Compressed Sensing Technology for Spectral Reconstruction Based on Maximum Entropy Criterion

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Abstract

In order to choose the related sampling ratio in the information-poor and information-rich spectral fragments, this paper attempts to recover the spectral reflectance by compressed sensing technology based on maximum entropy criterion. The maximum entropy threshold method is the criterion that the optimal threshold is determined to segment the information content of spectral curves. The spectral reflectance in each sub-spectral fragment is reconstructed by compressed sensing. The wavelet orthogonal matrix performs a sparse representation of each segmented spectral curve. Undersampling spectral curve be collected by random gaussian measurement matrix. The orthogonal matching pursuit algorithm recovers sparse original signals from undersampling observed signals. In this paper, the four standard color blocks of Munsell and the spectral curves of five types of ground objects in the hyperspectral data set are used as the experimental objects. The reconstructed results are evaluated by spectral curve reconstruction, root mean square error and information entropy difference. The experimental results show that our approach improves the reconstruction accuracy of spectral reflectance effectively, compared with the traditional method.

Introduction

The reflectivity is a fundamental characteristic of object surface, which is not affected by the external environment, the illumination and the detector^{1,2}. Spectral reflectance is associated with unique color information and rich texture information. Reconstruction of the spectral reflectance on every pixel can assist in the accurate reproduction of colors and object identification³. Spectral reconstruction technology is widely used in printed matter anti-counterfeiting, artwork restoration, construction industry and other fields.

There are some traditional strategies to reconstruct spectral reflectance⁴⁻⁷. Pratt and Mancill proposed pseudo-inverse method to reconstruct spectral reflectance⁸, the method can only have better reflectance reconstruction results when the number of camera response channels is large. Hardberg adopted principal eigenvector method which involves only the channel response that has a significant influence on the reconstructed spectral reflectance. Murakami proposed the wiener estimation method is based on the least square fitting algorithm to obtain the conversion matrix between the channel response value and the spectral reflectance, so that the difference between the reconstructed spectral reflectance and the original spectral reflectance is minimized⁹. Connah reconstructed the spectral reflectance by polynomial regression which employs different terms of higher-degree to determine the correlation of sampling points and the trend of interpolating points in the imaging response curve¹⁰.

Compressed sensing (CS) is the technology that the key idea is to adopt the sparsity of the signal to reduce the dimensionality of the signal with a far fewer samples than the traditional Nyquist–Shannon sampling theorem requires, and to restore the high-dimensional sparse vector by the optimization algorithm accurately¹¹⁻¹³. Compressed sensing has been widely used in remote sensing imaging,

medical imaging, digital communications and other fields because of the characteristics of greatly reducing the measurement time and samples^{14–16}.

In the early stage of our research group, compressed sensing has been applied to the spectral reflectance reconstruction, which achieved good reconstructed results¹⁷. Due to the smooth and non-smooth fragment in the spectral curve, the situation described as data rich but information poor will arise after random downsampling in the entire spectral fragment. It is necessary to adopt segmentation processing for the spectral fragment to improve the reconstructed accuracy in the non-smooth fragment and reconstructed efficiency in the smooth fragment. The traditional uniform compressed sensing segmentation method which does not depend on the distribution of information in the spectral fragment may result in insufficient sampling in the non-smooth spectral fragment and excessive sampling in the smooth spectral fragment^{18,19}. Aiming at the un-even distribution of information entropy in the spectral fragment, this paper proposes a compressed sensing technology for spectral reconstruction based on maximum entropy criterion.

First, we determine the optimal threshold by maximizing the entropy value of the spectral to find the most reasonable segment length of the smooth spectral fragment and the non-smooth spectral fragment. The spectral fragment with a large information entropy value is allocated more sampling values, and the spectral fragment with a small information entropy value is allocated with fewer sampling values. Finally, the spectral reflectance in each sub-spectral fragment is reconstructed by compressed sensing to improve the reconstruction accuracy in the overall spectral curve.

Methods

In order to verify the compressed sensing technology based on maximum entropy criterion, we first conducts experiments with four standard Munsell color blocks of red, green, blue and yellow. The evaluations are conducted by calculation of information entropy and root mean square error between the original and the reconstructed spectral reflectances.

