Stability for a Class of State Constrained Impulsive Nonlinear Systems: Barrier Lyapunov Functions Method

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Research Article

Keywords: State constrained, Impulsive systems, Barrier Lyapunov functions, Nonlinear systems, Stability

Posted Date: August 13th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-796093/v1>

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Stability for a class of state constrained impulsive nonlinear systems: Barrier Lyapunov Functions Method

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Received: date / Accepted: date

Abstract This paper studies the problem for a class of state constrained impulsive nonlinear systems. Firstly, we establish two sufficient conditions for the stability of invariant sets of state constrained hybrid systems. Secondly, we construct the symmetric and asymmetric barrier Lyapunov functions, respectively. A feedback method is presented to solve the stabilization problem of constrained hybrid systems. Introduce the auxiliary matrix, combining with inductive method and linear matrix inequality theory, some sufficient conditions are obtained to ensure stability for state constrained hybrid dynamical networks by the attractive ellipsoid method approach. Finally, one example with simulations is given to validate the effectiveness of the proposed criteria.

Keywords State constrained · Impulsive systems · Barrier Lyapunov functions · Nonlinear systems · Stability

1 Introduction

Input constraints and output constraints are existing extensively in nonlinear systems [1,2]. Input constraints are caused by the structure, material and motion characteristics of the physical system, including delays, dead zone [3] and saturation etc. And the states of the nonlinear system may be restricted. For example, in robot control, there is a maximum torque in the motor control, that is actuator saturation. It maybe led the robot

loses part of the control signal, which may make the robot deviate from the predetermined trajectory, or even cause a safety accident [4]. In the control system of a car, the movement of the wheel has a maximum speed, that is, the output signal of the system is limited within a specific range. If the influence of these nonlinear characteristic is ignored in the design of the control system, the whole nonlinear system may be unstable, and even damage the whole system. Therefore, it is necessary to solve the nonlinear system control problem with constraints in the field of nonlinear system control.

In many practical engineering and natural systems, the state of the system changes abruptly at certain points in time for some reason. That is the mutation or jump process can be regarded as happening at a moment. These systems can't be solved by traditional continuous systems or discrete systems alone. Such as population growth in ecology [5], prevention and treatment of infectious diseases, digital communication systems, optimal control in economics(economic stimulus package) [6], and so on. This phenomenon can also be found in many engineering fields, such as automatic control, computer networks, supply chain systems, and communication systems [7]. A system in which the state suddenly changes at some moment cannot be described as a single continuous dynamical system or a discrete-time dynamical system, so it is natural that people put forward an impulsive system to describe the dynamic system with impulsive phenomenon. In recent decades, many researchers have focused on stability or control of impulsive systems or hybrid systems, include Lyapunov stability of impulsive systems [8–12], impulsive control for synchronization of complex networks [13], application of hybrid systems [14,15]. Impulsive systems have been studied for a long time, but still less referred to the state constrained impulsive nonlinear systems.

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In conclusion, some researchers have attempted to solve the problem of constrained systems [16–29]. In [16], Rao et al. investigated moving horizon estimation as an online optimization strategy for constrained state estimation for nonlinear discrete-time systems. Harris et al. analyzed the problem of finite horizon optimal control with mixed non-convex and linear state constraints [17]. Recently, barrier Lyapunov function is proposed to solve the input or output constraint system [18, 19]. In [19], Dehaan and Guay considered a class of nonlinear systems with unknown dynamical parameters, whose states are subject to the unknown state-constrained minimizer with unknown parameters. Liu and Tong [24] studied a class of uncertain nonlinear parametric systems by an adaptive control technique and barrier Lyapunov functions. Tee et al. [25] considered the output tracking control for strict feedback delayed nonlinear systems with output constraint. In [26], a reinforcement learning control method is proposed for air-breathing hypersonic vehicles based on barrier Lyapunov functions. In [28], the authors considered the issue of the uncertain switched multi-input and output-constrained nonlinear systems by adaptive neural tracking control. However, although the aforementioned results account for the state constraints, the approaches cannot cope with the problem of the state constrained impulsive nonlinear systems. This key point motivates us to solve the problem. Based on this, this paper proposes a special type of feedback control with disturbance, which addresses the state constrained system with impulsive effects. The matrix theory and convex analysis method show that the state track for the closed loop nonlinear dynamic systems are stable. Finally, one instance with numerical simulations is offered to demonstrate the availability of our results.

This paper is organized as follows: In the following section, model and some theorem are presented. The robust control applications are established in Section 3. Simulation example is reported in Section 4, and the conclusion is drawn in Section 5.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the n -dimensional Euclidean space and the set of $n \times m$ matrices, respectively. $\mathbb{R}^+ = (0, +\infty)$. $\mathbb{N} = \{1, 2, \dots\}$. $A/B = \{x | x \in A, x \notin B\}$. ∂S denotes the boundary of set S . Denote $\|u\|$ as a vector norm of the vector $u \in \mathbb{R}^n$. $C(\mathbb{X}, \mathbb{Y})$ denotes the space of continuous mappings from the topological space $\mathbb{X}$ to the topological space $\mathbb{Y}$. The notation A^T and A^{-1} mean that the transpose of A and the inverse of a square matrix A , respectively. $X > 0$ ($X \geq 0$) means that X is a real symmetric and positive definite (positive semi-definite) matrix. I denotes the identity matrix.

