Fixed-Time Trajectory Tracking Control for Nonholonomic Mobile Robot Based on Visual Servoing

OU Meiyang (✉ oumeiyang@163.com)

Chuzhou University <https://orcid.org/0000-0003-1043-4243>

Haibin Sun

Qufu Normal University

Zhenxing Zhang

Southeast University

Shengwei Gu

Chuzhou University

Research Article

Keywords: Fixed-time stability, tracking control, mobile robot, uncalibrated camera

Posted Date: September 1st, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-795447/v1>

License: This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Fixed-time trajectory tracking control for nonholonomic mobile robot based on visual servoing

Meiying Ou^{1*}, Haibin Sun², Zhenxing Zhang³, Shengwei Gu⁴

¹ School of Mechanical and Electrical Engineering, Chuzhou University, Chuzhou 239000, P.R.China

² School of Engineering, Qufu Normal University, Rizhao 276826, P.R.China

³ School of Instrument Science and Engineering, Southeast University, Nanjing, 210096, P.R.China

⁴ School of Computer and Information Engineering, Chuzhou University, Chuzhou 239000, P. R. China

Abstract

This paper aims to discuss fixed-time tracking control problem for a nonholonomic wheeled mobile robot based on visual servoing. At first, by making use of the pinhole camera model, the robot system model with uncalibrated camera parameters is given. Then, the tracking error system between the mobile robot and desired trajectory is proposed. Thirdly, on the basis of fixed-time control theory and Lyapunov stability analysis, fixed-time tracking control laws are proposed for the robot, which can make the robot achieve the desired value in a fixed time. Simulation results are given at the end.

Keywords: Fixed-time stability; tracking control; mobile robot; uncalibrated camera.

1 Introduction

In recent years, visual servoing technologies play an important role in the robotic systems to accomplish various practical functions, such as hovering control for unmanned aerial vehicle [1], target tracking [2], target capture [3] and so on. Visual servo is an effective method to employ the visual information to establish the relationship between the robot and the environment, and generates control commands through visual information feedback. Generally, visual servoing can be classified into three categories: including position-based visual servoing [4], image-based visual servoing [5] and hybrid approach [6]. For position-based visual servoing control, the reconstructed Euclidean information is used in the feedback loop to control the robot, whereas image-based visual servoing control directly deals with visual information on the image plane. A combination of the these two schemes is named hybrid approach.

During the past few decades, the tracking control of wheeled mobile robots with nonlinear constraints has become the focus of researches owing the inherent nonlinearity in robot dynamics and the usefulness in many applications. It is well known that to calibrate the intrinsic and extrinsic parameters of the camera is an important, inevitable task in visual servoing control. However camera calibration is a repetitive and costly work. In order to avoid camera calibration, the uncalibrated visual servoing are studied by many researchers [7]-[9]. So far, the visual tracking control problem for nonholonomic mobile robot has been widely discussed. For a monocular camera system mounted on an underactuated wheeled mobile robot, the visual servoing tracking controller was developed in [10]. In [11], by using an uncalibrated fixed camera, a novel adaptive tracking controller was designed for a nonholonomic mobile robot system. In [12], the image-based tracking problem of wheeled mobile robots with uncertain camera parameters was studied, and adaptive trajectory tracking control laws were designed.

With the development of automatic control technology and the increasing demand for stability accuracy of nonlinear systems, the finite-time control method has attracted increasing attention of many researchers [13]-[15], which has the characteristics of faster convergence rate, higher accuracy, better disturbance rejection properties and robustness

¹ * Corresponding author. E-mail address: oumeiying@163.com

against uncertainties [16]-[18]. Numerous works have been proposed related to the finite-time tracking control problem of nonholonomic mobile robot systems. In [19], a finite time tracking controller was proposed for the nonholonomic systems with extended chained form. The authors of [20] studied finite-time tracking control problem of multiple nonholonomic wheeled mobile robots with unknown parameters and external disturbances, finite-time disturbance observers and finite-time tracking control laws were designed for each mobile robot. In [21], finite-time tracking control problem were discussed for a nonholonomic mobile robot system with uncalibrated camera parameters. Continuous nonsingular finite-time tracking control laws were given for underwater robot manipulators with lumped disturbances in paper [22].

Although the previously listed finite-time control algorithms can guarantee that the closed-loop system convergence in a finite time, the expressions of the finite settling-time are usually dependent on the initial conditions, which can not be obtained easily in the practical applications. So it would be useful if the settling time could be predetermined no matter whether the initial conditions are known or not. Motivated by this, fixed-time stability conception was proposed in [23]. This soon attracted a lot of scholars' attention. Fixed-time control is more preferable than finite-time control in practical applications since the fixed-time approach can generate a control law prescribing a transition time which is independent of the operation domain [24]. Based on the fixed-time stability conception, some new results are reported. For some second-order and high-order systems, the fixed-time control problem were studied in [25-28]. The authors of paper [29] discussed fixed-time tracking control problem for nonholonomic mobile robot, and fixed time control algorithm was designed by proposing a new integral terminal sliding mode surface. Fixed-time trajectory tracking control problem for a nonholonomic mobile robot was discussed in [30], on the basis of the fixed-time control method and Lyapunov stability analysis, continuous fixed-time tracking controllers were developed for the mobile robot. However, the papers [29,30] didn't consider visual servoing for the mobile robot system. The authors of paper [9] studied fixed-time trajectory tracking control problem for a wheeled mobile robot with uncalibrated camera parameters, and designed the fixed-time adaptive tracking controllers for the mobile robot system.

Motivated by the aforementioned discussion, the paper aims to consider the fixed-time trajectory tracking problem for a nonholonomic mobile robot using an uncalibrated, fixed (ceiling-mounted) camera system. Since the robot system based on visual servoing consists of nonholonomic constraint and uncalibrated camera parameters, it is more harder to design the fixed-time tracking control laws. The key contributions of this paper can be summarized as following: (1) The camera-objective visual servoing kinematic model is given and the tracking error system between camera-objective visual servoing model and the desired reference trajectory is introduced. (2) Fixed-time trajectory tracking controllers are proposed for the nonholonomic mobile robot based on visual servoing, which guarantees the desired trajectory can be tracked by the mobile robot in a fixed time. (3) The settling time is independent of the system initial conditions and only determined by the controller parameters.

The rest of this paper is organized as following arrangements: Section 2 gives some useful lemmas and problem formulation is provided. The tracking error dynamic system is considered and fixed-time tracking controllers are designed for the nonholonomic mobile robot in Section 3, the validity of the controllers for the system is proved. Section 4 presents the simulation results to illustrate the controller's performance. Section 5 gives some conclusions.

