The multivariate analysis of variance as a powerful approach for circular data

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Research Article

Keywords: MANOVA, Rayleigh test, directional data, orientation, periodicity

Posted Date: August 13th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-810559/v1>

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Abstract

A broad range of scientific studies involve taking measurements on a circular rather than linear scale (often times or orientations). For linear measures there is a well-established statistical toolkit based on linear modelling to explore the associations between this focal variable and potentially several explanatory factors and covariates. However, most statistical testing of circular statistics is much simpler: often involving either testing whether variation in the focal measurements departs from circular uniformity, or whether a single explanatory factor with two levels is supported. Here we demonstrate that a MANOVA approach based on the sines and cosines of the circular data is as effective as the most-commonly used tests in these simple situations, while additionally it offers extension to multi-factorial modelling that these conventional tests do not. This, in combination with recent developments in Bayesian approaches, offers a substantial broadening of the scientific questions that can be addressed statistically with circular data.

Introduction

Some scales of measurement in science are inherently periodic rather than linear. Data on compass orientations, times of day and times of year are obvious examples of this. Such data is often called circular, since it is easy to imagine the data being mapped onto the circumference of a circle. As soon as one considers that 355° is closer to 5° than it is to 340° , it is clear that such circular data needs different statistical treatment from linear variables (like mass and age), and a body of statistical theory has developed to allow investigation of circular data (for example: Batschelet (1981), Fisher (1995), Pewsey et al. (2013) and Landler et al. (2018)).

Since Lord Rayleigh (1880) formulated his seminal Rayleigh test, circular statistics as an academic discipline has constantly developed and expanded possible approaches, nowadays including a myriad of test alternatives and software (Berens 2009; Kovach 2011; Agostinelli and Lund 2017), second order analysis (Moore 1980) as well as Akaike and Bayesian Information criterion methods (Fitak and Johnsen 2017; Cremers and Klugkist 2018; Cremers 2021). All of which have the goal to help researchers understand and correctly interpret data distributed on a circle. However, recently we confirmed that despite all the developments, the Rayleigh test is still one of the most powerful tests for using a single sample to test the null hypothesis that the underlying population is uniformly distributed with no preferred direction (Landler et al. 2018). In contrast to some alternative tests, it also controls type I error rates for binned data (e.g. if directions are recorded to the nearest degree, or even ten degrees) and can be used to detect a variety of deviations from uniformity, with the exception of multimodal symmetrical distributions (Landler et al. 2019a, 2020).

One feature all commonly-used tests for circular statistics based on null-hypothesis testing have in common is the inability to use multiple covariates and/or multiple grouping factors, if the circular variable is the response. That is, investigation is limited to a single independent variable (although this

variable can be discrete, continuous and circular, or continuous and linear). This is in stark contrast to analyses of linear dependent variables, where multiple dependent variables are analyzed routinely.

In a recent analysis we compared a suite of available tests designed to compare two samples to test the null hypothesis that they come from the same underlying population and found that also in this testing situation a very common test, the Watson U^2 test (Watson 1962), showed superior power to many other standard options (Landler et al. 2021). Within this study, we also experimented with a new approach, which exploits the possibility to linearize circular data using sines and cosines of the angles (see for example Pewsey et al. 2013). In order to use two linear factors as dependent variables, we needed to switch from univariate linear models to a multivariate analysis of variance. This approach allows the use of multiple response variables (Krzanowski 2000). In our study we used MANOVA to test for a difference of two circular distributions by using the group as factor (see Pail et al. (2020) for one earlier study that used a MANOVA to analyze effects of a linear variable on a directional response). Surprisingly, this novel approach performed similarly well to established tests, for instance, the Watson U^2 test (Landler et al. 2021). It is important to add that the sine and cosine of the angle, while derived from the same number, are independent of each other (they are orthogonal terms by definition), which is an important criterion for the dependent variables in such approach.

It is clear that the MANOVA approach allows the use of more than two levels per factor, multiple grouping factors as well as linear covariates (Johnson and Field 1993; Weinfurt 1995; Iversen 2004). However, it is less clear how this would relate to circular distributions, or in other words, how p-values obtained in such analyses would relate to specific null hypotheses. While we already showed that differences between groups can be reliably tested, we wanted to also use this approach to test for significantly clustered (i.e. non-random) unimodal distributions. We hypothesized that the intercept of the respective model could be used to derive p-values related to significant clustering, at least if no other terms are provided. We compared this MANOVA intercept approach with the two most powerful tests for unimodal deviations from uniformity, the Rayleigh and Hermans-Rasson test and then simulated some example situations with known underlying distributions to explore its generally usefulness including grouping and linear factors.

