The Role of Harvesting in Population Control in Presence of Multiple Stochastic Noise Sources

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The role of harvesting in population control in presence of multiple stochastic noise sources

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Abstract

In this paper, we compare the role of constant and Michaelis-Menten type harvesting in single species population control in presence of stochastic noises sources. Steady state probability distributions and stationary potentials of the population for the two types of harvesting are determined by Fokker-Planck equations. Stochastic bifurcation analysis and mean first passage times have been computed. Noise induced critical transitions are observed depending on the strength of the noises. The extinction possibility of population in stochastic control with Michaelis-Menten type harvesting is higher than the constant rate of harvesting. One of the findings is the transition of the population from bistable to tristable for weak noise and Michaelis-Menten type harvesting. Another finding is noise enhanced stability phenomenon for negatively correlated noises. In case of population control, constant rate of harvesting is better in deterministic case whereas Michaelis-Menten type harvesting is better in stochastic case. The stochastic control is more efficient than deterministic control as average population size in stochastic case is lower than the deterministic case. The results obtained in this study can throw light on toxic phytoplankton blooms and its control in marine ecosystem. Moreover, the study can be useful to explain wild prey population outbreak and its control in deep forest.

Keywords: Predation, Harvesting, Critical transition, Regime shift, Stochastic bifurcation, Control.

1 Introduction

Ecological interference of forests caused by insect pests are an important threat to both natural ecosystems and human welfare. The insect outbreaks in agricultural system are more sensible to economic, social and cultural issues. Moreover, maintenance of bio-diversity is one of the greatest challenge for ecologist since it is essential for long term survival of human society. Damages and losses caused by insects outbreak have an increasing trend in recent decades due to global climate change and related natural disturbances. In a theoretical ecologist's point of view the question of bio-diversity of an ecological system is directly

related to stability of the system. Results of stability analysis of various deterministic population models have been reported by many authors [1, 2]. The study of models with noise are becoming increasingly important in various branches of science [3–8]. In real world ecosystem various environmental noises have significant impact on the stability of the system. The study of population models in a stochastically fluctuating environment has also received considerable interest [9–14] as it captures the real world scenarios. The control and management of outbreaks of pests and insect in forest, agro-ecosystem and other ecological systems draw attention of many researchers [15, 16] and studied widely as an outbreak has a deleterious influence on human survival and well being.

Population dynamics of an species is influenced by growth, predation, competition, mutualism, variations in food availability and many other ecological factors [17–20]. Harvesting is another source of mortality apart from the natural mortality which also can change a population dynamics significantly. The main aim of harvesting theory is to provide general principles to control exploited species. Harvesting theory is very useful in managing many ecological systems namely in fishery [21], forestry [22], wildlife management [23], etc. It has motivated the researchers to investigate the effects of harvesting in many population models [24–27] and reported the fact that harvesting processes have the potential to modulate the dynamics of exploited populations in many ways. There are various types of harvesting strategies eg. constant rate of harvesting [28], Michaelis-Menten type harvesting [29], density dependent harvesting [30], quadratic harvesting [31], etc. in theoretical ecology.

The predator management plays a crucial role in population control as the predator might lead the prey to decrease and even become extinct. The predator management [32, 33] is frequently observed in the common practice of agricultural pest control. Generally, in agriculture, pest is controlled through natural enemies and poisoning.

The critical need of comparative study in ecological systems is necessary to guide the choice of parameters for refinement and verification of theoretical results. A comparative study of effects of various types of harvesting strategies on stochastic control of population dynamics are not done yet as per as our knowledge. It has motivated us to investigate and compare the effects of various types of harvesting strategies on the population control in presence of stochastic noises. Our objective is to determine a statistical correlation between the noises in some model parameters and population control. These kind of correlations may be helpful for designing population control strategies. In this paper, we analyze and compare the effects of different types of harvesting strategies in stochastic control of a population.

In particular, constant harvesting and Michaelis-Menten type harvesting strategy are considered here. In this study, we have designed population control through modification of a logistic growth model with predators and two different types of harvesting strategies. Introducing the noises in predation and harvesting term we formulate stochastic models. Our goal is to investigate the effects of positively and negatively correlated noises on population control. Moreover, we are motivated to compare the impacts of constant rate of harvesting and Michaelis-Menten type harvesting in population control.

The paper is organized as follows. In Sect. 2, single species population control is designed in presence of harvesting and stochastic noise sources. In particular, we have modeled population control for constant rate of harvesting and Michaelis-Menten type harvesting independently by considering correlated white noises in predation and harvesting. In Sect. 3, Fokker-Planck equations are derived. Steady-state probability distributions and stationary potentials are calculated for the proposed models. The variation of the depth of potential wells with respect to noises strength are depicted to understand the possibility of noise induced transition behavior of the steady states of the population. In Sect. 4, stochastic

bifurcation of two proposed models are computed. In Sect. 5, the mean first passage times are computed with the variation of the noises and cross correlation strength. In Sect. 6, stochastic simulation and early warning signals have been calculated to ensure about critical transition and abrupt regime shift. In Sect. 7, quantitative analysis of the models have been done to compare the performance of two types of harvesting strategies and conclusion of the study is presented in Sect. 8.

2 Model

Limited resources enhance competition among individuals with the increase of population size and simultaneously resources become increasingly scarce as a population reaches the carrying capacity of the ecosystem. Growth rate of a species become very very small when population increasingly reaches to carrying capacity and growth rate become negative if population become larger than the carrying capacity. Mathematically this kind of growth is described by the well known logistic growth model given bellow,

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{\kappa}\right), \quad (1)$$

where x is the population biomass of insect at time t , r is the intrinsic growth rate of the population, κ is the carrying capacity.

The success of a population control model depends on predation, parasitism, herbivory, and other natural mechanisms. Almost all ecological systems are deeply perturbed by human exploiting activities. Harvesting has a considerable effect on the dynamical evolution of the harvested species, the severity of which depends on the different harvesting strategies. Generally, the population can be controlled by harvesting and predation. Harvesting and predation can be considered as two other sources of mortality apart from the natural mortality. So, a mathematical model of population control can be formulated considering the effect of different types of harvesting strategies and predation effects. In this study, the predation term is chosen as Holling type-III functional form. The mathematical model in case of constant rate of harvesting [28] can be written as following,

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} - \rho_c, \quad (2)$$

where α is half-saturation level for predation, β is the predation rate and ρ_c is the constant absolute rate of harvesting.

Similarly, the mathematical model in the case of Michaelis-Menten [29] type harvesting can be written as,

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} - \frac{\rho_m x}{\gamma + x}, \quad (3)$$

where ρ_m is the product of catchability coefficient and harvesting effort. Since harvesting is a population control mechanism therefore, the above two models can be interpreted as deterministic insect outbreak control models.

