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Research Article

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Posted Date: November 1st, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-809289/v1>

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Forecasting CO₂ Emissions Using a Novel Conformable Fractional Order Discrete Grey Model

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Abstract: Accurate and scientific forecasting of carbon dioxide emissions will help make better industrial carbon emission planning so as to promote low-carbon industrial development and achieve sustainable economic growth. For depressing the disturbance of various elements, grey system-based models play an important role in forecasting science. In this paper, we extend the cumulative order from integer order to fractional order based on the discrete gray model, which we call CFDGM (1,1). After introducing the free quantity of the model order, the accuracy of the convenient grey-based models can be further enhanced. We selected the data for carbon dioxide production by Germany, Japan, and Thailand for modeling. To obtain the optimal order of our grey model, we selected four optimizers to search for the order. The results show that although the search history of the four types of optimizers is different, the search results are the same, which proves that the four types of optimizers are stable and reliable, and the order for which we searched is reliable. By substituting the optimal order into CFDGM (1,1), we obtained the fitting and prediction error of the proposed model. The final results show that a satisfactory fitting effect and forecasting effect is obtained by our proposed model.

Keywords: Grey forecasting model; Carbon emission forecasting; Environmental management; Fractional calculus

1 Introduction

As global warming becomes more and more severe, the living environment of human beings is facing severe challenges (Frölicher et al. 2018). Carbon dioxide is the main component of greenhouse gases, and its increase is also the main cause of global warming. In the world's carbon dioxide emissions, industrial enterprises have the largest emissions, so it is urgent to accelerate the process of carbon emission reduction of industrial enterprises (Pourakbari-Kasmaei et al. 2020). Therefore, carbon dioxide emission reduction and low-carbon development has become a top priority, and it is also the shared mission and consensus of all countries for sustainable development.

Lots of countries have incorporated carbon emission standards as binding indicators into their national economic and social development mid- and long-term plans, balancing national economic growth with carbon emission reduction requirements through total carbon emission control. And many countries make a detailed study of various industries in the industrial sector, observe the characteristics of their carbon emissions, and put forward practical solutions to solve the huge amount of carbon emissions in the industry (Qi et al. 2020). Therefore, a method that can accurately predict the total amount of carbon dioxide emissions is needed to provide a theoretical basis and decision support for reducing carbon dioxide emissions in response to global warming issues and regional sustainable development. In this article, we will use Germany, Japan, and Thailand as cases to examine the validity of our model. Fig. 1 shows the basic situation of CO₂ emissions in the three countries (data from British Petroleum (BP) Statistical Review of World Energy 2018, available at the website of BP Statistical Review of World Energy <https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html>).

The contributions of this paper are as follow: we put forward a novel discrete grey model with conformable

fractional accumulation (CFA) and conformable fractional difference (CFD). In particular, the time response of the model is directly derived by the difference equation to eliminate the error caused by the jump from differentiation to difference and enhance the prediction accuracy of the model. And we forecast the CO₂ emissions from three countries from 2019 to 2023 by applying this model after verifying the effectiveness of the proposed model, the forecasting results will help make better industrial carbon emission planning so as to promote low-carbon industrial development.

2 Literature review

2.1 Research on Forecasting Carbon Dioxide

With the growth of global industry and energy consumption, CO₂ emissions are also on the rise, therefore an increasing number of researchers are focusing on predicting them in order to better plan industrial carbon emissions, promote industrial low-carbon development and achieve sustainable economic growth. Nyoni and Bonga used annual time series data for India's carbon dioxide emissions from 1960 to 2017, and used the Box-Jenkins Arima method for modeling and forecasting (Nyoni and Bonga 2019). Ho used the grey model and the discrete grey forecasting model to predict carbon dioxide emissions in Vietnam (Ho 2018). Xu et al. presented non-equigap GM(1,1) model with conformable fractional accumulation to analyze the relationship between energy consumption and CO₂ emissions (Xu et al. 2021). Zhou et al. take the ratio of CO and CO₂ as a measurement of the carbon efficiency to accurately predict the carbon efficiency in sintering process (Zhou et al. 2021). Chen et al. presented two grey interval forecasting methods, named interval GM(1, 1) and interval NGBM(1, 1), for few and uncertain time series data (Chen et al. 2019). Mustaffa and Shabri combined the nonlinear grey Bernoulli prediction model of the general reduced gradient nonlinear optimization method to predict CO₂ emissions (Mustaffa and Shabri 2020).

Additionally, some researchers have put forward targeted measures and schemes for industrial low-carbon development on the basis of CO₂ emission prediction research. Kanchev et al. designed the microgrid central energy management system in order to reduce the economic cost and CO₂ equivalent emissions (Kanchev et al. 2014). Guo et al. proposed a group multi-criteria decision-making approach based on combined weights and the hesitant fuzzy VIKOR method to evaluate the risk of CO₂ transmission pipelines (Guo et al. 2019). Yao et al. proposed a computational framework for integrating wind power uncertainty and carbon tax in economic dispatch (ED) model (Yao et al. 2012). Lee et al. proposed a seasonal auto regressive integrated moving average–support vector machine (SARIMA–SVM) time series analysis algorithm to improve pollution forecast accuracy (Lee et al. 2018). When conducting research based on time series data for CO₂ emission, greyscale prediction is a model that can achieve more accurate predictions with less data, which is quite favored by researchers for carbon emissions prediction. Considering the uncertainty, imperfection, and small sample of CO₂ emissions, we adopted the grey prediction model and optimized it in this study to realize the forecasting.

