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## Research Article

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# **Energy Consumption Modelling in Milling of Variable Curved Geometry**

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# Energy Consumption Modelling in Milling of Variable Curved Geometry

## Abstract

The accurate estimation of energy consumption is beneficial to manufacturing enterprises economically as well as to overcome global energy crisis. The present work concentrates on developing an energy consumption model in milling of variable curved geometries where magnitudes and directions of workpiece curvature vary along tool contact path of a component. The current work deals with estimation and analysis of energy consumption in peripheral milling of variable curved surfaces where cutting forces differ along tool contact path in the presence of workpiece curvature. The proposed hybrid model developed in MATLAB involves process mechanics, cutting forces and energy consumption and have modules for idle, auxiliary and cutting power. The proposed model is validated by the experimental work. The model is generic and versatile in nature and is useful for milling of straight, circular and curved surfaces. In addition to it, the influence of workpiece curvature on power consumption has been investigated to realize the variation of power consumption along the tool contact path. The developed model offers a basic platform to understand and characterize the energy consumption for general peripheral milling considering workpiece geometry. The comparison of predicted and measured results indicate that the model is capable to estimate the power consumption accurately. The proposed model will be used by the practitioners to find the optimum cutting conditions to reduce power consumption during the machining of curved geometries; a pragmatic condition but not much researched condition in machining.

Keywords: Energy modelling, Peripheral milling, Curved geometry, Energy consumption, Curvature

## **1. Introduction**

Manufacturing enterprise is one of the leading consumer of electricity which is majorly produced by burning fossil fuels. It not only generates huge carbon footprints but also changes global climate. The IEA (International Energy Agency) has announced that the generation of energy is the largest man-made source of air pollutants and the major energy demand is associated with the manufacturing sector [1]. As various machine tools used in manufacturing industries consume a huge amount of energy, a drive has been taken by ISO (International Organization for Standardization) on diminishing of power consumption as well as environmental impact of machine tools. In addition to it, the soaring cost of energy has become a prime concern for various industries. Therefore, the saving of power consumption in machining is one of the significant approaches to curtail the environmental impact [2, 3].

To get an in-depth comprehension about proper use of energy in a machining process, the key initiative is to realize and characterize energy distribution in a process. The accurate estimation of energy consumption for peripheral milling is a big challenge to research community due to varying chip load and continuous alteration of cutting forces along the helical teeth of a cutter based on depths of cut selected during the process. The machining situation becomes more complicated in the presence of workpiece curvature as magnitude of feed per tooth changes continuously along tool contact path. As the cutting process is fast and complex, it is very difficult to understand tool-workpiece interaction and concerned energy consumption in the presence of complex cutter engagement. Therefore, it necessitates modeling and computational study of energy consumption in milling of curved surfaces for accurate estimation

of energy consumed during material removal process. In the current work, an attempt is made to investigate the modelling of power consumption in peripheral milling of curved workpieces. The proposed model will be helpful for planning of machining process effectively so that manufacturer can develop an energy efficient and sustainable machining system.

There are significant amount of earlier research work dealing with various approaches of modelling of energy consumption in machining. These modelling approaches are broadly classified into four different groups namely machine motion state-based model (MSM), specific energy-based model (SEM), exponential function-based model (EFM) and cutting force-based model (CFM). The machine motion state-based model (MSM) approach considers various machine states such as machine start-up, standby, idle, air cut, spindle acceleration/deceleration, tool change, etc. to predict power consumption. As power consumption is different at various machine states, the MSM approach calculates total power consumption as a summation of individual power consumption at various machine states. Dietmair and Verl [4] concentrated to develop a generic model where the machining operation is divided into different operational states. The model is based on functionality for estimating energy consumption for machines and production systems considering each operational state. A similar type of model is developed by Avram and Xirouchakis [5] in which total energy is forecasted as a summation of energy requirements for the actuation drives by taken into account of steady and transient state regime. Mori et al. [6] decomposed total energy consumption into energy required for tool-workpiece positioning, spindle acceleration and deceleration, tool changing and workpiece cutting. He et al. [7] established mathematical relationship between energy consuming parts and NC codes and calculated total energy consumption by estimating energy consumed by each part such as spindle, axis feed, coolant pump, etc. Balogun and Mativenga [8] investigated on modelling of

total energy demand by classifying machine tool states into three groups such as basic, ready and cutting states.

Moradnazard and Unver [9] suggested an energy consumption model for turn-mill machining centers by splitting the total power into various modules such as power due to main spindle, sub spindle, milling spindle etc. Altıntaş et al. [10] introduced modelling of total energy consumption by dividing it into three types: basic, auxiliary and material removal.

Edem and Mativenga [11] focused on an improved prediction model of energy requirements by incorporating weight factor of feed drive. In their work, it is concluded that toolpath with longer linear path segments should be selected to reduce energy demand in machining. An energy consumption model for overall machining process is proposed by Gu et al. [12] by decomposing the full machining process into a series of activities and activity transitions for accurate estimation of energy demand. Recently, an improved energy consumption model is developed by Yu et al. [13] for CNC milling of stainless steel in addition to prediction of surface roughness of the part. The MSM models proposed by various researchers are able to predict energy consumption with decent accuracy for various machining operations. But, there are a large number of machine states during any machining process which need to be considered during modelling of energy consumption. Therefore, the implementation of MSM approach is not very convenient for every machining situation. Apart from it, these models are dependent on an assumption that power consumption for any machining component will remain same in every state. Since, machining conditions may not be same in every state, the power consumption for a machined component may be different in each state.

The SEM approach estimates power consumption using the concept of specific cutting energy which is described as an amount of energy desired for a unit volume of removed material.

As material removal rate (MRR) is product of axial & radial immersions and feed rate, the SEM approach calculates cutting power by multiplying specific cutting energy to material removal rate. Gutowski et al. [14] presented an energy consumption model in this category for the first time. The total power consumption is computed as a summation of constant idle power and variable cutting power which is directly proportional to MRR. This model provides a strong thermodynamic exergy framework but the major limitations are that idle power does not remain constant with spindle speed and the model lacks the experimental verifications. In case of turning operation, Li and Kara [15] developed an power consumption model based on experimental data and conclude that specific energy consumption is inversely proportional to MRR. Later on, Diaz et al. validated this model in milling operation. For milling process, Li et al. [16] introduced an improved energy consumption model that depends on MRR and spindle RPM. Aramcharoen and Mativenga [17] investigated the influence of tool wear in machining and eventually, it is concluded that dull tool requires higher specific cutting energy. Zhou et al. [18] developed specific energy consumption model by establishing relationship among spindle RPM, cutting conditions, MRR with specific energy consumption based on experimental results. A hybrid approach is suggested by Nguyen [19] for diminishing specific cutting energy and improving MRR on the basis upon cutting parameters for a stipulated surface roughness. Yuan et al. [20] investigated on identification and determination of energy efficiency for milling based on specific cutting energy.

The SEM models proposed by various researchers are considered as powerful tools for energy analysis through establishing mathematical relationship among energy consumption and various cutting parameters. In actual machining situation, the SEM models are very difficult to

obtain based on huge numbers of process variables. In addition to it, the value of specific cutting energy is not fixed rather it is affected by workpiece properties and machining environment.

The EFM approach predicts power consumption by using empirical formula derived based on machining experimental database. It uses various tools such as design of experiments and curve fitting to establish empirical relationship among these cutting parameters and concerned power consumption. Based on a second order regression model, Yoon et al. [21] focused on a model using feed, depth of cut, spindle speed. Later on, linear relationship between material removal power and tool flank wear was established using surface response methodology. Sealy et al. [22] developed a power regression model for estimation of resultant cutting specific energy including various cutting parameters. From this study, it is observed that tool wear had more effect on resultant cutting specific energy accompanied by feed rate and spindle RPM. For milling process, Xie et al. [23] developed a mapping technique for calculating loading loss coefficients in main driving system. Cutting power is calculated as exponential function of depth of cut, cutting width, feed rate and spindle RPM. Lv et al. [24] followed similar methodology to obtain cutting power for turning process and additionally to improve the accuracy of energy consumption model considering non-cutting motions such as spindle rotation, standby, cutting fluid spray and feeding operations. They also investigated the effect of cutting and non-cutting related specific energy consumption and found that cutting related SEC diminishes rapidly than non-cutting related SEC with raise in MRR. Zhang et al. [25] developed multistage approach to model power consumption. First, the total power consumption is broken down into three stages such as basic energy consumption, sum of power consumption of feeding process & spindle rotation process and the sum of power consumption of material removal and further load loss. After that, total power consumption is predicted in all these stages by using

sliding filter method, multiple linear regression model and gene expression algorithm respectively. Wang et al. [26] developed an empirical model based on power consumption characteristics curves. After this, algorithm to find cutting parameters was developed for low power consumption and short machining time. Advantages of EFM are simple, easy to apply and require less theoretical background. But, these models treating the machine tools as black box rely on machining experimentations and require large number of machining experiments to develop empirical relationships which make them expensive and time consuming.

The CFM approach estimates cutting power consumption using cutting force as a product of cutting constants and chip load. The values of cutting constants determined by conducting a separate set machining experiments. Later on, cutting power is estimated as a product of cutting force and spindle speed for machining operation. Liu et al. [27] developed hybrid power consumption model in slot milling for improving energy efficiency in production scheduling. For turning process, Xie et al. [28] proposed a model for estimating the specific energy consumption (SEC) considering effect of machine tools, process variables and workpiece material. Cutting force is calculated from an empirical model from mechanical engineering manual. Shi et al. [29,30] concentrated on developing an efficient energy model by taken into account the effect of tool wear. A polynomial approximation technique was used to model the dynamic condition of the tool successfully. In addition they also proposed an enhanced power consumption model on basis upon earlier model established by Liu et al. [27] for slot milling. They considered a statistical relationship among idle power and spindle RPM in place of constant idle power. Yang et al. [31] focused on an analytical cutting energy model for estimating total cutting power based on the power required to form chip and frictional power. For drilling process, Wang et al. [32,33] offered an energy consumption model dealing with idle power, cutting power and auxiliary

power for drilling. They also investigated on power consumption model in milling by taken into account of auxiliary load loss. An optimization based technique is followed by Rao [34] to cut down power consumption in case of micro-ball end milling for predefined surface roughness and vibration.