2 Model description and some preliminaries

We consider the nonlinear dynamic systems described by the following hybrid differential equations:

$$\begin{cases} \dot{x}(t) = f(x(t)), & a.e. \ t \geq 0, \ x(t) \in \mathbb{D}, \\ \Delta x(t) = S(x(t)), & x(t) \in M \subset \partial(\mathbb{R}^n/\mathbb{D}), \\ x(0) = x_0 \in \mathbb{D}, \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ corresponds to the state at time t , $\mathbb{D}$ is a bounded domain in $\mathbb{R}^n$. $f(x(t)) \in C(\text{mathbbD}, \mathbb{R}^n)$. M presents the impulsive point set. Assume that if there exists a $x(t_k) \in M$, note that $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = x(t_k)$ and $x(t_k^-) = \lim_{t \rightarrow t_k^-} x(t)$. Where $S(x(t))$ is a continuous single valued mapping. For system (1), its initial conditions are given by $x(0) = x_0 \in \mathbb{D}$.

The set T_0 is called an invariant set of system [30, 31] if it follows from the inclusion $x_0 \in T_0$ that $x(t, x_0) \in T_0$ for $t \geq 0$. We establish some sufficient conditions for the stability of invariant sets of state constrained hybrid systems.

Theorem 1. Let $\mathbb{D}$ is a bounded domain in $\mathbb{R}^n$, for any positive constants $\alpha, \mu_k \leq 1$, there exists a positive function $V(x)$ satisfies the following conditions:

$$\begin{cases} D^+V(x) |_{(1)} \leq -\alpha V(x), & x \in \mathbb{D}, \\ V(x + S(x)) \leq \mu_k V(x), & x \in M \subset \partial. \end{cases} \quad (2)$$

Then x of system (1) remains in the open set $\mathbb{D}$.

Proof. In the set of all trajectories of system, we consider the following classes:

1): The collection of trajectories that do not have common points with the set M . That is, for a certain x_0 and $t \geq 0$, we have $x(t_0, x_0, t) \notin M$, the $t = +\infty$. From the fact that $x_0 \in \mathbb{D}$ and $V(x_0)$. Since $V(x)$ is positive definite and $D^+V(x) \leq -\alpha V(x) \leq 0$, we obtain that $V(x) \leq V(x_0)$ for $t = +\infty$. This implies that the conclusion is true.

2): The collection of trajectories that have common points with the set M . Assume that the sequences $\{t_k^{x_0}\}$, $k = 1, 2, \dots$. Assumed that $t_{k+1}^{x_0} > t_k^{x_0}$ and $t_k^{x_0} \rightarrow +\infty$ as $k \rightarrow +\infty$.

For $t \in [0, t_1^{x_0})$, and $D^+V(x) \leq -\alpha V(x)$. Then $V(t) \leq V(t_0)e^{-\alpha t}$ for $t \in [0, t_1^{x_0})$.

Combining with $\mu_1 \leq 1$ and (2), when $t = t_1^{x_0}$, then $V(t_1^{x_0+}) \leq \mu_1 V(t_1^{x_0}) \leq \mu_1 V(t_0^-)e^{-\alpha t_1^{x_0-}} \leq V(t_0)$.

By deduction method, then for $t \in [t_k^{x_0}, t_{k+1}^{x_0})$, from (2), and $V(t_k^{x_0+}) \leq \mu_k V(t_k^{x_0-}) \leq \mu_k V(t_{k-1}^{x_0-})e^{-\alpha t_k^{x_0-}} \leq V(x_0)$.

Then $V(t) \leq V(x_0)$ for $t \in [t_k^{x_0}, t_{k+1}^{x_0})$.

This completes the proof. That is the x remains in the open set $\mathbb{D}$. $\square$

Fig. 1. Bouncing ball dynamics(model).

Fig. 2. Bouncing ball dynamics(trjectory).

Theorem 2. Let $\mathbb{D}$ is a bounded domain in $\mathbb{R}^n$, there exists a positive function $V(x)$ satisfy the following conditions:

$$\begin{cases} D^+V(x) |_{(1)} \leq 0, & x \in \mathbb{D}, \\ V(x + S(x)) \leq V(x), & x \in M. \end{cases} \quad (3)$$

Then x of system (1) remains in the open set $\mathbb{D}$.

Remark 1. In Theorem 1 and 2, for a given initial value $x_0 \in \mathbb{D}$, the state of system (1) will remain in the set $\mathbb{D}$. The theory can be used to solve the bouncing ball system and the car collision problem [32,33]. These are typical state constrained hybrid systems. Such as the bouncing ball system:

Example 1. Let $x(t)$ be the displacement of the ball, $y(t)$ be the displacement of the oscillating platform, $e_1(t)$ is relative displacement, $e_2(t)$ is relative velocity, g is gravitational acceleration, m be the mass of the ball and much less than that of the platform. When no collision occurs, the ball is only affected by gravity or moves with the platform. Assume that all collisions are instantaneous.

