Discoveries at High Redshifts: Strong Evidence for Consecutive Expansion Epochs of the Universe that Preceded the Current Epoch of Cosmic Expansion

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DISCOVERIES AT HIGH REDSHIFTS PROVIDE EVIDENCE FOR CONSECUTIVE EXPANSION EPOCHS OF THE UNIVERSE THAT PRECEDED THE CURRENT EPOCH OF COSMIC EXPANSION

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ABSTRACT

The comparison of redshift-distance relationship for high and low-redshift supernovae revealed the surprising transition of the Universe's expansion from deceleration to acceleration. As compared to local supernovae, remote supernovae appear 10% to 25% dimmer as they are further away than expected. The expansion rate obtained for local supernovae is higher with low redshifts as compared to the expansion rate obtained for remote supernovae with high redshifts. Since observed redshifts (z) in an expanding Universe provide an estimate of recession velocities (the more shifted a spectral line is towards the red end of the spectrum (redshift), the faster that object is receding from us), therefore, it is very disturbing to find that low recession velocities ($z = 0.01$) indicate a faster rate of expansion (acceleration or speeding up), whereas high recession velocities ($z > 1$) indicate a slower rate of expansion (deceleration or slowing down). In this paper, I unravel an undiscovered aspect that perfectly mimics cosmic acceleration. Rather than "cosmic deceleration that preceded the current epoch of cosmic acceleration", I show in this paper, that "consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion" were responsible for placing remote supernovae further away than expected. As a consequence of consecutive expansion, expansion began for remote structures in preceding expansion epochs before it did for local structures in the current (or more recent) expansion epoch; remote supernovae, quasars, and gamma-ray bursts are therefore not only further away than expected, but they also happen to yield a slower rate of expansion, thereby suggesting their deceleration even with "superluminal expansion". As a result of consecutive expansion, preceding expansion epochs "appear to be decelerating or slowing down" as compared to the expansion epoch that succeeds them. The analysis is based on the redshift-distance relationship plotted for 580 type Ia supernovae from the Supernova Cosmology Project, 7 additional high-redshift type Ia supernovae discovered through the Advanced Camera for Surveys on the Hubble Space Telescope from the Great Observatories Origins Deep Survey Treasury program, and 1 additional very high-redshift type Ia supernova discovered with Wide Field and Planetary Camera 2 on the Hubble Space Telescope. The results obtained by the High-Z Supernova Search Team through observations of type Ia supernovae have also been analysed. Studies incorporating quasars and gamma-ray bursts to determine how the expansion of the Universe has changed over time have been taken into consideration as well. The results obtained in this paper have been confirmed by plotting velocity-distance relationship, expansion rate vs. time relationship, expansion factor vs. time relationship, scale factor vs. time relationship, scale factor vs. distance relationship, distance-redshift relationship, distance modulus vs. redshift relationship, and Hubble residuals, moreover, the deceleration parameter (q_0) is also found to be negative ($q_0 < 0$). An additional aspect unravelled in this paper based on "expansion epochs of the Universe "succeeding" the current (or more recent) epoch of cosmic expansion" helps alleviate the Hubble tension and an additional perplexing problem that makes the Universe appear to us as if it is not only expanding faster than before, but is also younger than before.

Key words: accelerating Universe – cosmology: observations – cosmology: theory – dark energy.

1. INTRODUCTION

The research conducted by the High-Z Supernova Search Team (Riess et al. 1998) and by the Supernova Cosmology Project (Perlmutter et al. 1999) by using type Ia supernovae as standard candles resulted in a very surprising discovery that made the teams win the 2011 Nobel Prize in Physics. By comparing the brightness of the very distant, remote supernovae with the brightness of the nearby, local ones, remote supernovae were found to be 10% to 25% dimmer than the nearby, local supernovae – a direct indication that remote supernovae were further away than expected. Based on this direct observation, a surprising feat was found being displayed by the Universe, a feat so extraordinary that the remarkable results obtained were not even expected. It was the startling discovery of Universe expanding at an accelerating rate. A research that was aimed at observing the expected deceleration of the Universe under the gravitational influence of matter was welcomed by something completely unexpected.

Incorporating 16 type Ia supernovae in their study, the High-Z Team measured not only the distances to those supernovae, but also the velocities at which those supernovae were receding from us. Based on these measurements, the team was stunned to find that the Universe was expanding 10% to 15% more slowly in the past than can be accounted for without a cosmological constant (Science News 1998).

Here is an excerpt describing this astounding discovery from "Written in the stars" by The Nobel Committee for Physics – The Royal Swedish Academy of Sciences, "By establishing the distance to the supernovae and the speed at which they are moving away from us, scientists hoped to reveal our cosmic fate. They expected to find signs that the expansion of the Universe was slowing down, which would lead to equilibrium between fire and ice. What they found was the opposite – the expansion was accelerating".

A mysterious and hypothetical energy of unknown origin, having no explanation in fundamental physics, rightfully termed as "dark energy" is considered responsible for accelerating the Universe's expansion. The simplest explanation for dark energy is the cosmological constant (denoted by Λ) that was introduced by Einstein in 1917 into his gravitational field equations to account for what matched well with observations back then – a "static Universe" – a Universe "suspended between gravity pulling inward and the cosmological constant making the universe expand" (Kirshner 2004). Cosmological constant was essentially introduced to help keep the Universe distended by providing the necessary repulsion against the gravitational attraction of matter since it was not known back then that the Universe was expanding as observations were confined to the stars of our home galaxy.

However, in 1929, the revolutionizing discovery by Hubble from his observations of distant galaxies through the 100-inch Hooker telescope atop Mount Wilson in California showed that the Universe was expanding; it was not at all "static" as was being considered. This observation of expanding Universe led to the banishment of the cosmological constant. However, the startling discovery of Universe expanding at an accelerating rate has caused the dramatic return of the discarded constant.