Nature controls the prey population by the predator. The traditional techniques used to control population have targeted entire populations through the use of random harvesting. The recent literature has focused on controlling population size by random harvesting [34, 35]. The random rate of predation has a significant impact on prey population [36, 37]. We incorporate noise terms in the parameters representing harvesting and predation which can

be interpreted as the phenomenon of poaching as random harvesting and predation. It is well known that predation activity is random in nature.

The parameters of constant rate of harvesting model are modified by noises as $\beta + \Xi(t)$, $\rho_c + \Gamma(t)$ respectively. And the parameters in case of Michaelis-Menten type harvesting model are modified by noises as $\beta + \Xi(t)$, $\rho_m + \Gamma(t)$ respectively. Sometimes, these two kinds of noises may have a same source and thus they are not independent. Physically it means that the noises may be correlated with each other. Therefore, the mathematical model of population control governed by constant rate of harvesting and noises can be described by the following stochastic model,

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{\kappa}\right) - \frac{[\beta + \Xi(t)]x^2}{\alpha^2 + x^2} - [\rho_c + \Gamma(t)]. \quad (4)$$

Similarly, the mathematical model of the population control governed by Michaelis-Menten type harvesting and noises can be described by the following stochastic model,

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{\kappa}\right) - \frac{[\beta + \Xi(t)]x^2}{\alpha^2 + x^2} - [\rho_m + \Gamma(t)] \frac{x}{\gamma + x}. \quad (5)$$

The symbols $\Xi(t)$ and $\Gamma(t)$ represent Gaussian white noises with mean zero and correlations given by the following relations,

$$\langle \Xi(t)\Xi(t') \rangle = 2\sigma\delta(t-t'), \quad (6a)$$

$$\langle \Gamma(t)\Gamma(t') \rangle = 2D\delta(t-t'), \quad (6b)$$

$$\langle \Gamma(t)\Xi(t') \rangle = \langle \Xi(t)\Gamma(t') \rangle = 2\lambda\sqrt{\sigma D}\delta(t-t'), \quad (6c)$$

where σ and D are the strength of the noises $\Xi(t)$ and $\Gamma(t)$ respectively, λ ($-1 \leq \lambda \leq 1$) is cross correlation strength between $\Xi(t)$ and $\Gamma(t)$.

It can be noticed that the noise in harvesting rate in model (4) is additive whereas in model (5) it is multiplicative. Clearly, the stochastic model (4) and model (5) can be ecologically interpreted as the stochastic control of population. On the other hand, deterministic model (2) and model (3) can be considered as the deterministic control of population. Impacts of the noises and two different types of harvesting strategies on the population will be analyzed and compared analytically as well as numerically.

The deterministic potential of model 4 is,

$$V_c(x) = \frac{r}{3\kappa}x^3 - \frac{r}{2}x^2 + (\beta + \rho_c)x - \alpha\beta \arctan \frac{x}{\alpha}. \quad (7)$$

Also, the same for model 5 is,

$$V_m(x) = \frac{r}{3\kappa}x^3 - \frac{r}{2}x^2 + (\beta + \rho_m)x - \alpha\beta \arctan \frac{x}{\alpha} - \gamma\rho_m \ln(x + \gamma). \quad (8)$$

It is noted that $V_c(x)$ and $V_m(x)$ are the potential functions of the population controlled deterministically for constant rate of harvesting and Michaelis-Menten type harvesting respectively.

The potentials related to the deterministic forces are plotted in figure 1. It is observed that the potential is double-well for both the deterministic models. That means the potentials of the deterministically controlled population are double well irrespective of the harvesting process. The population has two stable steady states (low population biomass state

and high population biomass state) and an unstable intermediate population state, which separates the stable states when it is controlled deterministically. The high population state sometimes is named as outbreak state.

3 Fokker-Planck equations

The Fokker-Planck equation (FPE) corresponding to Langevin equation (4) with eqs. (6) is given by (following [38]),

$$\frac{\partial P_c(x, t)}{\partial t} = -\frac{\partial}{\partial x} A_c(x) P_c(x, t) + \frac{\partial^2}{\partial x^2} B_c(x) P_c(x, t), \quad (9)$$

where $P_c(x, t)$ is the probability density of the population followed by eq. (4) at time t , $A_c(x)$ and $B_c(x)$ are drift and diffusion coefficient respectively.

Therefore the probability density distribution of the population at time t maintains the equation eq. (9) when it is controlled stochastically with constant rate of harvesting. The drift and diffusion coefficients are given by,

$$A_c(x) = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} + \frac{2\lambda\alpha^2\sqrt{D\sigma}x}{(\alpha^2 + x^2)^2} + \frac{2\sigma\alpha^2x^3}{(\alpha^2 + x^2)^3} - \rho_c, \quad (10a)$$

$$B_c(x) = \frac{\sigma x^4}{(\alpha^2 + x^2)^2} + \frac{2\lambda\sqrt{D\sigma}x^2}{\alpha^2 + x^2} + D. \quad (10b)$$

In case of mathematical model of the stochastically controlled population with Michaelis-Menten type harvesting (eq. (5)), the FPE can be obtained as,

$$\frac{\partial P_m(x, t)}{\partial t} = -\frac{\partial}{\partial x} A_m(x) P_m(x, t) + \frac{\partial^2}{\partial x^2} B_m(x) P_m(x, t), \quad (11)$$

where $P_m(x, t)$ denotes the probability density distribution of the population controlled stochastically with Michaelis-Menten type harvesting at time t , $A_m(x)$ is drift coefficient and $B_m(x)$ is diffusion coefficient. The expressions of the coefficients are as follows,

$$A_m(x) = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} + \frac{2\sigma\alpha^2x^3}{(\alpha^2 + x^2)^3} - \frac{\rho_m x}{\gamma + x} + \frac{\gamma Dx}{(\gamma + x)^3} + \frac{2\lambda\alpha^2\sqrt{D\sigma}x^2}{(\gamma + x)(\alpha^2 + x^2)^2} + \frac{\lambda\gamma\sqrt{D\sigma}x^2}{(\alpha^2 + x^2)(\gamma + x)^2}, \quad (12a)$$

$$B_m(x) = \frac{\sigma x^4}{(\alpha^2 + x^2)^2} + \frac{2\lambda\sqrt{D\sigma}x^3}{(\alpha^2 + x^2)(\gamma + x)} + \frac{Dx^2}{(\gamma + x)^2}. \quad (12b)$$

3.1 Steady state probability distribution (SSPD) and stationary potential (SP) of the stochastically controlled population for constant rate of harvesting

The steady-state probability distribution (SSPD) [38] $P_c^{st}(x)$ of FPE (9) is given by,

$$P_c^{st}(x) = \frac{N_c}{B_c(x)} \exp \left[\int^x \frac{A_c(y)}{B_c(y)} dy \right], \quad (13)$$

$$= N_c \exp \left[-\phi_c^{st}(x) \right], \quad (14)$$

where N_c is the normalization constant, $\phi_c^{st}(x)$ is stationary potential (SP) of FPE (9).