The bouncing ball dynamics can be modeled by

$$\begin{cases} \dot{e}_1(t) = e_2(t), \\ \dot{e}_2(t) = -g - \ddot{y}(t), & (e_1(t), e_2(t)) \in \mathbb{D}; \\ e_2(t^+) = -\mu e_2(t^-), & (e_1(t), e_2(t)) \in M = \partial\mathbb{D}. \end{cases} \quad (4)$$

Where $\mathbb{D} = \{(e_1(t), e_2(t)) \in \mathbb{R}^2 | 0 \leq e_1(t) \leq h\}$, $M = \{(e_1(t), e_2(t)) \in \mathbb{R}^2 | e_1(t) = 0 \text{ or } h\}$. Consider a positive function $V(e(t)) = \frac{1}{2}e_2^2(t) + ge_1(t)$, when $(e_1(t), e_2(t)) \notin M$, the energy is decaying, then $\dot{V}(e(t)) |_{(4)} \leq 0$. When $(e_1(t), e_2(t)) \in M$, there has a collision and energy loss, that is $\mu \leq 1$, then $V(e(t^+)) = \frac{1}{2}e_2^2(t^+) + ge_1(t^+) = \frac{1}{2}\mu^2 e_2^2(t^-) + ge_1(t^-) \leq \frac{1}{2}2e_2^2(t^-) + ge_1(t^-) = V(e(t^-))$. From Theorem 2, we can get that the ball will moved steadily with the platform (see fig. 2).

Remark 2. The bounce ball system is simple and fundamental, naturally associated with physics and engineering problems. In fact, it has been used as a simplified model for vibrating granular materials [34], heart-beat model. Moreover, the bouncing ball model is relevant to periodic excitation, such as moored ships driven by steady ocean waves [32,33]. The bounce ball system belongs to a typical class state constrained impulsive nonlinear systems.

3 Control design for state constrained impulsive nonlinear dynamical networks

In this paper, we consider the following nonlinear dynamical networks with linearly couplings:

$$\dot{x}_i(t) = Ex_i(t) + Ag(x_i(t)) + c \sum_{j=1, j \neq i}^N b_{ij} \Gamma(x_j(t) - x_i(t)), \quad (5)$$

where $i = 1, 2, \dots, N$. $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \subset \mathbb{R}^n$ is the state vector of the i -th node. And the state is constrained in $\mathbb{R}^n$ inside a ellipsoid Θ . $g(x_i(t)) = (g_1(x_{i1}(t)), \dots, g_n(x_{in}(t)))^T$ and $g_j : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a smooth nonlinear continuous functions. It is defined by $\mathcal{L} = \{g_i(\cdot) | g_i \in C(\mathbb{R}^n, \mathbb{R}^n), \exists L_i > 0, 0 < sg_i(s) \leq L_i s^2, \forall s \in \mathbb{R}, i = 1, 2, \dots, n\}$. Let $L = \text{diag}\{L_1, L_2, \dots, L_n\}$. c is alterable coupling strength. $E, A \in \mathbb{R}^{n \times n}$. Γ is a positive definite diagonal matrix which describes the individual couplings between node i and j . $B = (b_{ij})_{n \times n}$ and b_{ij} is defined as follows: if there is a connection from node j to node i ($i \neq j$) then $b_{ij} > 0$; otherwise, $b_{ij} = 0$, and $b_{ii} = -\sum_{j=1, j \neq i}^N b_{ij}$. For dynamical systems (5), its initial conditions are given by $x_i(t_0) = \phi_i(t_0) \in \Theta$. In addition, too much cohesion and the system suffers all the ills of complexity, such as the impulsive influence. In this paper, we assume that the impulses are produced under certain environment and condition.

Then the system can be re-described by

$$\begin{cases} \dot{x}(t) = I_N \otimes Ex(t) + I_N \otimes Ag(x(t)) \\ + c(B \otimes \Gamma)x(t) + U(t), \quad x(t) \in \Theta, \\ \Delta x(t) = S(x(t)), \quad x(t) \in M \subset \partial(\mathbb{R}^{nN}/\Theta), \\ x(t_0) = \phi(t_0). \end{cases} \quad (6)$$

Where $U(t)$ is the control input. $S(x(t))$ denotes the change at each impulsive instant.

It is not difficult to see that there exists a family of ellipsoids and set a positive definite matrix R such that $\Theta := \{x | x^T R x \leq 1\}$. In this paper, design the controller

$$U(t) = \begin{cases} -Kx(t), & x(t) \in \Theta, \\ 0, & x(t) \in M \subset \partial\Theta. \end{cases}$$

where K is a gain matrix. Also, we need the following assumption and definition.

H1. Assume that there exists a constant $\mu \leq 1$ such that

$$(x(t) + S(x(t)))^T (x(t) + S(x(t))) \leq \mu x^T(t) x(t).$$

In order to better study its properties, we introduce the definition of barrier Lyapunov functions in the following.

Definition 1. [25] A BLF is a scalar function $V(x)$, defined with respect to the system $\dot{x} = f(x)$ on an open region $\mathbb{D}$ containing the origin, that is continuous, positive definite, has continuous first-order partial derivatives at every point of $\mathbb{D}$, has the property $V(x) \rightarrow \infty$ as x approaches the boundary of $\mathbb{D}$, and satisfies $V(x) \leq b$, $\forall t \geq 0$ along the solution $\dot{x} = f(x)$ for $x(0) \in \mathbb{D}$ and some positive constant b .

Remark 3. In fact, by definition, it is difficult to construct the appropriate barrier Lyapunov functions for the explicit model, especially for the impulsive model. In the following, the further analysis will be done for the state constrained impulsive systems.

Lemma 1. For any $\varepsilon > 0, a \in \mathbb{R}^n, b \in \mathbb{R}^n$, the inequality $2a^T b \leq \varepsilon a^T a + \varepsilon^{-1} b^T b$ holds.

In the following, we will present some stable criteria for systems (6) by different barrier Lyapunov functions.

