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Whirl Analysis of an Overhung Disk Shaft System Mounted on Non-rigid Bearings

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Abstract. Eigenvalues of a simple rotating flexible disk-shaft system are obtained using different methods. The shaft is supported radially by non-rigid bearings, while the disk is situated at one end of the shaft. Eigenvalues from a finite element and a multi-body dynamic tool are compared against an established analytical formulation. The Campbell diagram based on natural frequencies obtained from the tools differ from the analytical values because of oversimplification in the analytical model. Later, detailed whirl analysis is performed using AVL Excite multi-body tool that includes understanding forward and reverse whirls in absolute and relative coordinate systems and their relationships. Responses to periodic force and base excitations at a constant rotational speed of the shaft are obtained and a modified Campbell diagram based on this is developed. Whirl of the center of the disk is plotted as an orbital or phase plot and its rotational direction noted. Finally, based on the above plots, forward and reverse whirl zones for the two excitation types are established.

Keywords. Vibration, Disk-shaft system, Campbell diagram, whirl analysis

1. Introduction

A disk-rotor system is a basic building block of many mechanical systems that find application in automotive transmissions, differentials, and accessory drives, electrical motor output shafts, etc., to name a few. Understanding vibration characteristics of a rotating disk-rotor system can help us refine the system level design by addressing certain resonance conditions or *NVH* issues. Before personal computers became a widespread commodity, researchers mainly relied on analytical formulation to solve vibration analysis problems of various disk-rotor systems. However, this came at the cost of accuracy due to certain assumptions and/or simplifications.

In literature, vibration analysis was addressed for systems consisting of a flexible and/or rigid shaft carrying flexible or rigid single or multiple disks. Some of the earlier methods used lumped parameter approximation. This approach mainly used transfer matrix methods [1, 2]. Lumped parameter approach simplifies the problem by dividing the system into a series of mass/inertia (nodes) connected by spring-damper elements. However, a true physical system having infinite degrees of freedom cannot be accurately represented by this approach. Natural frequencies and critical speeds of even a simple disk-shaft system obtained by this approach is slightly off compared to those obtained by more

accurate methods. The problem of a continuous flexible non-rotating shaft carrying a rigid disk was solved by Srinath et al. [3]. Similar system for the non-rotating case was solved by Eshleman et al. [4]. In both the cases, the thickness of the disk was ignored, and characteristic equation method was used to solve the system eigenvalues. Influence of disk flexibility on the natural frequency of bending and critical speeds of a rotating disk-shaft system was solved by Chivens et al. [5]. Nataraj [6] developed a mathematical model that investigated the interaction between the torsional and flexural deflections of a uniform shaft rotating at a constant speed. Chiu et al. [7] analytically solved coupled vibration among shaft torsion and blade bending of a multi-disk rotor system with grouped blades by ignoring shaft bending in the formulation. Chowdhury et al. [8, 9] analytically examined the coupled vibration of high speed geared and pulley systems mounted on short, rotating, flexible shafts. Galerkin's method was used to discretize the discrete-continuous systems. For a low speed geared shaft system, Chowdhury et al. [10] used assumed modes method to find the effect of shaft flexibility and gear mesh stiffness on the coupled vibration modes and responses.

Upon enhancement of digital computers, dynamic finite element-based methods have become popular to solve such systems. Taplak and Parlak [11] used a program named Dynrot based on beam elements to analyze the dynamic behavior of a small gas turbine rotor. They obtained the critical speeds, Campbell diagram and the response of the rotor to the center of mass imbalance in the compressor. A more generalized approach was developed by Szolc [12] by resorting to a discrete-continuous modeling of the rotating systems under coupled lateral torsional vibrations. This is useful for industrial applications and avoids disadvantages and restrictions typical for the transfer matrix and finite element methods. Here, the rotor shaft segments are represented by the rotating cylindrical flexurally and torsionally deformable continuous viscoelastic elements. Hili et al. [13] used finite element method to solve the eigenvalue problem of a spinning flexible disk-shaft system. Natural frequency comparison between systems with and without disk flexibility was performed to show that both disk flexibility and shaft boundary conditions affect the system behavior. Verma et al [14] studied effect of the disc position on the natural frequencies for the simply supported bearings and cantilever types rotor.