It is already mentioned that the CFM approach estimates cutting power consumption based on cutting force and cutting velocity. This approach provides the real nature of cutting force profile and reflects the actual power profile during machining operation. But, the major problem in CFM approach is estimation of idle and auxiliary powers. No single model available in earlier work can highlight energy consumption modelling issue due to complicity and level of difficulty. Combining analytical models with empirical ones and intervention of human experience gives a cutting edge to the hybrid energy consumption model. The aim is to establish a single model for explaining machine tool behaviour as well as to provide a generic predictive tool framework for machining operation. It will be helpful to fulfill the basic requirements of machining operation for each particular case. In this paper, an attempt is made to develop a hybrid model combining more than one approaches to analyze energy consumption.

During machining of a variable curved geometry, total energy consumption needs to be calculated as a summation of instantaneous energy during the operation periods. Therefore, a hybrid approach is adopted for investigation of energy consumption in milling of variable curved surfaces. The earlier research work concentrated the modelling of energy consumption in milling for straight geometry where feed rate is constant along the tool path. Therefore, the constant power profile is observed during cutting. However, in case of machining of variable curved workpiece, the magnitude and direction of curvature vary continuously along the entire cutter contact path. Therefore, process geometry of milling changes continuously along the entire tool

path in presence of curvature of workpiece. It results into variation in instantaneous cutting forces and respective instantaneous cutting power along the entire tool path. Hence, it necessitates how energy consumption can be estimated in case of milling of curved workpieces along the entire tool path. In the developed energy consumption model, the idle and auxiliary power are also included along with cutting power to obtain total energy consumption. Idle power was computed based on spindle RPM and cutting power was determined based on cutting force which was predicted from mechanistic cutting force model. The auxiliary power was computed from the cutting power by deriving its mathematical relationship to the cutting power experimentally. The performance of the developed model is assessed through the validation by carrying out experiments.

The rest of the work is structured as follows. The section 2 explains about modelling of total energy consumption in milling consisting of idle, cutting and auxiliary power components. The experimental setup and machining conditions have been described in the section 3. The section 4 explains the predicted results with experimental ones along with in-depth discussions. The major contributions of the present work are reported and concluded in section 5.

## **2. Energy consumption model**

Before carrying out any machining operation, machine tool must be ready for performing metal cutting operation. In order to calculate various components of power consumption, various motion states of a machine tool need to be taken into account. Based on the machine tool motion state, the entire machining operation can be classified into three operational states such as standby, idle and cutting state. The power profile for a typical milling process is shown in Fig.1 consisting of cutting and non-cutting operations for a constant spindle RPM. During machining,

the total energy consumption of the machine is broadly categorized into the idle power ( $P_{idle}$ ), the cutting power ( $P_{cutting}$ ), and the auxiliary power ( $P_{auxiliary}$ ), which are given below:

$$P_{total} = P_{idle} + P_{cutting} + P_{auxiliary} \quad (1)$$

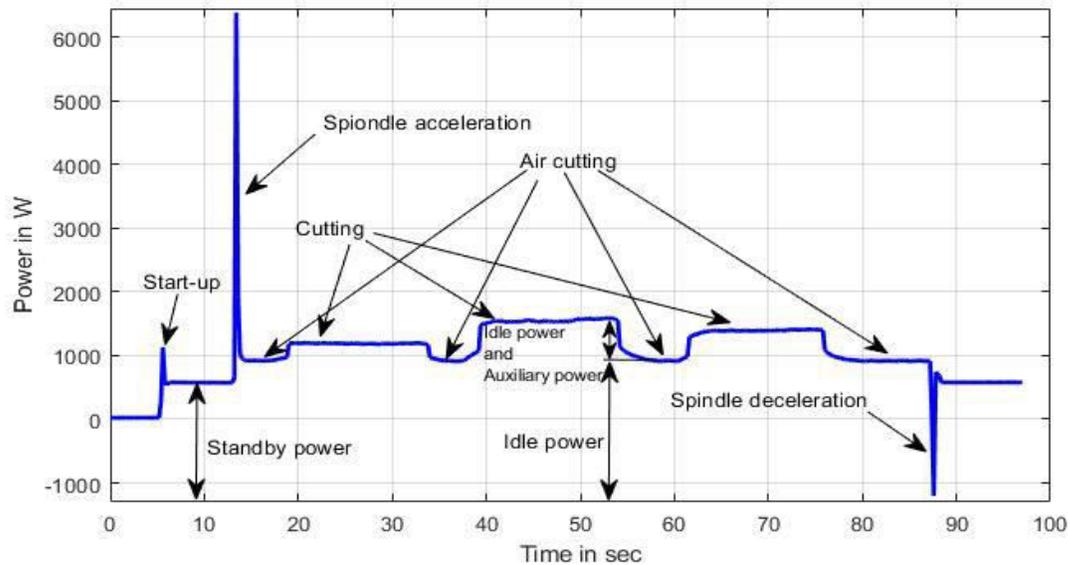


Fig.1 Power profile of a typical milling process

## 2.1 The idle power consumption

The idle power includes the power consumed by auxiliary components and machine spindle while the spindle rotates without any feed and cutting motion. On the basis upon power profile analysis, the idle power can be calculated as a summation of standby power demand and power required to rotate the spindle[14]. The idle power is mostly affected by rotation of spindle due to its mass moment of inertia [5]. The idle power can be expressed as:

$$P_{idle} = f(n) \quad (2)$$

where  $n$  is machine spindle RPM.

## 2.2 The cutting power consumption

For cutting power calculation, the tool tip energy is required during metal cutting operation. In case of peripheral milling, generally cutting teeth are helical in nature, process is intermittent and produces chips of varying thickness making the milling operation more complex and difficult to predict tool tip energy. Therefore, basic input for modeling of cutting power is instantaneous cutting forces acting along the helical cutting edge. As process geometry in milling is prerequisite for developing cutting force model, identification and determination of process geometry variables are essential which are discussed briefly in the following subsection.

### **2.2.1 Process geometry for milling of a curved workpiece**

The milling process geometry takes care of process mechanics along with complex interaction of tool and workpiece during metal cutting operation. It discusses about several process geometry variables i.e. feed per tooth, entry and exit angle, engagement angle etc. In case of straight workpiece, the magnitude of curvature is zero and thus, the radius of curvature is infinite in nature. But, workpiece curvature remains non-zero and constant for circular geometry. In addition, the curvature may be either positive or negative depending upon concavity and convexity of workpiece. The process geometry of milling for straight and circular workpieces has been discussed in details by Pawar et al.[35] . For variable curved geometry, the magnitude of workpiece curvature varies from point to point along tool contact path. Therefore, the process geometry of milling changes continuously along the circumferential length due to workpiece curvature as shown in Fig.2. This makes the milling of curved workpieces challenging for the modelling.

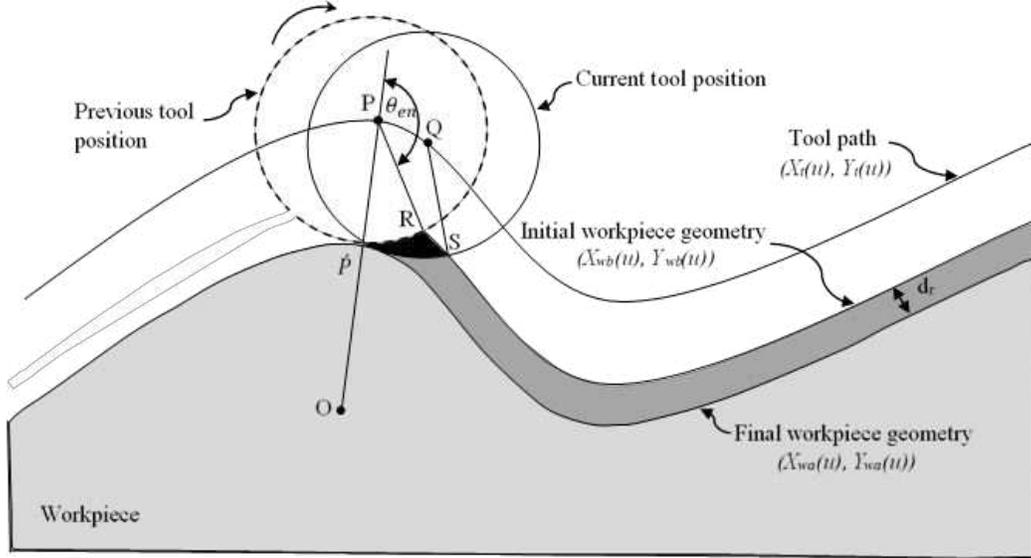


Fig.2. Milling for variable curved geometry

In Fig.2, there are three parametric curves  $(X_{wb}(u), Y_{wb}(u))$ ,  $(X_{wa}(u), Y_{wa}(u))$  and  $(X_t(u), Y_t(u))$  representing before cut, after cut workpiece geometries and locus of tool center respectively. Considering the contour of after cut workpiece geometry (machined surface) as reference, the contour of before cut workpiece geometry and locus of tool center is expressed mathematically as:

$$\begin{aligned} X_{wb}(u) &= X_{wa}(u) - d_r \cdot \frac{Y'(u)}{\sqrt{X'(u)^2 + Y'(u)^2}} \\ Y_{wb}(u) &= Y_{wa}(u) + d_r \cdot \frac{X'(u)}{\sqrt{X'(u)^2 + Y'(u)^2}} \end{aligned} \quad (3)$$

$$\begin{aligned} X_t(u) &= X_{wa}(u) - r \cdot \frac{Y'(u)}{\sqrt{X'(u)^2 + Y'(u)^2}} \\ Y_t(u) &= Y_{wa}(u) + r \cdot \frac{X'(u)}{\sqrt{X'(u)^2 + Y'(u)^2}} \end{aligned} \quad (4)$$