Theorem 3. Under (H1), if there exists a matrix $0 < P \in \mathbb{R}^{n \times n}$ and some constants $\gamma, \varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$ such the following inequation holds:

$$\Omega = \begin{pmatrix} \Pi_{11} + \gamma I_N \otimes P & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{pmatrix} \leq 0. \quad (7)$$

Then, any state of the systems (6) start in the ellipsoidal set and remain in the set Θ , and converge to zero. Where $\Pi_{11} = 2I_N \otimes PE + \varepsilon_1 I_N \otimes PAA^T P - 2I_N \otimes PK + \varepsilon_1^{-1} L^2 I_{N \times n} + 2c(B \otimes P\Gamma) + \varepsilon_2^{-1} L^2 I_{N \times n} - \gamma I_N \otimes R$. $\Pi_{12} = I_N \otimes R - I_N \otimes P + I_N \otimes E^T R^T + c(B^T \otimes \Gamma R)$. $\Pi_{22} = -2I_N \otimes RK - 2I_N \otimes R + \varepsilon_2 I_N \otimes RAA^T R$.

Proof. Consider the following BLF candidate

$$V(x(t)) = \log\left(\frac{1}{1 - x^T(t) I_N \otimes R x(t)}\right).$$

In order to introduce the matrix P , which characterize the attractive ellipsoid, instead of substituting $\dot{x}$ as usual we use the descriptor method. Add the term

$$\begin{aligned} \Phi(x(t), \dot{x}(t)) &= \frac{1}{1 - x^T(t) I_N \otimes R x(t)} (2x^T(t) I_N \otimes P \\ &\quad + 2\dot{x}^T(t) I_N \otimes R) (I_N \otimes Ex(t) + I_N \otimes Ag(x(t)) \\ &\quad + c(B \otimes \Gamma)x(t) + U(t) - \dot{x}(t)). \end{aligned}$$

We select the control input as linear input feedback $U(t) = -Kx(t)$. When $x(t) \in \Theta$. The time derivative of $V(x)$ with respect to t using (6) is given by $\dot{V}(x(t))|_{(6)}$

$$\begin{aligned} &= \frac{2x^T(t) I_N \otimes R \dot{x}(t)}{1 - x^T(t) I_N \otimes R x(t)} \\ &= \frac{1}{1 - x^T(t) I_N \otimes R x(t)} [2x^T(t) I_N \otimes R \dot{x}(t) \\ &\quad + (2x^T(t) I_N \otimes P + 2\dot{x}^T(t) I_N \otimes R) (I_N \otimes Ex(t) \\ &\quad + I_N \otimes Ag(x(t)) + c(B \otimes \Gamma)x(t) + U(t) - \dot{x}(t))] \\ &= \frac{1}{1 - x^T(t) I_N \otimes R x(t)} [2x^T(t) I_N \otimes R \dot{x}(t) \\ &\quad + 2x^T(t) I_N \otimes P Ex(t) + 2x^T(t) I_N \otimes P Ag(x(t)) \\ &\quad + 2cx^T(t) (B \otimes P\Gamma)x(t) - 2x^T(t) I_N \otimes PKx(t) \\ &\quad - 2x^T(t) I_N \otimes P \dot{x}(t) + 2\dot{x}^T(t) I_N \otimes R Ex(t) \\ &\quad + 2\dot{x}^T(t) I_N \otimes R Ag(x(t)) + 2\dot{x}^T(t) (B \otimes R\Gamma)x(t) \\ &\quad - 2\dot{x}^T(t) I_N \otimes RKx(t) - 2\dot{x}^T(t) I_N \otimes R \dot{x}(t)] \\ &\leq \frac{1}{1 - x^T(t) I_N \otimes R x(t)} [2x^T(t) I_N \otimes R \dot{x}(t) \\ &\quad + 2x^T(t) I_N \otimes P Ex(t) + \varepsilon_1 x^T(t) I_N \otimes PAA^T P x(t) \\ &\quad + \varepsilon_1^{-1} g^T(x(t)) g(x(t)) + 2cx^T(t) (B \otimes P\Gamma)x(t) \\ &\quad - 2x^T(t) I_N \otimes PKx(t) - 2x^T(t) I_N \otimes P \dot{x}(t) \\ &\quad + 2\dot{x}^T(t) I_N \otimes R Ex(t) + \varepsilon_2 \dot{x}^T(t) I_N \otimes RAA^T R \dot{x}(t) \\ &\quad + \varepsilon_2^{-1} g^T(x(t)) g(x(t)) + 2\dot{x}^T(t) (B \otimes R\Gamma)x(t) \\ &\quad - 2\dot{x}^T(t) I_N \otimes RKx(t) - 2\dot{x}^T(t) I_N \otimes R \dot{x}(t)] \end{aligned}$$

Let

$$\begin{aligned} &2x^T(t) I_N \otimes R \dot{x}(t) + 2x^T(t) I_N \otimes P Ex(t) + \varepsilon_1 x^T(t) I_N \otimes \\ &PAA^T P x(t) + \varepsilon_1^{-1} L^2 x^T(t) x(t) + 2cx^T(t) (B \otimes P\Gamma)x(t) - \\ &2x^T(t) I_N \otimes PKx(t) - 2x^T(t) I_N \otimes P \dot{x}(t) + 2\dot{x}^T(t) I_N \otimes \\ &R Ex(t) + \varepsilon_2 \dot{x}^T(t) I_N \otimes RAA^T R \dot{x}(t) + \varepsilon_2^{-1} L^2 x^T(t) x(t) + \\ &2\dot{x}^T(t) (B \otimes R\Gamma)x(t) - 2\dot{x}^T(t) I_N \otimes RKx(t) - 2\dot{x}^T(t) I_N \otimes \\ &R \dot{x}(t) \leq -\gamma(1 - x^T(t) I_N \otimes R x(t)) \end{aligned}$$

holds. That is, we can obtain that $\dot{V}(x) \leq -\gamma$, if $\delta^T \Omega_0 \delta + \gamma \leq 0$, where

$$\Omega_0 = \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{pmatrix}. \quad (8)$$