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None of the above works used a multi-body simulation tool that has gained momentum in recent years, especially, in terms of development of electrical vehicles [15] or balance shaft systems of IC engines [16].

In the present work, natural frequencies (Campbell diagram plot) of a rotating cantilever disk-shaft system supported by two bearings are compared using three different methods – (A) a simplified but established analytical formulation using lumped system formulation, (B) a well-known finite element tool (ANSYS), and (C) preprocessor (shaft modeler) of a comprehensive multi-body simulation tool (Excite Power Unit). Later, using the multi-body tool, full time domain dynamic simulation is performed to do the response analyses under two different loading conditions and based on these, a refined Campbell diagram is generated, which can be useful for real life vibration mitigation. Bearing clearances and non-linearity are incorporated in the full Excite Power Unit model and response analyses to force and base excitations are performed.

2. Mathematical model

Fig.1 shows the schematic of a simplified cantilever rotor bearing system analyzed in [17]. This is a lumped parameter-based model and is presented here mainly to introduce the concept of forward and reverse whirls. The models (B) and (C) are continuous system models and hence are significantly different from this model. In the experimental setup of Fig. 1(a), a small motor is suspended at its center of gravity by very soft springs.

Figure 1. Set up of the disk-shaft system in [17] explains the gyroscopic effect.

In the schematic (Fig. 1(b)), the point of support, ‘O’, is assumed to remain at rest, while the shaft with the disk attached to its end whirls about ‘O’. The natural frequencies are given by [17]:

$$\omega_n = \pm \frac{I_2}{2I_1} \pm \sqrt{\left(\frac{I_2\Omega}{2I_1}\right)^2 + \frac{k}{I_1}} \quad (1)$$

As ω_n cannot be negative, only the ‘+’ sign need to be retained before the square root. Natural frequency for forward whirl:

$$\omega_n = \frac{I_2}{2I_1} + \sqrt{\left(\frac{I_2\Omega}{2I_1}\right)^2 + \frac{k}{I_1}} \quad (2)$$

Natural frequency for reverse whirl:

$$\omega_n = -\frac{I_2}{2I_1} + \sqrt{\left(\frac{I_2\Omega}{2I_1}\right)^2 + \frac{k}{I_1}} \quad (3)$$

The Campbell diagram is described in [17] as given below (Fig. 2):

Figure 2. Campbell diagram from [17] showing forward and reverse whirl natural frequencies. With the forward whirl frequency, the precision ω has the same direction as the rotation Ω ; with the reverse whirl frequency, these directions are opposite.

A similar system is built using the finite element tool (Ansys) (Fig. 3(a)), as well as multi-body dynamic tool (AVL Excite Power Unit) (Fig. 4). However, for practicality, the shafts needed to be supported by high bearing stiffness (1e9 N/mm) are used in both the models. The disk end of the shaft behaves in the similar fashion as in the system of Fig. 1. The ‘Coriolis, on’ option is selected to enable the gyroscopic effect in Ansys. In Ansys, the frequencies are calculated in frequency domain. This is sufficient to build Campbell diagram. However, response analysis is not performed in Ansys. Excite is the tool of choice for response calculation.

AVL Excite Power Unit is a 3D multi-body dynamic analysis tool that solves a nonlinear dynamic system in time-domain [18]. In Excite, gyroscopic effect is inherently included in the analysis. The process of model-building in Excite is more elaborate than in Ansys. In Excite, while the model building and preliminary frequency domain analysis is done in the pre-processing tool called *shaft modeler*. The time domain response analysis is performed using the main solver.

(a)

(b)

Figure 3. Relative coordinate system along with the disk-shaft system. (a) Schematic of the disk-shaft system, (b) Circle diagram of the relative coordinate system.

Three different analyses are performed:

- Free Vibration Analysis with the shaft mounted on bearings (Based on a given bearing spring stiffness with no-clearance) – this is obtained in the preprocessor called *shaft modeler* [18].
- Force Excitation Analysis (Periodic force applied on the shaft)
- Base Excitation Analysis (Support is given a simple harmonic motion)

Timoshenko beam-based shaft model is built in AVL Excite Power *shaft modeler* as shown in Fig. 4. This is a linear model where there are no clearances at the rigid bearings.