Nomenclature			
$D_t^{(1)}(y)$	The first derivative of y	GM(1,1)	First order cumulative grey model
$\Delta y(k)$	The first difference of y	DGM(1,1)	Discrete grey model
r	The order of accumulation generation	MAPE	The mean absolute percentage error
${}^{CF}D_t^r(y)$	r -th order conformable derivative of y	CFGM(1,1)	Conformable fractional accumulated grey model
$\Delta_{CF}^r y(k)$	r -th order conformable difference of y	CFDGM(1,1)	Conformable fractional accumulated discrete grey model
$\nabla_{CF}^r y(k)$	r -th order conformable accumulation of y	NGM(1,1)	Non-homogeneous grey model
$\hat{x}^{(0)}$	CFDGM output	LR	Linear regression model

(a). Area map of CO₂ emissions from Germany, Japan, and Thailand.
Fig. 1. CO₂ emissions from Germany, Japan, and Thailand.

(b). A pie chart of CO₂ emissions from Germany, Japan, and Thailand.

2.2 Research on the Grey Forecasting Model

The grey-based modeling theory was proposed by Deng in 1982, who was a pioneer in the field of Grey System Theory (Ju-Long 1982). In particular, the grey forecasting model is a predictive method that can establish mathematical models and make predictions through a small amount of incomplete information. In fact, when we make predictions in certain fields, such as weather forecasting (Li et al. 2015), earthquake forecasting (Vijay and Nanda 2019), and pest forecasting (Wei 2016), the data provided are often a small amount, and sometimes it is not even possible to provide sufficient data so that modeling can be achieved. Therefore, it becomes crucial to identify a fairly proper model for small samples in practical applications.

With the grey-based methods, a prediction model and effective evaluation of the development trend can be achieved through the processing of data under the background of uncertainty with less data and less information (Li et al. 2020). In addition, the common grey model is considered to be a fundamental model of the grey system model, abbreviated as GM (1,1). For the last 30 years, a large number of generalized and ameliorated models based on GM (1, 1) have been widely used. Some examples are GMC (1,n), NGBM(1,1), DGM (1,1), FAGM (1,1), NGM (1,1), and CFGM ((Wu et al. 2020),(Ma et al. 2020),(Xie and Liu 2009),(Mirjalili 2015)). Moreover, based on the grey prediction model that is constantly optimized and improved, there have been widespread applications in different fields such as engineering, economics, and especially energy (see e.g., (Xu et al. 2019),(Fu et al. 2021),(Es and Hamzacebi 2020),(Hu 2020),(Ye et al. 2020)). It has been proved that the grey model possesses satisfactory predictive ability for small samples. The grey model has been proved to be a satisfactory fit for small sample data, and because DGM (1,1) possesses good properties, we further optimized it. Therefore, in this article, we put forward a new fractional order discrete grey-based method to optimize the model and attempt to predict the emission of carbon dioxide.

3 Conformable Fractional Order Discrete Grey Model

In this subsection, we put forward the conformable fractional discrete grey-based model CFDGM(1,1).

3.1 Conformable Difference and Conformable Accumulation

It is well known that the classical grey model is constructed according to Newton-Leibniz calculus. To set the function $y(t) \in C(a,b)$, its first derivative and first difference of $y(t)$ in the interval (a,b) are respectively:

$$D_t^{(1)}(y) = \lim_{h \rightarrow 0} \frac{y(y+h) - y(y)}{h}, \quad \Delta y(k) \approx \lim_{h \rightarrow 1} \frac{y(t) - y(t-h)}{h} \Big|_{t=k} = y(k) - y(k-1) \quad (1)$$

The first derivative and the first difference are widely used in grey system models. In recent years, some researchers have proposed a new fractional derivative that has the characteristics of simple calculation. Second, many of its characteristics are consistent with the classic derivative, and therefore, it has many applications. The definition is as follows:

Definition 1 Suppose $y(t) : [0, \infty) \rightarrow R$, then the r -th order conformable fractional order derivative of $y(t)$ can be defined as:

$${}^{CF}D_t^r(y) = \lim_{h \rightarrow 0} \frac{y(t + ht^{[r]-r}) - y(t)}{h} = t^{[r]-r} \frac{dy(t)}{dt} \quad (2)$$

in which $[r]-1 < r < [r]$, $t > 0$, where ${}^{CF}D_t^r(y)$ represents the r -th order conformable fractional derivative. In particular, when $r \in (0, 1]$, Equation (2) has the following definition, ${}^{CF}D_t^r(y) = \lim_{h \rightarrow 0} \frac{y(t + ht^{1-r}) - y(t)}{h}$, where $r \in (0, 1]$ represents the order.