Cosmological constant denotes the intrinsic energy associated with empty space or vacuum. According to Riess (2012), "A contemporary particle physicist would call it vacuum energy, that is, "the zero point energy summing all possible particles in the vacuum," and then would complain that calculating it yields a nonsensically enormous answer". The theoretically-calculated value of the cosmological constant according to the quantum field theory results in a colossal discrepancy – a mismatch between the observed and the theoretically-calculated value by 120 orders of magnitude (10^{120}).

According to Durrer (2011), "our single indication for the existence of dark energy comes from distance measurements and their relation to redshift. Supernovae, cosmic microwave background anisotropies and observations of baryon acoustic oscillations simply tell us that the observed distance to a given redshift z is larger than the one expected from a Friedmann–Lemaître universe with matter only and the locally measured Hubble parameter". Also, according to Mohayaee et al. (2021), "it was inferred (based on type Ia supernovae observations) that the Hubble expansion rate is accelerating as if driven by a positive Cosmological Constant Λ in Einstein's theory of gravity. This is still the only *direct* evidence for the 'dark energy' that is the dominant component of today's standard Λ CDM cosmological model. Other data such as baryon acoustic oscillations (BAO) in the large-scale distribution of galaxies, temperature fluctuations in the cosmic microwave background (CMB), measurement of stellar ages, the rate of growth of structure, *etc* are all 'concordant' with this model but do not provide independent evidence for accelerated expansion".

The expansion history of the Universe is depicted by the Hubble diagram as shown in Figure 1 and Figure 5 (plotted by using the Supernova Cosmology Project data for 580 type Ia supernovae from Union 2 (Amanullah et al. 2010) and Union 2.1 (Suzuki et al. 2012), 7 additional high-redshift type Ia supernovae discovered through the ACS (Advanced Camera for Surveys) on the Hubble Space Telescope from the GOODS (Great Observatories Origins Deep Survey) Treasury program (joint work conducted by Giavalisco et al. 2004 and Riess et al. 2004), and 1 additional very high-redshift type Ia supernova discovered with WFPC2 (Wide Field and Planetary Camera 2) on the Hubble Space Telescope (Gilliland et al. 1999)).

The observed deviation from redshift-distance linearity in Figure 5 indicates an accelerating Universe since the distances to the remote supernovae are larger than expected with respect to the nearby, local ones. The value of slope (or the expansion rate measured in $\text{km s}^{-1} \text{Mpc}^{-1}$) is higher for the local structures and lower for the remote structures, suggesting that the Universe is accelerating now and was decelerating in the past. “A purely kinematic interpretation of the SN Ia sample provides evidence at the greater than 99% confidence level for a transition from deceleration to acceleration or, similarly, strong evidence for a cosmic jerk” (Riess et al. 2004).

By comparing the redshift-distance relationship for remote and local type Ia supernovae, cosmologists have presented ground-breaking conclusive evidence regarding the transition of the Universe’s expansion that favours a recent accelerated expansion and a previous decelerated expansion. “Observations of Type Ia supernovae (SNe Ia) at redshift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be *accelerating*” (Riess et al. 2004). It is believed that the Universe was decelerating in the past due to the gravitational attraction of matter (Riess et al. 2001, Riess 2012). “A single SN Ia at $z \approx 1.7$, SN 1997ff, discovered with WFPC2 on the *Hubble Space Telescope* (HST) (Gilliland et al. 1999), provided a hint of past deceleration” (Riess et al. 2004).

Why does it appear that the Universe was decelerating in the past and is accelerating now? Why do remote supernovae appear 10% to 25% dimmer than the nearby, local supernovae, thereby making us believe that they are further away than expected? Could distant supernovae be appearing dim due to intervening dust? Or could it be that those distant supernovae have different properties as compared to the nearby, local supernovae? These possibilities have already been taken into account and have therefore been addressed – dust is not a factor, similarly, the brightness of local and remote supernovae differing due to property mismatch brought about by evolution is also not a factor (Riess 2000, Coil et al. 2000, Leibundgut 2001, Sullivan et al. 2003, Riess et al. 2004).

Some possible ideas that can help explain the accelerated expansion of the Universe have succinctly been introduced to us by Durrer (2011), these include, replacing cosmological constant by some other form of ‘dark energy’ like an ordinary or a tachyonic scalar field (Linder 2008), modifying the left-hand side of Einstein’s equation, that is, modifying gravity (Durrer & Maartens 2008, 2010), modifying Einstein–Hilbert action by $R \rightarrow f(R)$ (Capozziello & Francaviglia 2008), theories with extra dimensions (Maartens 2004, Sahni 2005, Durrer 2005, Dvali et al. 2000, 2000). However, according to Durrer (2011), “All these models are quite speculative. Furthermore, even if one of these models is realized in nature, the question of why we do *not* measure a cosmological constant gravitationally remains”.

To answer this question, a model based on massive gravity (de Rham et al. 2011, de Rham 2014) makes modification to gravity by assuming that the graviton (a massless, hypothetical particle representing quantum of energy exchanged in a gravitational interaction) has mass; such an assumption eliminates the need for dark energy and explains cosmic acceleration by rendering gravitational influence weaker on large scales. Since massive gravity imparts mass to a graviton and makes modification to gravity, therefore, in such a case, a gravitational wave is expected to travel at a velocity less than the velocity of light. However, the “detection of the binary neutron star merger GW170817 and its associated electromagnetic counterparts have shown almost coincident observation of both signals” (Ezquiaga & Zumalacárregui 2017), therefore, many theories based on modified gravity as alternatives to dark energy have been ruled out. In other words, massive gravity and additional models that are closely related to it, such as bigravity (Hassan & Rosen 2012, Akrami et al. 2013, 2015) and multi-gravity (Hinterbichler & Rosen 2012) “remain viable as long as the graviton mass is small” (Ezquiaga & Zumalacárregui 2017).

Another possibility that could mimic cosmic acceleration deals with inhomogeneities on small scales; we are probably located in a region where the expansion is “literally faster” than the remote background due to inhomogeneities (Räsänen 2008, Clarkson & Maartens 2010, Colin et al. 2011, Räsänen 2011, Colin et al. 2019). Instances holding bulk flow responsible as a possible contaminant while determining the expansion rate of the Universe have been noted in literature. Kashlinsky et al.