Figure 4. AVL Excite *shaft modeler* model of the disk-shaft system.

Each shaft segment consists of three nodes. Journal on each bearing is represented by a single node. Journal on bearing # 1 is represented by node 301 and that on bearing # 2 is represented by node 501 on the shaft. Although, in Exacite power unit, these nodes are connected via revolte joints (non-linear spring damper connections) to the corresponding nodes on the rigid stationary support, in the pre-processor shaft modeler, the model is simplified as a linear model with no bearing nonlinearity.

Right hand rule is followed according to the axis system. This means that

Ω, ω	Positive clockwise when viewed along the x-axis
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Two types of coordinate systems are defined: (a) Absolute coordinate system that is fixed to the ground or the rigid support and (b) Relative coordinate system (coordinate system $\{XYZ\}$ shown in the figure above as well as Fig. 3(b)) fixed to the shaft. Accordingly, for free vibration analysis, two different Campbell diagrams are obtained in two different coordinate systems: Absolute and Relative. For forced vibration analysis, orbital diagram of the center of the disk is plotted in both the absolute and relative coordinate systems. For forced vibration analysis, two different kinds of excitations are separately applied:

- Periodic force (of magnitude 10 kN) applied in-between the bearings on the rotating shaft,
- Periodic displacement (of magnitude 0.035 mm) applied to the rigid support.

The main purpose of obtaining response to force and base excitation is to validate the Campbell diagram and observe if the resonance frequency deviates from it. Note that, the Campbell diagram was obtained from a pre-processor tool (*shaft modeler*)

in frequency domain that assumed no-play between the shaft and the support nodes. On the other hand, in Excite Power Unit time-domain analysis, a non-zero clearance, however small, needs to be applied between the shaft and support nodes at the bearings. Hence, a corollary of the above exercise is identification of difference of the solution for a dynamic system between frequency and time domain analyses.

Accordingly, three different models were built using AVL Excite Power Unit.

1) Model corresponding to the free vibration analysis (Fig. 5):

Figure 5. AVL Excite Power Unit model for free vibration analysis.

This model is part of the Excite power Unit where the shaft-disk system is built using a pre-processor tool (*shaft modeler*). After assembling the shaft segments and the disk, the natural frequencies can be obtained for an unconstrained shaft with no bearing support constraints as well as for a constrained shaft with a specified bearing support stiffness. For obtaining the Campbell diagram of a gyroscopic shaft-disk system, shaft speed is specified over a range at a defined interval. Note that, the analysis in this tool is quick as it is done in frequency domain. No Excite Power Unit run is necessary for obtaining the Campbell diagram. As mentioned previously, this Campbell diagram can be obtained in both the absolute and relative coordinate systems.

The response analysis is done in the following two Excite Power Unit models where the response of the disk center node is obtained in time-domain to a periodic force excitation on the shaft or base excitation of the support. The shaft-disk system is supported by revolute joints (node-to-node connection) at the bearings. The shaft is rotated at a constant speed through an external rigid body using a rotational coupling (ROTX joint).

2) Model corresponding to the force excitation analysis (Fig. 6):

Figure 6. AVL Excite Power Unit model for force excitation analysis.

In this model, the response of the disk center node is obtained to a periodic vertical force excitation applied in-between the bearings on the shaft. The frequency of the forcing function of magnitude 10 kN is varied from 40 Hz to 150 Hz . The response is the radial displacement of the disk center node and is obtained as orbital diagram (horizontal z-displacement vs vertical y-displacement). As mentioned previously, this is obtained in both the absolute and relative coordinate systems. As the origin of the relative coordinate system is fixed to the bearing node on the shaft, magnitude of radial displacement of the disk center obtained from the orbital diagrams in either of the coordinate system is the same.

3) Model corresponding to the base excitation analysis (Fig. 7):

Figure 7. AVL Excite Power Unit model for base excitation analysis.