First we discuss the role of noise in harvesting. So, we assume $\sigma = 0$. For $\sigma = 0$, eqs. 10 can be reduced to,

$$A_c(x) = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} - \rho_c, \quad (15a)$$

$$B_c(x) = D. \quad (15b)$$

Therefore, the SP has the following form,

$$\begin{aligned} \phi_c^{st}(x) &= \frac{1}{2} \ln D - \frac{1}{D} \int^x \left[ry \left(1 - \frac{y}{\kappa}\right) - \frac{\beta y^2}{\alpha^2 + y^2} - \rho_c \right] dy, \\ &= \frac{1}{2} \ln D + \frac{V_c(x)}{D}, \end{aligned}$$

where $V_c(x)$ is the deterministic potential of model 4. Noticeably, in absence of predation noise, stationary and deterministic potential have similar forms.

In case of noise in predation, we fix $D = 0$. For $D = 0$, eq. 10 can be transformed to,

$$A_c(x) = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} + \frac{2\sigma\alpha^2 x^3}{(\alpha^2 + x^2)^3} - \rho_c, \quad (16a)$$

$$B_c(x) = \frac{\sigma x^4}{(\alpha^2 + x^2)^2}. \quad (16b)$$

So, SP has the following form,

$$\begin{aligned} \phi_c^{st}(x) &= \ln \frac{\sqrt{\sigma} x^2}{\alpha^2 + x^2} + \int^x \frac{(\alpha^2 + y^2) \left[\alpha^2 \rho_c \kappa - \alpha^2 \kappa r y + \alpha^2 r y^2 + \beta \kappa y^2 + \rho_c \kappa y^2 - \kappa r y^3 + r y^4 \right]}{\kappa \sigma y^4} dy, \\ &= \ln \frac{\sqrt{\sigma} x^2}{\alpha^2 + x^2} + \frac{\zeta_1}{x^3} + \frac{\zeta_2}{x^2} + \frac{\zeta_3}{x} + \zeta_4 x + \zeta_5 x^2 + \zeta_6 x^3 + \zeta_7 \ln x, \end{aligned}$$

where,

$$\begin{aligned} \zeta_1 &= -\frac{\alpha^4 \rho_c}{3\sigma}, \\ \zeta_2 &= \frac{\alpha^4 r}{2\sigma}, \\ \zeta_3 &= -\frac{\alpha^4 r + \alpha^2 \beta \kappa + 2\alpha^2 \rho_c \kappa}{\kappa \sigma}, \\ \zeta_4 &= \frac{2\alpha^2 r + \beta \kappa + \rho_c \kappa}{\kappa \sigma}, \\ \zeta_5 &= -\frac{r}{2\sigma}, \\ \zeta_6 &= \frac{r}{3\kappa \sigma}, \\ \zeta_7 &= -\frac{2\alpha^2 r}{\sigma}. \end{aligned}$$

Fig. 1 Deterministic potential vs. population density plot. In (a), deterministic potential for constant rate of harvesting. In (b), deterministic potential for Michaelis-Menten type harvesting. The parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01, \rho_m = 0.01, \gamma = 0.1$. The high and low density stable population state are indicated by red and magenta colored point respectively. The green colored point corresponding to unstable intermediate population state.

Fig. 2 SSPD $P_c^{st}(x)$ vs. population density x plot of stochastically controlled population for constant rate of harvesting. In (a) $\sigma = 0$. In (b) $D = 0$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01$.

We have calculated SSPD numerically using finite difference method [39] with step size 0.001. In Fig. 2, we have plotted SSPD of the population controlled by the noises in predation and harvesting separately for constant rate of harvesting. The figure exhibits bimodal structure of the population. So, the population has two stable states [40]. It can be noticed that probability peaks get ups and downs keeping invariant the modal values of the population distribution with variation of both the noises strength. It is observed that the finite difference scheme for constant rate of harvesting is in well agreement with analytical results. In Fig. 2a, we have plotted SSPD of the population controlled by the noise in harvesting only, i.e., $\sigma = 0$. It is observed that with increase of harvesting noise strength, probability peak at $x = 7.28$ (high density population biomass state or outbreak state) is suppressed together with the excitement of the peak at $x = 0.48$ (low density population

biomass state). So, population size is reduced with increase in intensity of harvesting noise. Therefore, increase in strength of harvesting noise with constant rate of harvesting strategy can prevent population expansion. In Fig. 2b, SSPD is drawn with $D = 0$, i.e, the population is controlled by only predation noise. In case of Fig. 2b, same behavior is observed. So, both the noises act as a significant role in controlling of the population for constant rate of harvesting. Biologically it is quite natural since the increase of strength of noise on predation will ultimately increase the mean predation rate which results the reduction of high density state of the population.

Fig. 3 SSPD $P_c^{st}(x)$ vs. population density x plot of stochastically controlled population for constant rate of harvesting. In (a) $\lambda = 0.5$. In (b) $\lambda = -0.5$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, D = 0.03, \rho_c = 0.01$.

In Fig. 3, the effects of cross correlated predation and harvesting noise in the control of the population is observed. In Fig. 3a, b, SSPDs are plotted with $\lambda = 0.5$ and $\lambda = -0.5$ respectively. It is observed that increase in strength of cross correlation strength of the noises is harmful for population enrichment. The population size is greatly reduced if both the positively correlated noises are applied together (Fig. 2, 3a). So, combination of positively correlated harvesting and predation noises with constant rate of harvesting acts an better effective role in the control of population than the single one.

It is well known fact that the stationary points of the potential corresponds to the equilibrium points of the system. The local minima of the potential correspond to stable equilibrium and local maxima correspond to unstable equilibrium point of the system. The consequence of predation noise, harvesting noise and strength of cross correlation between the noises in stochastic control with constant rate of harvesting can be noticed from the Fig 4a, b, c respectively. The figure shows that the stationary potential is double wellled. The shape of the potential is same for deterministic as well as stochastic control. The noise terms can drive random flips to the system point lying in one of the two minima. A series of kicks can push the system state point to quit a local minimum well. So, the system state point is bound to leave one potential well and to enter a new one for the noises. It is observed that both the noises can easily push system state point from higher density to lower one with increase in their intensities. So, increase in both the noises is harmful for population development. Also the same is observed in case of increment in cross correlation strength of the noises. So, the noises and their associationship play a significant role in outbreak control of the population. Let the SP has local minima at $x = \tilde{s}_l$ and $x = \tilde{s}_r$ ($\tilde{s}_l < \tilde{s}_r$). Let this two points are separated by local maxima at $x = \tilde{u}$. We define the depth of the left

Fig. 4 Three dimensional plot of SP of stochastically controlled population for constant rate of harvesting . In (a) $D = 0$. In (b) $\sigma = 0.01, \lambda = 0$. In (c) $D = 0.05, \sigma = 0.05$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01$.

well as $d_- = \phi_c^{st}(\tilde{u}) - \phi_c^{st}(\tilde{s}_l)$ and depth of right well as $d_+ = \phi_c^{st}(\tilde{u}) - \phi_c^{st}(\tilde{s}_r)$. It is obvious that d_+ and d_- are the functions of noises and cross correlation strength. So, the depth of wells of stationary potential will vary with the variation of the noises. The mechanism of noise and cross correlation induced transition can be explained through the study of depth of SP wells.

