New 3-D Combined Inversion Scheme Using Response Functions Free From Galvanic Distortion

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2 **Galvanic Distortion**

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8

9 **Abstract**

10 The combined inversion using distortion-free response functions is an effective
11 approach to robustly estimate the 3-D electrical resistivity structure against the
12 distortions caused by near-surface resistivity anomalies. However, previous combined
13 inversion analyses have presented a significant dependency of the inversion results on
14 initial and prior models. Therefore, in this study, we evaluated the effectiveness of the
15 following two new types of 3-D combined inversion using distortion-free response
16 functions: one uses the phase tensor and the vertical and inter-station horizontal
17 magnetic transfer functions, while the other uses the Network-MT response functions,
18 in addition to the former. Because long dipoles are used, the Network-MT response
19 function is negligibly affected by galvanic distortion. To access the combined inversion
20 approach, we developed a novel 3-D inversion scheme combining the response
21 functions of the usual magnetotelluric measurements and the Network-MT response
22 function. The synthetic inversion analysis demonstrated that both of the proposed
23 combined inversions can recover the characteristic resistivity distributions of the target
24 model without a significant dependence on the initial models, at least in the shallow

part. These results demonstrate that the combined inversions using only distortion-free response functions have the potential to estimate subsurface resistivity more robustly than what was previously thought. Furthermore, we confirmed that the combined inversion using the Network-MT response function can make the resultant resistivity structure closer to the actual one and enhance the stability of the inversion. This result suggests that the combined use of the Network-MT response function is the preferred approach.

Keywords

magnetotelluric inversion, Network-MT method, galvanic distortion, finite element method, phase tensor, magnetic transfer function, tetrahedral mesh

1. Introduction

The three-dimensional inversion of magnetotelluric (MT) data has become common in recent decades. This is because of the increase in computational resources and the widespread use of practical 3-D MT inversion codes, such as WSINV3DMT

(Siripunvaraporn et al. 2005; Siripunvaraporn and Egbert 2009) and ModEM (Egbert and Kelbert 2012; Kelbert et al. 2014). However, the galvanic distortion of the MT impedance tensor has long been recognized as a major problem in estimating subsurface electrical resistivity structures. Galvanic distortion, which is caused by lateral small-scale inhomogeneities near the Earth's surface, distorts the electrical field and, hence, the estimated impedance tensor (e.g., Ogawa 2002; Simpson and Bahr 2005).

One effective approach to robustly estimate the subsurface electrical resistivity structure against the galvanic distortion is to use the response functions free from the galvanic distortion as datasets in the MT inversion (e.g., Patro et al. 2013; Tietze et al. 2015; Campanyà et al. 2016). Patro et al. (2013) developed a 3-D inversion scheme using the phase tensor (PT) (Caldwell et al. 2004) as the input data. They demonstrated that the PT inversion can provide the proper electrical resistivity structure by performing synthetic inversion tests. Later, Tietze et al. (2015) developed a 3-D combined inversion using PT and the vertical magnetic transfer function (VMTF) and demonstrated that the integration of VMTF improves the recovery of the actual resistivity structure. In

addition, Campanyà et al. (2016) assessed the sensitivity of the impedance tensor, VMTF, and the inter-station horizontal magnetic transfer function (HMTF) to subsurface resistivity anomalies by performing synthetic inversions with different combinations of response functions. Campanyà et al. (2016) demonstrated that the combined use of VMTF and HMTF with the impedance tensor recovers the resistivity structure more accurately than using the impedance tensor alone. However, no study has yet examined the combined inversion of PT, VMTF, and HMTF.

The previous studies using distortion-free response functions presented a deficiency in robustness. Patro et al. (2013) and Tietze et al. (2015) showed that the resultant electrical resistivity structure of the inversion using either or both PT and VMTF depends significantly on the initial and prior models. They concluded that a sensible selection of the prior and initial models is essential for recovering a reliable resistivity structure. Campanyà et al. (2016) demonstrated that the inversion only using VMTF and HMTF failed to recover the subsurface resistivity variation in the vertical direction and the absolute resistivity values. They suggested that the inversion using either VMTF or

HMTF is reliable only when additional information defining the background resistivity structure is provided. As discussed in the previous studies, the dependency on the prior and initial models should be caused by the low sensitivity of PT, VMTF, and HMTF to the absolute values of the subsurface electrical resistivity.