In this model, the response of the disk center node is obtained to a periodic base excitation of magnitude 0.035 mm . The frequency of excitation is varied from 40 Hz to 150 Hz . The response is obtained as orbital diagrams of the disk center node (horizontal z-displacement Vs vertical y-displacement) in both the absolute and relative coordinate systems. Note that, in both the Excite Power Unit models, unlike the *shaft modeler* constraints, a non-

zero clearance (0.02 *micron*) must be specified at the bearings to avoid numerical error and avoiding model divergence. As the bearing node attached to the supporting structure is non-stationary unlike the previous model, obtaining magnitude of radial displacement of the disk center is not straightforward in this case. This is obtained by subtracting y- and z- components of the bearing radial displacements from the corresponding components of the disk radial displacements in relative coordinate system and summing them up vectorially. To remind us of this procedure, the orbital diagram of the bearing node is also added to orbital diagram of the disk center.

3. Results and discussion

3.1 Campbell Diagram

Due to induced gyroscopic effects, natural frequencies depend on the rotation rates. Campbell diagram is a well-known plot in rotor dynamics representing natural frequency sensitivity to the rotational speeds. Campbell diagram have been derived in geared rotor systems [8, 9], turbine systems [11], etc. Mode splitting is one of the most common phenomena observed in a gyroscopic system when natural frequencies at zero speed (ω_0) degenerate into two frequencies at higher speeds. These frequencies are higher (ω_1) and lower (ω_2) than the frequency at zero speed (ω_0). The shaft axis does not remain straight at a higher speed. It bends resulting in a plane consisting of the bent shaft and the axis of rotation. At ω_1 , called the forward whirl frequency, this plane rotates in the direction of shaft rotational speed. At ω_2 , called the reverse whirl frequency, the direction of rotation is opposite to the shaft rotational direction.

For the present system, Campbell diagram is plotted using three different methods (Fig. 8): (a) using the first model that utilizes the preprocessor tool called *shaft modeler* of AVL Excite Power Unit, (b) using FE tool, Ansys after activating the ‘Coriolis, on’ feature (applicable only to axisymmetric structures such as the present system), (c) using analytical expression from [17], after calculating the shaft bending stiffness from the FE model in (b).

Figure 8. Campbell diagram from AVL Excite *shaft modeler* (preprocessor) along with that from Ansys as well as analytical expression.

The natural frequencies from methods (a) and (b) are very close, while those obtained by method (c) are off. The main difference comes from inaccurate calculation of the shaft bending stiffness that is built-in within the analytical model of method (c). The slope of the shaft is continuously changing in a real shaft bending situation that is not captured by the analytical expression of [17]. This is, however, captured by the finite element tools that could account for the distributive mass-elastic properties.

The Campbell diagram above is plotted in the Absolute coordinate system. This is the most common type of plot we see in literature. Here, the observer is standing on the ground. Another type of Campbell diagram is possible when the observer is attached to the shaft coordinate system, i.e., rotating with the shaft itself. Under such a circumstance, the observer would see the forward whirl lag behind by the frequency corresponding to shaft speed (i.e., $\omega_1 - \Omega/60$) and the reverse whirl advance by the same amount (i.e., $\omega_2 + \Omega/60$), assuming that Ω is expressed in revolutions per minute (*rpm*).

Out of the three methods, method (a) could provide Campbell diagram in the Relative coordinate system directly. This is shown in Fig. 9.

(a)

(b)

Figure 9. Campbell diagram from AVL Excite *shaft modeler* (preprocessor) in (a) Absolute coordinate system (b) Relative coordinate system. At shaft speed of 4000 rpm (66.67 Hz), in absolute coordinate system, forward whirl frequency = 125.466 Hz and reverse whirl frequency = 71.8504 Hz. In relative coordinate system, forward whirl frequency = 125.466 – 66.667 Hz = 58.799 Hz and reverse whirl frequency = 71.8504 + 66.667 Hz = 138.52 Hz. Also shown in Fig. (b) are the reverse whirl frequencies at 1000 rpm.