(2008), Watkins et al. (2009), and Lavaux et al. (2010) found bulk flow that extends out to at least 300 Mpc, 100 Mpc, and 120 Mpc respectively. Study conducted by Colin et al. (2011) found that at low redshifts ($z < 0.05$), an isotropic model such as Λ CDM is barely consistent with the SNe Ia data. According to them, there is a bulk flow of 260 km s^{-1} extending out to $z \sim 0.06$, which disagrees with Λ CDM. However, they suggest that at higher redshifts ($z > 0.15$), the agreement between the SNe Ia data and the Λ CDM model does improve.

Keeping this in mind that inhomogeneities are suggested to exist at $z < 0.15$, Kenworthy et al. (2019) conducted a study by using a sample of 1295 type Ia supernovae over a redshift range $0.01 < z < 2.26$ that helped them conclude that local structure does not impact measurement of the Hubble constant (H_0); the study conducted by them therefore rejects the dipole/bulk flow interpretation and reconfirms cosmic acceleration. Study conducted using 740 spectroscopically-confirmed type Ia supernovae covering the redshift range $0.01 < z < 1.2$ (Betoule et al. 2014) also confirmed cosmic acceleration. Another study incorporating 1048 spectroscopically-confirmed type Ia supernovae in the redshift range $0.01 < z < 2.3$ (Scolnic et al. 2018) also confirmed cosmic acceleration; according to them, the evidence for an accelerating Universe using type Ia supernovae alone is $> 6\sigma$.

If this is the case, then taking into account those type Ia supernovae observations that alone provide $> 6\sigma$ evidence for an accelerating Universe, the aim of this paper is to shed light on an undiscovered aspect that mimics cosmic acceleration so perfectly well that it tricks us into believing that the Universe was decelerating in the past at high redshifts and is accelerating now at low redshifts. The undiscovered aspect unravelled in this paper deals with “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”. Since this undiscovered aspect is perfectly analogous to “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), therefore, it is most likely that it was this undiscovered aspect that got interpreted as cosmic acceleration.

2. THE SURPRISING TRANSITION OF THE UNIVERSE’S EXPANSION FROM DECELERATION TO ACCELERATION: ANALYSING REMOTE AND LOCAL OBJECTS

Figure 1. Distance modulus versus redshift relationship for 588 type Ia supernovae (from $z = 0.015$ to $z = 1.7$) showing the transition of the Universe’s expansion from deceleration to acceleration (red curve) (as measured from past to present – high redshift to low redshift – remote to local) (580 type Ia supernovae plotted by using the data (Union 2 and Union 2.1) from the Supernova Cosmology Project, 7 additional high-redshift type Ia supernovae discovered through the ACS (Advanced Camera for Surveys) on the Hubble Space Telescope from the GOODS (Great Observatories Origins Deep Survey) Treasury program, and 1 additional very high-redshift type Ia supernova discovered with WFPC2 (Wide Field and Planetary Camera 2) on the Hubble Space Telescope).

“The observed redshift of a distant galaxy or quasar gives a direct estimate of the universal expansion (i.e. the “recession velocity”). For instance, a redshift of $z = 0.1$ corresponds to a velocity of $30,000 \text{ km s}^{-1}$ ” (ESO 1998). Once the redshifts and the distances are known, “The relation between redshift and distance is then used to determine the expansion rate of the Universe” (Riess 2012).

Distance modulus versus redshift relationship for near and far type Ia supernovae plotted in Figure 1 is the new Hubble

diagram. Supernovae observations ($z > 1$) confirmed the result that the Universe was decelerating or slowing down in the past before it began accelerating (Riess 2012). Therefore, according to Figure 1 that clearly depicts the transition of the Universe's expansion from deceleration to acceleration (red curve), a nearby, local supernova at $z = 0.015$ is expanding at a faster rate as compared to a supernova which is further away than expected at $z = 0.479$, or as compared to another supernova which is even further away than expected at $z = 1.7$. Therefore, by just looking where the objects lie on the Hubble diagram (Figure 1 and Figure 5), one gets to know if the Universe is accelerating or decelerating over time. High-redshift remote objects that are further away than expected are expanding at a slower rate (decelerating) as compared to the low-redshift local objects, and this is where a paradoxical trend gets uncovered.

We are well aware that only superluminal expansion can cause a distant object to exhibit an observational redshift (z) of 1.7. According to Davis and Lineweaver (2004), "Recession velocities exceed the speed of light in all viable cosmological models for objects with redshifts greater than $z \sim 1.5$ ". Therefore, it is very disturbing to accept that superluminal remote expansion ($z = 1.7$) indicates "slowing down" or deceleration as compared to subluminal local expansion ($z = 0.015$) – completely counterintuitive. Moreover, based on the velocity versus luminosity-distance relationship for type Ia supernovae (Figure 6 and Figure 8) as well as the plot by the High-Z Supernova Search Team (Figure 7), it is very disturbing to find that further-away-than-expected remote supernovae even with high recession velocities (60% of speed of light) are yielding a slower rate of expansion (deceleration) as compared to the nearby, local supernovae that are yielding a faster rate of expansion (acceleration) even with low recession velocities (just 1% of speed of light). Confidently enough, remote expansion at 60% of speed of light is way faster than local expansion at just 1% of speed of light. Then why should such a paradoxical trend even be observed wherein remote expansion at 60% of speed of light indicates a slower rate of expansion (deceleration), whereas local expansion at just 1% of speed of light indicates a faster rate of expansion (acceleration)?

Such a paradoxical trend as discussed above is not limited only to type Ia supernovae observations. To study how the expansion of the Universe has changed over time, Risaliti and Lusso (2015) used quasars as standard candles (up to $z \approx 6$) as quasars are available in large numbers especially at much higher redshifts as compared to type Ia supernovae. The Hubble diagram (distance modulus vs. redshift relationship) obtained by them for those quasars (extending up to $z > 6$) (Figure 2) is in perfect agreement with the Hubble diagram for type Ia supernovae in the redshift (z) range of 0.01 to 1.4. Therefore, the study conducted by them using quasars is also consistent with past deceleration at high redshifts and recent acceleration at low redshifts.