2 Preliminaries

For convenience, we give the following definition

$$sig^\alpha(X) = |X|^\alpha sgn(X), \quad (1)$$

where $\alpha > 0$ and $sgn(\cdot)$ represents the standard symbolic function, and it is easy to verify that

$$\frac{d}{dy} sig^{1+\alpha}(y) = (1 + \alpha)|y|^\alpha. \quad (2)$$

2.1 Some useful Lemmas

Lemma 2.1 [23] Consider a system as follows

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \quad (3)$$

if there exists a positive definite continuous function $V(x) : R^n \rightarrow R$ satisfies $\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x)$ for all $x \in U_0$, where $\alpha > 0, \beta > 0, 0 < p < 1, q > 1$, then the system is fixed-time stable and the fixed settling time T can be upper bounded by $T \leq \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)}$.

Lemma 2.2 [31] The inequality $(\sum_{i=1}^n |x_i|)^p \leq \sum_{i=1}^n |x_i|^p$ holds for any $x_i \in R, i = 1, 2, \dots, n$, where p is a real number satisfying $0 < p \leq 1$.

Lemma 2.3 [32] The inequality $n^{1-p}(\sum_{i=1}^n |x_i|)^p \leq \sum_{i=1}^n |x_i|^p$ holds for any $x_i \in R, i = 1, 2, \dots, n$ and $p > 1$.

Lemma 2.4 [31] If $0 < l \leq 1$, then $|x^l - y^l| \leq 2^{1-l}|x - y|^l$, where l is a ratio of two odd integers.

Lemma 2.5 [33] The inequality $|x|^c |y|^d \leq \frac{c}{c+d} \gamma |x|^{c+d} + \frac{d}{c+d} \gamma^{-\frac{c}{d}} |y|^{c+d}$ holds for any $x, y \in R$ and $\forall c, d, \gamma > 0$.

2.2 Problem formulation

2.2.1 Kinematic model with monocular camera

A nonholonomic mobile robot system is given in Fig. 1, it has two steering wheels, one castor wheel. Letter P denotes the center of mass of the robot. If we assume the centroid coincides with the geometric center, then the nonholonomic constraint model of the robot is as below

$$\dot{x} \sin \theta - \dot{y} \cos \theta = 0, \quad (4)$$

where (x, y) represents P , the angle between X axis and X_1 axis is θ , and the counterclockwise direction is defined as the positive direction for angle θ . By this formula, the kinematic system can be written as:

$$\dot{x} = v \cos \theta, \quad (5a)$$

$$\dot{y} = v \sin \theta, \quad (5b)$$

$$\dot{\theta} = \omega, \quad (5c)$$

where v and ω are the translational velocity and angular velocity of the mobile robot respectively.

Fig. 1. Wheeled mobile robot.

Fig. 2 shows the robot-camera system. In this paper, we assume that the robot presented in Fig. 1 moving under a pinhole camera that fixed to the ceiling, where the camera plane and the robot plane are parallel. There are three coordinate systems, inertial coordinate system $X - Y - Z$, the camera coordinate system $x - y - z$ and the image coordinate system $u_1 - o_1 - v_1$. The $x - y$ plane of the camera coordinate system is parallel to the plane of image coordinate plane. Based on paper [34], camera model yields

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} R(\theta_0) \left[\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} c_x \\ c_y \end{pmatrix} \right] + \begin{pmatrix} O_{c1} \\ O_{c2} \end{pmatrix}, \quad (6)$$

where (x_m, y_m) is the coordinate of (x, y) relative to the image coordinate system, (x, y) is the coordinate of mass center P of robot with respect to $X - Y$ plane, (c_x, c_y) is the coordinate of the crossing point C relative to $X - Y$ plane, (O_{c1}, O_{c2}) is the coordinate of the intersection of the optical axis with the image plane, α_1 and α_2 are positive constants, which are determined by the depth information, scalar factors, and focal length.

$$R(\theta_0) = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix}, \quad (7)$$

where θ_0 means the angle between y axis and X axis with a positive anticlockwise orientation.

For parameters α_1, α_2 and θ_0 , the following assumption is required for controller design.

Assumption 1. θ_0 is unknown, $\alpha_1 = \alpha_2 = \alpha$ are unknown, $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$, where $\underline{\alpha}$ and $\bar{\alpha}$ are positive known constants.

Remark 1[35]. $\alpha_1 = \alpha_2 = \alpha$ shows that the scalar factor along u_1 axis is the same as that one along v_1 axis. Some Charge Coupled Device cameras (CCD cameras) are made like this. Commonly, the upper and lower bounds of the scalar factor, the depth and the focal length can be estimated in advance.

Fig. 2. Wheeled mobile robot with monocular camera.

The tracking control problem is considered in this paper. Firstly, the kinematic model of the mobile robot in the image coordinate system $u_1 - o_1 - v_1$ is assumed as below:

$$\dot{x}_m = v_m \cos \theta_m, \quad \dot{y}_m = v_m \sin \theta_m, \quad \dot{\theta}_m = \omega_m, \quad (8)$$

where (x_m, y_m) is the path of the mass center (x, y) in the image coordinate system, θ_m is the angular in the image coordinate system. v_m and ω_m are the translational velocity and the angular velocity of the robot, respectively.

According to equation (5), calculating the time derivative of system (6), the camera-object visual servoing model yields as follows

$$\dot{x}_m = v\alpha_1 \cos(\theta - \theta_0), \quad \dot{y}_m = v\alpha_2 \sin(\theta - \theta_0), \quad \dot{\theta} = \omega. \quad (9)$$

Combining systems (8) with (9), we have

$$v\alpha \cos(\theta - \theta_0) = v_m \cos \theta_m, \quad v\alpha \sin(\theta - \theta_0) = v_m \sin \theta_m. \quad (10)$$

From (10), it has

$$v\alpha[\sin(\theta - \theta_0) \cos \theta_m - \cos(\theta - \theta_0) \sin \theta_m] = v\alpha \sin(\theta - \theta_0 - \theta_m) = 0. \quad (11)$$