In Fig. 9, while the frequency of forward whirl in Absolute coordinate system is lower than that in the Relative coordinate system, the order is reversed for the Campbell diagram in Relative coordinate system. The reason is explained in the previous paragraph. More elaboration is given by choosing a specific shaft speed. Let us choose a shaft rotation speed of 4000 rpm. Here, revolutions per second is considered as Hertz. This is equivalent to a frequency of $(4000/60) \text{ Hz} = 66.667 \text{ Hz}$. At this speed, as shown in Fig. 9(a), the forward whirl frequency (ω_1) = 125.466 Hz and the reverse whirl frequency (ω_2) = 71.8604 Hz. The corresponding frequencies in Relative coordinate system would be $(125.466 - 66.667) \text{ Hz} = 58.799 \text{ Hz}$ and $(71.8504 + 66.667) \text{ Hz} = 138.52 \text{ Hz}$ (Fig. 8(b)).

Natural modes of the disk-shaft system can be animated within *shaft modeler*. This gives a more comprehensive visual representation of the forward and reverse whirls. However, the modes only in the Relative coordinate system are animated. Fig. 10 shows the snapshots of the animations for forward and reverse whirls at a shaft speed of 1000 rpm. From Fig. 9(b), natural frequency of forward whirl is 87 Hz, while that for reverse whirl is 106 Hz. The animation snapshots (Fig. 10) clearly depicts that forward whirl (frequency = 87 Hz) is in the direction of shaft rotation, while reverse whirl (frequency = 106 Hz) is opposite to the direction of shaft rotation.

Animation Obtained in Excite *Shaft Modeler* (Relative Coordinated system)

AVL Shaft Modeler - Modal Analysis (susp.) - Freq. 3 [87 Hz]

(a)

AVL Shaft Modeler - Modal Analysis (susp.) - Freq. 4 [106 Hz]

(b)

Figure 10. Natural mode shapes at shaft speed of 1000 rpm obtained from Excite *shaft modeler* (in relative coordinate system). (a) Forward whirl frequency (87 Hz), (b) reverse whirl frequency (106 Hz).

In Absolute coordinate system, the reverse whirl frequency (ω_2) = $106 - 16.67 (=1000/60) = 89.33 \text{ Hz}$ and forward whirl frequency (ω_1) = $87 + 16.67 = 103.67 \text{ Hz}$. These can be tentatively estimated from Fig. 9(a).

3.2 Response Analysis

For response analysis, the rotational speed of the shaft is chosen to be at 4000 rpm. Responses are obtained to **a periodic vertical force of magnitude 10 kN** applied to the shaft span in-between the bearings (model 2) as well as to **a base excitation of magnitude 0.035 mm** (model 3). In both the cases, frequency of excitation is varied from 40 Hz to 150 Hz. The resonance frequencies as obtained from the Campbell diagram from the preprocessor are included within this frequency range. As both the direction of the applied force and base excitation are fixed in the Absolute coordinate system, resonance frequencies for forward and reverse whirls are 125.466 Hz and 71.8604 Hz (Fig. 9(a)). These frequencies are included in the range of the

excitation frequencies. Responses are obtained as the magnitude of radial displacements as well as orbital diagrams of the center of the disk for various excitation frequencies. Orbital diagrams are plotted as the horizontal z-displacement Vs vertical y-displacement of the center of the disk.

3.2.1 Convergence Criteria

Excite Power Unit solves nonlinear differential equations using iterative methods. It takes a few shaft revolutions for the solution to stabilize. In the present case, a convergence study of the disk radial displacement is performed before gathering the results data. The simulation is performed for around twenty revolutions of the shaft (7200 degrees) and results are plotted for the whole range of shaft rotation. In Fig. 11, the y- displacement of the center of the disk is plotted against the shaft rotation angle for different excitation frequencies. As seen, angle to convergence is different for different frequencies. However, after 6000 degrees of shaft rotation, results are pretty much stabilized.

Figure 11. Response (displacement of the center of the disk) in time domain at different excitation frequencies. (a), (b) Force excitation, (c), (d) Base excitation.