Note $\delta = (x^T(t), \dot{x}^T(t))^T$. We have

$$\delta^T \Omega \delta + \gamma(1 - x^T(t)I_N \otimes Px(t)) \leq 0. \quad (9)$$

That is, if (7) holds, then the inequality (9) is implicit in the subset of $\mathbb{R}^n$ described by Θ , then $\dot{V}(x) \leq -\gamma < 0$ for $x(t) \notin M$.

When $x(t) \in M$, under (H1), we have $V(x(t_k^+))$

$$\begin{aligned} &= \log\left(\frac{1}{1 - x^T(t_k^+)I_N \otimes Rx(t_k^+)}\right) \\ &= \log\left(\frac{1}{1 - [x(t_k^-) + S(x(t_k^-))]^T I_N \otimes R[x(t_k^-) + S(x(t_k^-))]\right)} \\ &\leq \log\left(\frac{1}{1 - \mu x^T(t_k^-)I_N \otimes Rx(t_k^-)}\right) \\ &\leq \log\left(\frac{1}{1 - x^T(t_k^-)I_N \otimes Rx(t_k^-)}\right) \\ &= V(t_k^-). \end{aligned} \quad (10)$$

That is

$$\begin{cases} D^+V(x)|_{(6)} < 0, & x \in \Theta, \\ V(x(t_k^+)) \leq V(t_k^-), & x \in M \subset \partial\Theta. \end{cases}$$

Combining with Theorem 2, the solution of the system (4) will attracted to zero. We get the conclusions. This completes the proof. $\square$

Remark 4. In the existing literature [35–40], there were few researches on the state constrained impulsive systems by barrier Lyapunov functions. In Theorem 3, symmetric bounded region are considered by appropriate symmetric barrier Lyapunov functions. However, for the asymmetric bounded region, we need consider the other different scaling functions. In the following, we assume that $x_{11}(t) \in (-d_2, d_1)$, in fact, in Theorem 2, $d_1 = d_2$, the bound of state can be as a special supra-sphere $\|x(t)\| < \alpha$. For the corresponding, there exist some positive constants η_1, η_2 such that

$$x^T(t)Mx(t) \leq \eta_1, \quad \forall x_{11}(t) \in [0, d_1].$$

$$x^T(t)Mx(t) \leq \eta_2, \quad \forall x_{11}(t) \in [-d_2, 0].$$

Lemma 2. Let

$$V_2(z) = q(z) \log\left(\frac{a}{a - \|z\|}\right) + (1 - q(z)) \log\left(\frac{b}{b - \|z\|}\right),$$

then the Lyapunov function candidate V_2 is positive definite and $z \in (-b, a)$. Where a, b are some positive constants, and $q(z) = \begin{cases} 1, & \text{if } z > 0, \\ 0, & \text{if } z \leq 0. \end{cases}$

Proof. For the sake of analysis, we can rewrite $V_2(x)$ as

$$V_2(z) = \begin{cases} \log\left(\frac{a}{a - \|z\|}\right), & \text{if } 0 < z < a, \\ \log\left(\frac{b}{b - \|z\|}\right), & \text{if } -b < z \leq 0. \end{cases} \quad (11)$$

For $-b < z < a$, we have that $V_2(z) \geq 0$ and that $V_2(z) = 0$ if and only if $z = 0$, that is $V_2(z)$ is positive definite and is a BLF.

Additionally, $V_2(z)$ is piecewise smooth within each of the two intervals $z \in (-b, 0)$ and $z \in [0, a)$. Together with the fact that $\lim_{z \rightarrow 0^+} \frac{dV_2(z)}{dz} = \lim_{z \rightarrow 0^-} \frac{dV_2(z)}{dz} = 0$, we conclude that $z \in (-b, a)$. $\square$

Theorem 4. Under (H1), if there exists a matrix $0 < P \in \mathbb{R}^{n \times n}$ and some constants $\gamma, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in \mathbb{R}^+$ such the following inequations hold:

$$\Omega_1 = \begin{pmatrix} \tilde{H}_{11} + \gamma_1 I_N \otimes P & \tilde{H}_{12} \\ \tilde{H}_{12}^T & \tilde{H}_{22} \end{pmatrix} \leq 0, \quad (12)$$

$$\Omega_2 = \begin{pmatrix} \tilde{\tilde{H}}_{11} + \gamma_2 I_N \otimes P & \tilde{\tilde{H}}_{12} \\ \tilde{\tilde{H}}_{12}^T & \tilde{\tilde{H}}_{22} \end{pmatrix} \leq 0. \quad (13)$$