Maximum Entropy Threshold Segmentation Method Of Spectral Reflectance

Pre-segmentation processing on the spectral curve was performed to calculate the information entropy on the cases of different segmentation thresholds. When the information entropy reaches the maximum, it is determined as the optimal threshold. The original spectral domain is divided into two spectral fragments. The symbol A denotes the spectral fragment where the spectral wavelength is smaller than the threshold. The symbol B denotes the spectral fragment where the spectral wavelength is bigger than the threshold.

The spectral wavelength i ranges from 400nm to 700nm $\{i | 400 \leq i \leq 700, i \in \mathbb{N}\}$. The threshold t_i is selected in the same domain with i . Once t_i is determined, the Spectral curve is divided into X_A and X_B .

$X = (R_{400}, R_{401}, R_{402}, \dots, R_{700})$. R_i represents the spectral reflectance at wavelength i nm.

$$X_A = (R_{400}, R_{401}, R_{402}, \dots, R_{t_i}), A = [400, t_i].$$

AA shows the probability distribution of X_A . BB shows the probability distribution of X_B .

$$AA: \frac{P_{400}}{P_A}, \frac{P_{401}}{P_A}, \frac{P_{402}}{P_A}, \dots, \frac{P_{t_i}}{P_A}$$

(1)

$$P_A = \sum_{i=400}^{t_i} P_i$$

(2)

$$BB: \frac{P_{t_i+1}}{P_B}, \frac{P_{t_i+2}}{P_B}, \frac{P_{t_i+3}}{P_B}, \dots, \frac{P_{700}}{P_B}$$

(3)

$$P_A + P_B = 1$$

(4)

P_i represents the probability value of the spectral reflectance R_i at the wavelength i nm.

The entropy related to the two probability densities of A and B are expressed by:

$$H(A) = - \sum_{i=400}^{t_i} \frac{P_i}{P_A} \log_2 \frac{P_i}{P_A}$$

(5)

$$H(B) = - \sum_{i=t_i+1}^{700} \frac{P_i}{P_B} \log_2 \frac{P_i}{P_B} = - \sum_{i=t_i+1}^{700} \frac{P_i}{1 - P_A} \log_2 \frac{P_i}{1 - P_A}$$

(6)

The maximum entropy of spectral fragments is calculated to determine the optimal threshold t_0 of the spectral curve:

$$HH(t_0) = \max[H(A) + H(B)]$$

(7)

$HH(t_o)$ indicates the maximum sum of the information entropy of the spectral segments divided by different thresholds t_i . t_o is the optimal threshold for dividing the smooth spectral segment and the non-smooth spectral segment. The next step of the experiment is to reconstruct segmented spectral by compressed sensing.

Compressed Sensing

The maximum entropy threshold segmentation method is that the optimal threshold t_o is determined to find the most reasonable segment length of the smooth spectral fragment and the non-smooth spectral fragment. The total fragment length of the spectral reflectance is $N = 301$. The total dimensionality reduction sample number is $M = 100$. There are two conditions to be met, $M = M_a + M_b$, $N = N_a + N_b$. N_a is the length of the spectral fragment A , N_b is the length of the spectral fragment B . M_a is the downsampling size of the spectral fragment A , M_b is the downsampling size of the spectral fragment B .

The original signal is $X = 301 \times 1$ dimensional column vector, $X = \{R_{400}, R_{401}, R_{402}, \dots, R_{700}\}^T$. By using a set of wavelet orthogonal matrix $\varphi = \{\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_{301}\}$, the sparse representation of the original signal will be obtained.

$$X = \varphi \alpha$$

(8)

Where α is the linear projection of the original signal X on the wavelet orthogonal basis φ . The number of non-zero elements in the sparse coefficient α is $K(K \leq N)$. The original signal X is undersampled by the measurement matrix $\psi_{M \times N} = \{\psi_1, \psi_2, \psi_3, \dots, \psi_M\}$ for reduced dimensionality, namely:

$$Y = \psi X = \psi \varphi \alpha = C \alpha$$

(9)

Where C is the perception matrix. The signal Y is an M -dimensional column vector. After the linear projection of the signal X on the measurement matrix, the undersampling measured value Y is obtained. The measurement matrix selected in this experiment is gaussian random matrix. The gaussian random matrix obey standard normal distribution with mean 0 and variance 1. Gaussian random measurement matrix complies with the restricted isometry property (RIP). $M \ll N$, X is derived from Y by an underdetermined equation. The norm l_1 is selected to solve the optimal solution α' . The reconstruction for original signal adopts the orthogonal matching pursuit algorithm (OMP).