One possible technique to overcome the problem by adding information regarding the absolute values of the subsurface resistivity structure for inversion is the use of the response function of the Network-MT (NMT) measurement (Uyeshima 2007), where metallic telephone line networks are used to measure the electrical potential difference between two distant electrodes (Uyeshima 2007). Because dipoles longer than several kilometers are used in the NMT measurement, the response functions of the NMT measurement (NMTRF) are negligibly affected by the galvanic distortion (Uyeshima et al. 2001; Uyeshima 2007). In addition, NMTRF is significantly sensitive to the absolute values of the subsurface resistivity structure, like the impedance tensor. Thus, it is expected that the combined inversion with NMTRF allows the estimation of the electrical resistivity structure with the correct scale. Recently, Usui et al. (2021)

developed a 2-D combined inversion method using the usual MT response functions (apparent resistivity and phase) and NMTRF. Usui et al. (2021) confirmed that the combined inversion provides the correct resistivity structure despite the response functions of the usual MT stations being significantly affected by galvanic distortion.

In addition, the combined use of NMTRF is advantageous because it enhances the sensitivity to deep structures. Long dipole measurements of the NMT method make the signal-to-noise ratio of the observed electrical field higher than that of the standard MT measurement. In addition, well-maintained telephone lines enable us to perform long-term observations from several months to several years (Kinoshita et al. 1989; Uyeshima et al. 1989), allowing us to increase the signal-to-noise ratio by stacking the data. Consequently, we can estimate NMTRF with small errors, including periods longer than several thousand seconds, which assists in estimating a reliable electrical resistivity down to the upper mantle. Siripunvaraporn et al. (2004) developed a 3-D inversion scheme using NMTRF. However, no one has proposed a 3-D inversion scheme that combines the response functions of the standard MT measurements and

105 NMTRF.

106

107 In this study, we assessed the effectiveness of two new combined 3-D inversions using
108 response functions free from galvanic distortion, one of which is the combined
109 inversion of the PT, VMTF, and HMTF. All the response functions can be estimated
110 from the electromagnetic field observed by standard MT measurements. The second
111 inversion combines the PT, VMTF, HMTF, and NMTRF. To assess the second
112 combined inversion, we developed a novel 3-D inversion scheme that uses both
113 NMTRF and the response function of the standard MT measurements because no such
114 3-D inversion scheme exists. We performed synthetic inversions utilizing the same
115 model used in a previous study (Tietze et al. 2015) and compared the inversion results.
116 In the following sections, we first present the algorithm of the combined inversion
117 scheme used in this study, including the treatments of NMTRF in the inversion. Next,
118 we describe the synthetic inversion tests to evaluate the performance of the combined
119 inversion approaches and discuss the results of the combined inversions.

120

2. Algorithm of the combined inversion scheme

We used the inversion scheme based on the 3-D MT inversion code of Usui (2015) and Usui et al. (2017). This section summarizes the basic inversion algorithm and the treatments of NMTRF in the forward calculation and inversion. The finite element method with a tetrahedral mesh is used to obtain the electromagnetic field of the computational domain. The governing equation is the vector Helmholtz equation for the electric field with $e^{-i\omega t}$ time dependence.

$$\nabla \times \nabla \times \mathbf{E} = i\omega\mu_0\sigma\mathbf{E} \quad (1)$$

In (1), $\mathbf{E} \in \mathbb{C}^3$ denotes the electric field, ω is the angular frequency, μ_0 is the magnetic permeability of free space, and σ is the electrical conductivity. The computational domain Ω indicates a rectangular geometry, and all the boundary planes $\partial\Omega$ of Ω are perpendicular to the orthogonal coordinate axes. The Dirichlet boundary condition is set on the entire boundary, $\partial\Omega$. On the top of Ω , a uniform source electric field is specified, while the tangential electric field is designed to be zero at the bottom of Ω , which corresponds to the condition that the perfect conductor locates below the bottom. The electric field calculated by the 2-D forward calculation is provided on the

136 two sides parallel to the source field. The 2-D forward problems (TM mode) are solved
 137 by the Galerkin FEM using the triangular edge-based element with the lowest order (Jin
 138 2002). On the other hand, the tangential electric field is forced to be zero on the sides
 139 perpendicular to the source field. The boundary value problem is solved by the Galerkin
 140 FEM (Jin 2002) using the edge-based tetrahedral element. Within each element, the
 141 electric field can be approximated as follows:

$$\mathbf{E}^e = \sum_{k=1}^6 \mathbf{N}_i^e E_i^e, \quad (2)$$

142 where $\mathbf{N}_i^e \in \mathbb{R}^3$ and $E_i^e \in \mathbb{C}$ denote the vector basis function (Jin 2002) and the
 143 tangential component of the electric field on the i -th edge of the element, respectively.
 144 As shown in Usui (2015), we can obtain a linear equation for each element by using the
 145 Galerkin method as follows:

$$\mathbf{K}^e \mathbf{x}^e = \mathbf{b}^e, \quad (3)$$

146 where $\mathbf{x}^e = (E_1^e, \dots, E_6^e)^T \in \mathbb{C}^6$ is the solution vector, $\mathbf{b}^e \in \mathbb{C}^6$ denotes the right-
 147 hand-side vector including the boundary condition, and the i -th row and j -th column of
 148 the coefficient matrix $\mathbf{K}^e \in \mathbb{C}^{6 \times 6}$ is expressed as follows:

$$\iiint_{V^e} [(\nabla \times \mathbf{N}_i^e) \cdot (\nabla \times \mathbf{N}_j^e) - i\omega\mu_0\sigma \mathbf{N}_i^e \cdot \mathbf{N}_j^e] dV. \quad (4)$$

149 In Equation (4), V^e denotes the volume occupied by the e -th element. By assembling
150 all the element equations, we can obtain the following linear equation.

$$\mathbf{K}\mathbf{x} = \mathbf{b} \quad (5)$$

151 The dimension of (5) is the number of unknown tangential components of the electric
152 field at the element edges, and the coefficient matrix $\mathbf{K}$ is a symmetrical sparse
153 complex matrix. MKL PARDISO is used to solve the linear equation. To calculate the
154 response functions as indicated below, forward computation is performed twice for each
155 frequency by appending a source field in the x-direction (E_x -polarization) and y-
156 direction (E_y -polarization). After obtaining the electric field by solving the linear
157 equation above, the magnetic field $\mathbf{H}$ is calculated from the electric field using
158 Faraday's law.

$$\nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H} \quad (6)$$

159 From the results of the E_x -polarization and E_y -polarization, we can calculate the
160 frequency response function at each observation station. VMTF $\mathbf{T}_v \in \mathbb{C}^2$ is calculated
161 as follows:

$$\mathbf{T}_v = \begin{pmatrix} T_{zx} \\ T_{zy} \end{pmatrix} = \begin{pmatrix} H_x^{E_x-pol} & H_y^{E_x-pol} \\ H_x^{E_y-pol} & H_y^{E_y-pol} \end{pmatrix}^{-1} \begin{pmatrix} H_z^{E_x-pol} \\ H_z^{E_y-pol} \end{pmatrix}. \quad (7)$$

162 Meanwhile, HMTF $\mathbf{T}_h \in \mathbb{C}^{2 \times 2}$ is calculated from the horizontal magnetic field

163 components at an observation station and those at a reference station as follows:

$$\mathbf{T}_h = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} = \begin{pmatrix} H_x^{E_x-pol} & H_x^{E_y-pol} \\ H_y^{E_x-pol} & H_y^{E_y-pol} \end{pmatrix} \begin{pmatrix} H_{r,x}^{E_x-pol} & H_{r,x}^{E_y-pol} \\ H_{r,y}^{E_x-pol} & H_{r,y}^{E_y-pol} \end{pmatrix}^{-1}, \quad (8)$$

164 where the characters with subscript r denote the magnetic field components at the

165 reference station. Further, PT $\Phi \in \mathbb{R}^{2 \times 2}$ is calculated from the real and imaginary parts

166 of the impedance tensor as follows:

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} = \begin{pmatrix} \text{Re}(Z_{xx}) & \text{Re}(Z_{xy}) \\ \text{Re}(Z_{yx}) & \text{Re}(Z_{yy}) \end{pmatrix}^{-1} \begin{pmatrix} \text{Im}(Z_{xx}) & \text{Im}(Z_{xy}) \\ \text{Im}(Z_{yx}) & \text{Im}(Z_{yy}) \end{pmatrix}, \quad (9)$$

167 where Z_{xx} , Z_{xy} , Z_{yx} , and Z_{yy} are the components of the impedance tensor $\mathbf{Z} \in \mathbb{C}^{2 \times 2}$, which

168 is computed as follows:

$$\begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} = \begin{pmatrix} E_x^{E_x-pol} & E_x^{E_y-pol} \\ E_y^{E_x-pol} & E_y^{E_y-pol} \end{pmatrix} \begin{pmatrix} H_x^{E_x-pol} & H_x^{E_y-pol} \\ H_y^{E_x-pol} & H_y^{E_y-pol} \end{pmatrix}^{-1}. \quad (10)$$