Figure 2. "Hubble Diagram for the quasar sample (small gray points) and supernovae (cyan points) from the Union 2.1 sample (Suzuki et al. 2012). The large red points are quasar averages in small redshift bins". Credit: Risaliti G., Lusso E., "A HUBBLE DIAGRAM FOR QUASARS", *ApJ*, Vol. 815, 33, page 7, year 2015. © AAS. Reproduced with permission. <https://dx.doi.org/10.1088/0004-637X/815/1/33>

Studies incorporating gamma-ray bursts (GRBs) have gone even beyond by focusing at much higher redshifts (up to $z \approx 8$ and $z \approx 9$ (Dirirsa et al. 2019 and Demianski et al. 2017 respectively)). GRBs, just like type Ia supernovae, have also

been used as standard candles to probe the expansion history of the Universe. The results obtained using GRBs, just like quasars and type Ia supernovae observations, are also consistent with cosmic acceleration, that is, past deceleration at high redshifts and recent acceleration at low redshifts as shown below in Figure 3 (red curve).

Figure 3. Distance modulus versus redshift relationship for 588 type Ia supernovae (from $z = 0.015$ to $z = 1.7$) from Figure 1 and for 216 GRBs (from $z = 0.414$ to $z = 9.3$).

Now, according to Riess et al. (2001), "If the cosmological acceleration inferred from SNe Ia is real, it commenced rather recently, at $0.5 < z < 1$. Beyond these redshifts, the universe was more compact and the attraction of matter dominated the repulsion of dark energy. At $z > 1$, the expansion of the universe should have been decelerating".

Quasars, as depicted by the Hubble diagram (Figure 2) which extends up to $z > 6$, are way beyond, GRBs, as depicted by the Hubble diagram (Figure 3) which extends up to $z > 9$, are even way beyond, at these redshifts, the Universe was more compact, and the expansion of the Universe should have been decelerating according to Riess et al. (2001). However, we know that only superluminal expansion can cause distant objects to exhibit such high observational redshifts ($z \approx 6$ for a quasar, and $z \approx 9$ for a GRB), furthermore, we measure how fast the Universe is expanding now, and how fast the Universe was expanding in the past based on the distance and the redshift of remote and local objects, for instance, we compare the distance and the redshift of a remote object with the distance and the redshift of a local object before coming to this conclusion that the Universe is expanding faster now than it was in the past, in other words, we are literally comparing superluminal remote expansion with subluminal local expansion, therefore, advocating that very distant, remote expansion (which happens to be superluminal) is an indication of cosmic deceleration or slowing down under the gravitational attraction of matter (since $z > 1$) sounds very disturbing as it is completely counterintuitive (a paradox) – as compared to a minuscule redshift of 0.01 (SN Ia) that indicates acceleration, a high redshift of 1.7 (SN Ia) indicates deceleration, an extreme redshift of 6 (QSO) also indicates deceleration, and surprisingly, a whopping redshift of 9.3 (GRB) also indicates deceleration – this sounds weird; it makes no sense that every superluminal remote expansion can simultaneously be an indication of deceleration or slowing down under the gravitational influence of matter as compared to subluminal local expansion (a mere redshift of 0.01). One should answer, how can superluminal remote expansion even be scientifically justified as deceleration or "slowing down" under gravity as compared to subluminal local expansion?

Being unable to answer this question, one would readily put forward a lame excuse to get this paradoxical trend swept under the rug by suggesting that recession velocities are meaningless and so is superluminal expansion. However, as pointed out by Davis and Lineweaver (2004), "Expansion has no meaning without well-defined concepts of velocity and distance. If recession velocity were meaningless we could not refer to an 'expanding universe' and would have to restrict ourselves to some operational description such as 'fainter objects have larger redshifts'". It is a well-known observational fact, and also according to Davis and Lineweaver (2004), "higher redshift galaxies are more distant from us and receding faster than lower redshift galaxies". Clearly, "expanding Universe", "accelerating Universe", "Universe's expansion is *not* slowing down with

time, as expected, *but is speeding up*”, “remote supernovae, quasars, and GRBs are *not receding as quickly* as expected from observations of the local ones” (as per the discovery of accelerating Universe), and then, the unit of expansion rate “ $\text{km s}^{-1} \text{Mpc}^{-1}$ ”, all these necessarily require a velocity description to make sense in an expanding Universe.

Therefore, with respect to the very surprising discovery of accelerating Universe (the apparent transition of the Universe’s expansion from deceleration to acceleration), how can subluminal local expansion ($z = 0.01$) even be scientifically justified as an expansion faster than superluminal remote expansion ($z = 1.7$, $z = 6$, and $z = 9.3$)?

Figure 4. Distance modulus versus redshift relationship for type Ia supernovae and GRBs (from $z = 0.162$ to $z = 9.3$). All objects ($z < 0.15$) have been discarded since inhomogeneities on small scales (that are suggested to exist at $z < 0.15$) could mimic cosmic acceleration.

Perhaps we are located in a region where the expansion is “literally faster” than the remote background due to inhomogeneities, therefore, as a test, here in Figure 4, we have plotted the distance modulus versus redshift relationship by taking into account only those objects with redshifts (z) ranging from 0.162 to 9.3 (≈ 724 Mpc onwards). By doing so we have ensured that inhomogeneities on small scales ($z < 0.15$) would get washed away and we would be left only with those redshifts that happen to arise solely due to Universe’s expansion.