Based on definition 1, Zheng et al. (Zheng et al. 2019) first proposed CFD and CFA, and then Parsopoulos and Vrahatis (Parsopoulos and Vrahatis 2002) proposed unified expression of the CFA.

Definition 2 Suppose $y(t) : [0, \infty) \rightarrow R$, $y(k)$ represents the discrete form under $y(t)$ sampling at equal intervals, then the r -th order conformable fractional difference and accumulation of $y(k)$ are respectively (Wu et al. 2020):

$$\Delta_{CF}^r y^{(r)}(k) = k^{[r]-r} \sum_{j=1}^k \frac{(-1)^{k-j} \Gamma([r]+1)}{\Gamma(k-j+1) \Gamma([r]-k+j+1)}, y^{(0)}(j), \nabla_{CF}^r y^{(r)}(k) = \sum_{j=1}^k \frac{\Gamma(k-j+[r])}{\Gamma(k-j+1) \Gamma([r])} \cdot \frac{y^{(0)}(j)}{j^{[r]-r}} \quad (3)$$

3.2 Proposal of Discrete Conformable Fractional Grey System Model

Ma et al. (Ma et al. 2020) put forward a conformable fractional order grey-based model and achieved good results. To further enhance the accuracy of the model, we propose a new grey model based on the conformable fractional difference and accumulation operator. The time response formula of this model is directly derived by the difference equation to eliminate the error attributed to the jump from differentiation to difference, which we referred to as CFDGM model.

Definition 3 Suppose $y^{(r)}(k)$ is the cumulative sequence of order r of $y^{(0)}(k)$. Then, the expression of the conformable discrete fractional grey models is:

$$y^{(r)}(k+1) = \xi_1 y^{(r)}(k) + \xi_2, k > 0, y^{(r)}(k) > 0 \quad (4)$$

Among them, ξ_1 and ξ_2 represent the parameters to be estimated. When $r = 1$, Equation (4) degenerates to the classic first-order discrete grey model:

$$y^{(1)}(k+1) = \xi_1 y^{(1)}(k) + \xi_2 \quad (5)$$

By solving Eq. (4) recursively, we obtain the expression of the response function:

$$\hat{y}^{(r)}(k) = \left(y^{(0)}(1) - \frac{\xi_2}{1 - \xi_1} \right) \xi_1^{k-1} + \frac{\xi_2}{1 - \xi_1}, k = 2, 3, \dots, n \quad (6)$$

If $r = 1$, we can obtain the analytical expression of the DGM(1,1) model (Xie and Liu 2009):

$$\hat{y}^{(r)}(k) = \left(x^{(0)}(1) - \frac{\xi_2}{1 - \xi_1} \right) \xi_1^{k-1} + \frac{\xi_2}{1 - \xi_1}, k = 2, 3, \dots, n \quad (7)$$

In order to obtain model parameters ξ_1 and ξ_2 , we use the least squares algorithm. Suppose $\xi = (\xi_1, \xi_2)$, then, $\xi = (\Lambda^T \Lambda)^{-1} \Lambda^T \eta$, where:

$$\Lambda = \begin{bmatrix} \sum_{j=1}^1 \frac{\Gamma([r]+1-j)}{\Gamma(1-j+1)\Gamma([r])} \frac{y^{(0)}(j)}{j^{[r]-r}} & 1 \\ \sum_{j=1}^2 \frac{\Gamma([r]+2-j)}{\Gamma(2-j+1)\Gamma([r])} \frac{y^{(0)}(j)}{j^{[r]-r}} & 1 \\ \vdots & \vdots \\ \sum_{j=1}^{n-1} \frac{\Gamma([r]+n-1-j)}{\Gamma(n-1-j+1)\Gamma([r])} \frac{y^{(0)}(j)}{j^{[r]-r}} & 1 \end{bmatrix}, \quad (8)$$

$$\eta = \begin{bmatrix} \sum_{j=1}^2 \frac{\Gamma([r]+2-j)}{\Gamma(2-j+1)\Gamma([r])} \frac{y^{(0)}(j)}{j^{[r]-r}} \\ \sum_{j=1}^3 \frac{\Gamma([r]+3-j)}{\Gamma(3-j+1)\Gamma([r])} \frac{y^{(0)}(j)}{j^{[r]-r}} \\ \vdots \\ \sum_{j=1}^n \frac{\Gamma([r]+n-j)}{\Gamma(n-j+1)\Gamma([r])} \frac{y(j)}{j^{[r]-r}} \end{bmatrix}$$

The predictive fit and predictive sequence can be achieved by conformable fractional difference:

$$\hat{y}^{(0)}(k) = k^{[r]-r} \sum_{j=1}^k \frac{(-1)^{k-j} \Gamma([r]+1)}{\Gamma(k-j+1)\Gamma([r]-k+j+1)} \hat{y}^{(r)}(k), \quad (9)$$

$k = 1, 2, 3, \dots, n+N$

where n denotes the number of fitted data, and N denotes the number of forecasting data.