169 NMTRF $\mathbf{Y} = (Y_x, Y_y)^T \in \mathbb{C}^2$ is defined as the frequency response function between the

170 voltage difference $V \in \mathbb{C}$ for the dipole of an NMT station and the horizontal magnetic

171 field components at a reference station (Uyeshima et al. 2001).

$$V = Y_x H_x + Y_y H_y \quad (11)$$

172 In the developed combined inversion scheme, voltage differences were computed using
 173 an algorithm similar to that proposed by Siripunvaraporn et al. (2004). Because long
 174 dipoles of the NMT method generally extend over multiple finite elements of the
 175 computational mesh, dipoles are divided into segments by element edges (Figure 1).
 176 Each segment is located within an element surface, which is triangular. The voltage
 177 difference along a dipole can be obtained by summing the voltage differences of the
 178 small segments. According to Faraday's law with the $e^{-i\omega t}$ time dependence, the voltage
 179 difference $dV \in \mathbb{C}$ along each segment (illustrated as a red dashed line in Figure 1) can
 180 be calculated as follows:

$$dV = -E_1 L_1 - E_2 L_2 - i\omega\mu_0 H_n S, \quad (12)$$

181 where E_1 and E_2 are the electric field components along the triangle edges, L_1 and L_2
 182 are the lengths of the edges, i is the imaginary unit, H_n is the magnetic field
 183 component normal to the triangle, and S is the area of the triangle. The magnetic field
 184 component normal to triangle H_n is calculated from the electric field components of the
 185 element edges on the Earth's surface using Faraday's law (Equation (6)). Based on the

186 voltage differences and the horizontal magnetic field of the two polarizations, NMTRF

187 can be calculated as follows:

$$\mathbf{Y} = \begin{pmatrix} Y_x \\ Y_y \end{pmatrix} = \begin{pmatrix} H_x^{E_x-pol} & H_y^{E_x-pol} \\ H_x^{E_y-pol} & H_y^{E_y-pol} \end{pmatrix}^{-1} \begin{pmatrix} V^{E_x-pol} \\ V^{E_y-pol} \end{pmatrix} = \mathbf{H}^{-1} \mathbf{V}. \quad (13)$$

188 In the combined inversion, we find the model parameters (subsurface electrical

189 resistivities) that minimize the objective function $\phi(\mathbf{m}) \in \mathbb{R}$:

$$\phi(\mathbf{m}) = \|\mathbf{W}\mathbf{d} - \mathbf{W}\mathbf{F}(\mathbf{m})\|^2 + \alpha^2 \|\mathbf{R}\mathbf{m}\|^2, \quad (14)$$

190 where $\mathbf{W}$ is the diagonal matrix of the diagonals that are reciprocals of the errors in the

191 observed data; $\mathbf{d}$ and $\mathbf{F}(\mathbf{m})$ are the real vectors containing the observed and calculated

192 response functions, respectively; and $\mathbf{R}$ denotes the roughening matrix. Vectors $\mathbf{d}$ and

193 $\mathbf{F}(\mathbf{m})$ contain the real and imaginary parts of the complex frequency response functions

194 separately. The roughening matrix provides the differences in the log-resistivities of the

195 adjacent parameter cells $\mathbf{r}$ as follows:

$$\mathbf{R}\mathbf{m} = \mathbf{r} = (r_1 \quad \cdots \quad r_i \quad \cdots \quad r_M)^T \quad (15)$$

$$r_i = F_i m_i - \sum_{j=1}^{F_i} m_{j,i}, \quad (16)$$

196 where M is the number of model parameters; F_i and m_i are the number of faces and

197 the log-resistivity of the i -th parameter cell, respectively; and $m_{j,i}$ is the log-resistivity

198 of the parameter cell adjacent to the i -th parameter cell through its j -th face. It should
199 be noted that the objective function (Equation (14)) does not include the prior model,
200 unlike the previous inversion schemes that use the response functions free from
201 galvanic distortion (Patro et al. 2013; Tietze et al. 2015; Campanyà et al. 2016).
202 Therefore, our inversion scheme can estimate subsurface electrical resistivity without
203 dependency on the prior model. After linearizing Equation (14), the model parameters
204 are calculated by the data-space Gauss–Newton method, as described in Usui et al.
205 (2017).
206
207 The Gauss–Newton method requires a sensitivity matrix (Jacobian matrix), which
208 comprises the partial derivatives of the response functions with respect to the model
209 parameters. The sensitivity matrix is computed using the reciprocity property of the
210 electromagnetic field (Rodi 1976). Because the voltage difference and the horizontal
211 magnetic field used for calculating NMTRF can be expressed as linear combinations of
212 the electric field along the element edges, which are the unknowns in Equation (5), the
213 sensitivity matrix for NMTRF can be calculated using the reciprocity approach (Rodi