where  $d_r$  is radial depth of cut,  $r$  is cutter radius and  $X'(u)$  and  $Y'(u)$  are differentiation of parametric curve with respect to curve parameter. From Fig.2, it is also seen that the points P  $(X_t(u_a), Y_t(u_a))$  and Q  $(X_t(u_b), Y_t(u_b))$  are succeeding tool center positions along the locus of tool

center at a distance of nominal feed per tooth ( $f_t$ ). The points R( $X_w(u_c), Y_w(u_c)$ ) and S ( $X_w(u_d), Y_w(u_d)$ ) are tooth entry points corresponding to point P and point Q, respectively. The point (O) is center of radius of curvature of workpiece for the exit point of cutter ( $\dot{P}$ ) lying on machined surface. Using the known parameter  $u_a$  of point P, parameter  $u_b$  of Q can be obtained by

$$f_t^2 = (X_w(u_a) - X_w(u_b))^2 + (Y_w(u_a) - Y_w(u_b))^2 \quad (5)$$

The coordinates of the cutter entry point (R) corresponding to the tool center position (P) is calculated by following equations:

$$X_w(u_c) = X(u_c) - d_r \cdot \frac{Y'(u_c)}{\sqrt{X'(u_c)^2 + Y'(u_c)^2}} \quad (6)$$

$$Y_w(u_c) = Y(u_c) + d_r \cdot \frac{X'(u_c)}{\sqrt{X'(u_c)^2 + Y'(u_c)^2}}$$

$$(X_t(u_a) - Y_w(u_c))^2 + (Y_t(u_a) - Y_w(u_c))^2 = r^2 \quad (7)$$

The coordinates of the cutter entry point S corresponding to tool center position Q is obtained in similar manner. The distance RS that denotes the feed per tooth along tool contact is determined by

$$f_{cc} = \sqrt{(X_w(u_c) - X_w(u_d))^2 + (Y_w(u_c) - Y_w(u_d))^2} \quad (8)$$

The feed per tooth along tool contact path and their relationship with feed per tooth for straight and circular geometries can be obtained from Table 1.

Table1. Process geometry variables for straight and circular geometry [36]

Workpiece geometry	Feed per tooth along tool contact path ( $f_{cc}$ )	Entry angle ( $\theta_{en}$ )	Exit angle ( $\theta_{ex}$ )
Straight	$f_i$	$\pi - \cos^{-1} \left[ 1 - \frac{d_r}{r} \right]$	$\pi$
Circular concave	$f_i \left[ \frac{(R - d_r)}{(R - r)} \right]$	$\cos^{-1} \left[ \frac{(R - r)^2 + r^2 - (R - d_r)^2}{2r(R - r)} \right]$	$\pi$
Circular convex	$f_i \left[ \frac{(R + d_r)}{(R + r)} \right]$	$\pi - \cos^{-1} \left[ \frac{(R + r)^2 + r^2 - (R + d_r)^2}{2r(R + r)} \right]$	$\pi$

The curvature and radius of curvature for the curve  $(X_{wa}(u), Y_{wa}(u))$  at point  $\dot{P}$  can be obtained by Eq. (9) & (10).

$$K(u_a) = \frac{X'(u_a).Y''(u_a) - Y'(u_a).X''(u_a)}{(X'(u_a)^2 + Y'(u_a)^2)^{3/2}} \quad (9)$$

$$R(u_a) = \frac{1}{K(u_a)} \quad (10)$$

where  $K$  is curvature and  $R$  is radius of curvature.

During developing of process geometry model for milling of curved surface, it is required to determine the varying uncut chip thickness for calculation of chip load. For determination of chip load in milling, the uncut chip thickness is computed using the Eq. (11) based on the mathematical relationship derived as:

$$t_c(\beta(z)) = f_{cc} \sin(\beta(z)) \quad (11)$$

where  $t_c(\beta(z))$  is instantaneous uncut chip thickness realized by  $i^{th}$  flute of  $j^{th}$  disk element. The parameter  $\beta(z)$  represents the angular position of helical tooth at any instant in the immersion zone known as instantaneous tooth positioning angle. The value of  $\beta(z)$  can be obtained from Eq. (12) with respect to outward normal to workpiece as follows:

$$\beta(z) = \varphi - (i-1) \cdot \phi_p - \frac{\tan(\alpha)}{r} \cdot z \quad (12)$$

where  $\varphi$  is cutter rotation angle,  $\phi_p$  is pitch angle,  $\alpha$  is helix angle of milling cutter and  $r$  is radius of milling cutter. In order to obtain chip load for a given axial depth of cut, the instantaneous uncut chip thickness is calculated after every incremental rotation of cutter.

### 2.2.2 Cutting force model

On the basis upon process geometry model, the cutting force model is developed to predict the forces for milling of curved geometry [37, 38]. In this force model, milling cutter is discretized into a finite number of thin disk elements along the axis of the cutter. It is assumed that cutting forces act on each axial disk element.

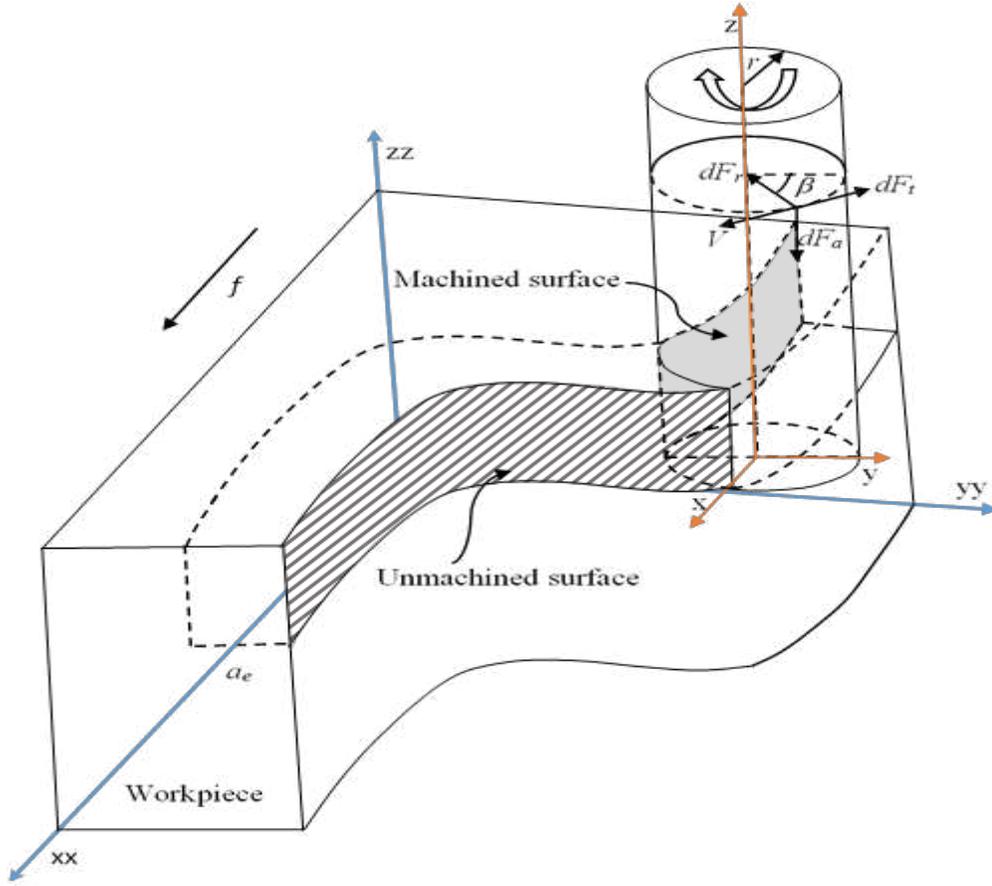


Fig.3 Milling forces for variable curved geometry

$$dF_t(i, j) = K_t t_c(\beta(z)) z_{i,j} \quad (13a)$$

$$dF_r(i, j) = K_r t_c(\beta(z)) z_{i,j} \quad (13b)$$

Where  $dF_t$  is tangential force acting on  $i^{th}$  flute,  $K_t$  is tangential cutting constants and  $K_r$  is radial cutting constant. The elemental tangential and radial forces acting on disk elements are resolved into local normal forces and feed forces which are expressed as follows:

$$dF_f(\varphi) = -dF_t \cos \beta(z) - dF_r \sin \beta(z) \quad (14a)$$

$$dF_n(\varphi) = dF_t \sin \beta(z) - dF_r \cos \beta(z) \quad (14b)$$

### 2.2.3 Cutting power calculation

The cutting power at the tool tip is the amount of power required at tip point during the machining process to remove workpiece material. The relative movement between tool and workpiece causes power consumption during a machining process. Rotational motion due to spindle drive and feed motion due to feed drive are two types of relative movements. These two motions are considered separately to calculate power consumption. In rotational motion of tool, the radial force and axial force are mutually perpendicular to the cutting speed. Therefore, their contribution in computation of power consumption is zero. But, the tangential force component coincides with the direction of cutting speed which contributes to the determination of power consumption. The differential powers consumed at each disc element along the helical tooth in tangential, radial, and axial directions is expressed as:

$$dP_{rotational,t} = dF_t \cdot V \quad (15a)$$

$$dP_{rotational,r} = 0 \quad (15b)$$

$$dP_{rotational,a} = 0 \quad (15c)$$

where  $V$  is cutting speed which is a function of spindle RPM and diameter of milling cutter.