Then, any state of the systems (6) start in the ellipsoidal set and remain in the set, and converge to zero. Where $\tilde{H}_{11} = 2I_N \otimes PE + \varepsilon_1 I_N \otimes PAA^T P - 2I_N \otimes PK + \varepsilon_1^{-1} L^2 I_{N \times n} + 2c(B \otimes P\Gamma) + \varepsilon_2^{-1} L^2 I_{N \times n} - \gamma_1 I_N \otimes R$. $\tilde{H}_{12} = q(x_1)I_N \otimes R - I_N \otimes P + I_N \otimes E^T R^T + c(B^T \otimes \Gamma R)$. $\tilde{H}_{22} = -2I_N \otimes RK - 2I_N \otimes R + \varepsilon_2 I_N \otimes RAA^T R$. $\tilde{\tilde{H}}_{11} = 2I_N \otimes PE + \varepsilon_3 I_N \otimes PAA^T P - 2I_N \otimes PK + \varepsilon_3^{-1} L^2 I_{N \times n} + 2c(B \otimes P\Gamma) + \varepsilon_4^{-1} L^2 I_{N \times n} - \gamma_2 I_N \otimes R$. $\tilde{\tilde{H}}_{12} = (1 - q(x_1))I_N \otimes R - I_N \otimes P + I_N \otimes E^T R^T + c(B^T \otimes \Gamma R)$. $\tilde{\tilde{H}}_{22} = -2I_N \otimes RK - 2I_N \otimes R + \varepsilon_4 I_N \otimes RAA^T R$.

Proof. Consider the following BLF candidate

$$\begin{aligned} V(x) &= q(x) \log\left(\frac{\eta_1}{\eta_1 - x^T I_N \otimes Rx}\right) \\ &+ (1 - q(x)) \log\left(\frac{\eta_2}{\eta_2 - x^T I_N \otimes Rx}\right), \end{aligned}$$

where

$$q(x) = \begin{cases} 1, & \text{if } x_{11} > 0, \\ 0, & \text{if } x_{11} \leq 0. \end{cases} \quad (14)$$

For easy of analysis, we can rewrite $V(x)$ as

$$V(x) = \begin{cases} \log\left(\frac{\eta_1}{\eta_1 - x^T I_N \otimes Rx}\right), & \text{if } 0 < x_{11} < d_1, \\ \log\left(\frac{\eta_2}{\eta_2 - x^T I_N \otimes Rx}\right), & \text{if } -d_2 < x_{11} \leq 0. \end{cases} \quad (15)$$

In order to introduce the matrix P , which characterize the attractive ellipsoid, instead of substituting $\dot{x}$ as usual we use the descriptor method. Add the term

$$\begin{cases} \frac{1}{\eta_1 - x^T I_N \otimes Rx} (2x^T(t)I_N \otimes P + 2\dot{x}^T(t)I_N \otimes R)(I_N \otimes Ex(t) \\ + I_N \otimes Ag(x(t)) + c(B \otimes \Gamma)x(t) + U(t) - \dot{x}(t)), \\ \frac{1}{\eta_2 - x^T I_N \otimes Rx} (2x^T(t)I_N \otimes P + 2\dot{x}^T(t)I_N \otimes R)(I_N \otimes Ex(t) \\ + I_N \otimes Ag(x(t)) + c(B \otimes \Gamma)x(t) + U(t) - \dot{x}(t)). \end{cases}$$

When $x(t) \in \Theta$, the time derivative of $V(x)$ is given by $\dot{V}(x) \big|_{(6)}$

$$\begin{aligned}
&= q(x) \frac{2x^T I_N \otimes R\dot{x}}{\eta_1 - x^T I_N \otimes R\dot{x}} + (1 - q(x)) \frac{2x^T I_N \otimes R\dot{x}}{\eta_2 - x^T I_N \otimes R\dot{x}} \\
&= \frac{1}{\eta_1 - x^T I_N \otimes R\dot{x}} [2q(x)x^T(t)I_N \otimes R\dot{x}(t) \\
&\quad + (2\dot{x}^T(t)I_N \otimes P + 2\dot{x}^T(t)I_N \otimes R)(I_N \otimes Ex(t) \\
&\quad + I_N \otimes Ag(x(t)) + c(B \otimes \Gamma)x(t) + U(t) - \dot{x}(t))] \\
&\quad + \frac{1}{\eta_2 - x^T I_N \otimes R\dot{x}} [2(1 - q(x))x^T(t)I_N \otimes R\dot{x}(t) \\
&\quad + (2\dot{x}^T(t)I_N \otimes P + 2\dot{x}^T(t)I_N \otimes R)(I_N \otimes Ex(t) \\
&\quad + I_N \otimes Ag(x(t)) + c(B \otimes \Gamma)x(t) + U(t) - \dot{x}(t))] \\
&\leq \frac{1}{\eta_1 - x^T I_N \otimes R\dot{x}} [2q(x)x^T(t)I_N \otimes R\dot{x}(t) \\
&\quad + 2x^T(t)I_N \otimes PEx(t) + \varepsilon_1 x^T(t)I_N \otimes PAA^T Px(t) \\
&\quad + \varepsilon_1^{-1} g^T(x(t))g(x(t)) + 2cx^T(t)(B \otimes P\Gamma)x(t) \\
&\quad - 2x^T(t)I_N \otimes PKx(t) - 2x^T(t)I_N \otimes P\dot{x}(t) \\
&\quad + 2\dot{x}^T(t)I_N \otimes REx(t) + \varepsilon_2 \dot{x}^T(t)I_N \otimes RAA^T Rx(t) \\
&\quad + \varepsilon_2^{-1} g^T(x(t))g(x(t)) + 2c\dot{x}^T(t)(B \otimes R\Gamma)x(t) \\
&\quad - 2\dot{x}^T(t)I_N \otimes RKx(t) - 2\dot{x}^T(t)I_N \otimes R\dot{x}(t)] \\
&\quad + \frac{1}{\eta_2 - x^T I_N \otimes R\dot{x}} [2(1 - q(x))x^T(t)I_N \otimes R\dot{x}(t) \\
&\quad + 2x^T(t)I_N \otimes PEx(t) + \varepsilon_3 x^T(t)I_N \otimes PAA^T Px(t) \\
&\quad + \varepsilon_3^{-1} g^T(x(t))g(x(t)) + 2cx^T(t)(B \otimes P\Gamma)x(t) \\
&\quad - 2x^T(t)I_N \otimes PKx(t) - 2x^T(t)I_N \otimes P\dot{x}(t) \\
&\quad + 2\dot{x}^T(t)I_N \otimes REx(t) + \varepsilon_4 \dot{x}^T(t)I_N \otimes \\
&\quad RAA^T Rx(t) + \varepsilon_4^{-1} g^T(x(t))g(x(t)) \\
&\quad + 2c\dot{x}^T(t)(B \otimes R\Gamma)x(t) - 2\dot{x}^T(t)I_N \otimes RKx(t) \\
&\quad - 2\dot{x}^T(t)I_N \otimes R\dot{x}(t)]
\end{aligned}$$