The depth of wells of the stationary potential of the stochastically controlled population for constant rate of harvesting (model (4)) is presented in Fig. 5. It is observed from Fig. 5a that the depth of left well exceeds the depth of right well for adequate strength of predation noise in absence of harvesting noise. So, the control by noise in predation and constant rate of harvesting could induce regime shift from high density population biomass state to a lower one. In population control, such type of state transition is an important factor as it push the population from one state to another by adding or reducing species through biological process such as breeding or mortality. In absence of predation noise, we do not observe any intersection of d_- and d_+ up to a reasonable value of D (Fig. 5b). So, the control by noise in harvesting and constant rate of harvesting can not shift the regime of the

Fig. 5 Plot of strength of the noises vs. depth of stationary potential wells for constant rate of harvesting. In (a) $D = 0$. In (b) $\sigma = 0$. Others are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01$.

population. Therefore, size of the population could be confined into a critically low value by the stochastic control with only predation noise and constant rate of harvesting.

3.2 Steady state probability distribution (SSPD) and stationary potential (SP) of stochastically controlled population for Michaelis-Menten type harvesting

The SSPD ($P_m^{st}(x)$) and SP ($\phi_m^{st}(x)$) of FPE (11) is given by

$$P_m^{st}(x) = \frac{N_m}{B_m(x)} \exp \left[\int^x \frac{A_m(y)}{B_m(y)} dy \right], \quad (17)$$

$$= N_m \exp [-\phi_m^{st}(x)], \quad (18)$$

where N_m is the normalization constant.

In absence of harvesting noise, i.e, $D = 0$, eq. (12) assume the form,

$$A_m(x) = rx \left(1 - \frac{x}{\kappa} \right) - \frac{\beta x^2}{\alpha^2 + x^2} + \frac{2\alpha^2 \sigma x^3}{(\alpha^2 + x^2)^3} - \frac{\rho_m x}{\gamma + x}, \quad (19a)$$

$$B_m(x) = \frac{\sigma x^4}{(\alpha^2 + x^2)^2}. \quad (19b)$$

So, SP takes the following form,

$$\phi_m^{st}(x) = \ln \frac{\sqrt{\sigma} x^2}{\alpha^2 + x^2} + \frac{\eta_1}{x^2} + \frac{\eta_2}{x} + \eta_3 x + \eta_4 x^2 + \eta_5 x^3 + \eta_6 \ln x + \eta_7 \ln(\gamma + x),$$

where,

$$\begin{aligned}
\eta_1 &= \frac{\alpha^4(\gamma r - \rho_m)}{2\gamma\sigma}, \\
\eta_2 &= \frac{-\alpha^4\gamma^2 r + \alpha^4\rho_m\kappa - \alpha^2\beta\gamma^2\kappa}{\gamma^2\kappa\sigma}, \\
\eta_3 &= \frac{2\alpha^2 r + \beta\kappa + \rho_m\kappa}{\kappa\sigma}, \\
\eta_4 &= -\frac{r}{2\sigma}, \\
\eta_5 &= \frac{r}{3\kappa\sigma}, \\
\eta_6 &= \frac{\alpha^2(\alpha^2\rho_m + 2\gamma^2\rho_m - 2r\gamma^3)}{\gamma^3\sigma}, \\
\eta_7 &= -\frac{\rho_m(\alpha^2 + \gamma^2)^2}{\gamma^3\sigma}.
\end{aligned}$$

In absence of predation noise, i.e, $\sigma = 0$, eq (12) assume the form,

$$A_m(x) = rx \left(1 - \frac{x}{\kappa}\right) - \frac{\beta x^2}{\alpha^2 + x^2} - \frac{\rho_m x}{\gamma + x} + \frac{\gamma D x}{(\gamma + x)^3}, \quad (20a)$$

$$B_m(x) = \frac{D x^2}{(\gamma + x)^2}. \quad (20b)$$

And SP takes the following form,

$$\phi_m^{st}(x) = \ln \frac{\sqrt{D}x}{\gamma + x} + \xi_1 x + \xi_2 x^2 + \xi_3 x^3 + \xi_4 \arctan \frac{x}{\alpha} + \xi_5 \ln x + \xi_6 \ln(\alpha^2 + x^2),$$

where,

$$\begin{aligned}
\xi_1 &= \frac{\beta\kappa + \gamma^2 r + \rho_m\kappa - 2\gamma\kappa r}{D\kappa}, \\
\xi_2 &= \frac{r(2\gamma - \kappa)}{2D\kappa}, \\
\xi_3 &= \frac{r}{3D\kappa}, \\
\xi_4 &= \frac{\beta(\gamma^2 - \alpha^2)}{\alpha D}, \\
\xi_5 &= \frac{\gamma(\rho_m - \gamma r)}{D}, \\
\xi_6 &= \frac{\beta\gamma}{D}.
\end{aligned}$$

The SSPD of the population controlled by noise in predation and harvesting separately for Michaelis-Menten type harvesting are shown in Fig. 6. It is observed that population is bistable when it is controlled by predation noise only and Michaelis-Menten type harvesting. But, population is tristable when it is controlled by only harvesting noise and Michaelis-Menten type harvesting. The high density population state (at $x = 7.28$) is suppressed

Fig. 6 SSPD $P_m^{st}(x)$ vs. population density x plot of stochastically controlled population for Michaelis-Menten type harvesting. In (a) $\sigma = 0$. In (b) $D = 0$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_m = 0.01, \gamma = 0.1$.

when it is controlled with increase in either of the noises and Michaelis-Menten type harvesting. Therefore, population size could be controlled by strengths of either of the noises and Michaelis-Menten type harvesting. In this case also results obtained by using finite difference scheme for Michaelis-Menten type harvesting is in well agreement with analytical results.

Fig. 7 SSPD $P_m^{st}(x)$ vs. population density x plot of stochastically controlled population for Michaelis-Menten type harvesting. In (a) $\lambda = 0.5$. In (b) $\lambda = -0.5$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, D = 0.03, \rho_m = 0.01, \gamma = 0.1$.