Surprisingly, even after discarding all such objects ($z < 0.15$), Figure 4 still continues to exhibit the similar trait (red curve) exhibited by Figure 1, Figure 2, and Figure 3 while considering all objects ($z < 0.15$); a supernova at $z = 0.162$ is still expanding comparatively faster than a supernova at $z = 1.7$, a quasar at $z = 6$, or as compared to the GRB at $z = 9.3$, therefore, even after discarding very local objects ($z < 0.15$) where inhomogeneities are believed to play a role in mimicking cosmic acceleration, we still continue witnessing the very disturbing paradoxical trend wherein subluminal local expansion ($z = 0.162$) still turns out to be faster as compared to superluminal remote expansion ($z = 1.7$, $z = 6$, and $z = 9.3$), and therefore, the question still remains open, why would further-away-than-expected remote object expanding superluminally end up yielding a slower rate of expansion, thereby suggesting its deceleration or “slowing down” as compared to a local object expanding subluminally?

Since “local structure does not impact measurement of the Hubble constant” (Kenworthy et al. 2019), redshifts as low as 0.01 (Betoule et al. 2014, Scolnic et al. 2018, Kenworthy et al. 2019) can also be used to determine Universe’s expansion rate to confirm recent acceleration at low redshifts and past deceleration at high redshifts, therefore, in such a case, the above question becomes even more serious and rightfully demands an answer, rather than being swept under the rug.

Since evidence for an accelerating Universe from type Ia supernovae-only sample is $> 6\sigma$ (Scolnic et al. 2018), therefore, we will conduct an analysis (hereafter) based on those type Ia supernovae observations (keeping QSOs and GRBs still in mind) and unravel how “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion” can mimic cosmic acceleration.

A very low-redshift local supernova ($z = 0.015166$) falling within the linear regime of the Hubble diagram (Figure 5) yields a slope of $2.2379 \times 10^{-18} \text{ m s}^{-1} \text{ m}^{-1}$ ($\approx 69 \text{ km s}^{-1} \text{ Mpc}^{-1}$) – this is the local expansion rate and it is significantly much higher than the remote measurement while incorporating further-away-than-expected remote supernova with very high-redshift ($z = 1.7$).

Figure 5. Redshift-distance relationship for 588 type Ia supernovae from Figure 1. The blue line indicates the linear redshift-distance relationship exhibited by low-redshift local structures. The deviation from linearity at high redshifts indicates an accelerating Universe since the distances to remote supernovae are larger than expected with respect to local ones. The slope is steeper for the local structures suggesting a faster rate of expansion (acceleration) and shallower for the remote structures suggesting a slower rate of expansion (deceleration).

However, since observed redshifts in an expanding Universe provide an estimate of recession velocities as shown in Figure 6, Figure 7, and Figure 8, therefore, the notion of deceleration or “slowing down” becomes very problematic at those high redshifts. As discussed earlier, “higher redshift galaxies are more distant from us and receding faster than lower redshift galaxies” (Davis and Lineweaver 2004), therefore, it becomes impossible to accept that further-away-than-expected remote supernova with high redshift ($z = 1.7$) indicates deceleration or “slowing down” as compared to a local supernova with low redshift ($z = 0.015166$).

Furthermore, it has also been acknowledged by Nugent (Research News 2001) while referring to this highly-redshifted supernova with a redshift (z) of 1.7, “highly redshifted objects are moving away from us so fast that time dilation is large”. However, something very strange that does not go hand in hand simultaneously with the above statement is then advocated while referring to this same highly-redshifted supernova with a redshift of 1.7, “the supernova allows us to glimpse an era when matter in the universe was still relatively dense and expansion was still slowing under the influence of gravity. More recently the dark energy has begun to predominate and expansion has started to speed up” (Nugent, Research News 2001).

Above statements clearly imply that the “highly-redshifted supernova with a redshift of 1.7 is moving away from us so fast (of course, due to expansion) that it is also providing us an indication of past deceleration or slowing down under the gravitational influence of matter” – this sounds paradoxical; how can “moving away from us so fast” simultaneously be an indication of “slowing down” under gravity?

3. ANALYSING THE SUPERNOVA SN 1995K AND ADDITIONAL TYPE Ia SUPERNOVAE OBSERVATIONS

SN 1995K was the most distant type Ia supernova discovered in April, 1995 by the High-Z Supernova Search Team. As compared to the nearby type Ia supernovae that happen to fall within the linear regime of the Hubble diagram as shown in Figure 6, SN 1995K happens to deviate from linearity as it is further away than expected. SN 1995K showed deceleration parameter (q_0) = -0.6 (Schmidt 2012). A negative value for the deceleration parameter clearly implied that the Universe was accelerating. However, SN 1995K was a single supernova that indicated cosmic acceleration; additional supernovae were therefore required to measure the expected cosmic deceleration. Surprisingly, further supernovae observations at even larger distances confirmed an accelerating Universe (Figure 7).

Figure 6. Velocity-distance relationship (Hubble Diagram of SNe Ia) showing SN 1995K at a redshift (z) of 0.479 from the proposal put forward by the High-Z Supernova Search Team. Credit: Schmidt B. P., Reviews of Modern Physics, Vol. 84, 1151, page 1158, year 2012, reprinted with permission, Copyright (2012) American Physical Society. <https://doi.org/10.1103/RevModPhys.84.1151>

SN 1995K plotted above in Figure 6 was the most distant type Ia supernova discovered in 1995 by the High-Z Team ($D = 9.7211$ Gly, $z = 0.479$) – this yields a slope of $1.5617 \times 10^{-18} \text{ m s}^{-1} \text{ m}^{-1}$ ($\approx 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Nearby, local supernovae that happen to fall within the linear regime of the Hubble diagram have also been plotted; consider one such local supernova ($D = 0.4877$ Gly, $z = 0.0333$) – this yields a slope of $2.1641 \times 10^{-18} \text{ m s}^{-1} \text{ m}^{-1}$ ($\approx 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$). The comparison of expansion rate ($\text{km s}^{-1} \text{ Mpc}^{-1}$ or $\text{m s}^{-1} \text{ m}^{-1}$) for these supernovae shows that SN 1995K is expanding at a slower rate (decelerating) as compared to this nearby supernova obeying the linear Hubble expansion.

