Algebra dissociates from arithmetic in the brain semantic network

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Research

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Abstract

Background

Mathematical expressions mainly include arithmetic (such as $8 - (1 + 3)$) and algebraic expressions (such as $a - (b + c)$). Previous studies shown that both algebraic processing and arithmetic involved the bilateral parietal brain regions. Although behavioral and neuropsychological studies have revealed the dissociation between algebra and arithmetic, how algebraic processing is dissociated from arithmetic in brain networks is still unclear.

Methods

Using functional magnetic resonance imaging (fMRI), this study scanned 30 undergraduates and directly compared the brain activation during algebra and arithmetic. Brain activations, single-trial (item-wise) interindividual correlation and mean-trial interindividual correlation related to algebra processing were compared with those related to arithmetic.

Results

Brain activation analyses showed that algebra elicited greater activation in the angular gyrus and arithmetic elicited greater activation in the bilateral supplementary motor area, left insula, and left inferior parietal lobule. Interindividual single-trial brain-behavior correlation revealed significant brain-behavior correlations in the semantic network, including the middle temporal gyri, inferior frontal gyri, dorsomedial prefrontal cortices, and left angular gyrus, for algebra. For arithmetic, the significant brain-behavior correlations were located in the phonological network, including the precentral gyrus and supplementary motor area, and in the visuospatial network, including the bilateral superior parietal lobules.

Conclusion

These findings suggest that algebra relies on the semantic network and arithmetic relies on the phonological and visuospatial networks.

Background

Mathematical expressions are frequently conceived and represented in arithmetic and algebraic expressions. Arithmetic is concretely represented as the combination of digits and operators (such as $8 - (1 + 3)$). Contrastingly, algebra includes abstract operations with letters and operators (such as $a - (b + c)$) and has more abstract representation than arithmetic in form [1]. Although behavioral and neuropsychological studies have reported that algebra can be dissociated from arithmetic [2-8], the neural

mechanisms underlying this dissociation remain unclear. The current study used fMRI to examine how algebraic processing dissociates from arithmetic processing in brain networks.

Arithmetic in the brain

In the arithmetic expression " $4 \times 5 - 9$," the sub-expression " 4×5 " is recursively combined with "9" by subtraction. It has been proposed that the arithmetic operations both involved numerical quantity calculation and order-rule of arithmetic operations [9]. Neuroimaging and neuropsychological studies have revealed that arithmetic processing is subserved by the visuospatial network, which typically includes the intraparietal sulcus (IPS) and inferior and superior parietal lobules [10-15]. Several studies have shown that both arithmetic and number processing are mediated by common regions in the parietal lobules [14, 16-18]. Approximately 73% of the behavioral variance of visuospatial magnitude processing has been attributed to the change in cortical thickness in the right superior parietal cortex, and approximately 55% of the behavioral variance of children's arithmetic abilities has been attributed to the change in cortical folding in the right IPS [19]. Patients with injuries in the parietal cortex typically experience arithmetic impairment [20, 21] and impaired visuospatial performance [22].

During arithmetic computation, the phonological system also supports the coding of visual or auditory numbers, as well as the temporary storage of intermediate results [23, 24]. Therefore, arithmetic processing also relies on the phonological network, which typically includes the bilateral supplementary motor areas and the precentral gyrus [15, 16, 25-28]. For example, Zhou et al. [28] reported that both addition and multiplication are supported by a broad neural system involving the supplementary motor areas, precentral gyrus, IPS, and middle frontal gyrus. One recent study showed that exact arithmetic involves more phonological or verbal processing localized in the left Rolandic operculum, precentral gyrus, and supplementary motor area than approximate arithmetic [26]. Neuropsychological investigations have also shown that patients with infarction in the left frontal lobe (including the precentral gyrus) have difficulty retrieving arithmetic facts [29, 30].

Algebra in the brain

Algebraic expressions, such as $a^2 + b^2 = c^2$, have a nested hierarchical structure of arithmetic procedural knowledge [9, 31]. Previous imaging studies have revealed that regions in the parietal lobes respond to both algebra and arithmetic. The brain organization of algebra is dissociated from language processing [32-34], which is similar to the dissociation between basic numerical processing and language processing [12, 13, 35, 36]. Specifically, algebraic processing typically recruits the bilateral parietal brain regions around the IPS to represent numerical magnitude, whereas processing the syntax of language elicits activation in the classic language areas including the left inferior frontal gyrus and the posterior and anterior middle temporal gyrus [34].