Since Assumption 1 holds and v is arbitrary, we get

$$\sin(\theta - \theta_0 - \theta_m) = 0, \quad (12)$$

so we can get

$$\theta - \theta_0 - \theta_m = N\pi, \quad (13)$$

where N is a positive integer. According to (10), we obtain

$$v\alpha \cos(\theta - \theta_0) \cos \theta_m = v_m \cos^2 \theta_m, \quad v\alpha \sin(\theta - \theta_0) \sin \theta_m = v_m \sin^2 \theta_m, \quad (14)$$

hence

$$v_m = v\alpha \cos(\theta - \theta_0 - \theta_m). \quad (15)$$

From (13), we can also obtain $\dot{\theta} - \dot{\theta}_m = 0$, it means that $\dot{\theta} = \dot{\theta}_m$, thus,

$$\omega_m = \omega. \quad (16)$$

On the basis of (15) and (16), we can rewrite system (8) as follows

$$\dot{x}_m = v\alpha \cos(\theta - \theta_0 - \theta_m) \cos \theta_m, \quad (17a)$$

$$\dot{y}_m = v\alpha \cos(\theta - \theta_0 - \theta_m) \sin \theta_m, \quad (17b)$$

$$\dot{\theta}_m = \omega. \quad (17c)$$

From (13), we have $\cos(\theta - \theta_0 - \theta_m) = \pm 1$. Because the inertial coordinate system and camera coordinate system are fixed, it is easy to see that $\theta - \theta_m$ and θ_0 are constants, it means that $\cos(\theta - \theta_0 - \theta_m)$ is a fixed value. In this paper, we let $\cos(\theta - \theta_0 - \theta_m) = 1$, then system (17) can be written as

$$\dot{x}_m = v\alpha \cos \theta_m, \quad (18a)$$

$$\dot{y}_m = v\alpha \sin \theta_m, \quad (18b)$$

$$\dot{\theta}_m = \omega. \quad (18c)$$

2.2.2 Control problem description and control objective

The fixed-time tracking problem will be discussed in this paper, and fixed-time control laws v and ω for the non-holonomic robot system (18) will be designed. The model of reference robot is presented as below

$$\dot{x}_r = v_r \alpha_1 \cos \theta_r, \quad (19a)$$

$$\dot{y}_r = v_r \alpha_2 \sin \theta_r, \quad (19b)$$

$$\dot{\theta}_r = \omega_r, \quad (19c)$$

where (x_r, y_r) is the desired path of the mass center (x, y) in the coordinate system, θ_r is the desired direction. v_r and ω_r are the desired translational velocity and the angular velocity of the reference mobile robot, respectively. They satisfy the following Assumption:

Assumption 2. Suppose the velocities v_r, ω_r and the derivative $\dot{\omega}_r$ are bounded, $|v_r(t)| \leq v_r^{max}$, $0 < \omega_r^{min} \leq |\omega_r(t)| \leq \omega_r^{max}$ and $|\dot{\omega}_r(t)| < |\dot{\omega}_1^{max}|$ for any $0 \leq t_0 \leq t$. where v_r^{max} , ω_r^{min} , ω_r^{max} and $\dot{\omega}_1^{max}$ are known positive constants.

Under the Assumption 1, by using the results of paper [36], the tracking error dynamic equation can be described as follows

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{pmatrix} = \begin{pmatrix} \cos \theta_m & \sin \theta_m & 0 \\ -\sin \theta_m & \cos \theta_m & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x_m \\ y_r - y_m \\ \theta_r - \theta_m \end{pmatrix}. \quad (20)$$

Combining (18) with (19), calculating the time derivative of (20), we can have

$$\dot{x}_e = \omega y_e - v\alpha + v_r \alpha \cos \theta_e, \quad (21a)$$

$$\dot{y}_e = -\omega x_e + v_r \alpha \sin \theta_e, \quad (21b)$$

$$\dot{\theta}_e = \omega_r - \omega. \quad (21c)$$

From the transformation equation (20), if $x_e = y_e = \theta_e = 0$, which will mean that $x_m = x_r$, $y_m = y_r$ and $\theta_m = \theta_r$. In the following section, the object of this paper is to design control laws v and ω

$$v = v(t, x_e, y_e, \theta_e), \quad (22a)$$

$$\omega = \omega(t, x_e, y_e, \theta_e), \quad (22b)$$

to make the tracking error system (21) is fixed-time stable.

3 Control design

The main task in this part is to design fixed-time controllers v and ω such that system (18) can track the desired trajectory (19) in fixed time.

Obviously, the tracking error dynamic equation (21) consists of two subsystems: a first-order subsystem (21c), a second-order subsystem (21a) and (21b), therefore, in the following part, two subsections will be given to study these two subsystems respectively. Firstly, subsystem (21c) will be discussed and the angular velocity ω will be designed. Secondly, subsystem (21a) and (21b) will be investigated and the translational velocity v will be proposed.

3.1 Control law design of ω

Under Assumption 1, the subsystem (21c) will be discussed, angular velocity controller ω will be proposed, which can make error state θ_e achieve to zero in fixed time.

Theorem 3.1 Consider system (18), if Assumption 1 holds and the control law ω is designed as below

$$\omega = \omega_r + k_1 \text{sig}^{\beta_1} \theta_e + k_2 \text{sig}^{\beta_2} \theta_e, \quad (23)$$

where $k_1, k_2 > 0$, $0 < \beta_1 < 1, \beta_2 > 1$, then the control law (23) can guarantee the error state θ_e of subsystems (21c) converge to zero in fixed time, in other words, the desired trajectory θ_r can be tracked by θ_m in fixed time.

Proof. The Lyapunov function is chosen as

$$V(\theta) = \frac{1}{2} \theta_e^2. \quad (24)$$

According to system (23), computing the time derivative of $V(\theta)$, we obtain

$$\begin{aligned} \dot{V}(\theta) &= \theta_e \dot{\theta}_e = -k_1 \theta_e \text{sig}^{\beta_1} \theta_e - k_2 \theta_e \text{sig}^{\beta_2} \theta_e \\ &= -k_1 |\theta_e|^{1+\beta_1} - k_2 |\theta_e|^{1+\beta_2} \\ &= -k_1 2^{\frac{1+\beta_1}{2}} \left(\frac{1}{2} \theta_e^2\right)^{\frac{1+\beta_1}{2}} - k_2 2^{\frac{1+\beta_2}{2}} \left(\frac{1}{2} \theta_e^2\right)^{\frac{1+\beta_2}{2}} \\ &\leq -k_1 2^{\frac{1+\beta_1}{2}} (V(\theta))^{\frac{1+\beta_1}{2}} - k_2 2^{\frac{1+\beta_2}{2}} (V(\theta))^{\frac{1+\beta_2}{2}}. \end{aligned} \quad (25)$$

Noticing that $0 < \beta_1 < 1, \beta_2 > 1$, it can calculate that $0 < \frac{1+\beta_1}{2} < 1, \frac{1+\beta_2}{2} > 1$. By virtue of Lemma 2.1, it can be seen that $V(\theta)$ converges to zero in a fixed time

$$T_\theta \leq \frac{1}{T_1} + \frac{1}{T_2}, \quad (26)$$

where $T_1 = k_1 2^{\frac{1+\beta_1}{2}} \frac{1-\beta_1}{2}$, $T_2 = k_2 2^{\frac{1+\beta_2}{2}} \frac{1-\beta_2}{2}$. On the other hand, if $V(\theta) = 0$ in fixed time, then it means that $\theta_e = 0$ in fixed time, i.e., the state θ_m can converge to the desired value θ_r in fixed time.

3.2 translational velocity design

Under Assumptions 1-2, the subsystem (21a) and (21b) will be considered and the fixed-time control law v will be given, we also to prove that the controller v can make the error states x_e and y_e converge to zero in a fixed time.