Since observed redshift is providing an estimate of recession velocity as shown above in Figure 6 itself, therefore, advocating that SN 1995K is expanding at a slower rate as compared to the nearby supernova sounds completely counterintuitive – the recession velocity of SN 1995K is ≈ 14 times higher than the recession velocity of the nearby supernova. Therefore, why is it that SN 1995K even with high recession velocity appears to be expanding at a slower rate as compared to a local supernova with low recession velocity? Same question also arises while taking into account the following plot by the High-Z Team.

Figure 7. Observations of additional type Ia supernovae by the High-Z Supernova Search Team. The plot confirmed an accelerating Universe – remote supernovae that are further away than expected are expanding at a slower rate (decelerating) as compared to the nearby, local supernovae. Credit: High-Z Supernova Search Team.

Figure 7 depicts the result of additional type Ia supernovae observations carried out by the High-Z Team at even larger distances (high redshifts) that helped them confirm Universe's acceleration. Distant supernovae were dimmer than expected (as they were further away than expected) and the expansion rate for them was found to be lower than the expansion rate for the nearby, local supernovae. Figure 7 clearly shows the transition of the Universe's expansion from deceleration to acceleration – Universe was expanding slowly (decelerating) in the past and is expanding faster (accelerating) now.

However, if we look at the observed redshifts that provide an estimate of recession velocities as shown in Figure 7, then there seems to be a conundrum, it is very disturbing to find that local recession velocities ranging from just 1% to 10% of speed of light indicate a faster rate of expansion (acceleration), whereas remote recession velocities ranging from 30% to 60% of speed of light indicate a slower rate of expansion (deceleration).

Why is it that a remote object which happens to be further away than expected is yielding a lower value of slope (a slower rate of expansion or deceleration) even with high recession velocity (60% of speed of light) as compared to a nearby, local object that is yielding a higher value of slope (a faster rate of expansion or acceleration) even with low recession velocity (just 1% of speed of light)? Such an intriguing attribute is also encountered while taking into account the following plot.

Figure 8. “Velocity versus luminosity-distance for type Ia supernovae (filled circles), S-Z clusters (open circles) and gravitational lens time-delay systems (filled triangles), with $z > 0.05$ ”. Credit: Blanchard A., et al., A&A, Vol. 412, 35, page 39, year 2003, reproduced with permission © ESO. <https://doi.org/10.1051/0004-6361:20031425>

Figure 8 illustrated above from Blanchard et al. (2003) shows remote measurement yielding an expansion rate of $46 \text{ km s}^{-1} \text{ Mpc}^{-1}$ which is significantly lower than the local measurement of $72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ obtained from the Hubble Key Project determination (Freedman et al. 2001). The expansion rate measured for local objects with low recession velocities is significantly greater than the expansion rate measured for remote objects with high recession velocities.

“It has been noted by Zehavi et al. (1998) that the SNe Ia out to 7000 km s^{-1} exhibit an expansion rate that is 6% greater than that measured for the more distant objects” (Riess et al. 1998). One might say that local void is expanding faster than the remote expansion rate. According to Riess et al. (1998), “In principle, a local void would increase the expansion rate measured for our low-redshift sample relative to the true, global expansion rate. Mistaking this inflated rate for the global value would give the false impression of an increase in the low-redshift expansion rate relative to the high-redshift expansion rate”. However, according to Riess et al. (1998), “only a small fraction of our nearby sample is within this local void, reducing its effect on the determination of the low-redshift expansion rate”. Furthermore, the reanalysis carried out (Riess et al. 1998) by discarding the seven SNe Ia within 7000 km s^{-1} (108 Mpc for $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$) ruled out the possibility of local void and confirmed cosmic acceleration.

Here the observed redshift has clearly been interpreted as a recession velocity of 7000 km s^{-1} by the researchers to determine the expansion rate of the local objects ($65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ($7000 \text{ km s}^{-1}/108 \text{ Mpc}$)); the researchers then compare this local expansion rate with the expansion rate of more distant objects to find that the expansion rate for local objects is 6% greater than the expansion rate measured for the more distant objects. Local determination of the expansion rate incorporating a local recession velocity of 7000 km s^{-1} makes it very prudent to consider that the redshifts of the more distant objects might have also been interpreted in terms of recession velocities by the researchers to determine the expansion rate of remote

objects. However, there is a problem associated with this; a paradoxical problem that we have been focusing upon right from the very beginning of this paper. The recession velocities of local structures are not high to justify a faster rate of expansion (acceleration) as compared to the recession velocities of remote structures that are not low to justify a slower rate of expansion (deceleration or “slowing down”). Therefore, why should it be that a remote object with high recession velocity is not only further away than expected, but it is also yielding a lower value of slope (or a slower rate of expansion, thereby suggesting its deceleration) as compared to a nearby, local object with low recession velocity?

4. AN UNDISCOVERED ASPECT

If there can be an epoch of “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), then there can also be epochs of “consecutive expansion that preceded the current epoch of cosmic expansion”.

It remains undiscovered that an object that begins expanding before in the preceding epoch of cosmic expansion (as a consequence of consecutive expansion) will not only be further away than expected, but it will also be yielding a lower value of slope (or a slower rate of expansion) even with high recession velocity as compared to an object with low recession velocity that begins expanding comparatively later in the current (or more recent) epoch of cosmic expansion.

Such an interpretation does not place an observer in any special or privileged position within the Universe, because, to an observer in any expansion epoch, preceding expansion epochs will appear to be decelerating or “slowing down”, whereas the observer’s current epoch of cosmic expansion that happens to “succeed” those preceding expansion epochs will appear to be accelerating.

Logically, an object that begins expanding before in the preceding epoch of cosmic expansion has an utmost probability of being further away than expected; the observational fact, that such an object, which happens to be further away than expected, yields a lower value of slope (a slower rate of expansion or deceleration) even with high recession velocity as compared to an object with low recession velocity is the most compelling (independent) evidence in favour of this undiscovered aspect.

Plotting together the high-recession-velocity remote structures that began expanding before and the low-recession-velocity local structures that began expanding comparatively later causes the Hubble diagram to deviate from linearity, whereas the comparison of slope and thus the expansion rate obtained from such objects causes the high-recession-velocity remote structures to appear as if they are receding slowly (not as fast) as compared to the low-recession-velocity local structures.