4 Optimization and Validation of Model Parameters

In the above equations, we have provided the expression of CFDGM, but how to select the order r of our model is not given. In this section, we will introduce how to select the order r and provide the evaluation standard of model accuracy. In fact, we can build the following optimization model to determine the order r ,

$$\min_r \text{MAPE} = \sum_{j=1}^N \left| \frac{\hat{x}^{(0)}(j) - x^{(0)}(j)}{x^{(0)}(j)} \right| \times 100\%$$

$$s.t. \begin{cases} \xi = (\Lambda^T \Lambda)^{-1} \Lambda^T \eta \\ \xi_1 = \frac{\sum_{k=1}^{n-1} y^{(r)}(k+1)y^{(r)}(k) - \frac{1}{n-1} \sum_{k=1}^{n-1} y^{(r)}(k+1) \sum_{k=1}^{n-1} y^{(r)}(k)}{\sum_{k=1}^{n-1} (y^{(r)}(k))^2 - \frac{1}{n-1} \left(\sum_{k=1}^{n-1} y^{(r)}(k) \right)^2} \\ \xi_2 = \frac{1}{n-1} \left[\sum_{k=1}^{n-1} y^{(r)}(k+1) - \xi_1 \sum_{k=1}^{n-1} y^{(r)}(k) \right] \\ \hat{y}^{(r)}(k) = \left(y^{(0)}(1) - \frac{\xi_2}{1-\xi_1} \right) \xi_1^{k-1} + \frac{\xi_2}{1-\xi_1}, \quad k = 2, 3, \dots, n \\ \hat{y}^{(r)}(k) = k^{[r]-r} \sum_{j=1}^k \frac{(-1)^{k-j} \Gamma([r]+1)}{\Gamma(k-j+1)\Gamma([r]-k+j+1)} \hat{y}^{(0)}(k), \quad k = 1, 2, 3, \dots, n \end{cases} \quad (10)$$

In order to obtain the order r , we will use four optimization algorithms (PSO, GWO, WOA, and ALO((Parsopoulos and Vrahatis 2002),(Mirjalili 2015))) to solve the model simultaneously and compare the solutions.

4.1 Application

The application part considers the CO₂ emissions from Germany, Japan, and Thailand with the initial data shown in Table 1 and using gray models. The original sequences for the period 2008–2016 are employed for constructing the NGM(1,1) model, the GM (1,1) model, the DGM (1,1) model, the LR model, and the

CFDGM (1,1) model. The original sequences for the period 2017–2018 are applied to validate the precision of diverse gray models, and then, we will use these models for calculating the prospective development potential of the three countries. The data for these three countries are shown in Fig. 1.

Table 1 shows the carbon dioxide emissions from Germany, Japan, and Thailand. (Source: BP Statistical Review of World Energy 2018). In the specific modeling process, we will use the mean absolute relative error (MAPE) to evaluate our model and previous models.

Table 1: The Carbon Dioxide Emissions of Germany, Japan, and Thailand (in million tonnes)

Year	Germany	Japan	Thailand
2008	806.5	1274.9	237.4
2009	751	1112.5	236.5
2010	780.6	1183.8	248.7
2011	761	1194.7	253.5
2012	770.3	1285.6	270.9
2013	794.6	1273.6	273.9
2014	748.4	1239.6	280.7
2015	751.9	1197.4	289.4
2016	766.6	1178.5	295.5
2017	762.6	1171.8	299.9
2018	725.7	1148.4	302.4

4.2 The CO₂ emissions from Germany

In this section, we consider the carbon dioxide emissions from Germany using gray models for forecasting. We used the WOA, PSO, ALO, and GWO algorithms to search the MAPE and related optimized value r with the initial data within the corresponding period from 2008 to 2016. The data for 2017 were used to fit the model, and the original sequence of 2018 was used to test the validity of the model. Figure 2 shows the fitness curve of the four types of optimization algorithms to optimize CFDGM (the smallest MAPE searched in each generation in the German carbon dioxide prediction). The fitness curve after 100 iterations is shown in Fig. 2. Although the four types of algorithms have different search records, after 100 iterations, they all obtained the same parameter r , which is 0.94777. The specific r and corresponding MAPE can be seen in Table 2. This shows that the searched orders are true and reliable.

Fig. 2. The optimal fractional order and MAPE of the CFDGM(1,1) model for Germany using the ALO, PSO, WOA, and GWO algorithms.