Concurrent effects of both the noises in the control with Michaelis-Menten type harvesting are visualized in Fig. 7. It is observed that population size is suppressed with increase in strength of cross correlation of the noises. Comparing Fig. 6 and 7, it can be concluded that combination of positively correlated harvesting and predation noise with Michaelis-Menten type harvesting is a better stochastic control of the population. Also, the control by harvesting noise and Michaelis-Menten type harvesting has the potential to slaughter the population.

Significance of cross correlated predation and harvesting noise in the population control

Fig. 8 Three dimensional plot of SP of stochastically controlled population for Michaelis-Menten type harvesting. In (a) $D = 0, \lambda = 0$. In (b) $\sigma = 0, \lambda = 0$. In (c) $\sigma = 0.05, D = 0.05$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_m = 0.01, \gamma = 0.1$.

for Michaelis-Menten type harvesting strategy are presented by the plot of SP of the population. The role of predation and harvesting noise as well as cross correlation strength are shown in Fig. 8a, b, c respectively. The SP of the population is double wellled in presence of predation noise only (Fig. 8a) and triple wellled in presence of harvesting noise only (Fig. 8b). Let the wells of double wellled SP are at $x = s_l, x = s_r$ ($s_l < s_r$) and the wells are separated by the point $x = u$. In case of double well, let us define the depth of left well as $d_l = \phi_m^{st}(u) - \phi_m^{st}(s_l)$ and depth of right well as $d_r = \phi_m^{st}(u) - \phi_m^{st}(s_r)$. On the other side, let the triple wellled SP has local minimum at $x = s_j$ ($j = 1, 2, 3$) (assume $s_1 < s_2 < s_3$) and the minimums are separated by local maximum at points $x = u_i$ ($i = 1, 2$) (assume $u_1 < u_2$). Let us define the depth of three wells as $d_{ij} = \phi_m^{st}(u_i) - \phi_m^{st}(s_j)$. It is observed that the most deepest well (at s_r or s_3) becomes shallow with increase in the noises strength. So, system state easily leaves this deepest well and enters the well at an endanger one (at s_l or s_2) crossing the potential barrier (u or u_2) with increase in the noises strength. With again increment in noise strength, system state leaves the well at s_2 and enters the well at s_1 (in triple wellled case). It can be noticed that $s_1 = 0$. The same has been observed in

case of cross correlation strength. So, population biomass is reduced in control by increase in strength of the noises and their cross correlation strength with Michaelis-Menten type harvesting. Therefore, size of the population could be controlled by the noises with suitable chosen correlation strength and Michaelis-Menten type harvesting strategy.

Fig. 9 Strength of the noises vs. depth of stationary potential wells plot for Michaelis-Menten type harvesting. In (a) $D = 0$. In (b) $\sigma = 0$. The others are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_m = 0.01, \gamma = 0.1$.

The depths of SP wells of stochastically controlled population for Michaelis-Menten type harvesting are plotted in Fig. 9. It is observed that there is no intersection of the curves d_l and d_r . So, the control by noise in predation and Michaelis-Menten type harvesting can not shift the regime of the population from high density population state to lower one. But the structure of the population controlled by harvesting noise only may be bistable or tristable depending on strength of harvesting noise (Fig. 9b). With increase of strength of harvesting noise, depth of right well (d_{23}) and middle well (d_{12} or d_{22}) decrease and the depth of left well (d_{11}) increases and ultimately d_{12} becomes zero. So, the control by harvesting noise with Michaelis-Menten type harvesting may exhibit abrupt regime shift of the population. Also the control with increase in intensities of the noises and Michaelis-Menten type harvesting decrease the probability of the insect population spreading. Moreover, the control by harvesting noise and Michaelis-Menten type harvesting could induce state transition of the population from high density state to zero state. Which means the control by harvesting noise and Michaelis-Menten type harvesting could enhance the possibility of extinction of the species.

4 Stochastic bifurcation

It is well known fact that the state of a stochastic system can be characterized by its probability distribution. In this study, we have determined the steady probability distribution function using Fokker-Planck equation. Noise induced transition of a system can be identified by its structural shape changes in SSPD. The structural changes in SSPD is related to the variation of number of extrema with the variation of noise intensity or fluctuating parameters. Therefore, simplest noise induced stochastic bifurcation can be identified by the variation of number of extrema of SSPD. The equilibrium behavior of deterministic model can be visualized by a bifurcation curve showing the equilibrium species density (x) across a range of parameter values.

In order to determine stochastic bifurcation curve of the population for constant rate of harvesting, we determine $\frac{dP_c^{st}(x)}{dx} = 0$ and obtain the following equation,

$$\rho_c - rx \left(1 - \frac{x}{\kappa}\right) + \frac{\beta x^2}{\alpha^2 + x^2} + \frac{2\lambda\alpha^2\sqrt{D}\sigma x}{(\alpha^2 + x^2)^2} + \frac{2\sigma x^3}{(\alpha^2 + x^2)^3} = 0. \quad (21)$$

Similarly, the stochastic bifurcation curve of the population for Michaelis-Menten type harvesting is given by (making $\frac{dP_m^{st}(x)}{dx} = 0$)

$$x \left[-r \left(1 - \frac{x}{\kappa}\right) + \frac{\rho_m}{\gamma + x} + \frac{D\gamma}{(\gamma + x)^3} + \frac{\beta x}{\alpha^2 + x^2} + \frac{2\sigma x^2}{(\alpha^2 + x^2)^3} + \frac{2\lambda\alpha^2\sqrt{D}\sigma x}{(\gamma + x)(\alpha^2 + x^2)^2} + \frac{\lambda\gamma\sqrt{D}\sigma x}{(\alpha^2 + x^2)(\gamma + x)^2} \right] = 0. \quad (22)$$

Fig. 10 In (a) one-parameter stochastic bifurcation diagram with variations of predation rate β and (b) two-parameter stochastic bifurcation diagram with variations in noises strength D and σ for constant rate of harvesting. In (a) $D = 0$. In (b) $\beta = 1, \lambda = 0$. The other parameters are $r = 0.5, \kappa = 10, \alpha = 1, \rho_c = 0.01$. Solid and dotted line denote high density stable equilibria and low density stable equilibria state respectively. Dot-dashed line denote unstable equilibrium state. The dashed vertical line indicates the value of chosen parameter.