Let

$2q(x)x^T(t)I_N \otimes R\dot{x}(t) + 2x^T(t)I_N \otimes PEx(t) + \varepsilon_1 x^T(t)I_N \otimes PAA^T Px(t) + \varepsilon_1^{-1} g^T(x(t))g(x(t)) + 2cx^T(t)(B \otimes P\Gamma)x(t) - 2x^T(t)I_N \otimes PKx(t) - 2x^T(t)I_N \otimes P\dot{x}(t) + 2\dot{x}^T(t)I_N \otimes REx(t) + \varepsilon_2 \dot{x}^T(t)I_N \otimes RAA^T Rx(t) + \varepsilon_2^{-1} g^T(x(t))g(x(t)) + 2c\dot{x}^T(t)(B \otimes R\Gamma)x(t) - 2\dot{x}^T(t)I_N \otimes RKx(t) - 2\dot{x}^T(t)I_N \otimes R\dot{x}(t) \leq -\gamma_1(\eta_1 - x^T(t)I_N \otimes R\dot{x}(t))$ and $2(1 - q(x))x^T(t)I_N \otimes R\dot{x}(t) + 2x^T(t)I_N \otimes PEx(t) + \varepsilon_3 x^T(t)I_N \otimes PAA^T Px(t) + \varepsilon_3^{-1} g^T(x(t))g(x(t)) + 2cx^T(t)(B \otimes P\Gamma)x(t) - 2x^T(t)I_N \otimes PKx(t) - 2x^T(t)I_N \otimes P\dot{x}(t) + 2\dot{x}^T(t)I_N \otimes REx(t) + \varepsilon_4 \dot{x}^T(t)I_N \otimes RAA^T Rx(t) + \varepsilon_4^{-1} g^T(x(t))g(x(t)) + 2c\dot{x}^T(t)(B \otimes R\Gamma)x(t) - 2\dot{x}^T(t)I_N \otimes RKx(t) - 2\dot{x}^T(t)I_N \otimes R\dot{x}(t) \leq -\gamma_2(\eta_2 - x^T(t)I_N \otimes R\dot{x}(t))$ hold. That is, we can obtain that $\dot{V}(x) \leq -\gamma_1\eta_1 - \gamma_2\eta_2$, if $\delta^T \tilde{\Omega}_1 \delta + \delta^T \tilde{\Omega}_2 \delta + \gamma_1\eta_1 + \gamma_2\eta_2 \leq 0$, where

$$\tilde{\Omega}_1 = \begin{pmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} \\ \tilde{\Pi}_{12}^T & \tilde{\Pi}_{22} \end{pmatrix}, \quad \tilde{\Omega}_2 = \begin{pmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} \\ \tilde{\Pi}_{12}^T & \tilde{\Pi}_{22} \end{pmatrix}. \quad (16)$$

Note $\delta = (x^T(t), \dot{x}^T(t))^T$, we have

$$\begin{aligned}
&\delta^T \Omega_1 \delta + \delta^T \Omega_2 \delta + \gamma_1(\eta_1 - x^T(t)I_N \otimes Px(t)) \\
&\quad + \gamma_2(\eta_2 - x^T(t)I_N \otimes Px(t)) \leq 0.
\end{aligned}$$

That is, if (12) and (13) hold, then the above inequality is implicit in the subset of $\mathbb{R}^n$ described by Θ , then $\dot{V}(x) \leq -\gamma < 0$ for $x(t) \notin M$.