$$\alpha' = \operatorname{argmin} \| \alpha \|_1 \text{ s.t. } Y = C \alpha \quad (10)$$

$$X^{\{\prime\}}=\varphi \{\alpha ^{\prime}\} \quad (11)$$

On the promise that the total sampling number remains unchanged, different sampling numbers are distributed on the two spectral fragments A and B . The spectral fragment with the large information entropy value is allocated more sampling number, and the spectral fragment with the small information entropy value is allocated fewer sample number. The reconstruction of compressed sensing is performed on the two spectral fragments A and B respectively.

Spectral fragment A is reconstructed by CS. The sampling size of signal $\{X_A\}=\{\left\{ \{R_{400}\},\{R_{401}\},\{R_{402}\}, \cdots ,\{R_t\} \right\}^T$ is M_a . The signal $\{X_A\}$ is represented linearly by a set of orthogonal wavelet matrices $\{\varphi _a\}=\left\{ \{\varphi _1\},\{\varphi _2\},\{\varphi _3\}, \cdots ,\{\varphi _{Na}\} \right\}$. The sparsity coefficient is $\{\alpha _{\text{1}}\}$. The formula can be expressed as:

$$\{X_A\}=\{\varphi _a\}\{\alpha _1\} \quad (12)$$

Where $\{\alpha _{\text{1}}\}$ is also means linear projection of signal $\{X_A\}$ on $\{\varphi _a\}$ domain, the number of non-zero elements in sparse coefficient is $\{K_a\}\left(\{K_a\} \leqslant \{N_a\} \right)$. The signal $\{X_A\}$ is linearly projected by a random Gaussian measurement matrix for dimensionality reduction. The measurement matrix $\{\psi _{Ma \times Na}\}=\left\{ \{\psi _1\},\{\psi _2\},\{\psi _3\}, \cdots ,\{\psi _{Ma}\} \right\}$ is not related with the wavelet basis, then the compressed sensing problem can be formulated as follows:

$$\{Y_A\}=\{\psi _{Ma \times Na}\}\{X_A\}=\{\psi _{Ma \times Na}\}\{\varphi _a\}\{\alpha _1\}=\{C_1\}\{\alpha _1\} \quad (13)$$

Where $\{C_{\text{1}}\}$ is the perception matrix. $\{Y_A\}$ is an M -dimensional column vector. After the linear projection of the signal $\{X_A\}$ on the measurement matrix $\{\psi _{Ma \times Na}\}$, the undersampling measured value $\{Y_A\}$ is obtained, $\{M_a\} \ll \{N_a\}$. The orthogonal matching pursuit algorithm is selected for recovering the sparsest solution of under-determined system.

$$\{\alpha _1\}^{\{\prime\}}=\arg \min \{\left\| \alpha \right\| _1\} \text{ s.t } \{Y_A\}=\{C_1\}\{\alpha _1\} \quad (14)$$

$$\{X_A\}^{\{\prime\}}=\{\varphi _a\}\{\alpha _1\}^{\{\prime\}}=\{\left\{ \{R_{400}\}^{\{\prime\}},\{R_{401}\}^{\{\prime\}},\{R_{402}\}^{\{\prime\}}, \cdots ,\{R_t\}^{\{\prime\}} \right\}^T\} \quad (15)$$

The CS-based reconstruction of the spectral fragment A is obtained.

Spectral fragment B is reconstruct by CS. The sampling size of signal $\{X_B\}=\{\left\{ \{R_{t+1}\},\{R_{t+2}\},\{R_{t+3}\}, \cdots ,\{R_{700}\} \right\}^T$ is M_b . The spectral signal $\{X_B\}$ is represented linearly by a set of orthogonal wavelet matrices $\{\varphi _b\}=\left\{ \{\varphi _{184}\},\{\varphi _{185}\},\{\varphi _{186}\}, \cdots ,\{\varphi _{301}\} \right\}$. The sparsity coefficient is $\{\alpha _{\text{2}}\}$. The formula can be expressed as:

$$\{X_B\}=\{\varphi _b\}\{\alpha _2\} \quad (16)$$

Where $\{\alpha _{\text{2}}\}$ is also means linear projection of signal $\{X_B\}$ on $\{\varphi _b\}$ domain, the number of non-zero elements in sparse coefficient is $\{K_b\}\left(\{K_b\} \leqslant \{N_b\} \right)$. The signal $\{X_B\}$ is linearly projected by a random gaussian measurement matrix for dimensionality reduction. the measurement matrix $\{\psi _{Mb \times Nb}\}=\left\{ \{\psi _1\},\{\psi _2\},\{\psi _3\}, \cdots ,\{\psi _{Mb}\} \right\}$

ψ is not related to the wavelet basis, then the compressed sensing problem can be formulated as follows:

$$Y_B = \psi_{M \times N} X_B = \psi_{M \times N} \varphi_b \alpha_2 = C_2 \alpha_2 \quad (17)$$

Where C_2 is the perception matrix. Y_B is an M -dimensional column vector. After the linear projection of the signal X_B on the measurement matrix ψ_2 , the undersampling measured value Y_B is obtained, $M \ll N$. The orthogonal matching pursuit algorithm is used to solve the under-determined equations.

$$\alpha_2^{\prime} = \arg \min_{\|\alpha\|_1} \text{ s.t } Y_B = C_2 \alpha_2 \quad (18)$$

$$X_B^{\prime} = \varphi_b \alpha_2^{\prime} = \{R_{t+1}^{\prime}, R_{t+2}^{\prime}, R_{t+3}^{\prime}, \dots, R_{700}^{\prime}\}^T \quad (19)$$

The CS-based reconstruction of the spectral fragment B is obtained .

Integrating the reconstructed spectral reflectance $X_A^{\prime}$ and $X_B^{\prime}$, the result is $X^{\prime} = \{R_{400}^{\prime}, R_{401}^{\prime}, R_{402}^{\prime}, \dots, R_{700}^{\prime}\}^T$.

Table 1 shows the maximum entropy threshold segmentation results of the four standard color blocks (red, green, blue and yellow). The optimal thresholds are $t_0 = 639\text{nm}, 536\text{nm}, 461\text{nm}, 583\text{nm}$ for red, green, blue and yellow blocks respectively.

Table 1
The maximum entropy threshold segmentation results.

	Red	Green	Blue	Yellow
t_0	639	536	461	583
A	[400:639]	[400:536]	[400:461]	[400:583]
B	[640:700]	[537:700]	[462:700]	[584:700]

Taking the yellow spectral curve for example. The optimal threshold t_0 of the yellow spectral curve is 583nm. The spectral curve is divided into fragments $A = [400\text{nm}-583\text{nm}]$, $B = [584\text{nm}-700\text{nm}]$. The total fragment length of the spectral reflectance is $N = 301$. The total dimensionality reduction sample value is $M = 100$. Subject to the constrains $M_a > M_b$, $100 = M_a + M_b$, $301 = N_a + N_b$, we yeild the following parameters after segmentation.

$$N_a = 183, N_b = 118, M_a = 60, M_b = 40.$$

The reconstructed spectral reflectance for yellow is shown in Fig. 1(d).

Results

The spectral reconstruction for the standard color blocks

The spectral curves of the four standard Munsell color blocks (red, green, blue and yellow) are reconstructed with different method. Figure 1 shows the reconstructed spectral curve with the traditional approaches and our approach. Solid red line is the original spectral curve. Dot solid blue line, solid purple line, dashed–dotted black line and solid green line are the spectral curves reconstructed by compressed sensing, polynomial regression, pseudo-inverse and compressed sensing technology based on maximum entropy criterion (MECCS) respectively. It can be clearly observed that the two spectral curves which are reconstructed by polynomial regression and pseudo-inverse both deviate from the original curve. The solid green curve is closer to the original spectral curve.