Stochastic bifurcation diagrams of the population for constant rate of harvesting and Michaelis-Menten type harvesting are presented in Fig. 10 and 11 respectively. In subplot (a) of the figures, the unstable equilibrium demarks the boundary separating two basins of attraction. The intermediate equilibrium state fixes the threshold for switching between the stable states. In subplot (b) of the figures, the stationary potential function contains two minima in bistable region and three minima in tristable region. A bifurcation appears for the change of stability of an equilibrium state. It is observed that the bifurcations are of saddle-node type. The saddle-node bifurcation exhibits distinctive characteristic of abrupt regime shifts and also has the associated features of hysteresis. The critical slowing down closely associated to bifurcation points and at a bifurcation point, the potential has flat bottom. The deterministic bifurcation curve of the population can be obtained by putting $\sigma = 0$ and $D = 0$ in the equation 21 and 22. It can be noticed from Fig. 10a and 11a that the deterministic counterpart of the stochastic models (4) and (5) i.e, model (2) and

Fig. 11 In (a) one-parameter stochastic bifurcation diagram with variations of ρ_m and (b) two-parameter stochastic bifurcation diagram with variations in noises strength D and σ for Michaelis-Menten type harvesting. In (a) $\sigma = 0$. In (b) $\rho_m = 0.01, \lambda = 0$. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \gamma = 0.1$. Solid and dotted line denote high density stable equilibrium and medium density stable equilibrium state respectively. Dotted and dot-dashed line indicate the intermediate unstable equilibrium state. The dashed vertical line indicates the value of chosen parameter.

(3) are bistable in biological sense. It is observed that stochastically controlled population with constant rate of harvesting is always bistable. In contrast, the population in presence of Michaelis-Menten type harvesting may be bistable or tristable depending on nature and strength of the noises. The tristability of population for the control by weak noise in harvesting and Michaelis-Menten type harvesting is not a surprising fact, this is due to shift of the deterministic bifurcation point caused by the noises [40]. Also, Michaelis-Menten type harvesting model transits from tristable states to bistable states with the increase in noises strength (Fig. 11b). Therefore, due to the noises, there may be a critical transition in the models. In next section, we have discussed the certainty of critical transition through the study of early warning signals.

5 Mean first passage time

In case of stochastic system, the transient properties beyond its steady states is important. Mean first passage time (MFPT) defines the averaging time of individual crossing barrier from one potential well to another. Also, it indicates how long the system will remain in a given state of interest. Our interest is to calculate the average time the system stays in the basin of attraction of high density stable state in respect to outbreak control. It can be noticed that the basin of attraction of high density stable state, $\tilde{s}_r$, in case of the stochastically controlled population for constant rate of harvesting, is extended from $\tilde{u}$ to ∞ . Let $T_c(x)$ be the MFPT of the stochastically controlled population for constant rate of harvesting to exit from the basin of attraction of high density stable state starting at $x(t=0) = \tilde{s}_r$. Then, $T_c(x)$ satisfies the relation [41],

$$A_c(x) \frac{dT_c(x)}{dx} + \frac{B_c(x)}{2} \frac{d^2 T_c(x)}{dx^2} = -1, \quad (23)$$

with boundary conditions,

$$T_c(\tilde{u}) = 0 \text{ and } \frac{dT_c(\infty)}{dx} = 0.$$

In case of the stochastically controlled population for Michaelis-Mentan type harvesting, the basin of attraction of high density stable state, s_2 , is extended from u_2 to ∞ . The MFPT $T_m(x)$ to exit the basin initially starting at the position $x(t = 0) = s_2$ satisfies the relation,

$$A_m(x) \frac{dT_m(x)}{dx} + \frac{B_m(x)}{2} \frac{d^2 T_m(x)}{dx^2} = -1, \quad (24)$$

with boundary conditions,

$$T_m(u_2) = 0 \text{ and } \frac{dT_m(\infty)}{dx} = 0.$$

Fig. 12 MFPT vs. noise intensity plot of the stochastically controlled population for constant rate of harvesting. In (a) $\lambda = -0.5$. In (b) $\lambda = 0.5$. Other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01$.

The MFPTs $T_c(x)$ and $T_m(x)$ are plotted in Fig. 12 and Fig. 13 respectively. It can be noticed that the curves are decreasing in predation noise when both noises are positively correlated. But the curves are decreasing in harvesting noise irrespective of the association with predation noise. So, the exit time from the attraction of outbreak state is decrease with increase in harvesting and positively correlated predation noise strength. Therefore, population takes shorter time to reach low density population state with increase in positively correlated predation noise. Alternatively population leaves outbreak state in shorter time with increase in positively correlated predation noise. As higher value of MFPT indicate the grater stability of steady state from which exist occurs, so, increase in the noises suppress the population expansion in number. As a result, population outbreak could be extinguished by increasing the strength in positively correlated predation and harvesting noise. Additionally, increase in intensities of the positively correlated predation and harvesting noise enhances the probability of regime shift from high density population state to lower one. We can conclude with the comparison of Fig. 12 and 13 that the constant rate of harvesting strategy is better to control population quickly with extinguishing outbreak by weak noise. But there is no significant difference between the harvesting strategies in respect of time in

Fig. 13 MFPT vs. noise intensity plot of the stochastically controlled population for Michaelis-Mentan type harvesting. In (a) $\lambda = -0.5$. In (b) $\lambda = 0.5$. Other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_m = 0.01, \gamma = 0.1$.

extinguishing outbreak by strong noise. In case of negative cross correlation between the noises, the MFPT curves is increasing upto a threshold value and then decreasing in predation noise strength. Which indicates the noise enhance stability phenomenon (NES). The NES, resonance-like effect, contradicts the monotonic behavior predicted by the Kramer's formula [42, 43]. Therefore, one should chose strength of the noises and cross correlation between the noises as high as possible in stochastic control of outbreak as it can reduce the large number of population in relatively short time.

6 Stochastic simulation and early warning signals

Numerical simulation of the time evolution diagrams of the population controlled stochastically for constant and Michaelis-Menten type harvesting strategy are done using the Euler-Maruyama method [44] with a standard integration step size as 0.001 [45]. The time evolution diagrams of the population for the two types of harvesting strategies with different noise intensities, cross correlation strength and initial population size are presented in Fig. 14 and 15 respectively. We choose $\rho_c = 0.01$ and $\rho_m = 0.01$ for Fig. 14 and 15 respectively. In both the figure, $\beta = 1$.

In Fig. 14a, b, the parameters are $\sigma = 0.05, D = 0.05, \lambda = -0.5$ but initial population size is $x_0 = 0.5$ and $x_0 = 7$ respectively. In Fig. 14c, d, the parameters are $\sigma = 0.05, D = 0.05, \lambda = 0.5$, initial population size is $x_0 = 0.5$ and $x_0 = 5$ respectively. The Fig. 14e, f are drawn for $x_0 = 0.5, x_0 = 5$ respectively with $\sigma = 0.1, D = 0.05, \lambda = -0.5$. The Fig. 14g, h are calculated for $x_0 = 0.5, x_0 = 7$ respectively with $\sigma = 0.1, D = 0.05, \lambda = 0.5$. In Fig. 14i, j, initial population density and correlation strength of the noises are $x_0 = 2, \lambda = -0.5$ and $x_0 = 5, \lambda = 0.5$ respectively along with $\sigma = 0.1, D = 0.1$. The Fig. 14k, l are drawn with $x_0 = 2, \sigma = 0.15, D = 0.1$ but different cross correlation strength $\lambda = 0.5$ and $\lambda = -0.5$ respectively. The Fig. 14m is calculated with $\sigma = 0.15, D = 0.15, \lambda = 0, x_0 = 2$. The Fig. 14n, o are drawn with $\sigma = 0.2, D = 0.15, \lambda = 0$ but different initial population size as $x_0 = 7$ and $x_0 = 2$ respectively.