Theorem 3.2 For the system (18), if Assumptions 1-2 hold and control law v is given as below

$$v = v_r - \frac{1}{\omega_r \alpha} (\eta_3 \bar{\sigma}^{1+m_1}(y_e) + \bar{\sigma}_3(x_e, y_e) + 1) \xi^{r_1+m_1-1} - \frac{1}{\omega_r \alpha} (2^{1-r_1} \bar{\sigma}(y_e) + \eta_2 \bar{\sigma}^{1+r_1}(y_e) + \eta_1 + \bar{\sigma}_4(x_e, y_e) + \bar{\sigma}_1(y_e) + 1) \xi^{2r_1-1}, \quad (27)$$

where

$$\begin{aligned} \eta_1 &= \frac{r_1 2^{\frac{3}{r_1}-r_1}}{(1+r_1)^{1+\frac{1}{r_1}}}, \quad \eta_2 = \frac{r_1^{r_1} 2^{2+3r_1-r_1^2}}{(1+r_1)^{1+r_1}}, \quad \eta_3 = \frac{m_1^{m_1} 2^{2-r_1-r_1 m_1+4m_1}}{(1+m_1)^{1+m_1}}, \\ \bar{\sigma}(y_e) &= 2^{\frac{1}{r_1}} (2-r_1) (1+y_e^{m_1-r_1})^{\frac{1}{r_1}} + 2^{\frac{1}{r_1}} (2-r_1) \frac{m_1-r_1}{r_1} (1+y_e^{m_1-r_1})^{\frac{1}{r_1}-1} y_e^{m_1-r_1}, \\ \bar{\sigma}_1(y_e) &= \frac{2-r_1}{1+r_1} \left(\frac{8r_1-4}{1+r_1} \right)^{\frac{2r_1-1}{2-r_1}} \omega_r^{\frac{2(1+r_1)}{2-r_1}} |y_e|^{\frac{2(1-r_1^2)}{2-r_1}}, \\ \bar{\sigma}_2(x_e, y_e) &= \frac{2^{1+r_1}}{1+r_1} \left(\frac{4r_1}{1+r_1} \right)^{r_1} \left| \frac{\dot{\omega}_r}{\omega_r} \right|^{1+r_1} |\xi|^{1-r_1^2}, \\ \bar{\sigma}_3(x_e, y_e) &= \frac{2^{1+m_1}}{1+m_1} \left(\frac{2m_1}{1+m_1} \right)^{m_1} \left| \frac{\dot{\omega}_r}{\omega_r} \right|^{1+m_1} |\xi|^{1+m_1-r_1-r_1 m_1}, \\ \bar{\sigma}_4(x_e, y_e) &= \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{1-r_1} + \bar{\sigma}_2(x_e, y_e), \quad \xi = (-\omega_r x_e)^{\frac{1}{r_1}} - (-2y_e^{r_1} - 2y_e^{m_1})^{\frac{1}{r_1}}, \end{aligned}$$

moreover, $r_1 = 1 + \tau_1$, $m_1 = 1 + \tau_2$, $-\frac{1}{2} < \tau_1 = -\frac{q_1}{p_1} < 0$, $\tau_2 = \frac{q_2}{p_2} > 0$, and $q_i (i = 1, 2)$ are positive even integer, $p_i (i = 1, 2)$ are positive odd integer, then the control law (27) can make the error states x_e and y_e of subsystems (21a)-(21b) converge to zero in fixed time.

Proof. We let

$$e_1 = y_e, \quad e_2 = -\omega_r x_e, \quad \theta_e = \theta_e. \quad (28)$$

Combing system (21) with control laws (23) and (27), the time derivative of (28) is given

$$\dot{e}_1 = \frac{\omega}{\omega_r} e_2 + v_r \alpha \sin \theta_e, \quad (29a)$$

$$\begin{aligned} \dot{e}_2 &= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r \omega e_1 + \omega_r v \alpha - \omega_r v_r \alpha \cos \theta_e \\ &= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r \omega e_1 - \omega_r v_r \alpha \cos \theta_e + \omega_r v_r \alpha \\ &\quad - \frac{\alpha}{\omega_r} (\eta_3 \sigma^{1+m_1}(e_1) + \sigma_3(e_1, e_2) + 1) \xi^{r_1+m_1-1} \\ &\quad - \frac{\alpha}{\omega_r} (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1 + \sigma_4(e_1, e_2) + \sigma_1(e_1) + 1) \xi^{2r_1-1}, \end{aligned} \quad (29b)$$

$$\dot{\theta}_e = -k_1 \text{sig}^{\beta_1} \theta_e - k_2 \text{sig}^{\beta_2} \theta_e, \quad (29c)$$

where $\eta_1, \eta_2, \eta_3, m_1, r_1$ are defined as above and

$$\sigma(e_1) = 2^{\frac{1}{r_1}} (2-r_1) (1+e_1^{m_1-r_1})^{\frac{1}{r_1}} + 2^{\frac{1}{r_1}} (2-r_1) \frac{m_1-r_1}{r_1} (1+e_1^{m_1-r_1})^{\frac{1}{r_1}-1} e_1^{m_1-r_1},$$

$$\sigma_1(e_1) = \frac{2-r_1}{1+r_1} \left(\frac{8r_1-4}{1+r_1} \right)^{\frac{2r_1-1}{2-r_1}} \omega_r^{\frac{2(1+r_1)}{2-r_1}} |e_1|^{\frac{2(1-r_1^2)}{2-r_1}},$$

$$\sigma_2(e_1, e_2) = \frac{2^{1+r_1}}{1+r_1} \left(\frac{4r_1}{1+r_1} \right)^{r_1} \left| \frac{\dot{\omega}_r}{\omega_r} \right|^{1+r_1} |\xi|^{1-r_1^2},$$

$$\begin{aligned}\sigma_3(e_1, e_2) &= \frac{2^{1+m_1}}{1+m_1} \left(\frac{2m_1}{1+m_1} \right)^{m_1} \left| \frac{\dot{\omega}_r}{\omega_r} \right|^{1+m_1} |\xi|^{1+m_1-r_1-r_1m_1}, \\ \sigma_4(e_1, e_2) &= \left| \frac{\dot{\omega}_r}{\omega_r} \right| \xi^{1-r_1} + \sigma_2(e_1, e_2), \quad \xi = e_2^{\frac{1}{r_1}} - (-2e_1^{r_1} - 2e_1^{m_1})^{\frac{1}{r_1}}.\end{aligned}$$

Since Assumption 2 holds, the transformation (28) means that when $e_j = 0 (j = 1, 2)$ in fixed time, then $x_e = 0$ and $y_e = 0$ in fixed time must be met. Therefore, next, we will prove $e_j = 0 (j = 1, 2)$ in fixed time. Based on Theorem 3.1, there exists a fixed time T_θ satisfying $\theta_e(t) = 0$ for any $t \geq T_\theta$, i.e., $\omega_r = \omega$ for any $t \geq T_\theta$, due to this condition, system (29) can be simplified as follows

$$\dot{e}_1 = e_2, \tag{30a}$$

$$\begin{aligned}\dot{e}_2 &= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r \omega e_1 + \omega_r v \alpha - \omega_r v_r \alpha \\ &= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r^2 e_1 - \frac{\alpha}{\underline{\alpha}} (\eta_3 \sigma^{1+m_1}(e_1) + \sigma_3(e_1) + 1) \xi^{r_1+m_1-1} \\ &\quad - \frac{\alpha}{\underline{\alpha}} (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1 + \sigma_4(e_1, e_2) + \sigma_1(e_1) + 1) \xi^{2r_1-1}.\end{aligned} \tag{30b}$$

Two steps will be given to prove this theorem, fixed-time control theory and adding a power integrator technique will be employed.