However, behavioral [2, 6, 7] and neuropsychological [3-5, 8] studies further revealed the dissociation of algebraic abilities and arithmetic processing. For example, preschool children can complete three-term arithmetic inversion problems (such as $3 + 2 - 2$) but do not understand the inversion principle of algebra ($a + b - b$) (Rasmussen et al., 2003). One neuropsychological study reported that a patient with Fahr's disease with both basal ganglia and frontal lobe brain damage was able to solve most simple subtraction and addition problems but could not understand algebraic concepts (such as $a + b = b + a$) [8]. These findings imply that there may be dissociated brain neural bases between algebra and arithmetic.

Algebraic expressions also involves language symbolic and represents abstract mathematical cognitions, which requires the application of conceptual and semantic knowledge [12, 32, 37, 38]. For example, Geary et al. [38] found that algebraic cognition was related to semantic memory systems of addition facts in ninth graders. Neuroimaging studies further implied that the brain network's response to algebra is also associated with language semantic processing. Recent studies have shown that compared to simple numerical processing and arithmetical computation, mathematical semantic knowledge (e.g., arithmetic principles and algebraic terms) elicited greater activation in the typically semantic network such as left angular gyrus, left middle temporal gyrus, and left inferior frontal gyrus [12, 39]. These regions typically overlap with the semantic network, including the posterior inferior parietal lobule (angular gyrus), middle temporal gyrus, inferior frontal gyrus, dorsomedial prefrontal cortex, fusiform and parahippocampal gyri, ventromedial prefrontal cortex, and posterior cingulate gyrus [40].

The present study

Although behavioral [2, 6, 7] and neuropsychological [3-5, 8] studies have reported that algebra is dissociated from arithmetic, how algebraic processing dissociated from arithmetic processing in brain networks is unclear. This study uses fMRI to identify the specific neural bases for algebraic processing and arithmetic processing. The arithmetic and algebra tasks used in this study were matched in surface form and difficulty. In the present study, algebra used the operational principles for addition, subtraction, multiplication, and division (such as $a-(b+c)$). To match the algebraic tasks, we used tasks requiring arithmetic procedures (such as $8-(1+3)$) rather than tasks requiring the simple retrieval of arithmetic facts.

Prior studies have indicated that arithmetic involves numerical quantity operations and phonological processing [10-15]. Contrastingly, the application of semantic knowledge is involved in algebraic processing [12]. Therefore, we hypothesized that algebra relies more on the semantic network and arithmetic involves the phonological and visuospatial networks based on the aforementioned reports. To test this hypothesis, we compared the fMRI data regarding brain activation for algebra with that for arithmetic, and we conducted a single-trial (item-wise) interindividual correlation analysis and a traditional mean-trial interindividual correlation analysis to examine the brain-behavior correlations [26, 41].

Methods

Participants

Thirty right-handed undergraduate students (15 males and 15 females; mean age = 21.92 years, range: 18–25 years) were recruited to participate in this study from Beijing Normal University, China. They majored in a wide range of disciplines except for mathematics. These participants reported having no history of neurological disorders or head injuries. The experiment was fully explained to the participants before informed consent was obtained. This study was approved by the Institutional Review Board of the Institute of Cognitive Neuroscience and Learning at Beijing Normal University.

Materials

The algebraic task included the operational principles for addition, subtraction, multiplication, and division acquired by students in primary school. The arithmetic task included the numerical calculations for addition, subtraction, multiplication, and division. Each problem included two expressions, and the average length of the problems was controlled (Fig. 1). All expressions were presented in white against a black background (with a red, green, and blue value of 0, 0, 0). The participants were given 16 algebra and 16 arithmetic problems to complete.

The stimulus presentation and behavioral data recordings were programmed using E-prime software (Version 1.1, Psychology Software Tools, Inc., www.pstnet.com) on a Pentium 4 laptop. Stimuli were projected onto a translucent screen placed at the back of the magnet bore. Participants viewed the screen through a mirror mounted on the head coil at a distance of 30 cm from the eyes.

Procedure

Prior to scanning, the participants underwent a training session with the same type of materials as a formal experiment to ensure that they understood the instructions of this experiment. After that, participants were required to complete the experimental tasks in the scanner.