Root mean square error (RMSE) can be used to discuss the deviation between the original spectral reflectance and the reconstructed spectral reflectance. Original spectral reflectance is $X = \left\{ \{R_{400}\}, \{R_{401}\}, \{R_{402}\}, \dots, \{R_{700}\} \right\}^T$, the reconstructed spectral reflectance is. $\{X^{\prime}\} = \left\{ \{R_{400}\}^{\prime}, \{R_{401}\}^{\prime}, \{R_{402}\}^{\prime}, \dots, \{R_{700}\}^{\prime} \right\}^T$. The root mean square error is expressed as:

$$RMSE = \sqrt{\frac{\sum_{i=400}^{700} \left\{ \left(\{R_i\} - \{R_i\}^{\prime} \right)^2 \right\}}{N}} \quad (20)$$

Table 2 shows RMSEs for the aboved methods which reconstruct spectral reflectance of four standard color blocks. RMSE1 reveals the difference between the original spectral data and the reconstructed spectral data with the polynomial regression. RMSE2 reveals the difference between the original spectral data and the reconstructed spectral data with the pseudo-inverse method. RMSE3 reveals the difference between the original spectral data and the reconstructed spectral data with the compressed sensing method. RMSE4 reveals the difference between the original spectral data and the reconstructed spectral data with compressive sensing based on maximum entropy criterion. It is clearly that RMSE4 has the smallest value, i.e., the deviation between the original spectral reflectance data and the reconstructed spectral reflectance data is smallest.

Table 2
Comparison of root mean square error.

	Red	Green	Blue	Yellow
RMSE1	0.0504	0.1738	0.0326	0.0297
RMSE2	0.1299	0.1647	0.0733	0.0223
RMSE3	0.0045	0.0092	0.0033	0.0092
RMSE4	0.0012	0.0016	0.0013	0.0034

Information entropy is used to evaluate the uncertainty of information quantity. The greater the uncertainty of the information is, the greater the information entropy is. The smaller the difference in

information entropy is, the higher the stability of the two information is.

$$H = -\sum_{i=1}^N \frac{P_i}{P} \log \frac{P_i}{P} \quad (21)$$

$$\Delta_{2-1} = H(2) - H(1) \quad (22)$$

$$\Delta_{3-1} = H(3) - H(1) \quad (23)$$

$$\Delta_{4-1} = H(4) - H(1) \quad (24)$$

$$\Delta_{5-1} = H(5) - H(1) \quad (25)$$

Table 3 shows the information entropy between the original and reconstructed curves with different methods. Δ_{2-1} is the difference between the original spectral data and the reconstructed spectral data with the polynomial regression, Δ_{3-1} refers to the difference between the original spectral data and the reconstructed spectral data with the pseudo-inverse, Δ_{4-1} is the difference between the original spectral data and the reconstructed spectral data with the compressed sensing, Δ_{5-1} refers to the difference between the original spectral data and the reconstructed spectral data with the compressive sensing based on maximum entropy criterion. It is clear that Δ_{5-1} is 2–6 times less than Δ_{4-1} , Δ_{5-1} is 2–10 times less than Δ_{2-1} and Δ_{3-1} .

Table 3
The information entropy.

	Red	Green	Blue	Yellow
H (1)	7.9626	8.0639	8.1764	8.0057
H (2)	7.9698	8.0612	8.1725	8.0025
H (3)	7.9728	8.0319	8.1668	7.9721
H (4)	7.9701	8.0619	8.1731	8.0068
H (5)	7.9649	8.0649	8.1769	8.0055
Δ_{2-1}	0.0072	0.0027	0.0039	0.0032
Δ_{3-1}	0.0102	0.0032	0.0096	0.0336
Δ_{4-1}	0.0075	0.0020	0.0033	0.0011
Δ_{5-1}	0.0023	0.0010	0.0005	0.0002

The experiment result shows that our method is better than other methods. The performance that our method reconstructs the spectral reflectance is excellent.

The Spectral Reconstruction For Hyperspectral Image

The above experimental results confirm the feasibility of our proposed method. In this part of the work, we use the hyperspectral data from Pavia university to extend the application of our method. The size of

the data cube is 340*610*103. The wavelength range is 430nm-860nm and the original channel number is 115. The roof image with 56*56 size in the 30th channel is selected as the area of interest to complete reconstruction by compressed sensing and compressed sensing based on maximum entropy criterion, as shown in Fig. 2. The experimental results are shown in Fig. 3. Figure 3(a) is original image and the spectral curve; Fig. 3 (b) is image and curve reconstructed by CS; Fig. 3 (c) is image and curve reconstructed by MECCS. The difference on intensity of image pixel can be seen. Our method improves the spectral reconstruction.