The Fig. 15a, b are drawn with $\sigma = 0.5, D = 0.01, x_0 = 7$ but different cross correlation strength $\lambda = 0.5$ and $\lambda = -0.5$ respectively. In Fig. 15c, d, cross correlation strength

Fig. 14 Time evolution diagrams of the population for constant rate of harvesting strategy with different initial conditions and different noise intensities. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01$. The blue color indicates time evolution diagram and other colors indicate different positive stable equilibrium of the population.

and initial population size are $\lambda = 0.05, x_0 = 2$ and $\lambda = -0.05, x_0 = 5$ respectively also the strength of the noises are $\sigma = 0.15, D = 0.01$. In Fig. 15e, f, g, strength of the noises are $\sigma = 0.25, D = 0.01$, cross correlation strength are 0.5, -0.5 and 0 respectively, initial population size are 5, 5 and 2 respectively. In Fig. 15h, i, strength of the noises are $\sigma = 0.05, D = 0.05$, initial population size and cross correlation strength are $x_0 = 5, \lambda = 0.5$ and $x_0 = 4, \lambda = -0.5$ respectively. The Fig. 15j, k are drawn with $\sigma = 0.01, D = 0.05, x_0 = 7$ but different correlation strength $\lambda = 0.5$ and -0.5 respectively. The Fig. 15l, m are calculated with $\sigma = 0.15, D = 0.1$ also correlation strength and initial population size $\lambda = -0.5, x_0 = 2$ and $\lambda = 0.5, x_0 = 7$ respectively. In Fig. 15n, o, strength of the predation noises are $\sigma = 0.25$ and 0 respectively, strength of harvesting noise are $D = 0.05$ and 0.3 respectively, the initial population size and cross correlation strength of the noises are $x_0 = 6, \lambda = 0$.

It is observed that with low-intensity of the noises, the population dynamics visits high density attractors and remain in its vicinity. On the other hand, the transition rate between the various steady stable states increases with the increase of noises intensities and cross correlation strength between the noises. That is switching of the population between

Fig. 15 Time evolution diagrams of the population for Michaelis-Menten type harvesting with different initial conditions and different noise intensities. The other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_m = 0.01, \gamma = 0.1$. The blue color indicates time evolution diagram and other colors indicate different positive stable equilibrium of the population.

various population states depends on the intensities of the noises and their associationship. The population may be extinct for stochastic control by adequate strength of only harvesting noise or both the noises, predation and harvesting noise with Michaelis-Menten type harvesting (Fig. 15n, o).

Depending on the noise intensity rapid switching from one stable state to other stable state is observed in the models. The shifts between the stable states indicates the possibilities of critical transition or extinction of species. Therefore, an early warning of such transition may be very helpful for conservation of species. Several early warning signals of such transitions have recently [46, 47] been developed eg. changes in the asymmetry [48], change in autocorrelation [49], change in variance [50] etc. In this study, we use the changes in variance and change in autocorrelation as indicators of early warning signal. Dakos et al. [49] show that change in autocorrelation in the period before the shift suggest that the systems indeed slow down before the abrupt change. We used the segment of the time series before the shift to scan for slowing down. To ensure stationary residuals, we used Gaussian detrending on the time series before statistical analysis. We fix the bandwidth to 40 [14],

Fig. 16 Early warning signals of critical transition for constant rate of harvesting. The signals are calculated for the segment $t=760$ to $t=840$ of the time evolution diagram displayed in Fig. 14o, immediately precede a jump from higher state to lower state. The red graph denotes the smoothed time series used for filtering.

which is an adjustment between too narrow and too wide of bandwidth, which would not only remove the long-run patterns but also the short-run fluctuations which we expected to study. All statistical measures are calculated using a moving window. We set half the size of the simulated time series as the size of the moving window [51]. For each of the indicators, we calculated the temporal trend by estimating the non-parametric Kendall rank correlation τ_k .

The early warning signals before an impending shift from a steady state to another one of the stochastically controlled population for constant rate of harvesting and Michealis-Menten type harvesting strategy have been drawn in Fig. 16 and 17 respectively. In subplots (a) and (b) of the figures, we have drawn time series segment and residuals respectively. The autocorrelation at lag 1 and variance within each rolling window of the segment shown in subplot (a) of the figures have been drawn in subplot (c) and subplot (d) respectively. The trend of autocorrelation at lag 1 and variance have been measured by non-parametric Kendall rank correlation. We see that the trends of autocorrelation at lag 1 and variance for constant rate of harvesting strategy are 0.75 and 0.878 respectively. Also, the trends of autocorrelation at lag 1 and variance for Michealis-Menten type harvesting strategy are 0.683 and 0.699 respectively. As both the trends are positive so the systems are proceeding towards critical transition [48, 52].

Fig. 17 Early warning signals of critical transition for Michaelis-Menten type harvesting. The signals are estimated for the segment from $t=700$ to $t=800$ of the time evolution diagram in Fig. 15g. The red graph shows the smoothed time series used for filtering.

Fig. 18 Mean population size vs. noise intensity plot. In (a) $D = 0.2$. In (b) $\sigma = 0.2$. Other parameters are $r = 0.5, \kappa = 10, \beta = 1, \alpha = 1, \rho_c = 0.01, \rho_m = 0.01, \gamma = 0.1, \lambda = 0$. The blue and red colored line indicate mean size of the population controlled shochatically with constant rate of harvesting and Michaelis-Menten type harvesting respectively. The dashed and dot-dashed horizontal line indicate the mean population size in deterministic control with Michaelis-Menten type harvesting and constant rate of harvesting respectively.