The scanning session was organized into two runs, each lasting 192 s. Each run consisted of four experimental blocks (two experimental blocks for each task) and four baseline blocks (Fig. 1). The balanced Latin square design [42] was used to counterbalance the order effect of the two experimental tasks. Each block with four trials lasted for 24 s, and the presentation order of trials was random. There was a 1-min rest period after each run.

For each trial in the experimental blocks, two expressions were presented synchronously on the screen, one at the top and one at the bottom. Participants were asked to judge whether the two expressions were equal. For each trial in the baseline blocks, two arrows were presented synchronously on the screen, one at the top and one at the bottom. Participants were asked to judge whether the two arrows pointed in the

same direction. Each trial lasted 6 s, and a fixation cross was displayed to fill the remaining time if the participant did not use all 6 s. Half of the participants responded to the trials by pressing a key on a response box on their left with their left index finger when the two mathematical expressions or arrows were equal. The remaining participants responded to the trials by pressing a key on a response box on their right with their right index finger. Both accuracy and speed were emphasized.

fMRI data acquisition

Images were obtained using a Siemens (Munich, Germany) 3T Trio MRI scanner using a standard eight-channel head coil. After automatic shimming of the magnetic field, three-dimensional high-resolution T1 anatomical images were acquired for co-registration with the functional images. Next, functional volumes were acquired using a multiple slice T2*-weighted echo planar imaging (EPI) sequence with the following parameters: in-plane resolution = $3.125 \times 3.125 \text{ mm}^2$; repetition time = 2000 ms; echo time = 30 ms; flip angle = 90° ; field of view = $240 \times 240 \text{ mm}^2$; matrix dimensions = 64×64 ; field of view = 200 mm; and slice thickness = 4 mm. The entire brain was imaged in 32 slices.

Statistical analysis of the fMRI data

Individual MRI datasets were analyzed using SPM12 software (Wellcome Department of Imaging Neurosciences, University College London, UK; <http://www.fil.ion.ucl.ac.uk/spm>). All volumes were realigned to the first volume and spatially normalized to a common value to correct for whole brain differences over time. Images were then smoothed using an isotropic Gaussian kernel of 4 mm and high-pass filtered at a cut-off of 128 s.

Brain activation analysis. After preprocessing, parameter-estimated images for individual participants across the whole brain were calculated. Group analyses with random effects were conducted using the one-way analysis of variance (ANOVA) on the brain activation maps of all participants with material type as the independent variable. Then, the brain activation for each type of material relative to fixation was calculated. Brain activations for the two types of materials were compared. We used the thresholds from the lenient $p < .05$ to $p < .005$, to the stringent $p < .001$, uncorrected, with a minimum cluster size of 20 voxels.

Brain-behavior correlation analysis. A single-trial (item-wise) interindividual correlation analysis that has been reported in previous fMRI and event-related potential (ERP) studies was used to examine the brain-behavior correlations (Li et al., 2020; Zhou et al., 2018). First, the correlation between the brain activation maps and the reaction times (RTs) was determined for each trial. Then, a one-sample t -test on the correlation coefficients obtained for all trials of one type of processing against zero was performed. The analysis was performed separately for algebra and arithmetic tasks. The single-trial correlation has been reported as more effective than the traditional mean-trial correlation as it can filter out much of the noise that exists after the first step [41].

The traditional mean-trial interindividual correlation was also used, and the results were compared with those of the single-trial correlation. The mean brain activation map and the mean RT for each type of processing for each participant were determined. Then, a correlation analysis between the mean brain activation map and the RT for all participants was performed. We used the thresholds for two brain-behavior correlation analyses from the lenient $p < .05$ to $p < .005$, to the stringent $p < .001$, uncorrected, with a minimum cluster size of 20 voxels.