This paper is a first step in exploring this method and providing advice for people interested in using this for their own research. We show that the intercept of the MANOVA models is a reliable proxy of unimodal clustering and that grouping factors as well as linear covariates can considerably enhance model performance and interpretation of circular data.

Methods

Statistical tests used

All analyses were performed in the statistical software R (R Core Team 2020), R code used for simulations and R output can be found in the supplement. For all the analysis we performed the MANOVA

approaches using the summary function for the “manova” function in base R. The sine and cosine of the directional response variable (in radians) was used as the response matrix, independent variables were either missing (intercept-only approach) or different for each of the analyses. The intercept-only approach used no response variables, therefore, only a single p-value was reported. In the linear MANOVA approach, a linear variable was added, therefore reporting the p-value of the intercept and the covariate. In the grouped MANOVA approach a grouping factor was added, therefore, generating p-values for both the intercept and the grouping factor. In the third addition, linear and grouping factors were combined in one MANOVA analysis, allowing the analysis of significant intercept, linear and grouping effects. Throughout, we compared two MANOVA approaches, using both conventional theory and numerical simulation to evaluate p-values. For the simulation-based MANOVA approaches, the obtained P-values were adjusted by comparing them to 9999 iterations of the same analysis using samples from a uniform random distribution. Rayleigh test was calculated using the function `rayleigh.test` of the package *circular* (Agostinelli and Lund 2017). The Hermans-Rasson (HR) test was performed using the R package *CircMLE* (Fitak and Johnsen 2017) and function `HermansRasson2T` (Landler et al. 2019b).

Simulation tests

All tests described above were applied to the following samples, which were drawn from the given distribution (9999 iterations), power was defined as the proportion of significant test results ($p > 0.05$). The function `rcircmix` from the package *NPcirc* (Oliveira Pérez et al. 2014) was used to generate the samples.

First, in order to test type I error, we simulated samples from circular uniform distributions (type “unif” in `rcircmix`, sample sizes: 5, 10, 15, 25, 50 and 100), from a continuous as well as binned distribution (with data aggregated into 10° bins around the circle, for all binned data the minimum sample size used was 10). The binning was done to test for sensitivity to such rounding, which can cause issues with some circular tests (Landler et al. 2020).

In a next step, we tested samples drawn from a unimodal von Mises (type “vm”, $\kappa = 1$, mean direction: 0, sample sizes: 5, 10, 15, 25, 50 and 100) and wrapped skew normal (type “wsn”, dispersion parameter = 2, skewness: 30, location parameter: 0, sample sizes: 5, 10, 15, 25, 50 and 100) distribution. We then simulated samples from both a symmetrical (mean directions: 0° and 180°) and an asymmetrical (mean directions: 0° and 240°) bimodal von Mises distribution (type “vm”, $\kappa = 1$, sample sizes: 10,20,30,50,100,200). To complete this power comparison of basic distributions, we used samples from symmetrical (mean directions: 0°, 120° and 240°) and asymmetrical (mean directions: 0°, 120° and 270°) trimodal von Mises distributions (type “vm”, $\kappa = 1$, sample sizes: 15, 30, 45, 75, 150, 300).

In order to gain insights into the usefulness of the MANOVA approach in comparison we simulated data for a hypothetical scenario. In this scenario, we work on an animal population where we know the age and sex (called group 1 and group 2 herein) of the tested individuals. The assumption is that the homeward orientation after displacement becomes more clustered with age (which in this example

ranges from 1 year to 5 years) and that group 1 is either less clustered or directionally different oriented than group 2. To keep the presentation consistent, data were plotted according to the sample size of the potential treatment combination. Two groups and five ages resulted in 10 treatment combinations, hence the minimum sample size was 10.

In order to test the presented approach on real data, we used three publically available data sets from studies on animal behavior. The first one is a data set from Gagliardo et al. (2008) embedded in the R package *circular*. In this study a translocation experiment was performed on pigeons, with three groups: control, sectioned olfactory nerve and section of the ophthalmic branch of the trigeminal nerve. The expectations were a difference between groups, an overall orientation towards the home direction, and no directional clustering in the group with the sectioned olfactory nerve.

The second data set is from Lindecke et al. (2019) involving migratory bats (*Pipistrella pygmaeus*). The expectation was a difference between two treatment groups (with a 180° switch in preferred direction) and an effect of age on the treatment effect (interaction between age and treatment).