Total power consumed by each elemental helical cutting edge owing to rotational motion can be given by:

$$dP_{rotational} = dF_t \cdot V \quad (16)$$

During the feed motion of the cutter, the axial force component is also perpendicular to feed direction. Therefore, its contribution to the power consumption is zero. The differential powers are expressed as:

$$dP_{feed,t} = dF_t \cos(\beta) \cdot f_{cc} / 60000 \quad (17a)$$

$$dP_{feed,r} = dF_r \sin(\beta) \cdot f_{cc} / 60000 \quad (17b)$$

$$dP_{feed,a} = 0 \quad (17c)$$

where  $f_{cc}$  is feed rate along cutter contact path. The total power consumed by each disc element along the cutting edge owing to feed motion is expressed as:

$$dP_{feed} = dF_t \cos(\beta) \cdot f_{cc} / 60000 + dF_r \sin(\beta) \cdot f_{cc} / 60000 \quad (18)$$

According to Eq. (14a) and Eq.(18), it can be expressed as

$$dP_{feed} = -dF_f \cdot f_{cc} / 60000 \quad (19)$$

The instantaneous cutting power consumption at any given instant is given by

$$\begin{aligned} P_{cutting} &= P_{rotational} + P_{feed} = \int dP_{rotational} + \int dP_{feed} \\ &= V \cdot \int dF_t + f_{cc} / 60000 \cdot \int (-dF_f) \end{aligned} \quad (20)$$

In similar manner, average cutting power for one complete cutter rotation can be written as:

$$\bar{P}_{cutting} = \frac{1}{2\pi} \int_0^{2\pi} P_{cutting} \quad (21)$$

#### 2.2.4 Auxiliary power consumption

There are several factors contributing to the total power consumption excluding spindle rotation and metal cutting process. They may come from frictional forces between various moving elements of machine tool, heat generation and thermal effects of machine tool, resistivity of electrical components, etc. Auxiliary power consumption includes the extra power loss due to the movement of table axis drive, electrical load loss, magnetic power loss, etc. It may be any form of power loss excluding in the idle and cutting power consumption. Initially, the cutting

power and idle power is added together and later on, it is deducted from the total power consumption in order to compute auxiliary power consumption. It can be described as follows:

$$P_{auxiliary} = P_{total} - (P_{cutting} + P_{idle}) \quad (22)$$

The auxiliary power consumption is also expressed as a function average cutting power. Based on the research work suggested by Hu et al.[39], auxiliary power consumption is expressed as:

$$P_{auxiliary} = C_0 \cdot \bar{P}_{cutting}^2 + C_1 \cdot \bar{P}_{cutting} = f(\bar{P}_{cutting}) \quad (23)$$

where  $C_0$  and  $C_1$  are the coefficients used in the polynomial equations which need to be determined by conducting a set of machining experiments.

### 2.2.5 Total energy consumption

Total power consumption consists of three power components such as idle, cutting and auxiliary power. In order to calculate total power consumption, summation of these three components will be taken into account and it can be written as:

$$P_{total} = P_{idle} + \bar{P}_{cutting} + P_{auxiliary} \quad (24)$$

It is widely accepted that idle power depends on specific machine tool and the cutting power is influenced by workpiece material and cutting conditions. The machine tool power consumption (total power) is measured in-line using power logger with the help of current and voltage sensors. The idle power consumption is measured experimentally without axis movement and cutting operation. Therefore, the machine tool energy consumption reflects idle power component only considering standby power and spindle rotation. During milling operation, the

cutting force is measured directly using force sensor (dynamometer) for obtaining the spindle power and machine tool power consumption is measured in-line, simultaneously for accomplishing of total power which includes idle, cutting and auxiliary power components. Therefore, sum of idle and auxiliary power components is easily calculated experimentally by subtracting the cutting power from the machine tool power consumption. The auxiliary power is difficult to measure or model directly due to complexity of various time dependent and independent parameters [39]. Therefore, the auxiliary power is determined from the machine total power consumption by subtracting the idle and cutting power consumption. Eventually, an empirical relationship based on experimental results is established between auxiliary and cutting power consumption. Later on, the values of various coefficients of the mathematical equation are used in the proposed model for computing the auxiliary power component for varying machining conditions.

### **3. Experimental details**

Three sets of milling experiments have been conducted for calibration and validation of the proposed models. The first set of experiments has been performed to determine the values of cutting constants referred in the developed force model. The second set of experiments has been conducted to calibrate and determine the values of various coefficients used in the empirical power equations of the idle and auxiliary power models. The third set of experiments has been performed for validation and assessment of performance of the proposed cutting power model. The experimental setup and machining conditions are discussed in the following subsections.

#### **3.1 Experimental setup**

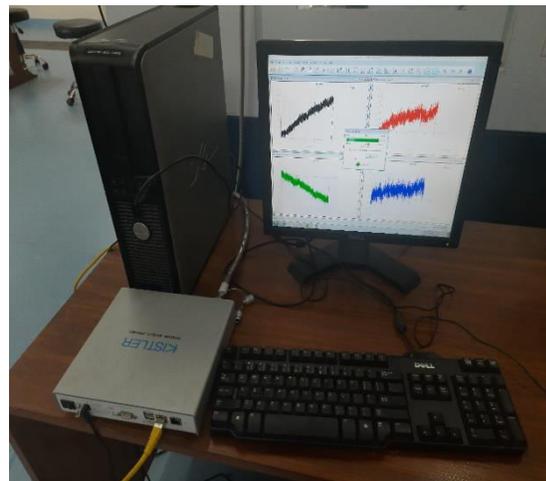
All the experiments have been performed on three axis LMW JV- 40 vertical machining center (VMC) fixing with a universal piezo-electric dynamometer (Type-9272) made by Kistler

and a power logger (National Instrument). The detailed specifications of the VMC machine tool along with spindle and feed motors are given in Table 2. The vertical machining center is equipped with a FANUC controller of series Oi-MF CNC. The FANUC controller has linear and circular interpolation facilities that helps to perform machining of curved geometry.

Table 2. Technical specifications of the machine tool, spindle and feed motors

Machine tool specifications:	
Manufacturer	LMW JV- 40
Power supply	3 phase 415V 50HZ
Table travel (X × Y × Z) in mm	500 × 400 × 450
Spindle speed	150-8000 RPM
ATC capacity	20
Maximum weight on table	300 kg
Spindle motor power	3/5.5 kW
Feed motor power (X)	3 kW
Feed motor power (Y and Z)	1.2 kW
Spindle motor specifications:	
Speed	800 rpm
Motor power	5.5/7.5 kW
Maximum Torque	32.3/58.7 Nm
Feed motor specifications:	
Feed motor power(X and Y axis)	1.2 kW
Feed motor power(Z axis)	1.8 kW
Feed motor Torque (X & Y axes))	7 Nm
Feed motor Torque (Z axis)	11 Nm

A multi-channel piezo-electric dynamometer has been used to measure various force components with a charge amplifier (Type-5070) made by Kistler and DAQ system. The dynamometer is fixed to the VMC machine table along with a dedicated flange type fixture with multiple holes as shown in Fig. 4(a). The force values have been collected with the help of data acquisition system (DAQ) and stored in a computer as shown in Fig. 4(b). A dedicated DynoWare software developed by Kistler is used for sampling and processing the force data with respect to time. A dedicated power logger was used for measurement of total power consumption with the help of another DAQ system as shown in Fig. 4(c) and 4(d). The setup for power measurement is composed of various components such as voltage sensors (LEM LV25-P), current sensors (LEM LA55-P), NI-9215 data collecting cards, a compact DAQ card, and a LabVIEW software interface, etc. The machine tool power consumption for all machining experiments have been measured in-line at power supply port of electrical panel box of the CNC machine using voltage and current transducers. The spindle power is measured directly by using the force sensor (dynamometer) which helps to determine the cutting power consumption.



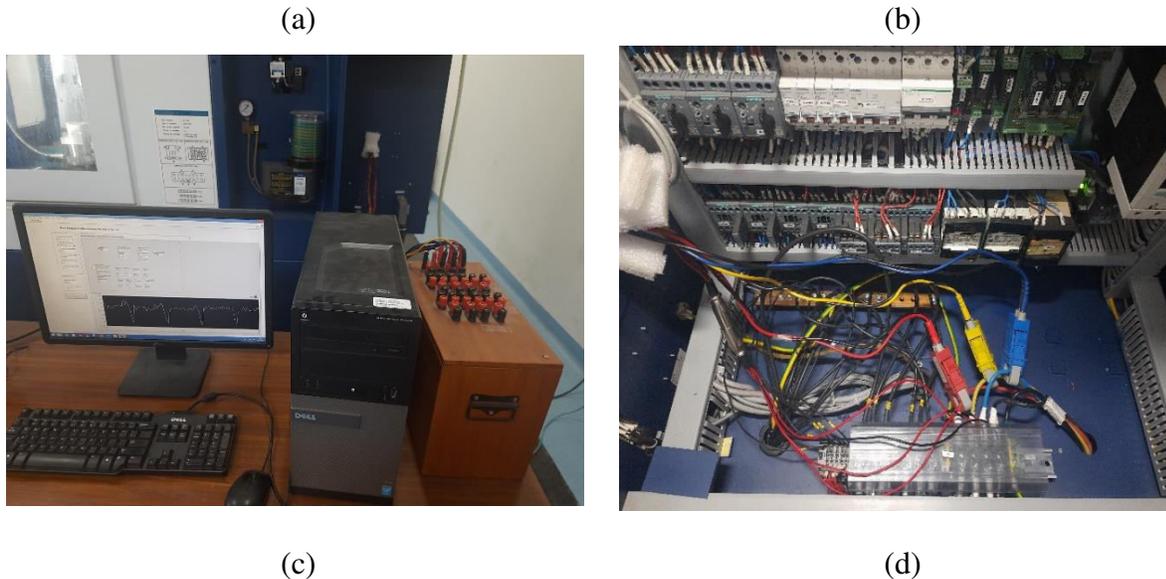
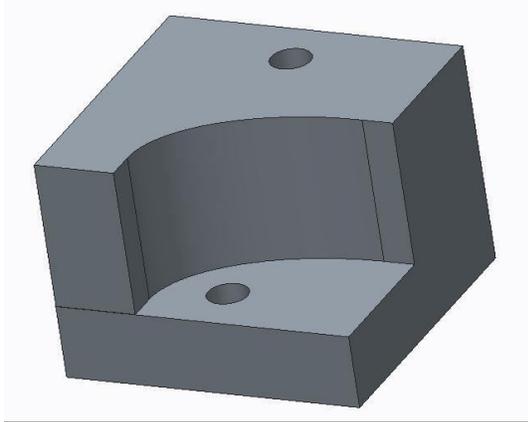


Fig.4 The setup for experiment: (a) and (b) Installation of force measurement setup ; (c) and (d) Installation of power measurement setup