Results for one load cycle is captured for calculation of displacement magnitude and plot of the orbital diagrams. To satisfy this, as excitation frequency is varied, requirement of shaft rotation angle differs. Table 1 shows the shaft rotation angle requirements and the chosen start and end angles of shaft rotation for one load cycle.

Table 1. Shaft rotational angle requirements for one load cycle.

Excitation Frequency (Hz)	Required shaft rotation angle for result (deg)	Start of shaft rotation angle (deg)	End of shaft rotation angle (deg)
40	600	6480	7080
50	480	6480	6960
58.8	408	6480	6888
66	364	6480	6844
71.9	334	6480	6814
80	300	6480	6780
95	253	6480	6733
110	218	6480	6698
118	204	6480	6684
125.466	192	6480	6672
132	182	6480	6662
138.53	173	6480	6653
150	160	6480	6640

3.2.2 Response Amplitude Vs Frequency

In Fig. 12, response amplitude is plotted against the excitation frequency to force and base excitations (Fig. 12(b)). Also plotted is the Campbell diagram from *shaft modeler* (Fig. 12(a)). It is evident that this Campbell diagram obtained from frequency domain with no bearing clearance is not the true representation of the system. This is because the response peaks did not appear exactly at the resonance or whirl frequencies from this diagram (i.e., 125.406 Hz and 71.8604 Hz). From the realistic Excite Power Unit time domain analysis, forward whirl frequency is somewhere around 118 Hz, the reverse whirl frequency is around 60 Hz. The difference can be attributed to the fact that in the Excite Power Unit model (unlike the shaft modeler), a realistic bearing clearance (vs zero clearance in shaft modeler) and bearing stiffness (vs infinite bearing stiffness in shaft modeler)

were used. Fig. 12(c) depicts this with the dotted lines in the modified Campbell diagram.

Figure 12. Response amplitude Vs Frequency superimposed with Campbell diagram. (a) Campbell Diagram from AVL Preprocessor (*shaft modeler*), (b) Response Amplitude Vs Frequency, (c) Response Amplitude Vs Frequency Campbell Diagram based on Response (Dotted Lines)

As concluded in [17], resonating condition for reverse whirl is extremely rare. Resonance during forward whirl is easy to explain compared to the reverse whirl. During resonance, deflection amplitude rises. In the first case, the centrifugal force of the unbalance due to the deflected center of the disk works on increasing the amplitude and essentially pumps energy into the system. It is difficult to find explanation where the energy for resonance comes from for the second case (reverse whirl). It is an extremely rare phenomenon [17]. In the present case, slight increase in displacement amplitude is observed for the reverse whirl frequency (~ 60 Hz) for both the excitations (base and force). The amplitude of excitation is nowhere near to the value at forward whirl condition.

For the magnitude of excitations chosen, response amplitude for base excitation is higher than the force excitation. The excitation values are not randomly selected. These values are typical for an internal combustion engine. While the firing force at the piston that comes on the crankshaft is of around 10 kN , based on the mount stiffness, displacement amplitude of the block can be around 0.035 mm . For this damped system (with material damping, damping at the REVO joints) maximum displacement of the disk center is close to 0.4 mm (Fig. 12(b)). Depending on the strength of the crankshaft material, this could affect the life of the crankshaft. Hence, the above study concludes that engine block vibration can be more detrimental for the durability of the crankshaft than high cylinder pressure.

3.2.3 Orbital Diagram of the Disk Center at Different Frequencies

3.2.3a Force Excitation Response

Orbital diagrams are plotted as the horizontal z-displacement Vs vertical y-displacement of the center of the disk for various frequencies of the exciting force. Displacements are obtained in both in the Absolute and Relative coordinate systems. These orbital diagrams are superimposed for different excitation frequencies of the force in Fig. 13.

Figure 13. Orbital diagrams for displacement of disk center due to the force excitation at various excitation frequencies.

In the orbital diagrams, arrows indicate the whirl direction. As the force is unidirectional in the Absolute coordinate system, the orbital diagrams in this system repeat themselves forming a closed loop. On the other hand, as the Relative coordinate system rotates with the shaft, repetition of orbits is not probable. Note that the z-axis is horizontal towards the right and the y-axis is vertical.