1976). While Siripunvaraporn et al. (2004) used only the derivatives of the voltage differences for calculating the sensitivity matrix, we consider the derivatives of the reference horizontal magnetic field components in addition to the derivatives of the voltage differences. Thus, the derivatives of NMTRF can be expressed as follows:

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{m}} = \frac{\partial \mathbf{H}^{-1}}{\partial \mathbf{m}} \mathbf{V} + \mathbf{H}^{-1} \frac{\partial \mathbf{V}}{\partial \mathbf{m}}. \quad (17)$$

3. Synthetic inversion analysis and discussion

We performed inversions using two different datasets. The first dataset contains PT, VMTF, and HMTF, whereas the second dataset contains PT, VMTF, HMTF, and NMTRF. Hereon, we refer to the former and latter cases as Comb-A and Comb-B, respectively. We performed a synthetic inversion analysis to assess whether the two new types of the combined inversion approaches can reproduce the appropriate resistivity structure robustly. The target model used in this study was the oblique conductor (OC) model used by Tietze et al. (2015). However, we exchanged the x- and y-axes to have the strike of the regional 2-D structure of the model parallel to the x-axis. PT and VMTF were calculated for the 100 observation stations with the exact locations used by

229 Tietze et al. (2015). The HMTF was also calculated for the same observation stations,
230 except for the bottom-right point in Figures 2 and 3 (the station at $x = -18$ km and $y = 18$
231 km). We used the bottom-right point as the reference station to calculate the HMTF. In
232 addition, we calculated NMTRF at the 36 NMT observation stations (Figure 3). The
233 NMT observation stations are represented as broken lines with circular ends, as shown
234 in Figure 3. The dipole length of each NMT station is 5.66 km. We used the horizontal
235 magnetic field at the bottom-right point as the reference magnetic field to calculate
236 NMTRF in the same manner as the HMTF. We calculated PT, VMTF, and HMTF for 16
237 logarithmically spaced periods from 0.01 to 1,000 s, as in Tietze et al. (2015).
238 Meanwhile, we calculated NMTRF for ten logarithmically spaced periods from 10 to
239 10,000 s because it is usually difficult to obtain the response functions at short periods
240 at NMT stations. Because NMTRF can be estimated with small errors at periods longer
241 than several thousand seconds, as indicated in the Introduction section, we set the
242 maximum period for NMTRF to be longer than the other response functions. Because it
243 is difficult to incorporate a thin 0.1 km near-surface layer ($100 \Omega\text{m}$) of the OC model
244 using a tetrahedral mesh, which requires a huge number of elements to represent such a

thin layer, we used a forward program using hexahedral mesh (Usui 2020; Usui et al. 2020) to calculate the synthetic response functions. The algorithm for the forward calculation using the hexahedral mesh is the same as that described in Section 2. By using different meshes, we can also avoid the so-called “inverse crime.” Before calculating PT from the impedance tensor, we provided a galvanic distortion to the calculated impedance tensor by multiplying a distortion tensor because the impedance tensor observed in the real world is generally distorted. The distortion tensors were calculated by randomly generated values of the twist angle (within $\pm 30^\circ$), shear angle (within $\pm 30^\circ$), anisotropy (within ± 0.5), and the gains (from 0.1 to 10.0 in the logarithmic axis) of the Groom–Bailey decomposition (Groom and Bailey 1989).

To generate synthetic data, we added Gaussian noise with a mean of zero to the response functions. The standard deviation of the noise for VMTF was 0.02, as used in Tietze et al. (2015). Similarly, the standard deviation of the noise for HMTF was 0.02. The standard deviations for NMTRF were 3% of their absolute values; that is, $0.03|Y_x|$ or $0.03|Y_y|$. For PT, we added Gaussian noise to the distorted impedance tensor and