Table 2: The order and MAPE obtained by four types of optimization algorithms (ALO, PSO, GWO, and WOA) for carbon dioxide prediction in Germany

ALO		GWO		PSO		WOA	
MAPE (%)	r	MAPE (%)	r	MAPE (%)	r	MAPE (%)	r
1.5234	0.9477	1.5234	0.94777	1.5234	0.94777	1.5234	0.94777

Table 3: For the German carbon dioxide prediction, the MAPE of GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models using the four algorithms

CFDGM	GM	DGM	NGM	LR
MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)
1.5234	1.6412	1.6421	7.4478	1.8689

Table 4: Comparison of the accuracy of the five models in the prediction stage for determining the German carbon dioxide

CFDGM	GM	DGM	NGM	LR
MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)
2.17741045	2.55842027	2.53979206	3.24655099	2.29969099

The fitness curves for the four types of algorithms for 100 generations appear in Fig. 3 (left) for comparison, and we can see that although the four types of algorithms are different at the beginning, they all converge in the end. Fig. 3 (right) and Table 3, Table 4 show the fitting error for CFDGM and four comparison models GM, DGM, NGM, LR and the relative error of prediction; the fitting errors for the five types of models are 1.5234%, 1.6412%, 1.6421%, 7.4478%, and 1.8689%, and the prediction order errors are 2.17741045%, 2.55842027%, 2.53979206%, 3.24655099%, and 2.29969099%, respectively. It follows that the fitting accuracy and forecast accuracy of the CFDGM model are relatively high on the issue of German carbon dioxide prediction, but the performance metrics of the NGM (1,1) model are relatively poor. It is interesting to know that more optimal results were obtained with CFDGM (1,1) than with the previous integer-order grey-based models. The specific fitting and prediction results for the five models are shown in Table 5, and the fitting and prediction curves for the five models are shown in Fig. 5.

Fig. 3. Fitness curve comparison of four types of optimization algorithms (ALO, PSO, GWO, and WOA) (left) and gray model error comparison (right).

In the problem of carbon dioxide prediction in Germany, the choice of accumulation order is particularly important. In order to further demonstrate the optimization process, we will search for the order of CFDGM using four types of optimization algorithms (ALO, WOA, PSO, and GWO). After 100 generations, we visualized the MAPE values obtained in the last 10 generations, as shown in Fig. 4.

Fig. 4. Search records for the four types of optimization algorithms in Germany's carbon dioxide prediction. The MAPE values corresponding to the parameters searched for all populations are shown, where we visualized the most stable results for the last 10 generations.

Table 5: The results for the CO₂ emissions from Germany obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models

Year	Data	CFDGM	GM	DGM	NGM	LR
2008	806.5	806.5000	806.5000	806.5000	806.5000	782.6133
2009	751	760.0167	767.5309	767.6280	420.8706	779.4850
2010	780.6	767.4335	766.9641	767.0333	728.5083	776.3567
2011	761	770.1657	766.3977	766.4391	763.3790	773.2283
2012	770.3	770.3043	765.8317	765.8454	767.3316	770.1000
2013	794.6	768.8004	765.2662	765.2521	767.7796	766.9717
2014	748.4	766.1704	764.7010	764.6593	767.8304	763.8433
2015	751.9	762.7267	764.1363	764.0670	767.8362	760.7150
2016	766.6	758.6736	763.5720	763.4751	767.8368	757.5867
2017	762.6	754.1521	763.0081	762.8836	767.8369	754.4583
2018	725.7	749.2638	762.4446	762.2926	767.8369	751.3300
2019	-	744.0843	761.8815	761.7021	767.8369	748.2017
2020	-	738.6715	761.3189	761.1121	767.8369	745.0733
2021	-	733.0705	760.7566	760.5225	767.8369	741.9450
2022	-	727.3175	760.1948	759.9333	767.8369	738.8167
2023	-	721.4417	759.6334	759.3446	767.8369	735.6883
2024	-	715.4669	759.0724	758.7564	767.8369	732.5600
2025	-	709.4130	758.5119	758.1686	767.8369	729.4317
2026	-	703.2967	757.9517	757.5813	767.8369	726.3033

Fig. 5. The results for the CO₂ emissions from Germany obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models.

To consider the fitting effect of the five types of models, we used the linear regression model to obtain the fitting results for the five types of models. This shows that the R^2 of the CFDGM model is 0.54628, which indicates the best fitting performance. Second, we analyzed the correlation between the fitting results for each model and the original data, and we can see that the fitting correlation coefficient for CFDGM is the most optimal at 0.74, as shown in Fig. 6.

Fig. 6. The results for the CO₂ emissions from Germany obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models.

4.3 The CO₂ emissions from Japan

We used an analogous argument in the study of Japan's carbon dioxide emissions. First, we applied the WOA, PSO, ALO, and GWO algorithms to ensure the total MAPE and preferred order r number. We used the 2017-2018 data to fit the model and test the accuracy. Figure 7 shows the fitness curves for the four types of optimization algorithms to optimize CFDGM (the smallest MAPE searched in each generation) in the Japanese carbon dioxide prediction. The fitness curve after 100 iterations is also shown in Fig. 7. Although the 4 types of algorithms have different search records, after 100 iterations, they all obtained the same

parameter r with a value of 0.61815. The specific r and corresponding MAPE is shown in Table 6, and indicates that the accuracy of the searched parameters is very high.

Fig. 7. The preferred order number and MAPE of the CFDGM (1,1) model for Japan using the ALO, PSO, WOA, and GWO algorithms.