Further, we defined seven functional regions of interest (ROIs) based on results from previous studies for the semantic, phonological, and visuospatial networks. Based on the meta-analysis of 120 functional neuroimaging studies of semantic processing (Binder et al., 2009), we defined four ROIs of the semantic network in the left hemisphere that included the middle temporal gyrus (Montreal Neurological Institute (MNI) coordinates $[-45, -21, -15]$), inferior frontal gyrus (MNI coordinates $[-51, 24, 0]$), dorsomedial prefrontal cortex (MNI coordinates $[-6, -48, 30]$), and angular gyrus (MNI coordinates $[-57, -45, 30]$). Based on the fMRI studies of phonological processing for arithmetic (Zhou et al., 2007), we defined two ROIs of the phonological network in the left hemisphere that included the precentral gyrus (MNI coordinates $[-45, 4, 30]$) and supplementary motor area (MNI coordinates $[-3, 3, 58]$). Based on the fMRI studies of visuospatial processing for arithmetic [43], we defined one ROI of the visuospatial network in the left superior parietal lobule (MNI coordinates $[-32, -68, 56]$). Each ROI was a sphere with a radius of 9 mm. These ROIs were used to compare the levels of brain-behavior correlations between algebra and arithmetic.

Results

Behavioral results

The mean RT was 2653 ms for algebraic task and 2855 ms for arithmetic task. The mean accuracy was 87.6% and 86.8%, respectively. RTs and accuracy were analyzed with a repeated measure ANOVA. The main effect of stimulus type was not significant for RTs ($F(1, 29) = 2.55, p = .116, \eta^2 = .042$) or accuracy ($F(1, 29) = 1.10, p = .754, \eta^2 = .002$).

Brain activations

The brain activations for algebra and arithmetic relative to baseline are displayed in Fig. 2. Algebra and arithmetic elicited significant activation in a broad array of brain regions, including the bilateral parietal, frontal, and occipital gyri. The differences in brain activation between algebra and arithmetic are shown in Fig. 2 and Table 1. Algebra relative to arithmetic had greater activation in the bilateral angular gyrus. In contrast, arithmetic elicited greater activation in the bilateral supplementary motor areas, left insula, and left inferior parietal lobule than algebra.

Brain-behavior correlations

The single-trial brain-behavior correlations for algebra and arithmetic are displayed in Fig. 3 and Table 2. Significant positive brain-behavior correlations ($p < .001$, uncorrected) were observed in the bilateral middle temporal gyri, bilateral inferior frontal gyri, bilateral dorsomedial prefrontal cortices, and left angular gyrus for algebra. Significant negative brain-behavior correlations ($p < .001$, uncorrected) were observed in the bilateral superior parietal lobules for algebra. For arithmetic, significant positive brain-behavior correlations ($p < .001$, uncorrected) were observed in the right inferior temporal gyrus and negative brain-behavior correlations ($p < .001$, uncorrected) were observed in the bilateral superior parietal lobules, bilateral middle occipital gyri, bilateral precentral gyri, left supplementary motor area, left insula, and bilateral cerebella. Fig. 3 shows the number of voxels of the single-trial brain-behavior correlations for algebra and arithmetic in the ROIs used in this study. Algebra elicited a larger number of voxels in four ROIs of the semantic network, including the middle temporal gyrus, inferior frontal gyrus, dorsomedial prefrontal cortex, and angular gyrus. Arithmetic elicited a greater brain-behavior correlation and a larger number of voxels in three ROIs of the phonological and visuospatial networks, including the precentral gyrus, supplementary motor area, and superior parietal lobule. The differences between algebra and arithmetic can be found based on the single-trial brain-behavior correlation approach under the threshold $p < .001$, uncorrected.

The traditional mean-trial brain-behavior correlations for algebra and arithmetic are displayed in Fig. 4. Significant positive brain-behavior correlations ($p < .05$, uncorrected) were observed in the bilateral middle temporal gyri and bilateral dorsomedial prefrontal cortices, and negative brain-behavior correlations ($p < .05$, uncorrected) were observed in the bilateral superior parietal lobules for algebra. Significant positive brain-behavior correlations ($p < .05$, uncorrected) were observed in the bilateral inferior temporal gyri, and negative brain-behavior correlations ($p < .05$, uncorrected) were observed in the bilateral superior parietal lobules, bilateral middle occipital gyri, bilateral precentral gyri, and bilateral cerebella for arithmetic. Fig. 4 shows the number of voxels of the mean-trial brain-behavior correlations for algebra and arithmetic for each ROI, which identifies weak differences between algebra and arithmetic under the threshold $p < .05$, uncorrected.