The third data set is from Obleser et al. (2016), where the flight direction of deer was investigated. This data set includes numerous potential explanatory variables, i.e. hour of the day, temperature, light intensity, wind speed, wind direction, sun direction, hide direction, previous alignment, distance of the hide, group size, sex, age, observer direction and vegetation height, which were included in a full model. We then eliminated non-significant variables until all variables (or variable combinations in the case of cosine and sine of directions) were significant. The original study observed a north-south flight direction of roe deer, thus the dependent variable was hypothesized to be axial. We, therefore, doubled the angles and reduced to modulo 360° (= 2π in radians) for our analysis. All independent variables were added as axial as well as non-transformed directions. Hence this data set represents a complex system of multiple variables. In the published paper this was overcome by performing several different graphical and statistical analyses, in our approach we can test the underlying hypothesis in one single model.

Results

Type I error

Type I error was slightly above 0.05 at very low sample sizes for the MANOVA approach but kept expected alpha values at sample sizes of at least 10–15. The simulation MANOVA approach controlled type I error throughout. The HR test showed slightly increasing type I error rates with increasing sample sizes with binned data, as demonstrated earlier (Landler et al. 2020).

Power analysis – basic distributions

For unimodal distributions MANOVA approaches and the Rayleigh-test performed equally well, with the HR test showing just slightly lower power (Fig. 2A&B). The standard MANOVA approach showed artificially increased power levels at a sample size of 5, due to increased type 1 error rate at such small

sample sizes. This was controlled for by the simulation MANOVA approach. When evaluating symmetrical bimodal distributions (Fig. 2C), the HR test was the only test with useful power to detect clustering, however, for asymmetrical bimodal distributions (Fig. 2D), the results were similar to the unimodal distribution, with the HR test performing slightly worse than all the other tests. None of the tests were able to detect clustering in symmetrical trimodal distributions (Fig. 2E) over the range of sample size used (15 to 300). All of the tests had very similar (low) power to detect clustering of asymmetrical trimodal distributions (Fig. 2F). Taken all the analyses together, for the standard distributions, the power of the MANOVA approach was almost identical to the very powerful Rayleigh test, and only shared the same weakness of low power with symmetrical distributions, which traditionally can be overcome by simple data transformation (Batschelet 1981).

Power analysis – hypothetical examples

All tests controlled type 1 error when all treatment combinations were sampled from the same distributions (Fig. 3A). The overall power to detect orientation non-uniformity was similar between tests when the mean direction was the same for all distributions, however, the MANOVA approach in addition appropriately identified a linear and/or grouping effect (Fig. 3B-D). In the case of differing mean directions between groups, the MANOVA approaches performed very well in detecting deviations from uniformity (Fig. 4). The exception was the case of two groups with mean orientations 180° apart, in this special case only the HR test showed moderate power to detect non-random orientation (Fig. 4C). However, the MANOVA approach showed good power to detect differences between groups, and hence an effect on orientation. The MANOVA approach performed almost as well as the HR test when there was a linear effect (different between groups) added to the two groups with opposite orientations (Fig. 4D).

Real data examples

The MANOVA performed on the pigeon data showed a highly significant treatment effect and an overall significant non-uniform orientation (intercept) (Table 1), which is in line with the conclusions made in the original study (Gagliardo et al. 2008).

Table 1
MANOVA table for the pigeon example,
showing the significant overall orientation
(intercept) and treatment effect.

<i>Factor</i>	<i>Pillai</i>	<i>approx. F</i>	<i>p</i>
Intercept	0.63	89.50	< 0.01
Treatment	0.30	9.31	< 0.01

For the bat example, the MANOVA analysis elegantly combined all possible separate analysis in one model and showed that the treatment as well as age by treatment interaction were both significant (Table 2). Hence, the treatment effect changes with age group. This corroborates the results already

presented by Lindecke et al. (2019). The overall orientation (intercept) was not significant, which is expected as the treatment groups showed opposite (overall axial) orientation.

Table 2
MANOVA table for the bat example, showing the significant treatment effect, as well as interaction with age.

<i>Factor</i>	<i>Pillai</i>	<i>approx. F</i>	<i>p</i>
Intercept	0.02	0.52	0.598
Age	0.05	1.23	0.302
Treatment	0.21	5.82	< 0.01
Age by treatment	0.15	3.82	0.029

For the deer example, our approach really showed the potential of the MANOVA approach to handle multiple potential explanatory variables. While in the original paper numerous separate analyses were made and effect strengths compared (Obleser et al. 2016), we can show in our rudimentary model selection approach that only the axis of the hide as well as the previous alignment of the deer axis influences the flight direction, i.e. was retained in the final model (Table 3). All other factors were eliminated from the model because they did not reach significance. This supports the idea brought forward in the paper by Obleser et al. (2016), that animals tend to align along the north-south axis, and (even more so) use the same axis as their preferred flight direction.