It must be noted that even with high recession velocity (no matter how high), an object that begins expanding before will never yield a value of slope, or the expansion rate that is higher than the value of slope, or the expansion rate for an object with low recession velocity (no matter how low) that begins expanding comparatively later. Comparing the slope and thus the expansion rate of such objects results in the apparent transition of the Universe’s expansion from deceleration to acceleration – an object with high recession velocity that began expanding before will be further away than expected and will appear to be decelerating, whereas an object with low recession velocity that began expanding comparatively later will appear to be accelerating.

It is merely this comparison that makes it appear that the Universe is accelerating now even with low recession velocities (low redshifts) and was decelerating in the past even with high recession velocities (high redshifts). Requiring mysterious dark energy of unknown origin to explain this apparent transition of the Universe’s expansion from deceleration to acceleration would inevitably complicate things to an unimaginable extent.

5. A SIMPLE NUMERICAL PROOF USING HIGH AND LOW-VELOCITY TEST PARTICLES

Let us consider two test particles – particle A and particle B. Particle A has an extreme recession velocity of 10^6 m s^{-1} , whereas particle B has a recession velocity of just 0.4 m s^{-1} .

Initially, particle A begins expanding, after 4 seconds, particle B begins expanding and is observed for 1 second. By the time particle B is observed for 1 second, particle A has already been expanding for 5 seconds.

Since particle A began expanding before, therefore, logically, as compared to particle B, particle A will undoubtedly be further away than expected.

The distance covered by particle A in 5 seconds with a recession velocity of 10^6 m s^{-1} is $5 \times 10^6 \text{ m}$, whereas the distance covered by particle B in 1 second with a recession velocity of 0.4 m s^{-1} is 0.4 m .

The slope or the expansion rate for these particles is obtained by using the relation,

$$H = \frac{v}{D} \quad (1)$$

where H is the slope or the expansion rate ($\text{m s}^{-1} \text{ m}^{-1}$), v is the recession velocity of the particles (m s^{-1}), and D is the distance covered by them (m). The inverse of slope or the expansion rate ($1/H$ or H^{-1}) gives back the time (t_H) in seconds.

The value of slope or the expansion rate for particle A with a whopping recession velocity of 10^6 m s^{-1} turns out to be $0.2 \text{ m s}^{-1} \text{ m}^{-1}$, whereas for particle B, the value of slope or the expansion rate with a mere recession velocity of 0.4 m s^{-1} turns out to be $1 \text{ m s}^{-1} \text{ m}^{-1}$.

The value of slope or the expansion rate for particle A even with an extreme recession velocity of 10^6 m s^{-1} is much lower (5 times lower) than the value of slope or the expansion rate for particle B even with a mere recession velocity of 0.4 m s^{-1} . Does this scientifically imply that particle A with high recession velocity of 10^6 m s^{-1} is decelerating, whereas particle B with low recession velocity of 0.4 m s^{-1} is accelerating?

10^6 m s^{-1} – recession velocity of particle A is 2.5×10^6 times higher than the recession velocity of particle B! Such a high recession velocity of particle A does not indicate in any way a low recession velocity, or a slower rate of expansion, or deceleration due to the gravitational attraction of matter!

Then why is particle A with a whopping recession velocity of 10^6 m s^{-1} yielding a lower value of slope or a slower rate of expansion, thereby suggesting deceleration as compared to particle B with a minuscule recession velocity of 0.4 m s^{-1} ?

There is absolutely no other reason for an object with high recession velocity to yield a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) and then be further away than expected, unless it began expanding before.

As already discussed, even with high recession velocity (no matter how high), an object that begins expanding before will never yield a value of slope, or the expansion rate that is higher than the value of slope, or the expansion rate for an object with low recession velocity (no matter how low) that begins expanding comparatively later.

Therefore, we should never compare the slope and thus the expansion rate of such objects, doing so, without any doubt, will result in the apparent transition of the Universe’s expansion from deceleration to acceleration – an object with high recession velocity that began expanding before will be further away than expected and will appear to be decelerating, whereas an object with low recession velocity that began expanding comparatively later will appear to be accelerating. Requiring mysterious dark energy of unknown origin to explain this apparent transition from deceleration to acceleration would unnecessarily complicate things to an unimaginable extent.

Comparing the slope and thus the expansion rate of high-recession-velocity object that began expanding before, with the slope and thus the expansion rate of low-recession-velocity object that began expanding comparatively later, causes the high-recession-velocity object to appear as if it is receding slowly (not as fast) as compared to the low-recession-velocity object. It is only the result of this comparison that particle A even with an extreme recession velocity of 10^6 m s^{-1} appears to be expanding at a slower rate (decelerating) as compared to particle B with a mere recession velocity of 0.4 m s^{-1} .

6. GRAPHICAL CONFIRMATION

To further confirm the credibility of this undiscovered aspect, it is necessary to plot some graphical relationships for such a scenario where an object with high recession velocity (high redshift) begins expanding before in the preceding expansion epoch, and an object with low recession velocity (low redshift) begins expanding comparatively later in the current (or more recent) expansion epoch. Therefore, we will consider 11 test particles that have been assigned random recession velocities. These test particles expand consecutively (one particle after another). Based on calculations, we will plot some graphical

relationships to verify if this undiscovered aspect perfectly mimics cosmic acceleration.

6.1. Velocity-distance relationship

Initially, particle A (3517.60 m s^{-1}) begins expanding, 0.1 second later, particle B (2983.93 m s^{-1}) begins expanding, the expansion of particle B is followed by the expansion of particle C (2648.64 m s^{-1}) after another 0.1 second. Consecutive expansion of particles continues in the same way for particle D (2496.43 m s^{-1}), particle E (2223.52 m s^{-1}), particle F (1676.20 m s^{-1}), particle G (1219.96 m s^{-1}), particle H (917.97 m s^{-1}), and particle I (768.62 m s^{-1}). Particle J (530.48 m s^{-1}) and particle K (257.85 m s^{-1}) are the last particles to expand, and they expand exactly at the same time and are observed for 1 second. By the time these last two particles expand and are observed for 1 second, particle A has already been expanding for 1.9 second, and particle B for 1.8 second, this becomes their respective observation/expansion time.