Discussion

The present study aimed to clarify the neural dissociation between algebra and arithmetic. Between-group brain activation analyses revealed that algebra resulted in greater activation in the bilateral angular gyrus, and arithmetic elicited greater activations in the bilateral supplementary motor area, left insula, and left inferior parietal lobule. Interindividual single-trial brain-behavior correlation analyses showed significantly positive brain-behavior correlations in the semantic network, including the bilateral middle temporal gyri, bilateral inferior frontal gyri, bilateral dorsomedial prefrontal cortices, and left angular gyrus for algebra. For arithmetic, the significantly negative brain-behavior correlations were observed in the phonological network, including the bilateral precentral gyrus and left supplementary motor area, and in the visuospatial network, including the bilateral superior parietal lobules. These findings suggest that algebra relies more on the semantic network and arithmetic relies more on the phonological and visuospatial networks.

Semantic network in algebra

The current study found that algebra relies more on the semantic network, including the bilateral angular gyrus, bilateral middle temporal gyri, bilateral inferior frontal gyri, bilateral dorsomedial prefrontal cortices, and left angular gyrus, than arithmetic. These regions overlap with the semantic network according to the meta-analysis of functional neuroimaging studies by Binder et al. that was based on word processing [40]. In algebra, formula derivations of the relationship between the letter and the operator (such as the commutative or associative laws) require knowledge of mathematical principles. Thus, algebra is similar to mathematical principles. Previous studies have shown that the mathematical principle processing based on conceptual knowledge is subserved by a semantic network [12, 39, 44, 45]. Liu et al. [45] reported that mathematical principles (e.g., exchanging the position of two operands in addition does not change their sum) elicited greater activation of the semantic network in the left middle temporal gyrus, left inferior frontal gyrus, and left angular gyrus than arithmetic computation (e.g., when the number 8 is first divided by 4 and then multiplied by 3, the final result is 6). The important role of the semantic network in mathematical principles suggests an important role of conceptual knowledge in algebra.

Furthermore, algebra requires abstract conceptual knowledge, and arithmetic involves concrete conceptual knowledge. Recent studies have suggested that concepts are organized in the brain according to semantic categories [46] such as animate versus inanimate [47] or concrete versus abstract [48] and that the classical semantic network is important for abstract conceptual knowledge [49-51]. A meta-analysis of studies regarding abstract concepts revealed that abstract concepts tend to produce stronger activation in the semantic network than concrete concepts [51]. By contrast, previous studies emphasized that abstract algebra concepts induced the non-linguistic cortical network dissociated from the classical semantic network [32, 33]. The subjective representation of the concreteness of algebra by expert mathematicians relied more on mental imagery, which differs from nonmathematical concepts in these studies [32, 33]. However, for laymen, algebra relies on the classical semantic network, suggesting a close association between algebra and language processing.

Phonological and visuospatial networks in arithmetic

This study found that arithmetic relied more on the phonological network, including the bilateral precentral gyri and supplementary motor areas, than algebra. Matching tasks of arithmetic associated with the retrieval of arithmetic facts are thought to be stored as verbal codes [11] and are related to cognitive and neural processes involved in phonological processing. These results are in line with models of arithmetic operations that outline a phonological processing network [15, 26, 52].

Our results also indicate that arithmetic results in greater activations in the visuospatial networks including the bilateral superior parietal lobules, which is consistent with the previously reported role of the visuospatial network in arithmetic [10-15]. These regions are responsible for visuospatial information

from the symbolic form of the arithmetic expression and reflect the visuospatial processing of concrete symbolic and quantitative representations in arithmetic. Numbers can elicit a visuospatial response, as the quantity of a number can be represented on a mental number line from left to right. This processing is evidenced by the spatial numerical association of response codes effect, which describes that people respond faster to small numbers with the left hand and large numbers with the right hand [53, 54]. Arithmetic can be completed using the visuospatial processing of a dynamic mental number line, as addition leads to a right shift and subtraction leads to a left shift in visuospatial processing [55-57].

Significance of the single-trial brain-behavior correlation

Neuroimaging researchers often use interindividual brain-behavior correlations to explore the associations between the brain and human behaviors [58-60]. The correlation analysis investigates how interindividual differences in brain functions relate to interindividual behavioral performances and can help provide an accurate and representative neural mechanism of cognitive processing [61].