Table 6: The order and MAPE obtained by the four types of optimization algorithms (ALO, PSO, GWO, and WOA) for the carbon dioxide prediction in

Japan

ALO		GWO		PSO		WOA	
MAPE (%)	r	MAPE (%)	r	MAPE (%)	r	MAPE (%)	r
1.3894	0.61815	1.3894	0.61815	1.3894	0.61815	1.3894	0.61815

Table 7: For the Japanese carbon dioxide prediction, the MAPE of GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models using the four algorithms

CFDGM	GM	DGM	NGM	LR
MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)
1.3894	3.3689	3.3751	8.6097	3.9001

Table 8: Comparison of the accuracy of the five models in the prediction stage for determining the Japanese carbon dioxide

CFDGM	GM	DGM	NGM	LR
MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)
3.3192199	7.4423636875	7.360079528	6.408979	5.268061312

The fitness curves for the four types of algorithms in the 100 generations are shown in Fig. 8 (left) for comparison. We can see that although the four types of algorithms are different at the beginning, they all converge in the end. Figure 8 (right) and Table 7, Table 8 show the fitting errors of CFDGM and the four comparison models GM, DGM, NGM, LR, and the prediction relative error; the fitting errors for the five types of models are 1.3894%, 3.3689%, 3.3751%, 8.6097%, and 3.9001%, respectively, and the prediction order errors are 3.3192199%, 7.4423636875%, 7.360079528%, 6.408979%, and 5.268061312%, respectively. This shows that the fitting accuracy and forecast accuracy of the CFDGM model are relatively

high regarding the Japanese dioxide prediction. We can see from the displayed results that some gray models are not fully in accordance with Japan's carbon dioxide emission trend. The MAPE values for all models are above 3% except for CFDGM (1,1), but the values of CFDGM (1,1) are much smaller. Obviously, in this case, the new model can achieve excellent results. The specific fitting and prediction results for the five models are shown in Table 9, and the fitting and prediction curves for the five models are shown in Fig. 10.

Fig. 8. Comparison of fitness curves of four types of optimization algorithms (ALO, PSO, GWO, and WOA)(left) and gray model error comparison (right).

The choice of cumulative order is particularly important in the problem of Japanese carbon dioxide prediction. In order to further demonstrate the optimization process, we will search for the order of CFDGM using the four types of optimization algorithms (ALO, WOA, PSO, and GWO). At the end of 100 generations, we visualized the MAPE values obtained for the last 10 generations, as shown in Fig. 9.

Fig. 9. The search records for the four types of optimization algorithms in Japan's carbon dioxide prediction. The MAPE values corresponding to the parameters searched for all populations are shown, where we visualized the most stable results for the last 10 generations.

Table 9: The results for the CO₂ emissions from Japan obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM(1,1) models

Year	Data	CFDGM	GM	DGM	NGM	LR
2008	1274.9	1274.9000	1274.9000	1274.9000	1274.9000	1211.6489
2009	1112.5	1098.5090	1182.0690	1182.7310	601.5355	1212.6422
2010	1183.8	1193.3576	1189.4470	1189.9273	1115.2758	1213.6356

2011	1194.7	1239.3846	1196.8711	1197.1674	1211.9282	1214.6289
2012	1285.6	1255.8548	1204.3415	1204.4515	1230.1119	1215.6222
2013	1273.6	1252.8625	1211.8586	1211.7800	1233.5328	1216.6156
2014	1239.6	1236.5052	1219.4225	1219.1530	1234.1764	1217.6089
2015	1197.4	1210.7912	1227.0337	1226.5709	1234.2975	1218.6022
2016	1178.5	1178.5040	1234.6924	1234.0340	1234.3203	1219.5956
2017	1171.8	1141.6499	1242.3989	1241.5424	1234.3246	1220.5889
2018	1148.4	1101.7122	1250.1534	1249.0966	1234.3254	1221.5822
2019	-	1059.8081	1257.9564	1256.6967	1234.3256	1222.5756
2020	-	1016.7888	1265.8081	1264.3430	1234.3256	1223.5689
2021	-	973.3076	1273.7088	1272.0359	1234.3256	1224.5622
2022	-	929.8679	1281.6588	1279.7755	1234.3256	1225.5556
2023	-	886.8574	1289.6584	1287.5623	1234.3256	1226.5489
2024	-	844.5731	1297.7080	1295.3965	1234.3256	1227.5422
2025	-	803.2411	1305.8078	1303.2783	1234.3256	1228.5356
2026	-	763.0307	1313.9582	1311.2080	1234.3256	1229.5289

Fig. 10. The results for the CO₂ emissions from Japan obtained by the GM(1,1), the NGM(1,1), the DGM(1,1), the LR, and the CFDGM(1,1) models.

To study the fitting effect of the five types of models, we used the linear regression model to obtain the fitting results for the five types of models. This shows that the R^2 of the CFDGM model is 0.85537, which indicates the best fitting performance. Second, we analyzed the correlation between the fitting results for each model and the original data, and we can see that the fitting correlation coefficient for CFDGM is the most optimal at 0.92, as shown in Fig. 11.