Step 1 The Lyapunov function is chosen as

$$V_1(e_1) = \frac{1}{2} e_1^2, \tag{31}$$

From (30), the derivative of $V_1(e_1)$ is

$$\dot{V}_1(e_1) = e_1 e_2 = e_1 e_2^* + e_1 (e_2 - e_2^*). \tag{32}$$

With the help of the backstepping design idea, let us design a virtual control law as follows

$$e_2^* = -2e_1^{r_1} - 2e_1^{m_1}, \tag{33}$$

which leads to

$$\dot{V}_1(e_1) \leq -2e_1^{1+r_1} - 2e_1^{1+m_1} + e_1 (e_2 - e_2^*). \tag{34}$$

Step 2 Consider the Lyapunov function $V_2(e_1, e_2)$, which can be described as

$$V_2(e_1, e_2) = V_1(e_1) + \int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}})^{2-r_1} ds. \tag{35}$$

For brevity, denote $\xi = e_2^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}}$. The derivative of $V_2(e_1, e_2)$ along systems (30) and (34) is

$$\begin{aligned}\dot{V}_2(e_1, e_2) &\leq -2e_1^{1+r_1} - 2e_1^{1+m_1} + e_1 (e_2 - e_2^*) + \xi^{2-r_1} \dot{e}_2 \\ &\quad + (2-r_1) \frac{d(-e_2^{*\frac{1}{r_1}})}{dt} \int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}})^{1-r_1} ds.\end{aligned} \tag{36}$$

Using Lemmas 2.4 and 2.5, one obtains

$$e_1 (e_2 - e_2^*) \leq |e_1| |(e_2^{\frac{1}{r_1}})^{r_1} - (e_2^{*\frac{1}{r_1}})^{r_1}| \leq 2^{1-r_1} |e_1| |\xi|^{r_1} \leq \frac{1}{4} |e_1|^{1+r_1} + \eta_1 |\xi|^{1+r_1}. \tag{37}$$

Noticing that

$$-e_2^{*\frac{1}{r_1}} = (2e_1^{r_1} (1 + e_1^{m_1-r_1}))^{\frac{1}{r_1}} = 2^{\frac{1}{r_1}} e_1 (1 + e_1^{m_1-r_1})^{\frac{1}{r_1}}, \tag{38}$$

which leads to

$$\begin{aligned}
(2-r_1) \frac{d(-e_2^{\frac{1}{r_1}})}{de_1} &= 2^{\frac{1}{r_1}} (2-r_1) (1+e_1^{m_1-r_1})^{\frac{1}{r_1}} \\
&\quad + 2^{\frac{1}{r_1}} (2-r_1) \frac{m_1-r_1}{r_1} (1+e_1^{m_1-r_1})^{\frac{1}{r_1}-1} e_1^{m_1-r_1} \\
&\triangleq \sigma(e_1).
\end{aligned} \tag{39}$$

In addition, based on Lemma 2.2, from (33) and the definition of ξ , we have

$$|e_2| = |\xi + e_2^{\frac{1}{r_1}}|^{r_1} \leq |\xi|^{r_1} + |e_2^*| \leq |\xi|^{r_1} + 2|e_1|^{r_1} + 2|e_1|^{m_1}. \tag{40}$$

By Lemma 2.4, we can also obtain that

$$\int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{\frac{1}{r_1}})^{1-r_1} ds \leq |\xi|^{1-r_1} |e_2 - e_2^*| = |\xi|^{1-r_1} |(e_2^{\frac{1}{r_1}})^{r_1} - (e_2^{\frac{1}{r_1}})^{r_1}| \leq 2^{1-r_1} |\xi|. \tag{41}$$

From (39), (40), (41) and Lemma 2.5, one obtains

$$\begin{aligned}
&(2-r_1) \frac{d(-e_2^{\frac{1}{r_1}})}{dt} \int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{\frac{1}{r_1}})^{1-r_1} ds \\
&= (2-r_1) \frac{d(-e_2^{\frac{1}{r_1}})}{de_1} \frac{de_1}{dt} \int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{\frac{1}{r_1}})^{1-r_1} ds \\
&\leq \sigma(e_1) (|\xi|^{r_1} + 2|e_1|^{r_1} + 2|e_1|^{m_1}) 2^{1-r_1} |\xi| \\
&\leq \frac{1}{4}|e_1|^{1+r_1} + \frac{1}{2}|e_1|^{1+m_1} + (2^{1-r_1} \sigma(e_1) \\
&\quad + \eta_2 \sigma^{1+r_1}(e_1)) |\xi|^{1+r_1} + \eta_3 \sigma^{1+m_1}(e_1) |\xi|^{1+m_1}.
\end{aligned} \tag{42}$$

Substituting (37) and (42) into (36) yields

$$\begin{aligned}
\dot{V}_2(e_1, e_2) &\leq -\frac{3}{2}|e_1|^{1+r_1} - \frac{3}{2}|e_1|^{1+m_1} + \eta_3 \sigma^{1+m_1}(e_1) |\xi|^{1+m_1} \\
&\quad + (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1) |\xi|^{1+r_1} + \xi^{2-r_1} \dot{e}_2 \\
&\leq -\frac{3}{2}|e_1|^{1+r_1} - \frac{3}{2}|e_1|^{1+m_1} + \eta_3 \sigma^{1+m_1}(e_1) |\xi|^{1+m_1} \\
&\quad + (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1) |\xi|^{1+r_1} + \xi^{2-r_1} \dot{e}_2.
\end{aligned} \tag{43}$$