7 Quantitative analysis

In order to quantically investigate the impacts two types of harvesting strategies in the population control by the noises in predation as well as harvesting, we plot the mean value

of the population density by $\langle x \rangle_{c/m} = \int_0^\infty x P_{c/m}^{st}(x) dx$ in Fig. 18. In more elaborative form, $\langle x \rangle_c$ and $\langle x \rangle_m$ denote the average size of the population controlled stochastically for constant rate of harvesting and Michaelis-Menten type harvesting strategy respectively. It is observed that the population density decreases with the increase in σ and D . So, increase in strength of noise on harvesting or predation reduce the population size in both harvesting strategies. The positive cross correlation strength between the noises helps to control population rapidly. Therefore, population could be controlled by predation and harvesting noise intensity. Moreover, Michaelis-Menten type harvesting is better for controlling population stochastically than constant rate harvesting for reducing population size significantly at faster rate. Alternatively, Michaelis-Menten type harvesting is better than constant rate harvesting for controlling population outbreak effectively by noises. Also, comparing slope of the curves, one can notice that size of the population is greatly reduced in control by increase in predation noise strength than harvesting noise strength. Again it is observed that stochastic control reduces average population size rapidly than deterministic control with suitable strength of the noises. In contrast to the stochastic case, the constant rate of harvesting reduces average population size greater than Michaelis-Menten type harvesting in deterministic control (inset). So, Michaelis-Menten type harvesting should be accepted with sufficient strong noises in respect to population control with reduction of population size.

8 Conclusion

In this paper, the roles of constant and Michaelis-Menten type harvesting in single prey species population control in presence of multiple noise sources are studied formulating stochastic models. Steady state probability distributions (SSPDs) and stationary potentials (SPs) of the stochastically controlled population for considered two types of harvesting strategies are calculated with help of the Fokker-Plank equations. Here, we analyze and compare the effects of different types of random harvesting in population control in presence of predation noise. The models can also be interpreted as mathematical formulations of harmful toxic phytoplankton blooms and its controls in marine ecosystem. Moreover, one can explain this as wild prey population outbreak and its controls in deep forest. In general this research can be considered as a prey species outbreak and its controls in any predator prey system. The considered method can be used to control prey population very rapidly.

The roles of cross correlated noises in predation and harvesting in population control have been investigated and compared through the study of SSPD and SP. Stochastic bifurcation diagrams with respect to noise induced parameters are presented. Effects of the different cross correlated noises on outbreak control have been investigated through the calculation of mean fast passage times. Early warning signals of critical transition are computed via Euler-Maruyama stochastic simulation to check the possibility of noise induced critical transition. Quantitative analysis of the population has been carefully evaluated to compare between two types of harvesting strategies and design effective population controlling strategy.

It is observed that the population is bistable irrespective of harvesting strategy in the control by strong noises. On the other hand, the structures of the population vary depending on harvesting strategy in control by weak noises. The population changes its structure from bistable to tristable for the change of harvesting strategy from constant rate of harvesting to Michaelis-Menten type harvesting with weak harvesting noise. This structural variation is due to shift of the deterministic bifurcation point caused by the noises [40]. Population size

is decreased with increase of noises in both predation and harvesting. The cross correlation strengths of the noises play a crucial role in reduction of population size. Combination of positively correlated harvesting and predation noise can reduce population size greatly than single one (Fig. 2, 3, 6, 7). So, the control becomes effective if it is done with the positively correlated noises rather than the single one. Population size can be reduced to a critically low value by the control with noise in predation and constant rate of harvesting strategy or noise in harvesting and Michaelis-Menten type harvesting strategy or noise in predation and harvesting as all can shift the regime of the population from high density population state to lower one (Fig. 5, 9, 16, 17). The predation noise can reduce population size faster than harvesting noise (Fig. 18).

The Michaelis-Menten type harvesting strategy is better to control population with reduction of population size since it can reduce the average population size very fast compare to constant rate of harvesting strategy (Fig. 18). Moreover, there is a possibility of extinction of the species in the control by adequate strength of predation and harvesting noise or only harvesting noise together with Michaelis-Menten type harvesting (Fig. 15). The population can not be easily restrained from low density population biomass state to high one through its development in control with suitable noises strength as the population approaches towards critical transition in this control (Fig. 16, 17). Therefore, one should choose positively strongest correlated highest possible strength of noises in predation and harvesting together as control with Michaelis-Menten type harvesting to reduce average population density rapidly and effectively (Fig. 3, 7, 18).

The constant rate of harvesting strategy is better than Michaelis-Menten type harvesting strategy to extinguish population outbreak quickly by weak noises. In contrast there is no significant difference between the harvesting strategies in extinguishing outbreak in respect of time in case of strong noises (Fig. 12 and 13). Michaelis-Menten type harvesting is better than constant rate harvesting for stochastic control of population outbreak effectively as average size of the population is smaller in Michaelis-Menten type harvesting. The increase in positive cross correlation strength between the noises accelerates the act of extinguishing outbreak. One should choose strength of the noises and positive cross correlation strength between the noises as high as possible to outbreak control rapidly and effectively as it can significantly reduce the mean first exit time from outbreak state and suppress the outbreak state. The MFPT shows a maximum value when noises are negatively correlated. The maximum in MFPT for noise strengths is an indication of the noise-enhanced stability (NES) phenomenon, which has been extensively investigated both theoretically and experimentally in various systems ranging from physics to biology [53, 54]. The NES, resonance-like effect, contradicts the monotonic behavior predicted by the Kramer's formula [42, 43]. This enhancement of stability was first noted by Hirsch et al. [55].

We can conclude that stochastic control is effective than deterministic control as the average population size reduced by stochastic control is larger than that by deterministic control (Fig. 18). The reduction of average population size by Michaelis-Menten type harvesting strategy is larger than by constant rate of harvesting in stochastic control. Whereas constant rate of harvesting strategy reduces average population size grater than Michaelis-Menten type harvesting in deterministic control. The stochastic control with sufficient strong positively cross correlated noises in predation and harvesting along with Michaelis-Menten type harvesting strategy should be accepted to control population with reduction of population size. The noise intensities can be artificially controlled in real world ecological systems in various ways such as introducing artificial rainfall, using pesticides etc.

Our work reveals that the correlation strength between the noises is the key factor in

controlling population outbreak. Depending upon the key factor different complex phenomena (such as regime shift, critical transition, noise enhance stability, etc.) can appear in the stochastic dynamics. This model based study is not only useful to predict insect outbreak and its control but also very useful to investigate wild prey outbreak and its control in deep forests. Moreover it can be applicable to predict outbreak of some particular type of phytoplankton population in marine ecosystem. More general point of view, this research can be interpreted as a method for predicting prey species outbreak or extinction and its control in any predator prey ecosystem. This study is very useful for the conservation of those species whose population are guided mainly by prey-predator interactions. There is no doubt that a natural extension of our work is the investigation with introduction of various types of interactions (like prey predator, mutualistic, etc.) with other species in presence of Gaussian as well as non-Gaussian noise. Moreover, the color of the external noise can influence the population dynamics strongly as population growth is often sensible to temporal autocorrelation. So, in this direction proper investigations should be done to get definite conclusion about the temporal correlation of noise. Findings of this study are clearly relevance to conservation biology.

Declarations

Data availability: All data generated or analyzed during this study are included in this article.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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