In this study, the traditional mean-trial brain-behavior correlations approach yielded correlations with $p < .05$, which is not considered significant in fMRI studies. The mean-trial correlation approach may not efficiently remove enough of the noise and does not result in a high correlation coefficient [62]. To improve the disadvantages of the traditional mean-trial correlation approach, we used the single-trial brain-behavior correlation approach previously reported in fMRI and ERP studies [41, 59]. The single-trial brain-behavior correlation approach avoids the influence of noise on the statistical analyses. This correlation approach first removes much of the noise (or residual activity) when determining the correlation for each trial and then conducts a t -test for the weak yet possibly stable correlation coefficients with the majority of the noise filtered. The results of this study showed significant brain-behavior correlations in the semantic network for algebra ($p < .001$) and significant brain-behavior correlations in the phonological and visuospatial networks for arithmetic ($p < .001$).

Conclusion

Algebra is the abstract form of arithmetic and activates the semantic network, including the bilateral middle temporal gyri, bilateral inferior frontal gyri, bilateral dorsomedial prefrontal cortices, and left angular gyrus. In contrast, arithmetic activates the phonological network, including the bilateral precentral gyri and supplementary motor areas, and the visuospatial network, including the bilateral superior parietal lobules. These results suggest that the semantic network in the brain supports algebra. It deepened our understanding of the relation between algebraic processing and arithmetic processing. Whereas previous studies suggested that algebraic processing activated the non-semantic cortical network in expert mathematicians [32, 33], our results highlighted the important role of the classical semantic network in algebraic processing. In school education in general, algebra learning might take advantage of semantic knowledge.

Abbreviations

IPS: intraparietal sulcus; fMRI: functional magnetic resonance imaging; EPI: echo planar imaging; ANOVA: analysis of variance; ERP: event-related potential; RTs: reaction times; MNI: Montreal Neurological Institute

Declarations

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Author Contributions

D.Z., M.Y. and X.L. conceived and designed the study. D.Z., M.Y. and L.Y. performed the experiment and analyzed the data. D.Z. and M.Y. interpreted the data and drafted the manuscript. N.Y. and X.L. critically revised the manuscript for important intellectual content. All authors have read and approved the manuscript.

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Availability of data and materials

The datasets are available from the corresponding author on formal and logical request.

Ethics approval and consent to participate

This study was approved by the Institutional Review Board of the Institute of Cognitive Neuroscience and Learning at Beijing Normal University. Consent to participate was obtained from each participant.

Consent for publication

Not applicable.

Competing interests

The authors declare that they have no competing interests.

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Tables

Table 1 Brain regions with significantly different activations for algebra and arithmetic.

Brain region	Volume	<i>t</i> value	Coordinates		
			x	y	z
Algebra > arithmetic					
Right angular gyrus	37	4.56	51	-57	36
Arithmetic > algebra					
Left supplementary motor area	1715	-3.47	-15	6	63
Right middle frontal gyrus	1715	-6.55	27	42	21
	1715	-6.34	27	15	12
	1715	-6.27	15	6	33
Left insula	629	-6.08	-33	21	9
	629	-4.78	-24	39	33
	629	-4.65	-24	3	18
Left thalamus	39	-5.40	-21	-12	0
Left cerebellum (9)	26	-5.14	-9	-42	-33
Left lingual	358	-5.05	-3	-72	6
	358	-3.89	-9	-96	-3
Left hippocampus	30	-4.92	-36	-30	3
Right postcentral gyrus	78	-4.39	48	-33	57
Left postcentral gyrus	68	-4.35	-33	-42	72
	68	-4.35	-21	-42	51
Right postcentral gyrus	21	-3.96	27	-36	66
Left inferior parietal lobule	24	-3.80	-42	-42	54

Coordinates are in Montreal Neurological Institute space. Extent threshold: $p < .001$, uncorrected, cluster size > 20.

Table 2 Brain regions with positive and negative brain-behavior correlations for algebra and arithmetic based on single-trial correlations.