Table 3
MANOVA table of the selected model, after eliminating non-significant factors, for the deer example, showing significant effects of the previous axial alignment as well as the hiding axis – expressed as cosine and sine of the axial directions in radians.

<i>Factor</i>	<i>Pillai</i>	<i>approx. F</i>	<i>p</i>
Intercept	0.07	6.67	< 0.01
Cosine of hide axis	0.08	8.30	< 0.01
Sine of hide axis	0.07	6.75	< 0.01
Cosine of previous alignment	0.06	5.90	< 0.01
Sine of previous alignment	0.02	1.51	0.224

Discussion

Our analysis shows that the MANOVA approach for circular data is as powerful to determine significant departure from uniformity as the Rayleigh-test. In addition, adding linear and grouping variables can vastly improve the power, but also enhance the range of hypothesis testing. If one suspected that a

certain linear variable or a grouping factor might be responsible for a change in orientation this can be tested explicitly without applying multiple tests (i.e., avoiding the issue of multiple hypothesis testing and inflating the type I error).

A potential weakness of the MANOVA approach is the increased type I error rate for very low sample sizes (i.e., over all of $n < 15$). However, in such cases the p-value can easily be adjusted by simulating from a random distribution, which retains power levels and controls alpha inflation.

If symmetrical multimodal orientations can be expected in a diversity of real-world situations generating circular data, the data can be transformed prior to analysis (see deer example). If cofactors (or groupings) are not needed, and the type of modality is not known, one can use the HR test, which is more powerful than most other available options in such a situation.

In contrast to other more sophisticated circular models, e.g. the Bayesian GLM approach (Cremers and Klugkist 2018), our approach provides insight into the circular response variable and its deviation from uniformity, in addition to testing for the significant contributions of groups and linear factors.

In principle, MANOVAs can be used to incorporate random variables as well, e.g. in a repeated measures MANOVA. While such approach still requires validation for the intercept approach described here, this opens the possibility to perform experiments and accompanying analyses without any of the limitations previously experienced in circular statistics. Thus, the MANOVA approach demonstrated here, in combination with recent developments in Bayesian approaches, offers a substantial broadening of the scientific questions that can be addressed statistically with circular data. The fact that it also performs well when testing very simple hypotheses that are currently evaluated by a collection of specialist tests, should make its wider adoption all the more attractive.

Declarations

Acknowledgement

LL is supported by the Austrian Science Fund (FWF, Grant Number: P32586). EPM receives funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No 948728).

Authors' contributions

LL, GDR and EPM conceptualized the problem and discussed the analytic approaches. LL prepared the code. LL, GDR and EPM interpreted the results. LL, GDR and EPM wrote the manuscript.

Availability of data and materials

Data and code are available in the supplementary material.

Competing interests

All authors declare that they have no competing interests.

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Supplementary & Figures

Figures

Figure 1

Type 1 error of the four approaches used (see legend) for continuous data (A) and binned data (B).

Figure 2

Power of the four approaches used. Distribution types (unimodal A, bimodal B and trimodal C) are shown in the plots.

Figure 3

Power of several hypothetical examples for the proposed approach. In a hypothetical 5-year study (linear variable) of population composed of two groups (grouping variable) orientation performance of subjects are measured. P-values for the MANOVA intercepts as well as Rayleigh and HR test are shown in the left panel, p-values for the linear variables in the center and grouping variables in the right panel. In the case of randomized data (A), all p-values of all measured tests remained at nominal levels. In all following

distributions the clustering of orientations continuously increased each year, however, mean orientation was identical between all groups. First, we only changed the yearly increase from 0-2 (B) to 2-4 (C). In a second step, we used different yearly clustering increases for group 1 and group 2 (D).

Figure 4

Continuation of our hypothetical example, analyzing the usefulness of the MANOVA approach for biological data. In addition to the yearly increase of clustering, we now manipulated the orientation

direction for each group, using either a 90° (A & B) or a 180° (C & D) difference. This was then either combined with a group difference in yearly clustering increase (B & D), or not (A & C). The left panel shows the power for the intercept of MANOVA approaches as well as Rayleigh and HR test, the center panel shows the power of detecting the linear effect, the right panel shows power for detecting group differences.

Supplementary Files

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