At a lower frequency (< 66 Hz), the whirl orbits are more or less vertical (Figs. 13(a)-(c)). At 66 Hz, the orbit is inclined at an angle of 45 degrees (Fig. 13(d)). Above 66 Hz, the orbits are closer to the horizontal axis (Figs. 13(e)-(g)). From the resonance frequency onwards (118 Hz, Fig. 13(i)-(m)), the orbits are circular and in Absolute and Relative coordinate systems these overlap. Note that, shaft rotational frequency is 66.67 Hz (4000

rpm). Also, the whirl direction is opposite to the shaft rotation direction for frequencies < 80 Hz (in accordance with the reverse whirl red curve of Fig 12(c)) and in the direction of shaft rotation for frequencies ≥ 80 Hz (in accordance with the forward whirl black curve of Fig 12(c)). Maximum displacement amplitude during reverse whirl occurs at 58.8 Hz and that for forward whirl occurs at 118 Hz. This was previously shown in Figs. 12(b) and (c). Figs. 13(o) and (p) overlay all the orbital diagrams together excluding that for 118 Hz as its magnitude is higher than the scale of the plot. At higher frequencies, there is a little difference between orbits in Absolute and Relative coordinate systems.

3.2.3b Base Excitation Response

For the base excitation, orbital diagrams show similar trend as the force excitation. However, as the supporting structure and the attached bearing nodes moves up and down along the vertical axis, the wobble of the disk-center is calculated in Relative coordinate system, which is not allowed to move vertically with the shaft. This is done by subtracting the y- and z- components of the bearing radial displacements from the corresponding components of the disk radial displacements in relative coordinate system and summing these components vectorially. To visualize this, orbital diagrams of the bearing journal node (the blue dotted curve, ‘Relative Coord – Journal Node’ – Fig. 14(a)-(m)) are included along with the orbital diagrams of the disk center (the blue solid curve, ‘Relative Coord – FW Node’ – Fig. 14(a)-(m)).

(g) 95 Hz

(k) 132 Hz

(h) 110 Hz

(l) 138.5 Hz

(i) 118 Hz

(m) 150 Hz

(j) 125.5 Hz

Base Excitation Direction

(n)

Figure 14. Orbital diagrams for displacement of disk center due to the base excitation at various excitation frequencies.

Note that like Fig. 13, in Fig. 14, the z-axis is horizontal towards the right and the y-axis is vertical.

As in the force excitation, for lower frequencies, the whirl orbits are aligned close to the vertical axis (Figs. 14(a)-(c)). However, unlike the force excitation, whirl orbits align horizontally (i.e., elliptical) only after the forward whirl resonance frequency of 118 Hz is reached. For force excitation, this happened at frequencies just above 66 Hz. The whirl direction is opposite to the shaft rotation direction for frequencies < 66 Hz and in the direction of shaft rotation for frequencies ≥ 66 Hz. For force excitation, this frequency was 80 Hz. Maximum displacement amplitude during reverse whirl occurs at 58.8 Hz and that for forward whirl occurs at 118 Hz.

Interesting facts are observed when comparing the orbits of the disk center and the bearing journal node in Relative coordinate system. For this, the whole frequency range is divided into three generic segments.

Frequencies < 66 Hz

At a low frequency (40 Hz), radial displacement of the journal node is similar (but lower) to the disk center node. As the frequency increases, the difference becomes larger becoming maximum at 58.8 Hz. At this frequency, radial displacement is close to double of the magnitude of base excitation ($2 \times 0.035 \text{ mm}$

= 0.070 mm). This also indicates that at the reverse whirl resonance condition, motion of the disk center is synchronous to the base excitation but in opposite phase. At 66 Hz, even though the orbital diagrams for journal node and disk center overlap, there is a phase difference. Hence, the difference in radial displacement is non-zero (as seen in a previous figure). Also, change in whirl direction of the shaft in takes place at this frequency (Fig. 14(d)).

66 Hz < Frequencies < 95 Hz

These frequencies are in-between the forward and reverse whirl frequencies (Fig. 12(c)). At these frequencies (i.e., 71.9 and 80 Hz), radial displacements of the disk-center are lower than that of the journal nodes and located at one side of map in Relative coordinate system (Fig. 14(e), (f)). This means that the shaft is bent while the disk-center makes small circular motion around it.