Based on Assumption 1, we can obtain that $1 \leq \frac{\alpha}{\underline{\alpha}}$. Combining (30b) with (43), yields

$$\begin{aligned}
\dot{V}_2(e_1, e_2) &\leq -\frac{3}{2}|e_1|^{1+r_1} - \frac{3}{2}|e_1|^{1+m_1} + \eta_3 \sigma^{1+m_1}(e_1) |\xi|^{1+m_1} \\
&\quad + (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1) |\xi|^{1+r_1} \\
&\quad + \xi^{2-r_1} (\frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r^2 e_1) - \frac{\alpha}{\underline{\alpha}} (\eta_3 \sigma^{1+m_1}(e_1) + \sigma_3(e_1) + 1) \xi^{m_1+1} \\
&\quad - \frac{\alpha}{\underline{\alpha}} (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1 + \sigma_4(e_1, e_2) + \sigma_1(e_1) + 1) \xi^{r_1+1} \\
&\leq -\frac{3}{2}|e_1|^{1+r_1} - \frac{3}{2}|e_1|^{1+m_1} + \eta_3 \sigma^{1+m_1}(e_1) |\xi|^{1+m_1} \\
&\quad + (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1) |\xi|^{1+r_1} \\
&\quad + \xi^{2-r_1} (\frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r^2 e_1) - (\eta_3 \sigma^{1+m_1}(e_1) + \sigma_3(e_1) + 1) \xi^{m_1+1} \\
&\quad - (2^{1-r_1} \sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1 + \sigma_4(e_1, e_2) + \sigma_1(e_1) + 1) \xi^{r_1+1} \\
&\leq -\frac{3}{2}|e_1|^{1+r_1} - \frac{3}{2}|e_1|^{1+m_1} - |\xi|^{1+m_1} - |\xi|^{1+r_1} + |\xi|^{2-r_1} |\frac{\dot{\omega}_r}{\omega_r}| |e_2| \\
&\quad + |\xi|^{2-r_1} \omega_r^2 |e_1| - \sigma_3(e_1, e_2) |\xi|^{1+m_1} - \sigma_4(e_1, e_2) |\xi|^{1+r_1} \\
&\quad - \sigma_1(e_1) |\xi|^{1+r_1}.
\end{aligned} \tag{44}$$

By Lemma 2.5, we obtain

$$|\xi|^{2-r_1} |\omega_r^2| |e_1| \leq |\omega_r^{\frac{2}{2-r_1}} e_1^{\frac{2-2r_1}{2-r_1}} \xi|^{2-r_1} |e_1|^{2r_1-1} \leq \sigma_1(e_1) |\xi|^{1+r_1} + \frac{1}{4} |e_1|^{1+r_1}. \quad (45)$$

Note that $\xi = e_2^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}}$, $e_2^* = -2e_1^{r_1} - 2e_1^{m_1}$, using Lemmas 2.2 and 2.5, we have

$$\begin{aligned} \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |e_2| &\leq \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |\xi + e_2^{*\frac{1}{r_1}}|^{r_1} \leq \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} (|\xi|^{r_1} + |e_2^*|) \\ &= \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |\xi|^{r_1} + \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |2e_1^{r_1} + 2e_1^{m_1}| \\ &\leq \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |\xi|^{r_1} + 2 \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |e_1|^{r_1} + 2 \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{2-r_1} |e_1|^{m_1} \\ &\leq \left| \frac{\dot{\omega}_r}{\omega_r} \right| |\xi|^{1-r_1} |\xi|^{1+r_1} + \frac{1}{4} |e_1|^{1+r_1} + \sigma_2(e_1, e_2) |\xi|^{1+r_1} \\ &\quad + \frac{1}{2} |e_1|^{1+m_1} + \sigma_3(e_1, e_2) |\xi|^{1+m_1} \\ &= \sigma_4(e_1, e_2) |\xi|^{1+r_1} + \frac{1}{4} |e_1|^{1+r_1} + \frac{1}{2} |e_1|^{1+m_1} + \sigma_3(e_1, e_2) |\xi|^{1+m_1}. \end{aligned} \quad (46)$$

According to (45) and (46), the equation (44) can be written as

$$\dot{V}_2(e_1, e_2) \leq -|e_1|^{1+r_1} - |e_1|^{1+m_1} - |\xi|^{1+m_1} - |\xi|^{1+r_1}. \quad (47)$$

Meanwhile, by using Lemma 2.4, from the Lyapunov function $V_2(e_1, e_2)$ defined in (35), it has

$$V_2(e_1, e_2) \leq \frac{1}{2} e_1^2 + 2^{1-r_1} \xi^2 \leq \lambda(e_1^2 + \xi^2), \quad (48)$$

where $\lambda = \max\{\frac{1}{2}, 2^{1-r_1}\}$. By Lemmas 2.2-2.3, it can be concluded that

$$(e_1^2 + \xi^2)^{\frac{1+r_1}{2}} \leq |e_1|^{1+r_1} + |\xi|^{1+r_1}, \quad (49)$$

and

$$(e_1^2 + \xi^2)^{\frac{1+m_1}{2}} \leq 2^{\frac{m_1-1}{2}} (|e_1|^{1+m_1} + |\xi|^{1+m_1}). \quad (50)$$

In view of inequalities (47), (48), (49) and (50), it yields

$$\dot{V}_2(e_1, e_2) \leq -\lambda^{-\frac{1+r_1}{2}} V_2^{\frac{1+r_1}{2}}(e_1, e_2) - 2^{-\frac{m_1-1}{2}} \lambda^{-\frac{1+m_1}{2}} V_2^{\frac{1+m_1}{2}}(e_1, e_2). \quad (51)$$

On one hand, according to the inequality equation (51) and by using Lemma 2.1, it can easily be obtained that there exists a fixed time constant $T_0 = \lambda^{\frac{1+m_1}{2}} \frac{2}{1-r_1} + 2^{\frac{m_1-1}{2}} \lambda^{\frac{1+m_1}{2}} \frac{2}{m_1-1} < \infty$ satisfies $V_2(e_1, e_2) = 0$ for any $t \geq T_0$. In addition, based on equation (35), $V_2(e_1, e_2) = 0$ indicates that $e_1 = 0$ and $e_2 = 0$. Therefore, it can be concluded that the subsystem (21a)-(21b) with the controller (27) is globally fixed-time stable.

Remark 2. The boundedness of states $e_j (j = 1, 2)$ in the time interval $[0, T_\theta]$ doesn't prove in Theorem 3.2, it is mainly because the closed-loop system consists of complex nonlinear items and is difficult to analysis. In order to illustrate the effectiveness of the theoretical results, with the controllers (23) and (27), a large number of simulations have been done for systems (18) and (19) in the following simulation section, we do not observe any divergence phenomenon. In many situations, we can assume the control law is a bounded value in the time interval $[0, T_\theta]$ to guarantee the boundedness of system states.

Based upon the above analysis, we can directly obtain the following theorem.

Theorem 3.3 If Assumptions 1-2 are satisfied and the control laws ω and v are designed as (23) and (27), then the mobile robot system (18) can globally track the desired reference trajectory (19) in a fixed time, where the control parameters used in (23) and (27) are chosen as those in above Theorems 3.1 and 3.2.