Table 4 shows PSNR and image entropy reconstructed by CS and MECCS method. We can observe that the result of MECCS is better than CS'. The image entropy reconstructed by MECCS is closer to the original image entropy, that is to say, these two images are more similar.

Table 4
PSNR and image entropy by CS and MECCS method

	Original image	Image reconstructed by CS	Image reconstructed by MECCS
PSNR		33.1411	35.0211
Image entropy	7.4713	7.4797	7.4694

The Spectral Reconstruction For The Objects On The Ground

We further reconstruct the spectral reflectance curves of the different ground objects in the hyperspectral data such as trees, shadows, meadows, bricks, metal. The evaluations are conducted by calculating the information entropy of spectral curve and RMSE between the original and the reconstructed spectral.

In Fig. 4, solid blue line is the original spectral curve. Solid black line and dashed–dotted red line are spectral curve reconstructed by CS and MECCS respectively. It is clear that the red curves are closer to the blue curves.

Table 5 shows RMSEs of CS and MECCS method. RMSE1 represents the difference between the original spectral data and the reconstructed spectral data with the compressed sensing. RMSE2 represents the difference between the original spectral data and the reconstructed spectral data with compressive sensing based on maximum entropy criterion. It is clear that RMSE2 is 2–4 times less than RMSE1.

Table 5
RMSE of the reconstructed spectral curve.

	Trees	Shadows	Meadows	Bricks	Metal
RMSE1	0.0088	0.0051	0.0166	0.0045	0.0097
RMSE2	0.0066	0.0028	0.0044	0.0023	0.0063

Table 6 shows information entropy of ground objects. Δ_{2-1} represents the difference between the original spectral data and the spectral data reconstructed by CS, Δ_{3-1} represents the difference between the original spectral data and the reconstructed spectral data with MECCS. Δ_{3-1} is 2–5 times less than Δ_{2-1} .

Table 6
RMSE of the reconstructed spectral curve.

	Trees	Shadows	Meadows	Bricks	Metal
H (1)	6.1693	6.5437	6.1318	6.6703	6.5948
H (2)	6.1735	6.5286	6.1217	6.6713	6.5929
H (3)	6.1655	6.5407	6.1272	6.6697	6.5945
Δ_{2-1}	0.0042	0.0151	0.0101	0.0010	0.0019
Δ_{3-1}	0.0038	0.0030	0.0046	0.0006	0.0003

Conclusion

A compressed sensing technology based on the maximum entropy criterion is proposed to reconstruct the spectral curve in this paper. Maximum entropy is the criterion for spectral signal segmentation. The wavelet base is a sparse matrix, and the gaussian random measurement matrix reduces the dimensionality of the spectral reflectance signal. The orthogonal matching pursuit algorithm solves the optimization problem to recover original spectral reflectance precisely. Hyperspectral cube of Pavia university and the four standard Munsell color blocks as the object are used to verify the excellent performance of the experiment. Compared with other methods, the reconstructed spectral curve with the CS based on maximum entropy is closer to the original spectral curve, Root mean square error and information entropy difference are both given to evaluate our method quantitatively.

Declarations

Data availability

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request

Competing interests (mandatory)

The authors declare no competing financial interests.

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Author contributions

S.Z. conceived the experiment and wrote the manuscript. M.Y.: conducted experiments. Y.W. and J.F. performed data analysis. All authors reviewed the manuscript.

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Figures

(a)

(b)

(c)

(d)

Figure 1

Spectral curves of four standard Munsell color blocks. (a) the spectral curve of the blue block; (b) the spectral curve of the green block; (c) the spectral curve of the red block; (d) the spectral curve of the yellow block.

Figure 2

The 30th channel image and the extracted region of interest.

Figure 3

Experimental results and images selected from hyperspectral data. (a) is original image and the spectral curve; (b) is image and curve reconstructed by CS; (c) is image and curve reconstructed by MECCS.

Figure 4

The reconstructed spectral reflectance curves of five types ground objects in the hyperspectral data set. (a) the reconstructed spectral curves of trees; (b) the reconstructed spectral curves of meadows; (c) the reconstructed spectral curves of metals; (d) the reconstructed spectral curves of shadows; (e) the reconstructed spectral curves of bricks.