Brain region	Volume	t value	Coordinates		
			x	y	z
Positive correlations for algebra					
Left superior frontal gyrus	56	6.38	-21	45	39
Right inferior frontal gyrus (Triangle)	34	6.28	45	33	0
Left middle temporal gyrus	51	6.14	-48	-21	-12
Left postcentral gyrus	67	6.02	-66	-21	24
	67	5.34	-63	-9	3
Left middle temporal gyrus	21	5.81	-66	-45	12
Right middle temporal gyrus	47	5.73	60	-18	-9
Left superior medial frontal gyrus	37	5.68	0	63	12
Left inferior frontal gyrus (Triangle)	201	5.65	-54	24	3
	201	5.57	3	33	-18
	201	5.47	-54	9	-12
Right superior medial frontal gyrus	93	5.60	6	45	42
Left superior medial frontal gyrus	93	5.11	-3	48	15
Left supramarginal gyrus	31	5.20	-63	-48	30
Left superior frontal gyrus	21	5.17	-21	63	15
	21	5.01	-33	0	-24
Left supplementary motor area	20	4.63	-3	21	60
Negative correlations for algebra					
Right superior parietal lobule	95	-8.33	24	-63	42
Left superior parietal lobule	47	-7.25	-24	-63	45
Left superior occipital gyrus	23	-6.59	-18	-90	6
Left fusiform gyrus	20	-5.15	-30	-60	-12
Positive correlations for arithmetic					
Right inferior temporal gyrus	20	5.82	48	12	-39
Negative correlations for arithmetic					
Left superior parietal lobule	575	-12.74	-27	-63	51

	575	-7.68	-39	-42	42
Left precentral gyrus	191	-9.79	-51	0	27
	191	-4.22	-60	18	27
Right middle occipital gyrus	666	-7.91	27	-87	15
	666	-7.71	27	-66	42
	666	-6.37	45	-42	42
Left middle occipital gyrus	735	-7.75	-30	-84	12
	735	-7.65	-36	-69	-9
	735	-6.97	-12	-81	-6
Right middle frontal gyrus	138	-7.34	36	3	63
Right precentral gyrus	182	-7.08	57	12	33
	182	-6.13	54	6	12
Left middle cingulum gyrus	34	-6.83	-9	21	39
Left precentral gyrus	124	-6.80	-36	0	57
Right cerebellum (7)	73	-6.57	30	-72	-51
Left cerebellum (8)	140	-6.34	-9	-69	-30
	140	-5.86	-18	-72	-48
Left supplementary motor area	52	-5.96	-6	9	57
Right fusiform	132	-5.84	27	-78	-15
	132	-5.41	42	-66	-9
Right inferior temporal gyrus	61	-5.71	51	-48	-9
Left insula	26	-5.35	-30	27	9
Right inferior frontal gyrus (Triangle)	40	-5.27	33	27	27
Right inferior frontal gyrus (Triangle)	42	-5.09	51	39	15
	42	-4.86	39	51	27
Left cerebellum (6)	30	-5.08	-30	-57	-33
Right calcarine	20	-5.07	15	-96	3

Coordinates are in Montreal Neurological Institute space. Extent threshold: $p < .001$, uncorrected, cluster size > 20.

Figures

Figure 1

Experimental procedure and examples of trials.

Algebra - baseline

Arithmetic - baseline

Algebra - arithmetic

Figure 2

Brain activations for algebra and arithmetic relative to baseline and the contrast between algebra and arithmetic (cluster size > 20).

Algebra

Arithmetic

ROIs

Figure 3

The single-trial brain-behavior correlations for algebra and arithmetic (cluster size > 20) and the number of voxels of the single-trial brain-behavior correlations for algebra and arithmetic in seven ROIs of the semantic, phonological, and visuospatial networks are shown. Semantic network: MTG = middle temporal gyrus; IFG = inferior frontal gyrus; dmPFC = dorsomedial prefrontal cortex; AG = angular gyrus. Phonologic network: PCG = precentral gyrus; SMA = supplementary motor area; visuospatial network: SPL = superior parietal lobule.

Algebra

Positive
 $p < .05$
 $p < .005$
 $p < .001$

Arithmetic

Negative
 $p < .05$
 $p < .005$
 $p < .001$

ROIs

Figure 4

The mean-trial brain-behavior correlations for algebra and arithmetic (cluster size > 20) and the number of voxels of the mean-trial brain-behavior correlations for algebra and arithmetic in seven ROIs of the semantic, phonological, and visuospatial networks are shown. Semantic network: MTG = middle temporal gyrus; IFG = inferior frontal gyrus; dmPFC = dorsomedial prefrontal cortex; AG = angular gyrus. Phonologic network: PCG = precentral gyrus; SMA = supplementary motor area. Visuospatial network: SPL = superior parietal lobule.