Frequencies ≥ 95 Hz

At and beyond 95 Hz, disk-center orbits in Absolute and Relative coordinate systems grow and become very similar at the resonance frequency of 118 Hz. Notice that the journal orbit is minuscule compared to the disk-center orbit at this frequency. Above 118 Hz, the orbits shrink indicating departure away from the resonating condition. Also, as stated before, the whirl orbits in Absolute frame changes alignment from vertical to horizontal above 118 Hz (Fig. 14(q)).

Based on the above discussion on whirl direction, the Campbell diagram of Fig. 12(c) can be segregated into forward and reverse whirl zone based on the type of excitation as shown in Fig. 15.

Figure 15. Segregated Campbell diagram based on the excitation type.

While for force excitation frequency up to 80 Hz, disk node rotational direction is opposite to the shaft rotation, the same frequency for base excitation is 66 Hz. Knowing true vibrational dynamic characteristics such as the above can help us design better rotor dynamic systems by avoiding resonance conditions.

4. Conclusions

Shaft speed is kept constant while the excitation frequency is varied.

- 1) For the gyroscopic disk shaft system, Campbell diagram obtained in frequency domain from FE tool, Ansys and dynamic analysis tool, AVL Excite (*shaft modeler*) is very similar and differ from plots based on simplistic analytical equations. The difference comes due to realistic calculation of shaft bending stiffness.
- 2) In Absolute coordinate system, for a degenerate mode, frequency of forward whirl mode is higher than the corresponding mode with a reverse whirl. The relationship gets reversed in Relative coordinate system. This is because, the frequency of the forward whirl gets reduced by the rotational frequency of the shaft while that of the reverse whirl gets increased by the same amount.
- 3) Campbell diagram based on response in time domain showed that resonance frequencies for both forward and reverse whirls are slightly lower than the values obtained in frequency domain. This is because in the frequency domain nonlinearities due to bearing clearance is ignored.
- 4) For both the force and base excitations, the highest peak observed corresponds to the forward whirl condition. Although extremely rare, a slight peak is observed for reverse whirl condition as well.
- 5) For the magnitude of excitations chosen, base excitation is more predominant than the force excitation. These values are representative of an IC engine. The magnitude of the force corresponds to the force coming on the crank pin while magnitude of the base displacement corresponds to the displacement of the engine on the mounts. As such, engine vibration can be more damaging than cylinder pressure to the durability of the crankshaft.
- 6) Even though the cut off frequencies are not exactly the same, the orbital diagrams show interesting trends for both the force and base excitations:
 - a) At lower frequencies, the orbital plots are aligned to the excitation direction and at higher frequencies, these are perpendicular to the direction of excitation.
 - b) At lower frequencies, the whirl direction is opposite to shaft rotation and at higher frequencies the direction is along to the shaft rotation.
 - c) At resonance, the orbit is circular and in Absolute and Relative coordinate systems these overlap.
- 7) Response to base excitation is more intricate than the force excitation.
 - a) At the resonance frequency for reverse whirl, the radial displacement is twice the magnitude of base excitation indicating that the motion of the disk-center is synchronous and opposite in phase to the base motion.
 - b) In-between the forward and reverse whirl resonance frequencies, the shaft is bent while the disk makes small wobble around it.
 - c) As the Relative coordinate is fixed translationally, unlike the force excitation, orbits in Absolute and Relative coordinate systems differ at higher frequencies.

Nomenclature

Ω	Angular velocity of disk rotation
ω_1, ω_2	Forward and reverse whirl frequencies
I_1	Moment of inertia of the rotating shaft-disk system about the diametric axis through O
I_2	Moment of inertia of the rotating shaft-disk system about the shaft axis
k	Bending stiffness of the cantilever shaft at the point of support assuming cylindrical shaft
Absolute coordinate system	Coordinate system that is fixed to the ground
Relative coordinate system	Coordinate system that rotates with the shaft. However, no translational motion is allowed.

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