261 then calculated the PT errors by error propagation, as indicated in Patro et al. (2013).
262 The standard deviation of the noise to the impedance tensor was 3% for each
263 component, along with a floor of 3% of $|Z_{xy}Z_{yx}|^{1/2}$ for diagonals, following Tietze et
264 al. (2015). We started the combined inversion from three different uniform half-space
265 models (10, 100, and 1,000 Ωm) to investigate the dependency of the inversion results
266 on the initial model. In all cases, the tradeoff parameter α was set to 10.
267
268 The resultant resistivity structures of Comb-A are shown in Figure 2. The estimated
269 resistivity structures have characteristic resistivity anomalies in the OC model.
270 Specifically, both the resistivity contrast along the y-direction and the oblique conductor
271 could be adequately imaged. However, the estimated resistivity values tended to be
272 larger than the actual values of the OC model. It should be noted that, despite using
273 significantly different initial models, no significant differences were observed in the
274 resultant resistivity structure down to 15 km below ground. Tietze et al. (2015)
275 performed a combined inversion of PT and VMTF with different prior and initial
276 models between 10 Ωm and 1000 Ωm (the prior model was identical to the initial model

in each case). Although the target model (i.e., OC model) was the same as that used in this study, Tietze et al. (2015) indicated a more significant dependency of the resultant resistivity structure on the prior and initial models, which is inconsistent with our results. Even if we performed the combined inversion of PT and VMTF, just like Tietze et al. (2015), the resultant resistivity structure down to 15 km below ground presented no significant dependency on the initial models as the Comb-A results (Figure S1 in Supporting Information). Therefore, this inconsistency was not attributed to the use of HMTF. Because the inversion scheme of this study does not use the prior model, unlike Tietze et al. (2015), the robustness of the results of our combined inversions is caused by not using the prior model. Other studies using PT and/or the magnetic transfer functions demonstrated a deficiency of robustness owing to the low sensitivity to the absolute values of the resistivity structure (Patro et al. 2013; Tietze et al. 2015; Campanyà et al. 2016). However, the results of our synthetic inversions suggest that not using the prior model can increase the robustness of the inversion using PT and the magnetic transfer functions.

293 Figure 3 presents the resultant resistivity structures of Comb-B down to 15 km below
294 ground. Similar to Comb-A (Figure 2), only a slight difference is observed among the
295 resultant resistivity structures starting from the different initial models. Characteristic
296 resistivity anomalies of the OC model can also be adequately recovered by Comb-B.
297 Upon comparing the results of Comb-A (Figure 2) and Comb-B (Figure 3), Comb-B
298 recovered the resistivity values at depths of 5 km and 15 km closer to the actual values
299 of the OC model. It is considered that the additional use of NMTRF increased the
300 sensitivity to the absolute values of the resistivity structure, allowing us to obtain a
301 more reliable resistivity structure.

302

303 Figure 4 illustrates the electrical resistivity variation down to a depth of 100 km at the
304 two points of ($x=10$ km, $y=-10$ km) and ($x=-10$ km, $y=10$ km). The locations of the two
305 points are symmetrical about the axis of the oblique conductor. The former point is
306 located at the resistive side, whereas the latter is located at the conductive side.

307 Although both Comb-A and Comb-B were unable to recover sharp resistivity changes at
308 a depth of 70 km, this was inevitable owing to the smoothing effect due to the

309 stabilizing term of the objective function (Rodi and Macki 2012). At depths greater than
310 15 km, apparent differences are observed between the resultant resistivity structures of
311 Comb-A and Comb-B. Figure 4 demonstrates that the resistivity values obtained by
312 Comb-A tend to be larger than the actual values of the OC model at the depths down to
313 100 km. Furthermore, the overestimation of the resistivity becomes more significant
314 with an increase in the initial resistivity value. This overestimation demonstrates that the
315 results of the combined inversion using PT and the magnetic transfer function depend
316 on the initial model, especially in the deep parts, where the sensitivity of the data is
317 relatively low. Compared to the Comb-A results, the resistivity values obtained by
318 Comb-B are closer to the actual values. Although the dependency on the initial model
319 can also be found in the Comb-B results, no significant bias exists from the actual
320 model, unlike the results of Comb-A. Especially at depths greater than 70 km, the
321 resistivity values of Comb-B converge to the actual value (100 Ωm) without depending
322 on the initial model. It is believed that the use of NMTRF increased the robustness
323 against the initial models and enabled us to estimate a more accurate resistivity structure
324 because NMTRF has a significant sensitivity to the absolute values of the subsurface

325 resistivity as the impedance tensor.

326

327 In addition, we found notable differences in the stabilization of the inversion between

328 Comb-A and Comb-B (Figure 5). Figure 5 compares the variations in the objective

329 functions during the combined inversions. At the 0-th iteration, the objective functions

330 of Comb-B varied according to the initial model, while those of Comb-A presented

331 similar values without a dependency on the initial model. Because the objective

332 function at the 0-th iteration was calculated from the initial model, the variation of the

333 objective function at the 0-th iteration reflects the high sensitivity of NMTRF to the

334 absolute values of the subsurface resistivity structure. In the Comb-B inversions, the

335 objective functions decreased rapidly and remained unchanged up to the 7-th iteration.