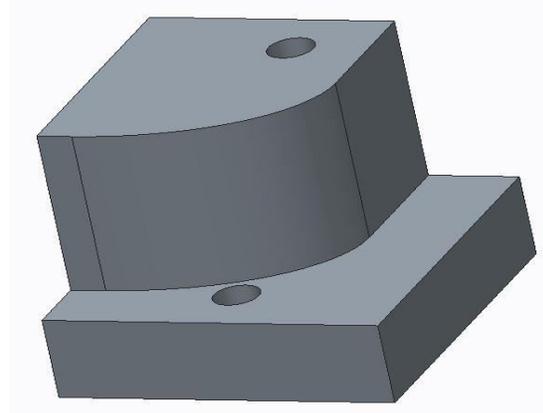
### 3.2 Machining conditions

The machining experiments have been carried out on rectangular and elliptical workpieces of aluminium 6351-T6 for calibration and validation of the proposed model. The aluminium 6351-T6 alloy is extensively used in the aerospace industry. The rectangular workpiece is used mainly for the calibration and determination of various coefficients used in the various mathematical equations. A concave and convex type elliptical workpieces as shown in Fig. 5 are selected for validation and assessment of the effectiveness of the models. The major and minor diameters of the unmachined surface for both concave and convex elliptical geometries are kept constant. The before cut surfaces for both these geometries are considered as reference surfaces. The chemical composition of the alloy and mechanical properties of aluminium 6351-T6 are given in Table-3 and Table-4 respectively. All geometric and dimensional details of workpieces are described in Table-5. The cutting tool used in the

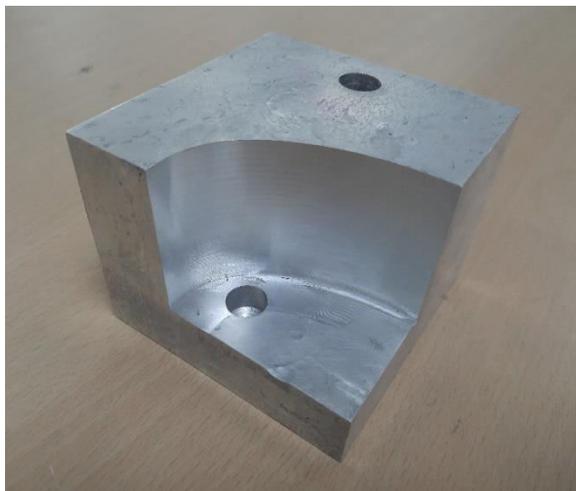
machining experiments is four flutes HSS end milling tool with 16 mm diameter and 30° helix angle. All the experiments are performed under dry cutting condition.



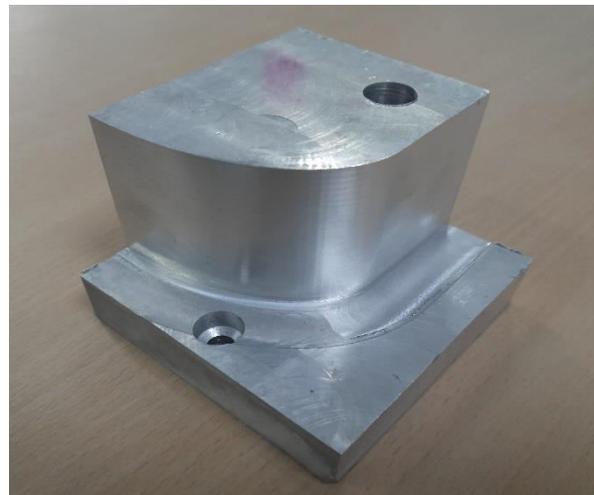
(a) 3D CAD model of concave ellipse



(b) 3D CAD model of convex ellipse



(c) Physical concave elliptical workpiece



(d) Physical convex elliptical workpiece

Fig. 5 The concave and convex elliptical workpieces.

Table 3. Chemical composition of Aluminum 6351-T6

Al	Cu	Fe	Mg	Mn	Si	Ti	Zn
95.9 – 98.5 %	≤ 0.1%	≤ 0.5%	0.4 – 0.8 %	0.4 – 0.8 %	0.7 – 1.3 %	≤ 0.2%	≤ 0.2%

Table 4. Mechanical properties of aluminum 6351-T6

Density	2.71 g/cm <sup>3</sup>
Brinell Hardness	95
Vickers hardness	107
Ultimate tensile strength	310 MPa
Yield tensile strength	283 MPa
Specific heat capacity	0.890 J/g °C
Thermal conductivity	176 W/(m.K)

Table 5. Dimensional details of elliptical workpiece

Dimensions of rectangular block	96 mm × 96 mm × 75 mm
Major diameter of concave and convex workpieces	120 mm
Minor diameter of concave and convex workpieces	73 mm
Height of the elliptical workpiece	50 mm
Square base dimensions	96 mm × 96 mm
Square base length	25 mm

During design of curved workpiece, a square base section is made for holding the elliptical part without interference of clamping force and additional damage of the part. The square base section is to be clamped to the dynamometer by using two Allen screws for transmitting cutting load to the dynamometer during material removal process. The pre-machined elliptical workpart is made from a rectangular bar by performing conventional machining operations.

The cutting parameters designed for the first set of experiment are given in Table 6. The machining parameters used for second set of experiment are mentioned in Table 7. A total of 27

combinations are selected with varying axial immersion, spindle RPM and feed rate in three levels. Table 8 describes about the machining condition for performing third set of experiment. The objective of this experiment is to validate and to assess the effectiveness of the proposed model. During these experiments, the sampling data of cutting force and the respective power distribution are measured and recorded with respect to time. Later on, the force and power profiles are used for analysis and assessment of the performance of the proposed model.

Table 6. Cutting conditions for cutting constant

$a_e$ [mm]	$a_p$ [rpm]	$f$ [mm/min]
3	2	100,150,200,250,300,350,400
5	2	100,150,200,250,300,350,400
7	2	100,150,200,250,300,350,400

Table 7. Cutting parameters for auxiliary power

$a_p$ [mm]	$n$ [rpm]	$f$ [mm/min]
2,3,4	2000	200,300,400
2,3,4	3000	200,300,400
2,3,4	4000	200,300,400

Table 8. Machining conditions for elliptical component

Workpiece geometry	Elliptical concave Elliptical convex
Spindle speed	2000 rpm
Nominal feed rate	400 mm/min
Nominal feed per tooth	0.05 mm/tooth
Axial immersion	3 mm
Radial immersion	5 mm
Length of machined surface	77 mm
Cutter diameter	16 mm

Cutter helix angle	30°
Cutter teeth number	4
Type of milling	Down milling without coolant

### 3.3 Energy consumption model calibration

It is already mentioned that the idle power consumption depends spindle rotation. The total power consumption ( $P_{total}$ ) equals to idle power ( $P_{idle}$ ) while the machine tool is ON without cutting and axis drive movement. Therefore, the power consumed by the machine spindle in such state is measured experimentally at various spindle RPM. A statistical polynomial relationship is fitted between idle power and spindle RPM using standard linear least square [LLS] method. Table 9 shows the results for measured idle power for various spindle rotational speed. The variation of idle power at different spindle RPM can be seen from the Fig. 6. A quadratic relationship is established between idle power and spindle RPM and it can be expressed as:

$$P_{idle} = 0.00002.n^2 - 0.0144.n + 678.18 \quad (25)$$

Table 9. Data for measured idle power

No.	Spindle rotation speed [rpm]	Idle power [W]
1	500	670
2	1000	687
3	1500	723
4	2000	746
5	2500	780
6	3000	844
7	3500	932
8	4000	980

9	4500	1080
10	5000	1205

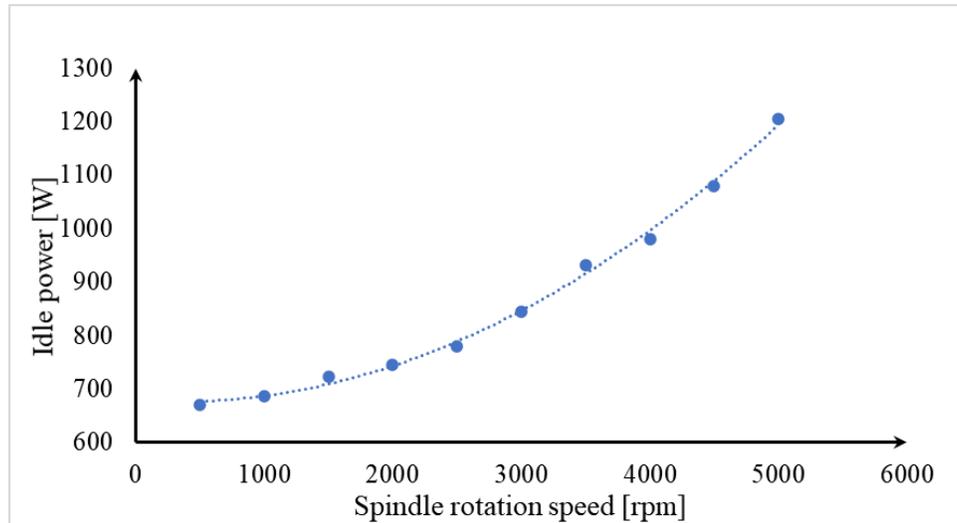


Fig.6. Variation of idle power at different spindle rotational speeds

In order to estimate cutting power consumption, the cutting force model is prerequisite in which cutting constants are to be determined from machining experiments for various feed rate and radial immersions. Therefore, the first set of cutting test is carried out at various uncut chip thickness values for determining cutting constants for the force model. The exponential relationship of  $K_t$  and  $K_r$  with average uncut chip thickness is formulated which is shown in Eq. (26) below.

$$K_t = 287.1(t_c)^{-0.478} \quad (26a)$$

$$K_r = 106.8(t_c)^{-0.659} \quad (26b)$$

The magnitudes of cutting constants mentioned in above equations are used further in the force model developed in MATLAB to estimate the accurate cutting forces. The Fig. 7 shows some of the predicted force results for variable curved geometry and later on, a comparison has been made between the estimated and measured forces. From the Fig. 7, it is noticed that the estimated cutting force results are good in agreement with their measured counterpart. Therefore, the cutting force model is acceptable and reliable to compute cutting power and analyze the power profile.