Proof. First of all, Theorems 3.1 and 3.2 show that control laws (23) and (27) can make the states θ_e and $e_j (j = 1, 2)$ in system (29) converge to zero in a fixed time. In addition, the state transformation equations (20) and (28) show that $\theta_e = 0$ and $e_j (j = 1, 2) = 0$ imply that $x_r = x_m$, $y_r = y_m$, $\theta_r = \theta_m$. Therefore, system (18) can globally track the desired trajectory (19) in a fixed time.

Remark 3. The authors of paper [21] has discussed finite-time tracking control problem for systems (18) and (19), and finite-time tracking controllers have been given as follows

$$\omega = \omega_r + \mathcal{K}_1 \text{sig}^\beta \theta_e, \quad (52a)$$

$$v = v_r - \frac{1}{\omega_r \underline{\alpha}} (\mathcal{K}_3 + \rho_1(y_e) + \rho_2(x_e, y_e)) (\mathcal{K}_2^p y_e - \omega_r^p x_e^p)^{\frac{2}{p}-1}, \quad (52b)$$

where $0 < \beta < 1$, $\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3$ denote positive constant control gains, $1 < p < 2$ is a ratio of two odd integers, $\rho_1(y_e) = \frac{2p-1}{1+p} \omega_r^{\frac{2(p+1)}{2p-1}} y_e^{\frac{2(p^2-1)}{p(2p-1)}}$, $\rho_2(x_e, y_e) = |\frac{\dot{\omega}_r}{\omega_r}| (\mathcal{K}_2^p y_e - \omega_r^p x_e^p)^{1-\frac{1}{p}} + \frac{2p-1}{1+p} (\frac{\dot{\omega}_r}{\omega_r} \mathcal{K}_2)^{\frac{1+p}{2p-1}} y_e^{\frac{p^2-1}{p(2p-1)}}$. Compared with the finite-time controllers (52), the main advantage of the proposed fixed-time control result lies in the convergence time can be predetermined but independent with any initial conditions, which will be illustrated in the next section.

4 Simulation results

For system (18), simulation examples will be presented in this section, and two cases are considered. In the first case, simulation results will be given to show the effectiveness of the proposed fixed-time control laws (23) and (27). In the second case, under different initial condition, we will compare the convergent performance between fixed-time control laws (23) and (27) and finite-time controllers (52).

Case 1: In this case, suppose the desired value of system (19) as $v_r = 1.5 - \frac{1.5t}{t+10} m/s$, $\omega_r = 1 + \frac{2t}{t+10} rad/s$. Let the initial value $[x_r(0), y_r(0), \theta_r(0)] = (-4, 1, 0)$, $[x(0), y(0), \theta(0)] = (2, 1.5, 0.5)$. Fixed-time parameters are taken as $k_1 = 1, k_2 = 2, \tau_1 = -\frac{2}{7}, \tau_2 = \frac{2}{435}, \beta_1 = 0.65, \beta_2 = 1.5, \underline{\alpha} = 10$ and $\underline{\alpha} = 8$. Simulation results are shown in Figs 3–6.

Fig. 3. Response state curves for x_r, y_r, θ_r and x, y, θ .

Fig. 3 shows state curves for x_r, y_r, θ_r and x, y, θ . Fig. 4 presents the tracking errors x_e, y_e and θ_e . Curves of desired and tracking trajectories are presented in Fig. 5. Fig. 6 gives the control outputs v and ω acted on the robot, respectively Figs 3–5 demonstrate that control laws (23) and (27) can make the mobile robot achieve to the desired value in a fixed time.

Case 2: In this case, based on remark 3, under different initial conditions for fixed-time control laws (23) and (27) and finite-time control law (52), the convergent time performance of these two kinds of control laws are compared. Fig. 7 shows that the convergent time is independent of initial state for fixed-time control laws, where the 2 norm of initial value is defined as $\delta(0) = \sqrt{(x_r(0) - x(0))^2 + (y_r(0) - y(0))^2}$.

Fig. 4. Response tracking errors curves for x_e , y_e and θ_e .

Fig. 5. Response desired trajectory and tracking curves.

Fig. 6. Control outputs v and ω acted on the robot.

Fig. 7. The convergence time for the different initial conditions.

5 Conclusions

For the nonholonomic wheeled mobile robot with uncalibrated camera parameters, fixed-time tracking control problem has been considered. Firstly, according to the pinhole camera model, the camera-objective visual model has been established, and tracking error system between the robot and desired trajectory has been given. Then, by using the fixed-time control theory and Lyapunov stability analysis, fixed-time control laws for the robot have been proposed, which can make the wheeled mobile robot track the desired trajectory in a fixed time. Finally, the simulation results have been given to verify the validity of the conclusions.

Acknowledgement

This work is supported by Natural Science Foundation of Anhui Province (1908085MF219), Natural Science Foundation of Shandong Province (ZR2020YQ48), Postdoctoral Science Foundation of China(2017M621590) and the Open Project Program of Ministry of Education Key Laboratory of Measurement and Control of CSE(MCCSE2018B03).

Compliance with ethical standards

Conflicts of interest: The authors declare that they have no conflict of interest.