336 Meanwhile, in Comb-A, the convergence of the inversion significantly depended on the

337 initial models. While the objective functions remained unchanged after the 5-th iteration

338 when the initial resistivity was 10 Ωm , it required up to the 16-th iteration to converge

339 when we used the initial model of 1,000 Ωm . When starting from the same initial

340 model, Comb-A required more iterations than Case B. The relatively low sensitivity to

the absolute values of the subsurface resistivity apparently led to the instability in Comb-A. Figure 5 demonstrates that the addition of NMTRF can improve the inversion stability.

Because both Comb-A and Comb-B were able to recover the characteristic features of the target resistivity structure, we can conclude that both combined inversion approaches are effective in estimating the subsurface resistivity structure. This study indicates that combined inversions using distortion-free response functions have the potential to estimate subsurface resistivity more robustly than what was previously thought. However, an overestimation bias in the resultant resistivity structure and poor convergence of the Gauss–Newton iteration was found in the results of Comb-A. The low sensitivity to the absolute values of the subsurface resistivity is thought to be the cause of these problems. Because both issues were improved by Comb-B, we confirmed that the combined inversion using NMTRF can enhance the reliability of the resultant resistivity structure and the stability of the inversion. Therefore, we believe that the combined use of NMTRF is the preferred approach for robustly estimating the

357 resistivity structure.

358

359 **4. Summary**

360 To evaluate the effectiveness of the two new types of 3-D combined inversion using

361 response functions free from galvanic distortion, we performed synthetic inversion

362 analyses using a newly developed 3-D combined inversion scheme; this can include

363 NMTRF as the datasets in addition to the response functions of the usual NMT

364 measurements. By comparing the estimated resistivity structures and histories of the

365 objective functions, we obtained the following results:

366 1. Despite using only PT, VMTF, and HMTF, the combined inversion recovered the

367 characteristic resistivity anomalies of the target model in the shallow part without a

368 significant dependence on the initial models. This suggests that the combined

369 inversion using only PT and the magnetic transfer functions has the potential to

370 recover the subsurface resistivity structure more robustly than what was previously

371 thought if the prior model is not used.

372 2. When we used NMTRF in addition to PT and the magnetic transfer functions, the

resultant resistivity structure became closer to the actual model. Therefore, we confirmed that the additional use of NMTRF enabled us to estimate a more accurate resistivity structure.

3. The objective functions stably decreased when NMTRF is included as data. In contrast, the convergence of the inversion was worse and depending significantly on the initial model when we used only PT, VMTF, and HMTF. This difference demonstrates that the use of NMTRF can improve the stability of the combined inversion.

Because PT, VMTF, and HMTF can be calculated from the observed data of the MT measurements, we can apply the combined inversion proposed in this study to existing data. In addition, in many areas of Japan, NMT measurements have been conducted in the same survey area as usual MT measurements. This enables us to perform the inversion by combining NMTRF with other distortion-free response functions. The aforementioned results suggest that the combined inversion using only the response function free of galvanic distortion has the potential to robustly recover the target resistivity structure, especially by using NMTRF. Therefore, we expect that the

developed combined inversion approach will help in robustly recovering resistivity structures by avoiding the distortions caused by near-surface anomalies.

Declarations

Ethics approval and consent to participate

Not applicable

Consent for publication

Not applicable

List of abbreviations

MT, magnetotelluric; NMT, Network-MT; PT, phase tensor; VMTF, vertical magnetic transfer function; HMTF, inter-station horizontal magnetic transfer function; NMTRF, response functions of the NMT measurement; RMS, root mean square

Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

405 **Competing interests**

406 The authors declare that they have no competing interests.

407 **Funding**

408 Not applicable

409 **Authors' contributions**

410 YU developed a combined inversion scheme and performed synthetic
411 inversion tests. MU is the inventor of the combined inversion of the
412 phase tensor, magnetic transfer functions, and Network-MT response
413 function. All authors read and approved the final manuscript.

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415 In this study, we used the computer systems of the Earthquake and
416 Volcano Information Center of the Earthquake Research Institute, the
417 University of Tokyo. We illustrated a few figures using generic mapping
418 tools (Wessel et al. 2013).