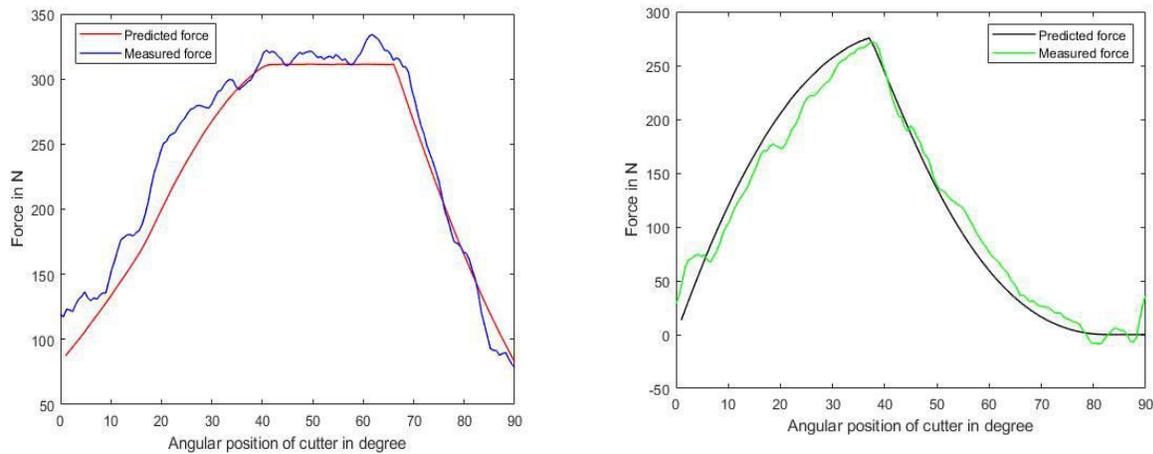


Fig.7 Prediction of cutting forces under different cutting conditions: (a)  $a_p = 16$  mm,  $a_e = 2$  mm,  $n = 3000$  rpm,  $f = 300$  mm/min; (b)  $a_p = 9$  mm,  $a_e = 2.4$  mm,  $n = 2000$  rpm,  $f = 400$  mm/min

In order to determine the coefficients used in auxiliary power consumption, a separate set of experiments has been performed. Based on the machining experiments, experimental results are given in Table 10. The machine tool total power consumption is measured using power measurement setup and the idle power and cutting power are calculated by using Eq. (25) and Eq. (21) respectively. Based on these results, auxiliary power is calculated using Eq. (22). Later on, the variation of auxiliary power with respect to cutting power has been studied using standard LLS method as shown in Fig. 8. It is observed that auxiliary power is gradually increasing with

respect to cutting power. A quadratic relationship between the auxiliary and cutting power is formulated based on measured results. Later on, the values of various coefficients are determined based on the quadratic relationship. After determining the values of coefficients, the auxiliary power can be expressed in terms of cutting power as shown below:

$$P_{auxiliary} = 0.0003.\overline{P_{cutting}}^2 + 0.2632.\overline{P_{cutting}} \quad (27)$$

Table 10. Measured total power ( $P_{total}$ ) and predicted cutting power ( $\overline{P_{cutting}}$ )

S. No.	$a_p$ [mm]	$n$ [rpm]	$f$ [mm/min]	$P_{total}$ [W]	$\overline{P_{cutting}}$ [W]	S. No.	$a_p$ [mm]	$n$ [rpm]	$f$ [mm/min]	$P_{total}$ [W]	$\overline{P_{cutting}}$ [W]
1	2	2000	200	1027	212.06	15	3	3000	400	1616	554.6
2	2	2000	300	1101	262.15	16	3	4000	200	1585	442.89
3	2	2000	400	1164	304.73	17	3	4000	300	1767	547.39
4	2	3000	200	1183	257.35	18	3	4000	400	1890	636.2
5	2	3000	300	1281	318.1	19	4	2000	200	1351	424.13
6	2	3000	400	1352	369.73	20	4	2000	300	1501	524.3
7	2	4000	200	1388	305.26	21	4	2000	400	1633	609.47
8	2	4000	300	1469	364.92	22	4	3000	200	1582	514.72
9	2	4000	400	1552	414.13	23	4	3000	300	1777	636.2
10	3	2000	200	1183	318.1	24	4	3000	400	1930	739.46
11	3	2000	300	1293	393.22	25	4	4000	200	1829	590.52
12	3	2000	400	1368	457.1	26	4	4000	300	2038	729.85
13	3	3000	200	1365	386.03	27	4	4000	400	2269	848.27
14	3	3000	300	1505	477.15						

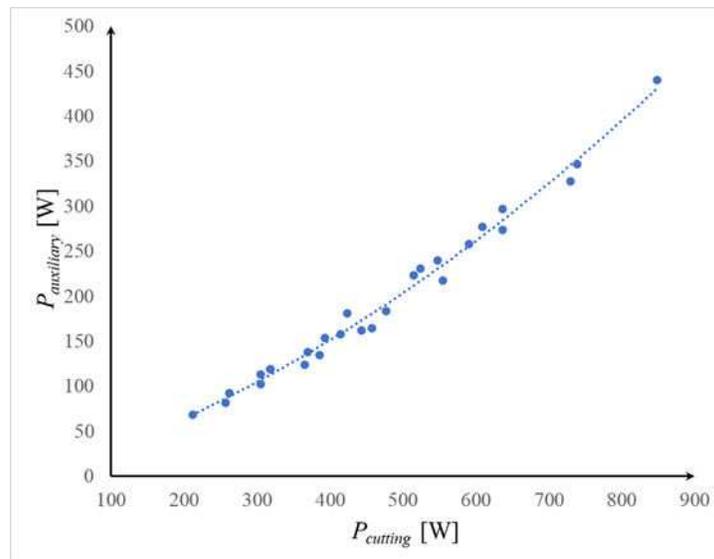


Fig.8. Co-relationship between auxiliary and cutting power

Finally, the mathematical relationship of total power consumption with the idle, cutting and auxiliary power can be established. The total power consumption can also be described as function of idle and cutting power which is written as:

$$P_{total} = P_{idle} + 1.2632\overline{P_{cutting}} + 0.0003\overline{P_{cutting}}^2 \quad (28)$$

There are various uncertainties in the power measurement process as well as in measuring instruments which affect the accuracy of the measured results [40, 41]. The possible precautions have been taken during selection of machine tool, workpiece and cutting tool for taking care of uncertainties factors. The reproducibility of the part manufacturing and development of power measurement setup account for uncertainties upto certain extent. An earlier study [42] on the same machine tool using same power measurement setup showed that 99.5% of the total variance during the unloaded spindle power is explained. Although it is very difficult to avoid all sources of errors and uncertainties, the results obtained from the machining experiments will not divert the main focus of the present study. The next section deals with results and discussion of the present work.

## **4. Results and discussion**

For assessment of the effectiveness of the model, the proposed model is first validated using the data obtained from the third set of experiments. Subsequently, the predicted results are plotted and compared with their experimental counterpart. In addition, the influence of workpiece curvature on cutting forces and cutting power consumption in milling of curved workpieces are investigated. The next section deals with validation and comparison of results achieved from experimental and computational investigations.

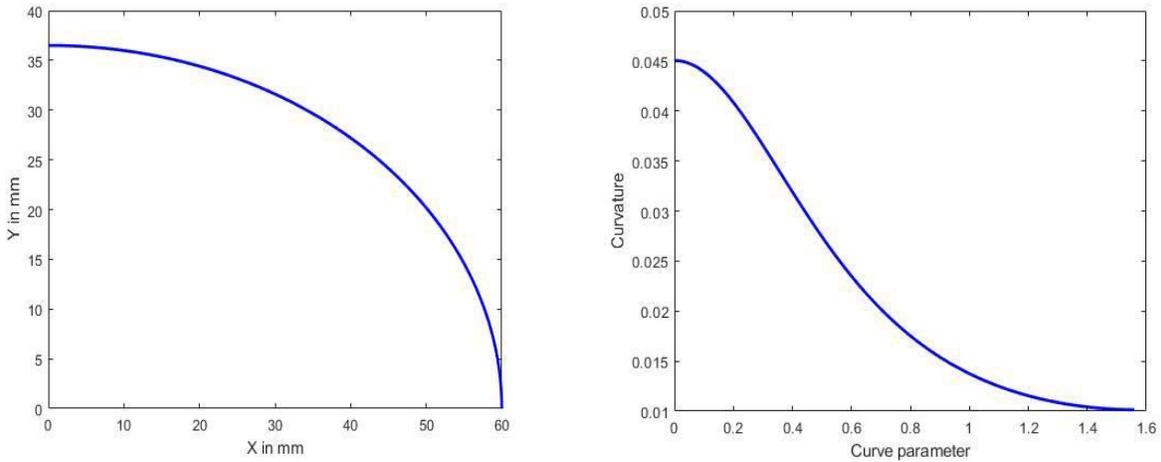
### **4.1 Validation and comparison of results**

This section deals with the model validation and comparison of computational results with their experimental counterpart results to assess performance of the model developed in MATLAB. In order to study the influence of workpiece curvature on power consumption, machining conditions for concave and convex elliptical geometries are kept identical. The parametric form of the elliptical geometry is written as:

$$X(t) = a.\cos(t) \quad (29a)$$

$$Y(t) = b.\sin(t) \quad (29b)$$

where  $a$  and  $b$  are the radius on the X and Y axes respectively and  $t$  is curve parameter which ranges from 0 to  $\frac{\pi}{2}$  for elliptical geometry. The values of  $a$  and  $b$  are taken as 60 mm and 36.5 mm respectively during design of workpiece geometry. Fig. 9 shows the details of elliptical workpiece geometry along with curvature variation from start to end of the profile.