Fig. 11. The results for the CO₂ emissions from Japan obtained by the GM(1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models.

4.4 The CO₂ emissions from Thailand

Similarly, in this section, we will explore carbon dioxide emissions from Thailand through diverse gray models. The four algorithms we used, as well as the training, fitting and validation procedures for data sets of different years, are the same as in the previous section. Figure 12 shows the fitness curves for four types of optimization algorithms to optimize CFDGM (the smallest MAPE searched in each generation) for the carbon dioxide prediction in Thailand. The fitness curve after 100 iterations is shown in Fig. 12. Although the four types of algorithms have different search records, after 100 iterations, they all obtained the same parameter r with a value of 0.9184. The specific r and corresponding MAPE are shown in Table 10, and indicates that the searched parameters are true and reliable.

Fig. 12. The preferred fractional order number and MAPE for the CFDGM(1,1) model for Thailand using the ALO, PSO, WOA, and GWO algorithms.

Table 10: The order and MAPE obtained by the four types of optimization algorithms (ALO, PSO, GWO, and WOA) for the prediction of carbon dioxide in Thailand

ALO		GWO		PSO		WOA	
MAPE (%)	r	MAPE (%)	r	MAPE (%)	r	MAPE (%)	r
0.5497	0.9184	0.5497	0.9184	0.5497	0.9184	0.5497	0.9184

Table 11: For Thailand's carbon dioxide prediction, the MAPE for the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models using the four algorithms

CFDGM	GM	DGM	NGM	LR
MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)
0.5497	0.8981	0.8979	8.6104	0.9549

Table 12: Comparison of the accuracy of the five models in the prediction stage for determining Thailand's carbon dioxide

CFDGM	GM	DGM	NGM	LR
MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)	MAPE (%)
1.713574	3.83133	3.835092	7.229005	2.521348

We compared the fitness curves for the four types of algorithms in the 100 generations in Fig. 13 (left). Although the four types of algorithms are different at the beginning, they all converge in the end. Figure 13 (right) and Table 11, Table 12 show the fitting errors for CFDGM and the four comparative models GM, DGM, NGM, and LR and prediction relative error; the fitting errors of the five types of models are 0.549692%, 0.898127%, 0.897931%, 8.610352%, and 0.954928%, and the prediction order errors are 1.713574%, 3.83133%, 3.835092%, 7.229005%, and 2.521348%, respectively. The fitting accuracy and forecasting accuracy of the CFDGM model are the most optimal in terms of the prediction of carbon dioxide in Thailand. Based on the above, the specific fitting and prediction results for the five benchmark models are shown in Table 13, and the fitting and prediction curves for the five models are shown in Fig. 15. Through observation, we can find an obvious law: compared with the other four gray models, which more or less overestimate emissions, the CFDGM (1,1) model is essentially the most similar to the raw sequence. In addition, the new model indicates that the CO₂ emissions have been steadily rising and will continue to rise over the next few years.

Fig. 13. Comparison of fitness curves for the four types of optimization algorithms (ALO, PSO, GWO, and WOA) (left) and gray model error comparison (right).

For carbon dioxide prediction in Thailand, the choice of the accumulation order is particularly important. In order to further demonstrate the optimization process, we will search for the order of CFDGM by using four types of optimization algorithms (ALO, WOA, PSO, and GWO). After 100 generations, we visualized the MAPE values for the last 10 generations, as shown in Fig. 14.

Fig. 14. The search records for the four types of optimization algorithms in Thailand's carbon dioxide prediction. The MAPE values corresponding to the parameters searched for all populations are shown, where we visualized the most stable results for the last 10 generations.

Table 13: The results for the CO₂ emissions from Thailand obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models

Year	Data	CFDGM	GM	DGM	NGM	LR
2008	237.4	237.4000	237.4000	237.4000	237.4000	233.4667
2009	236.5	236.5003	240.5183	240.5506	128.8386	241.3917
2010	248.7	248.0289	248.0572	248.0877	223.7037	249.3167
2011	253.5	257.6325	255.8324	255.8609	258.8011	257.2417
2012	270.9	266.2030	263.8513	263.8777	271.7861	265.1667
2013	273.9	274.1442	272.1216	272.1456	276.5902	273.0917
2014	280.7	281.6744	280.6511	280.6727	278.3676	281.0167
2015	289.4	288.9251	289.4480	289.4668	279.0252	288.9417
2016	295.5	295.9821	298.5206	298.5366	279.2684	296.8667
2017	299.9	302.9043	307.8776	307.8905	279.3585	304.7917
2018	302.4	309.7344	317.5278	317.5375	279.3918	312.7167
2019	-	316.5042	327.4806	327.4868	279.4041	320.6417
2020	-	323.2382	337.7453	337.7478	279.4086	328.5667
2021	-	329.9557	348.3317	348.3303	279.4103	336.4917
2022	-	336.6723	359.2500	359.2444	279.4109	344.4167
2023	-	343.4009	370.5105	370.5004	279.4112	352.3417
2024	-	350.1521	382.1240	382.1092	279.4113	360.2667
2025	-	356.9352	394.1014	394.0816	279.4113	368.1917
2026	-	363.7579	406.4543	406.4292	279.4113	376.1167

Fig. 15. The results for the CO₂ emissions from Thailand obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM(1,1)models.