Data availability statement

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

References

- [1] H. B. Shi, X. S. Li, K. S. Hwang, et al, Decoupled visual servoing with fuzzy Q-Learning, *IEEE Transactions on Industrial Informatics*, vol. 14, no. 1, pp. 241-252, 2016.
- [2] N. Djelal, N. Saadia and A. Ramdane-Cherif, Target tracking based on SURF and image-based visual servoing, In *Proceeding of the 2th IEEE International Conference on Communications, Computing and Control Applications*, Marseilles, France, pp. 1-5, 2013.
- [3] W. Pan, M. Lyu, K. S. Hwang, et al, A neuro-fuzzy visual servoing controller for an articulated manipulator, *IEEE Access*, vol. 6, no. 99, pp. 3346-3357, 2018.
- [4] S. Kim and S. Y. Oh, Hybrid position and image based visual servoing for mobile robots, *Journal of Intelligent and Fuzzy Systems*, vol. 18, no. 1, pp. 73-82, 2007.
- [5] A. Ghasemi, P. C. Li and W. F. Xie, Adaptive switch image-based visual servoing for industrial robots, *International Journal of Control, Automation and Systems*, vol. 18, no. 5, pp. 1324-1334, 2020.
- [6] X. B. Zhang, Y. C. Fang and X. Liu, Motion estimation based visual servoing of nonholonomic mobile robots, *IEEE Transactions on Robotics*, vol. 27, no. 6, pp. 1167-1175, 2011.
- [7] Y. C. Fang, X. Liu and X. B. Zhang, Adaptive active visual servoing of nonholonomic mobile robots, *IEEE Transactions on Industrial Electronics*, vol. 59, no. 1, pp. 486-497, 2012.
- [8] H. S. Wang, Y. H. Liu and D. X. Zhou, Dynamic visual tracking for manipulators using an uncalibrated fixed camera, *IEEE Transactions on Robotics*, vol. 23, no. 3, pp. 610-617, 2007.
- [9] P. F. Guo, Z. Y. Liang, X. Wang and M. W. Zheng, Adaptive trajectory tracking of wheeled mobile robot based on fixed-time convergence with uncalibrated camera parameters, *ISA Transactions*, vol. 99, pp. 1-8, 2020.
- [10] J. Chen, W. E. Dixon, D. M. Dawson and M. McIntyre, Homography-based visual servo tracking control of a wheeled mobile robot, *IEEE Transactions on Robotics*, vol. 22, no 2, pp. 407-416, 2006.
- [11] F. Yang, C. L. Wang and G. Z. Jing, Adaptive tracking control for dynamic nonholonomic mobile robots with uncalibrated visual parameters, *International Journal of Adaptive Control and Signal Processing*, vol. 27, no. 8, pp. 688-700, 2013.
- [12] X. W. Liang, and H. S. Wang, W. D. Chen, et al, Adaptive image-based trajectory tracking control of wheeled mobile robots with an uncalibrated fixed camera, *IEEE Transactions on Control Systems Technology*, vol. 23, no. 6, pp. 2266-2282, 2015.
- [13] S. P. Bhat and D. S. Bernstein, Finite-time stability of continuous autonomous systems, *SIAM Journal on Control and Optimization*, vol. 38, no. 3, pp. 751-766, 2000.
- [14] H. B. Sun, L. L. Hou and G. D. Zong, Continuous finite time control for static var compensator with mismatched disturbances, *Nonlinear Dynamics*, vol. 85, pp. 2159-2169, 2016.
- [15] H. B. Du, S. H. Li and C. J. Qian, Finite-time attitude tracking control of spacecraft with application to attitude synchronization, *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2711-2717, 2011.
- [16] S. P. Bhat, D. S. Bernstein, Geometric homogeneity with applications to finite-time stability, *Mathematics of Control Signals and Systems*, vol. 17, no. 2, pp. 101-127, 2005.
- [17] H. Chen, C. L. Wang, D. K. Zhang and F. Yang, Robust saturated finite-time stabilization for nonholonomic mobile robots based on visual serving feedback, In *Proceeding of the 24th Chinese Control and Decision Conference*, Taiyuan, China, pp. 1268-1272, 2012.
- [18] J. J. Xiong and G. B. Zhang, Global fast dynamic terminal sliding mode control for a quadrotor UAV, *ISA Transactions*, vol. 66, pp. 1233-1240, 2017.
- [19] Y. Q. Wu, B. Wang and G. D. Zong, Finite time tracking controller design for nonholonomic systems with extended chained form, *IEEE Transaction on Circuits Systems II: Express Briefs*, vol. 52, no. 11, pp. 798-802, 2005.

- [20] M. Y. Ou, H. B. Sun, S. W. Gu and Y. Y. Zhang, Distributed finite-time trajectory tracking control for multiple nonholonomic mobile robots with uncertainties and external disturbances, *International Journal of Systems Science*, vol. 48, no.15, pp. 3233-3245, 2017.
- [21] M. Y. Ou, S. H. Li and C. L. Wang, Finite-time tracking control for a nonholonomic mobile robot based on visual servoing, *Asian Journal of Control*, vol. 16, no. 3, pp. 679-691, 2014.
- [22] Z. C. Zhou, G. Y. Tang, R. K. Xu, et al, A novel continuous nonsingular finite-time control for underwater robot manipulators, *Journal of Marine Environmental Engineering*, vol. 9, no. 3, pp. 1-18, 2021.
- [23] A. Polyakov, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Transactions on Automatic Control*, vol. 57, no. 8 pp. 2106-2110, 2012.
- [24] A. Levant, On fixed and finite time stability in sliding mode control, In *Proceeding of the 52nd IEEE Annual Conference on Decision and Control*, Florence Italy, pp. 4260-4265, 2013.
- [25] J. P. Li, Y. N. Yang, C. C. Hua and X. Guan, Fixed-time backstepping control design for high-order strict-feedback nonlinear systems via terminal sliding mode, *IET Control Theory and Application*, vol. 11, no. 8, pp. 1184-1193, 2016.
- [26] Y. Huang and Y. M. Jia, Fixed-time consensus tracking control of second-order multi-agent systems with inherent nonlinear dynamics via output feedback, *Nonlinear Dynamics*, vol. 91, pp. 1289-1306, 2018.
- [27] L. Zhang, C. Z. Wei, L. Jing and N. G. Cui, Fixed-time sliding mode attitude tracking control for a submarine-launched missile with multiple disturbances, *Nonlinear Dynamics*, vol. 93, pp. 2543-2563, 2018.
- [28] Z. Y. Zuo, Non-singular fixed-time terminal sliding mode control of non-linear systems, *IET Control Theory and Applications*, vol. 9, no. 4, pp. 545-552, 2015.
- [29] W. Huang, Y. N. Yang and C. C. Hua, Fixed-time tracking control approach design for nonholonomic mobile robot, In *Proceeding of the 35th Chinese Control Conference*, Chengdu, China, pp. 3423-3428, 2016.
- [30] M. Y. Ou, H. B. Sun, Z. X. Zhang, et al, Fixed-time tracking control for nonholonomic mobile robot, *Kybernetika*, vol. 57, no. 2, pp. 220-235, 2021.
- [31] G. H. Hardy, J. E. Littlewood and G. Polya. *Inequalities*[M], Cambridge: Cambridge University Press, 1952.
- [32] Z. Y. Zuo and L. Tie, A new class of finite-time nonlinear consensus protocols for multi-agent systems, *International Journal of Control*, vol. 87, no. 2, pp. 363-370, 2014.
- [33] C. J. Qian and W. Lin, A continuous feedback approach to global strong stabilization of nonlinear systems, *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1061-1079, 2001.
- [34] W. E. Dixon, D. M. Dawson, E. Zergeroglu and A. Behal, Adaptive tracking control of a wheeled mobile robot via an uncalibrated camera system, *IEEE Transactions on Systems, Man and Cybernetics, Part B: Cybernetics*, vol. 31, no. 3, pp. 341-352, 2001.
- [35] F. Yang and C. L. Wang, Adaptive stabilization for uncertain nonholonomic dynamic mobile robots based on visual servoing feedback, *Acta Automatica Sinica*, vol. 7, no. 7, pp. 857-846, 2011.
- [36] Y. Kanayama, Y. Kimura, F. Miyazaki and T. Noguchi, A stable tracking control method for an autonomous mobile robot, In *Proceedings of the IEEE International Conference on Robotics and Automation*, pp. 384-389, 1990.