods instead of using one model in the nonlinear part of time series modeling. The combination methods for the integration of forecasts outperform individual models (Wang et al., 2020). All used combination methods improved the efficiency of the hybrid model. NSE improved by 0.18 with the improved hybrid model with GA in Rasht and 0.13 with the improved hybrid model with SVR in Tabriz relative to the hybrid model with GEP. Adhikari and Agrawal (2014) indicated that the combination procedure significantly outperformed all individual methods based on the evaluation scores of the weighting schemes. But it is hard to select one combination method as an inclusive method. The integration of SARIMA for the linear component of time series with a combination of SVR, GP and BP for nonlinear components improved the results of the forecasts (Mo et al., 2018). The forecasts combination aims to integrate the competing forecasts to produce an ensemble of forecasts, which is superior to each of the composing individual models (Wang et al., 2020). By combining all of the forecasts with a proper function, forecast combinations can fully use the information from each forecast model to improve the prediction accuracy and stability (Wang et al., 2019). The accuracy of combined forecasts depends on the weight of each forecast model. According to the evaluation metrics, the improved hybrid models with SVR, GA and error-based models had high performance. In the error-based methods, the type of error equation and the inversion method were different for each configuration. Therefore, there is a need to define a comprehensive error equation and an inversion method.

The average geometric standard deviation of the error for all improved hybrid models for configurations (one and two) for the Tabriz and Rasht time series was (1.47-1.36) and (2.12-1.41), respectively, which indicates forecasts overestimation. If the median of annual Tabriz time series were calculated in the verification period for all improved hybrid models, the median of all time series (observed, improved hybrid model) occurred in 2015. Based on the improved hybrid model using GA, the months with the high and low errors were calculated. The high error was related to May (RRMSE = 0.167), and low was related to July (RRMSE = 0.0044). Based on the forecasted monthly time series, the seasonal time series was calculated. The maximum observed precipitation occurred in autumn, and it was preserved in improved hybrid models.

5. Conclusion

Analysis of precipitation time series can provide insight into its spatial and temporal complexity. This study introduced a few hybrid models, which first separate linear and nonlinear components of precipitation and then extract the temporal patterns with high-performing machine learning methods for the nonlinear component of the precipitation in two weather stations in humid and semiarid climates of Iran. The hybrid model is a combination of stochastic approaches and one of the machine learning techniques. The type of artificial intelligence techniques can affect the forecasts of the hybrid model. We found that the improved hybrid models combined by SVR and GMDH showed better performance than GEP in Rasht and Tabriz weather stations.

In contrast to the traditional hybrid models used for precipitation forecast, a combination of forecasts by multiple nonlinear models was used in the structure of hybrid models used in this study instead of using only one nonlinear model. An ensemble of forecasts using an appropriate weighting method and using artificial intelligence techniques improved forecast errors for annual and monthly precipitation. This was shown by all of the forecast combination methods. Among combination methods, artificial intelligence techniques (SVR), optimization method (GA), and error based methods showed the highest performance. In the error-based methods, the type of error formulation and the structure of the inversion method had the most crucial role. In general, three factors have the main control on the precipitation ensemble forecast using data-driven methods used in this study: separating linear and nonlinear components and appropriately modeling the nonlinear component; Assigning appropriate weights to each ensemble member of the nonlinear

modeling; and finally, the type of nonlinear models. One of the advantages of the improved hybrid model is that it can extract information with high confidence. It uses an accurate forecast combination method embedded in its structure that can overcome the challenges in modeling nonlinear processes in real time series. The other advantage is that it uses optimization methods to estimate ANN and SVR parameters and assign weights to the ensemble members. The proposed improved hybrid model can be a powerful tool to increase precipitation forecast confidence across time scales by combining forecasts from sophisticated machine learning methods instead of relying on only individual models.

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Author's Contribution

L.P. and K.R. analyzed the data and wrote the paper. A.T.H., L.P. and K.R. Research conceptualization and methodology, supervision, reviewing and editing A.T.H.

Availability of data and material

The data that support the findings of this study is available from the corresponding author upon request.

Code availability

Not applicable.

Declarations

Ethics approval The authors confirm that this article is original research and has not been published or presented previously in any journal. This study does not include any animal experiments.

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