(a) Elliptical geometry in XY plane

(b) Curvature variations for ellipse

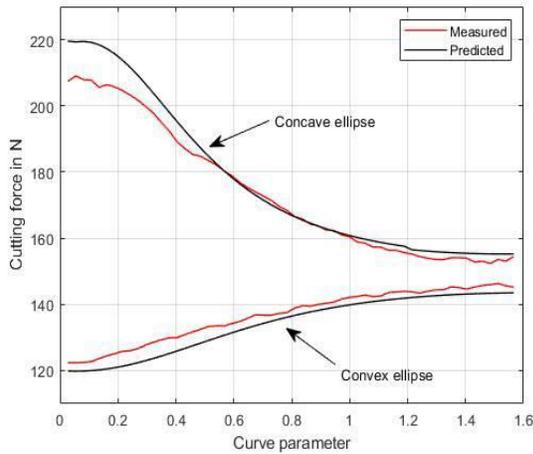
Fig.9 The details of elliptical workpiece geometry.

The third set of machining experiments given in Table 8 is performed to measure and collect experimental force and power consumption data. Later on, the predicted force and power

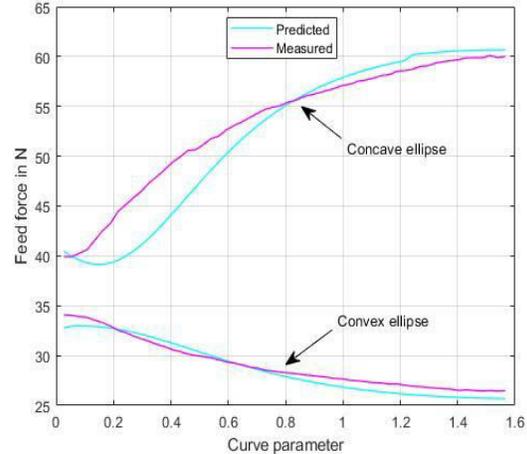
consumption results which are achieved from the proposed model are compared with their measured counterpart. On the basis upon the predicted results, the proposed model is validated with a good agreement. Initially, the variation of cutting force and feed force components are studied with respect to curve parameters along the tool contact path for concave and convex elliptical workpart. Next, the predicted results are plotted and compared with their measured counterpart results. The variation of power consumption are investigated with respect to curve parameters along the tool contact path for both type of geometries. A comparative study between concave and convex elliptical geometries has also been done on power profiles along the tool contact path.

Fig.10 shows the variation of cutting forces and feed forces with respect to curve parameter along the cutter contact path for concave and convex elliptical geometry. As cutting force and feed force components are major controlling parameters for power consumption in milling of curved geometry, the understanding about variation of these force profiles are essential along the tool contact length. The type of milling is same for both these geometries i.e. down milling is performed for both these geometries, the uncut chip thickness is varying from the maximum to minimum value. The start and end of cut are the extreme right and top points of the curved geometry respectively as shown in Fig. 9. The cutting force component is gradually increasing for convex elliptical workpiece along the tool contact path from the beginning to end of cut which can be seen from Fig.10a. But, it is gradually decreasing for concave geometry. The variation of cutting force profiles for both these geometries is opposite to each other. As the curvature value is gradually decreasing for concave and convex elliptical geometry, the magnitude of uncut chip thickness is gradually increases for convex and decreases for concave elliptical geometries. It results into variation in cutting force profiles for both these geometries.

From the Fig.10b, it is noticed that feed force component is gradually decreasing for convex workpiece and gradually increasing for concave workpiece from the beginning to end of cut. In the presence of workpiece curvature, the feed per tooth along tool contact path and cutter engagement angle vary that lead to change in uncut chip thickness and corresponding force value. As there is a change in instantaneous feed per tooth along the tool contact path in the presence of curvature, the uncut chip thickness varies in that feed station. Therefore, the feed force profile varies along the peripheral length for both geometries.



(a) Cutting force variation

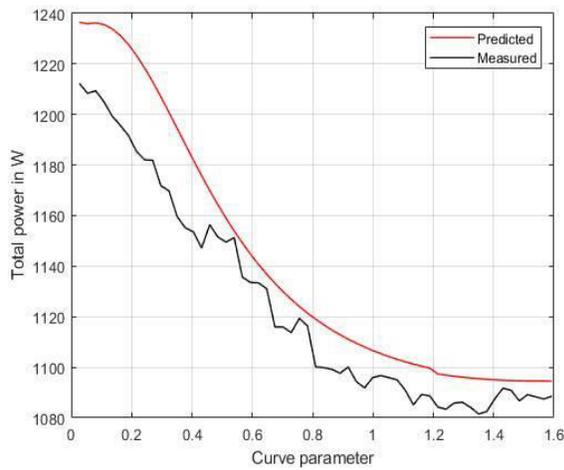


(b) Feed force variation

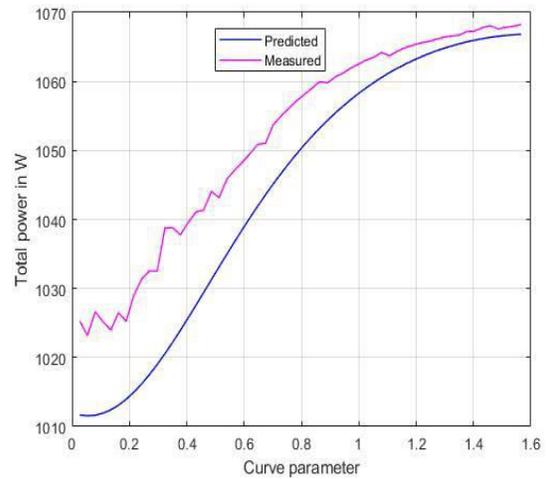
Fig.10 Force variation with respect to curve parameter for concave and convex geometries

Fig. 11 shows the variation of power consumption along the peripheral length for concave and convex elliptical workpiece. In case of concave elliptical geometry, the power profile is gradually decreasing from the start to end of cut as shown in Fig.11a. It happens due to resultant effect of gradual declination of cutting force and inclination of feed force components. In case of convex elliptical geometry, the reverse trend of power profile is observed as shown in Fig. 11b. It happens due to resultant effect of gradual increase of cutting forces and gradual

diminishing of feed force components. In addition to that, it is also observed that machining of concave geometries deals with larger cutting forces due to greater tool-workpiece engagement and respective higher chip load. Hence, the higher cutting power consumption is observed due to greater cutting forces. Table 11 shows the predicted and measured power along with calculation of error for both concave and convex elliptical workpart. The average prediction error is 1.63 % for concave elliptical geometry and 0.91 % for convex elliptical geometry which indicates the accuracy and acceptability of the developed model.



(a) Variation of power consumption for concave elliptical geometry



(b) Variation of power consumption for convex elliptical geometry

Fig.11 Comparison of experimental and computational power variations for concave and convex elliptical geometry

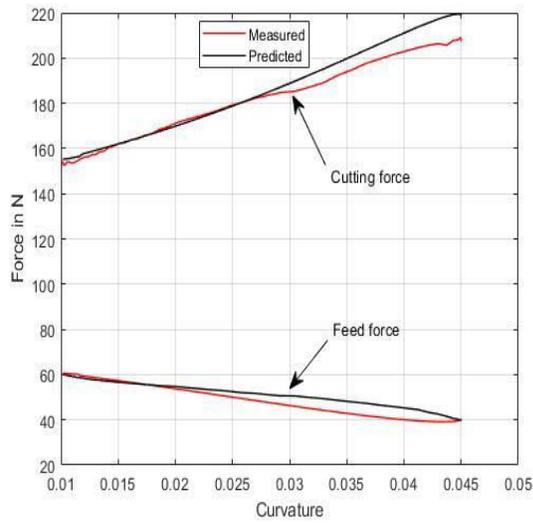
Table 11. Comparison of the predicted power with measured power at different positions along tool path for concave and convex elliptical geometries

No.	Concave elliptical geometry			Convex elliptical geometry		
	P (Measured)	P* (Predicted)	Prediction error (%)	P (Measured)	P* (Predicted)	Prediction error (%)
1	1235.9	1208.3	2.27	1011.6	1027.2	1.52
2	1181.9	1153.5	2.45	1025.7	1041.5	1.51
3	1119	1100.1	1.71	1050.8	1060.1	0.87
4	1097.3	1084.2	1.20	1063.5	1067.5	0.37
5	1094.5	1088.6	0.54	1066.8	1070.2	0.31

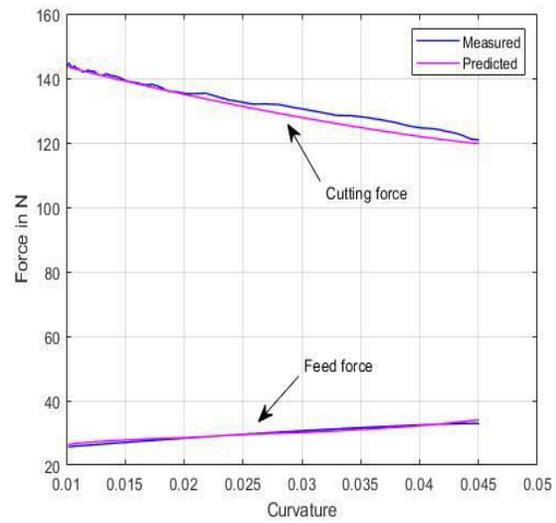
	Average prediction error	1.63		0.91
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#### **4.2 Influence of workpiece curvature on cutting force and feed force components**

This section deals with the influence of workpiece curvature on cutting force and feed force components for curved elliptical geometries. Fig. 12 shows the variation of simulated cutting forces and feed forces with respect to workpiece curvature for concave and convex elliptical workpieces. It is seen from the Fig. 12a that in case of concave elliptical geometry, the cutting force increases gradually as the value of curvature increases. It takes place due to higher cutter engagement angle and concerned uncut chip thickness that results into higher cutting force component for concave geometry. But, the reverse trend is noticed in case of feed force profile for concave geometry which can be seen from Fig.12a. As the magnitude of feed per tooth along the tool contact path decreases gradually with increase of curvature value, the uncut chip thickness is affected that leads to gradual declination in the feed force profile. In case of convex elliptical geometry, the cutting force decreases gradually as the value of curvature increases along the tool contact path which can be seen from Fig.12b. It happens due to reduction of cutter engagement angle and respective uncut chip thickness leading to decrease in cutting force component. But, the feed force profile gradually increases as the magnitude of feed per tooth along tool contact path increases due to increase of curvature value.