When $x(t) \in M$, under (H1), we have $V(x(t_k^+))$

$$\begin{aligned}
&= q(x(t_k^+)) \log\left(\frac{\eta_1}{\eta_1 - x^T(t_k^+)I_N \otimes Rx(t_k^+)}\right) \\
&\quad + (1 - q(x(t_k^+))) \log\left(\frac{\eta_2}{\eta_2 - x^T(t_k^+)I_N \otimes Rx(t_k^+)}\right) \\
&= q(x(t_k^+)) \times \\
&\quad \log\left(\frac{\eta_1}{\eta_1 - [x(t_k^-) + S(x(t_k^-))]^T I_N \otimes R[x(t_k^-) + S(x(t_k^-))]} \right) \\
&\quad + (1 - q(x(t_k^+))) \times \\
&\quad \log\left(\frac{\eta_2}{\eta_2 - [x(t_k^-) + S(x(t_k^-))]^T I_N \otimes R[x(t_k^-) + S(x(t_k^-))]} \right) \\
&\leq q(x(t_k^+)) \log\left(\frac{\eta_1}{\eta_1 - \mu x^T(t_k^-)I_N \otimes Rx(t_k^-)}\right) \\
&\quad + (1 - q(x(t_k^+))) \log\left(\frac{\eta_2}{\eta_2 - \mu x^T(t_k^-)I_N \otimes Rx(t_k^-)}\right) \\
&\leq q(x(t_k^+)) \log\left(\frac{\eta_1}{\eta_1 - x^T(t_k^-)I_N \otimes Rx(t_k^-)}\right) \\
&\quad + (1 - q(x(t_k^+))) \log\left(\frac{\eta_2}{\eta_2 - x^T(t_k^-)I_N \otimes Rx(t_k^-)}\right) \\
&= V(t_k^-).
\end{aligned}$$

This completes the proof. $\square$

Remark 5. Compared with traditional Lyapunov function [41–43], it is difficult to calculate theoretically for barrier Lyapunov function. For the asymmetric problems, this is a tough one to solve. Theorem 3 and 4 provide a reference in theoretical result and analyzed method for the constrained systems. In the future, we continue to find a better way to consider it.

Remark 6. In this paper, we focus on the state constraint systems with impulsive input effects. The general assumption for impulsive input is provided for a potentially large control effort is key to safeguarding against any constraint transgression. Nevertheless, the trajectory of system remains bounded for all time. By careful selection of control parameters, we can limit the control signal within a desirable operating range or stabilize the state of systems.

4 Examples

In this section, we will provide a numerical example to illustrate the effectiveness of the proposed criteria in this paper.

Example 1. Consider the following nonlinear dynamic networks with 3-nodes

$$\begin{cases} \dot{x}(t) = I_N \otimes Ex(t) + I_N \otimes Ag(x(t)) \\ + c(B \otimes \Gamma)x(t) + U(t), \quad x(t) \in \mathbb{D}, \\ \Delta x(t) = S(x(t)), \quad x(t) \in M \subset \partial\mathbb{D}, \\ x(t_0) = \phi(t_0). \end{cases} \quad (17)$$

Where

$$x(t) = (x_1(t), x_2(t), x_3(t))^T \in \mathbb{D},$$

$$\mathbb{D} = \{x_{ij}(t) \in \mathbb{R} \mid |x_{ij}(t)| < 1.2, \quad i = 1, 2, 3, \quad j = 1, 2\},$$

$$x_i(t) = (x_{i1}(t), x_{i2}(t))^T,$$

$$E = \begin{pmatrix} 1.1 & 0.2 \\ 0.1 & 1.3 \end{pmatrix}, \quad A = \begin{pmatrix} 1.5 & 1.1 \\ 0.9 & 1.4 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

$$M = \{x_{ij}(t) \in \mathbb{R} \mid |x_{ij}(t)| = 1.2, \quad i = 1, 2, 3, \quad j = 1, 2\},$$

$$g(s) = \frac{|s+1| + |s-1|}{2},$$

$$\Delta x(t) = -0.65x_1(t).$$

Here, the initial condition of systems (17) is chosen as: $x(0) = (1.05; 1.07; 1.15; -1.15; -0.9; -0.78)^T$. Fig. 3-5 show the state trajectories for system (17) without control input. From Fig. 3, we can see that when $x_{11}(t) = 1.2$, it will be triggered the impulsive mechanism. From Fig. 4, when $x_{21}(t) = -1.2$ or $x_{22}(t) = -1.2$, system (17) will be triggered the impulsive mechanism. The shape of Fig. 5 changed depending on Fig. 3 and 4. In conclusion, the state trajectories for systems (17) never going out of a certain range.

Design $U(t) = Kx(t) = -5.5x(t)$. Choose $\varepsilon_1 = 0.95$, $\varepsilon_2 = 0.95$, $R = 2I$. By numerical count software MATLAB LMI Toolbox, we can get that

$$P = \begin{pmatrix} 1.9479 & 0.7683 \\ 0.7683 & 1.8925 \end{pmatrix}.$$

So the conditions of Theorem 3 are all satisfied and dynamic systems (17) is stable. Fig. 6 shows the time response of states of (17) with controller.

Fig. 3. Transient response of state variable $x(t)$ of system (17) without controller.

Fig. 4. Transient response of state variable $x(t)$ of system (17) without controller.

Fig. 5. Transient response of state variable $x(t)$ of system (17) without controller.

Fig. 6. Transient response of state variable $x(t)$ of system (17) with controller.

5 Conclusions

This paper consider the problems of state constrained nonlinear systems via attractive ellipsoid method, auxiliary matrix and barrier Lyapunov method, some stability criteria have been derived. In addition, the usefulness of the proposed results has been demonstrated by one numerical example. In our future work, designing a state constrained impulsive control law for a class of delayed dynamic systems by using the proposed methodology (or other barrier Lyapunov functions) will be considered. In conclusion, there is always something more challenging for the problem of state constrained system.

Acknowledgements This study was funded by Natural Science Foundation Project of Anhui, and in part by the University Natural Science Research Project of Anhui Province under Grant KJ2020ZD008, in part by the National Natural Science Foundation of China under Grant 61873213, and in part by the Natural Science Foundation Project of Chongqing under grants Nos. cstc2018jcyjAX0588.

Data availability statement

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest

The authors declare that they have no conflict of interest.

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