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495

496 **Figure legends:**

497 Figure 1. Schematic view illustrating how to calculate the voltage difference along each
498 dipole of the NMT observation station. Here, E_1 and E_2 are the electric field
499 components along the edges of the triangle, L_1 and L_2 are the lengths of the edges, i is
500 the imaginary unit, μ_0 is the magnetic permeability of vacuum, H_n is the magnetic
501 field component normal to the triangle, and S is the area of the triangle. The solid lines
502 indicate the element edges, while a thick dash line denotes a dipole of the NMT
503 observation station. The black-filled circle denotes the endpoints of the dipole. The
504 voltage difference along each dipole is computed by summing the voltage differences
505 dV of the small segments.

506

507 Figure 2. Electrical resistivity structures obtained by Comb-A. (a) True resistivity
508 structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined
509 inversion with three different initial models. The lowermost panels are the vertical
510 cross-sections of the profile along the major axis of the oblique conductor, which is
511 shown as a white line in the upper-left panel. The inverted triangles indicate the

512 locations of observation stations. We used the bottom-right point (the station at $x = -18$
513 km and $y = 18$ km) as the reference station for calculating HMTF.

514

515 Figure 3. Electrical resistivity structures obtained by Comb-B. (a) True resistivity
516 structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined
517 inversion with three different initial models. The lowermost panels indicate the vertical
518 cross-sections of the profile along the major axis of the oblique conductor, which is
519 shown as a white line in the upper-left panel. The inverted triangles indicate the
520 locations of observation stations of PT, VMTF, and HMTF. The broken lines with
521 circular ends indicate the dipoles of the NMT stations. We used the bottom-right point
522 (the station at $x = -18$ km and $y = 18$ km) as the reference station for calculating HMTF
523 and NMTRF.

524

525 Figure 4. Electrical resistivity variation with a depth at (a) $(x, y) = (10 \text{ km}, -10 \text{ km})$ and
526 (b) $(x, y) = (-10 \text{ km}, 10 \text{ km})$. The thick black lines indicate the resistivity variation of
527 the true resistivity structure. The broken colored and solid-colored lines indicate the

528 resistivity variations of the resistivity structure obtained by Comb-A and Comb-B,

529 respectively.

530

531 Figure 5. Objective function versus the iteration number for combined inversions with

532 different initial models. The broken and solid lines indicate the objective function

533 changes during Comb-A and Comb-B inversions.

Figures

Figure 1

Schematic view illustrating how to calculate the voltage difference along each dipole of the NMT observation station. Here, E_1 and E_2 are the electric field components along the edges of the triangle, L_1 and L_2 are the lengths of the edges, i is the imaginary unit, μ_0 is the magnetic permeability of vacuum, H_n is the magnetic field component normal to the triangle, and S is the area of the triangle. The solid lines indicate the element edges, while a thick dash line denotes a dipole of the NMT observation station. The black-filled circle denotes the endpoints of the dipole. The voltage difference along each dipole is computed by summing the voltage differences dV of the small segments.

Figure 2

Electrical resistivity structures obtained by Comb-A. (a) True resistivity structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined inversion with three different initial models. The lowermost panels are the vertical cross-sections of the profile along the major axis of the oblique conductor, which is shown as a white line in the upper-left panel. The inverted triangles indicate the locations of observation stations. We used the bottom-right point (the station at $x = -18$ km and $y = 18$ km) as the reference station for calculating HMTF.

Figure 3

Electrical resistivity structures obtained by Comb-B. (a) True resistivity structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined inversion with three different initial models. The lowermost panels indicate the vertical cross-sections of the profile along the major axis of the oblique conductor, which is shown as a white line in the upper-left panel. The inverted triangles indicate the locations of observation stations of PT, VMTF, and HMTF. The broken lines with circular ends indicate the dipoles of the NMT stations. We used the bottom-right point (the station at $x = -18$ km and $y = 18$ km) as the reference station for calculating HMTF and NMTRF.

(a) $x = 10$ km, $y = -10$ km

(b) $x = -10$ km, $y = 10$ km

Figure 4

Electrical resistivity variation with a depth at (a) $(x, y) = (10 \text{ km}, -10 \text{ km})$ and (b) $(x, y) = (-10 \text{ km}, 10 \text{ km})$. The thick black lines indicate the resistivity variation of the true resistivity structure. The broken colored and solid-colored lines indicate the resistivity variations of the resistivity structure obtained by Comb-A and Comb-B, respectively.

Figure 5

Objective function versus the iteration number for combined inversions with different initial models. The broken and solid lines indicate the objective function changes during Comb-A and Comb-B inversions.

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