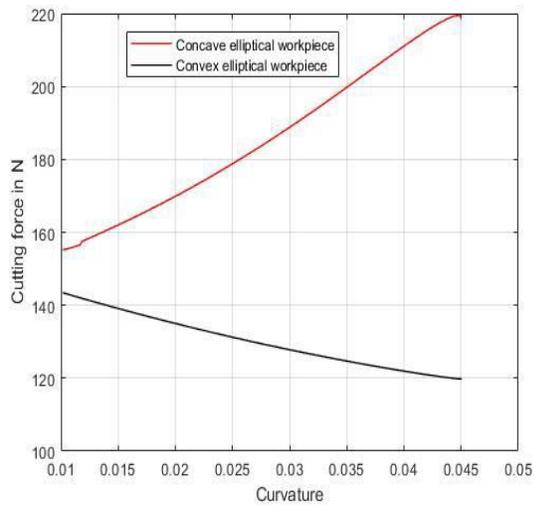
(a) Variation of cutting and feed forces for concave geometry



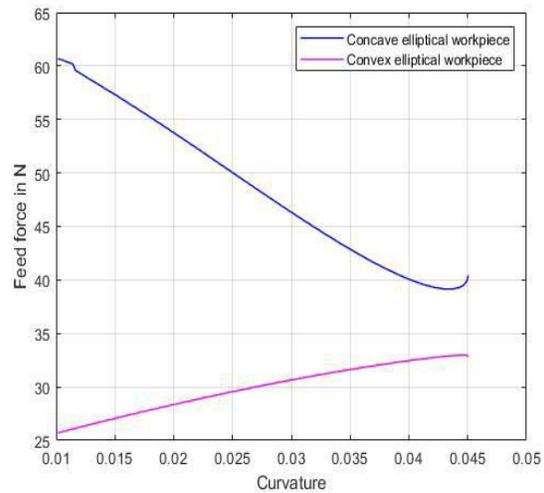
(b) Variation of cutting and feed forces for convex geometry

Fig.12 Variation of cutting forces and feed forces with respect to curvature for concave and convex elliptical geometries

Fig. 13 shows the variation of cutting force and feed forces for concave and convex elliptical geometry for performing comparative study between these geometries. From these graphs it is concluded that the workpiece curvature creates significant impacts on both the cutting force and feed force components during machining of variable curved geometry. Hence, the power consumption which is function of cutting force and feed force is influenced by workpiece curvature significantly which has been described in the following subsection.



(a) Cutting forces for elliptical geometry



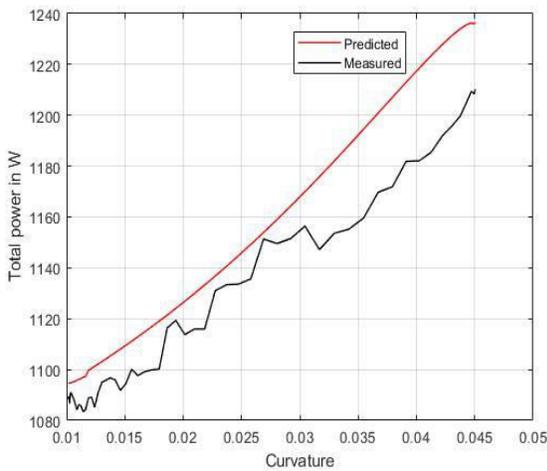
(b) Feed forces for elliptical geometry

Fig.13 Comparison of cutting and feed forces for concave and convex elliptical geometry

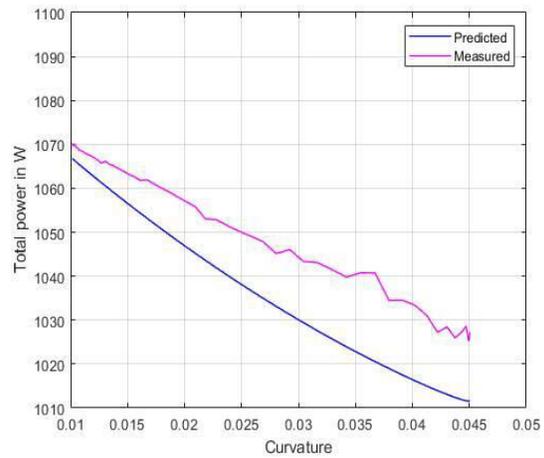
### 4.3 Influence of workpiece curvature on power consumption

This section deals with the influence of workpiece curvature on power consumption for both concave and convex elliptical geometries. Fig.14 shows the variation of total power consumption along the tool contact path for concave elliptical and convex elliptical geometry. In case of concave elliptical geometry, the power profile increases gradually along the tool contact path from the start to end of cut which can be seen from Fig.14a. Although for concave geometry, the feed force decreases but cutting force increases, their resultant effect decides the magnitude and nature of cutting power profile. Eventually, the power profile increases gradually along the peripheral length of workpiece for concave elliptical geometry. The reverse trend of cutting power profile is observed in case of convex elliptical geometry as shown in Fig.14b. Although, the feed force increases and cutting force decreases for convex elliptical geometry with increase of curvature value, their combined effect decides the magnitude and nature of

cutting power consumption for convex geometry. From the graph, it is concluded that cutting power consumption is function of cutting and feed forces along with cutting speed of the tool. As cutting power depends on both cutting force and feed force components, the workpiece curvature influences the cutting power consumption significantly during machining of variable curved geometry.



(a) Variation of cutting power for concave geometry



(a) Variation of cutting power for convex geometry

Fig.14 Variation of cutting power consumption with respect to curvature for concave ellipse and convex ellipse geometries

## 5. Conclusions

The current paper represents a hybrid model of cutting power in milling of variable curved workpieces. The analytical model for prediction forces and power is developed in MATLAB and validated using experimental work. It is found that the proposed model estimates the cutting forces and power within  $\pm 2\%$  in comparison with the experimental results. The model predicted and the experimental results are found to follow the similar trend for curved concave and convex geometries. The model also concentrates on the influence of workpiece curvature on

cutting power consumption during milling of variable curved geometry. The proposed hybrid model is validated by the milling experimental results. The proposed model is more generic in nature and can predict the power consumption irrespective of any workpiece geometry. It can estimate the power consumption for straight, circular and variable curved geometries. The proposed model needs calibration for only one machine with a tool-workpiece pair. In case of other machine tools, the calibration procedure needs to be performed to find out the values of different coefficients used in the model. The following conclusions are drawn basis upon the results discussed herein.

- In order to accurate estimation of power consumption, the role of workpiece curvature is significant and it needs to be incorporated in the model. Otherwise, erroneous results are expected for prediction of various force components and power consumption.
- In case of curved geometry, cutting force and cutting power vary along the cutter contact path due to continuous change in workpiece curvature resulting into varying process geometry variables and concerned chip load.
- In case of convex surface, the cutting power continuously decreases with increase in curvature value and an opposite trend is observed for concave surface. In other words, the cutting power increases from convex to concave geometries for identical cutting conditions.

The present study provides an excellent platform to process planners for estimation of cutting forces and power consumption in milling process. Without conducting actual expensive cutting experiments, the energy efficiency of the milling under various machining conditions can be accurately forecasted for any type of workpiece geometry. It can also be utilized as a reliable platform to the process planner for optimization of machining parameter towards cleaner

production. Hence, it will save both materials, energy and time. The outcomes of the present research work can contribute in developing energy-efficient and cleaner production system.

### **Acknowledgement**

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### **Nomenclature**

$P_{idle}$	Idle power [W]
$\bar{P}_{cutting}$	Cutting power [W]
$P_{auxiliary}$	Auxiliary power [W]
$P_{total}$	Total power [W]
$X_t(u), Y_t(u)$	Parametric curve of locus of tool center
$X_{wb}(u), Y_{wb}(u)$	Parametric curve of before cut workpiece trajectory
$X_{wa}(u), Y_{wa}(u)$	Parametric curve of after cut workpiece trajectory

$X'(u), Y'(u)$	Derivative of parametric w.r. parameter
$d_r$	Offset distance between before cut and after cut workpiece trajectory [mm]
$r$	Milling cutter radius [mm]
$f_{cc}$	Feed per tooth along tool contact path [mm]
$\theta_{en}$	Entry angle [radian]
$\theta_{ex}$	Exit angle [radian]
$R$	Radius of curvature [mm]
$t_c$	Uncut chip thickness
$a_e$	Radial immersion [mm]
$a_p$	Axial immersion [mm]
$\alpha$	Helix angle [radian]
$K_t, K_r$	Cutting constants
$dF_{i,j,f}(\varphi)$	Feed force acting on tooth $j$ at angular rotation $\varphi$
$dF_{i,j,n}(\varphi)$	Normal force acting on tooth $j$ at angular rotation $\varphi$
$P_{cutting}$	Instantaneous cutting power [W]

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### Ethical approval

The work is original and not have been published in any journal or conference in any form or language in best of our knowledge.

### Consent to participate

Not applicable

### Consent to publish

We are looking forward for possible publication in IJAMT.

### **Authors contributions**

Shrikant Shankarrao Pawar: Original draft writing, Investigation, Performing Experimentation, Validation, Resources, Software.

Tufan Chandra Bera: Conceptualization, Investigation, Methodology, Formal Analysis, Reviewing, Supervision.

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### **Declaration of competing interest**

I claim that there is no potential conflict of interest.

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### **Availability of data and materials**

The datasets used or analysed during the current study are available from the corresponding author on reasonable request.

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## **Highlights**

- Energy consumption model for milling of variable curved geometries is hybrid in nature.
- Cutting power consumption is expressed as a function of cutting and feed forces.
- Cutting power consumption is also function of workpiece curvature in milling.
- Proposed model is more generic and applicable for straight, circular and variable curved geometries.