To study the fitting effect of the five types of models, we used a linear regression model to obtain the fitting results for the five types of models. This shows that the R^2 of the CFDGM model is 0.98936, which indicates the most optimal fitting performance. Second, we analyzed the correlation between the fitting results for each model and the original data, and Fig. 16 shows that the fitting correlation coefficient of CFDGM is the most optimal at 0.99.

Fig. 16. The results for the CO₂ emissions from Thailand obtained by the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models.

4.5 Further discussions

From section A to section D, we have discussed the CO₂ emissions from three nations obtained with five diverse grey models, which are the GM (1,1), the NGM (1,1), the DGM (1,1), the LR, and the CFDGM (1,1) models. From the computational results, it is clear that the previous grey models NGM(1,1) and LR have the lowest performance measures, and the CFA-based grey model CFDGM(1,1) has the most optimal modeling index. This indicates that the CFA-based and CFD-based models have a strong ability to build grey-based models with satisfactory performance in CO₂ modeling of the three countries. Furthermore, Fig. 17 displays the future results of CO₂ emissions, and Table 14 gives the annual increase rates.

Fig. 17. The CO₂ emissions from Germany, Japan, and Thailand obtained by the CFDGM (1,1) model.

Table 14: The annual rate of increase for CO₂ emissions in the three countries (%)

Year	Germany	Japan	Thailand
2019	-0.6913	-3.8035	2.1857
2020	-0.7275	-4.0592	2.1276
2021	-0.7582	-4.2763	2.0782
2022	-0.7848	-4.4631	2.0356
2023	-0.8079	-4.6254	1.9985
2024	-0.8282	-4.7679	1.9660
2025	-0.8461	-4.8938	1.9372
2026	-0.8622	-5.0060	1.9115
mean value	-0.7883	-4.4869	2.0300

In Germany, the current CO₂ emissions fluctuate, but are generally stable. They will be slightly reduced, from 744.084 million tons in 2019 to 703.2967 million tons in 2026, with an average annual growth rate of approximately -0.7883% . Obviously, Germany's carbon dioxide has maintained negative growth, which proves that Germany has achieved success in environmental governance. At the same time, Germany's stable economic development in the last several years has led to only a small change in carbon dioxide emissions.

In Japan, CO₂ emissions in the next few years will decline at a fairly steady rate, with an average annual growth rate of -4.4869% , and it is expected to reach 763.0307 million tons by 2026. In recent years, Japan has worked hard to develop environmental sustainability practices and has maintained negative growth in carbon dioxide emissions, which proves that Japanese environmental policymakers are determined to support clean energy sources and control pollution.

In Thailand, carbon dioxide emissions will increase from 316.5042 million tons in 2019 to 363.779 million tons in 2025, with an average annual growth rate of 2.03% . Moreover, Thailand's annual growth rate of carbon dioxide emissions has been decreasing year by year. This proves that Thailand's environmental system is developing toward reduction of pollution, and its governance policies have significantly contributed to this achievement.

5 Conclusion

The production of carbon dioxide has always been a national concern. Accurate carbon dioxide forecasting helps decision-makers formulate reasonable environmental policies, adjusts the energy structure of industrial production, and effectively reduces carbon emissions and assists in realizing a country's sustainable development. There are many mature models for carbon dioxide prediction, and these models produce excellent results. However, these models are only suitable for environments with massive amounts of data. When there is a lack of data, it is difficult to meet the conditions of modeling because the data do not meet a certain distribution. However, a country's carbon dioxide emissions may be affected by environmental policies and other reasons, and the historical data may not be consistent with the current data, with only a small amount of data being suitable for use. To prevent this dilemma, we have put forward a new grey-based forecasting model for carbon dioxide prediction problems.

Grey system theory successfully solves the modeling problem of small samples and is a useful modeling

tool. To further enhance the forecast accuracy, we introduced CFA and CFD into the discrete gray model, and deduced the formula of the model. To prove the effect of the model, we selected carbon dioxide emissions data from Germany, Japan, and Thailand for analyses. The experimental results showed that the model we suggested is effective, and the parameters obtained by the optimized model are stable and reliable. Finally, we made predictions regarding the future carbon dioxide emissions from these three countries and formulated some valuable conclusions to assist policymakers in their decision-making about the implementation of sustainable development strategies.

Compliance with ethical standards

Funding This work was supported in part by the National Natural Science Foundation of China under Grant 71874082, and in part by the Humanities and Social Sciences Foundation of the Ministry of Education of China under Grant 18YJA870021.

Conflict of interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval This study does not involve any human participants or animals performed by any of the authors. Informed consent was obtained from all individual participants included in the study.

Authorship contributions

All authors contributed to the study conception and design. All authors contributed to material preparation, data collection and analysis. All authors wrote the first draft of the manuscript commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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