The velocity-distance relationship for these 11 test particles has been plotted in Figure 9. The plot is remarkably similar to the redshift-distance relationship for 588 type Ia supernovae (Figure 5), and to the velocity-distance relationship for 5 type Ia supernovae (Figure 21). The deviation from linearity in Figure 9 clearly indicates that remote particles are not only further away than expected, but they are also yielding a lower value of slope, or a slower rate of expansion (deceleration) even with high recession velocities as compared to the local particles that are yielding a higher value of slope, or a faster rate of expansion (acceleration) even with low recession velocities – apparent transition from deceleration to acceleration (past to present).

Figure 9. Velocity-distance relationship for 11 test particles (local and remote particles) expanding consecutively (one particle after another). Distances to remote particles are larger than expected with respect to local particles without acceleration. In other words, expansion initiated for remote particles in preceding expansion epochs before it did for local particles in the current (or more recent) expansion epoch.

The value of slope for the most distant, remote particle A in Figure 9 is $0.526315789 \text{ m s}^{-1} \text{ m}^{-1}$ (a lower value of slope, or a slower rate of expansion even with high recession velocity of 3517.60 m s^{-1}) – does this imply deceleration? The inverse of this slope gives the original observation/expansion time of 1.9 second.

For local particles, particle J and particle K, the value of slope (slope of the blue line) turns out to be $1 \text{ m s}^{-1} \text{ m}^{-1}$ (a higher value of slope, or a faster rate of expansion even with low recession velocities of 530.48 m s^{-1} and 257.85 m s^{-1} respectively) – does this imply acceleration? The inverse of this slope gives the original observation/expansion time of 1 second.

The recession velocity of particle A is 6.63 times higher than the recession velocity of particle J, and 13.64 times higher than the recession velocity of particle K. Particle A still happens to yield a lower value of slope, thereby suggesting a slower rate of expansion or deceleration as compared to these two particles (not to mention again that particle A is further away than expected as compared to these two particles).

Could there be any other scientifically-valid reason why an object with high recession velocity would be yielding a lower value of slope, thereby suggesting a slower rate of expansion or deceleration and then be further away than expected as compared to an object with low recession velocity?

There is absolutely no other reason for an object with high recession velocity to yield a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) and then be further away than expected, unless it began expanding before.

High recession velocities of remote objects yielding a lower value of slope do not indicate their deceleration. Similarly, low recession velocities of local objects yielding a higher value of slope do not indicate their acceleration. Requiring mysterious dark energy of unknown origin to explain such a transition would only complicate things to an unimaginable extent.

Since expansion began for remote particles before it did for local particles, therefore, remote particles are not only further away than expected, but they are also yielding a lower value of slope (or a slower rate of expansion) even with high recession velocities as compared to local particles that are yielding a higher value of slope (or a faster rate of expansion) even with low recession velocities. It therefore appears that local particles are expanding at a faster rate as compared to remote particles. One would therefore be forced into believing that local particles, as compared to remote particles, are accelerating.

6.2. Expansion rate versus time relationship

Although the observational fact that a remote object which happens to be further away than expected yields a slower rate of expansion even with high recession velocity as compared to a local object that yields a faster rate of expansion even with low recession velocity is the most compelling (independent) evidence to suggest that remote structures began expanding before the expansion got initiated for local structures, however, to further confirm upon this aspect, we will plot expansion rate versus time relationship for such a scenario where remote particles with high recession velocities (high redshifts) began expanding before the expansion got initiated for local particles with low recession velocities (low redshifts).

Figure 10. Plot of expansion rate versus time when expansion began (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 5) shows an accelerating expansion (expansion rate increasing with time). Expansion rate for remote supernovae that are further away than expected (see Figure 5 and Figure 21) is lower even with high recession velocities as compared to the expansion rate for nearby, local supernovae even with low recession velocities.

Here in Figure 10, we can see that expansion rate is increasing with time; expansion rate for remote supernovae is lower than the expansion rate for local supernovae – Universe is expanding slower in the past and is expanding faster now.

Now we need to plot the expansion rate versus time relationship when particles with high recession velocities (high redshifts) began expanding before the expansion got initiated for particles with low recession velocities (low redshifts).

As discussed previously in Section 6.1, initially, the high-recession-velocity (high-redshift) particle A began expanding, 0.1 second later, particle B began expanding, expansion of particle B was followed by the expansion of particle C after another 0.1 second. Consecutive expansion of particles continued in the same way for remaining particles. Low-recession-velocity (low-redshift) particles – particle J and particle K were the last particles to expand, and, they expanded exactly at the same time and were observed for 1 second. By the time these last two particles expanded and were observed for 1 second, particle A had already been expanding for 1.9 second, particle B for 1.8 second, particle C for 1.7 second, and so on.

Figure 11. Plot of expansion rate versus time when expansion began (measured from past to present) for 11 test particles (remote and local particles from Figure 9) mimics an accelerating expansion (expansion rate appears to be increasing with time) when remote particles with high recession velocities began expanding before the expansion got initiated for local particles with low recession velocities. Expansion rate for remote particles that are further away than expected (see Figure 9) is lower even with high recession velocities as compared to the expansion rate for nearby, local particles even with low recession velocities, similar to what we observe for type Ia supernovae in Figure 10.

In Figure 10 and Figure 11, the time when expansion began has been obtained by using the relation,

$$t_H = \frac{1}{H} \quad (2)$$

See H from Equation (1). The similarities encountered while plotting Figure 10 and Figure 11 (expansion rate increasing with time) are strong enough to indicate that remote structures with high recession velocities (high redshifts) were the ones that began expanding before the expansion got initiated for local structures with low recession velocities (low redshifts).

6.3. Expansion factor versus time relationship

Since redshift also helps indicating the size of the Universe now as compared to its size when the light was emitted, therefore, the study conducted here based on the redshifts of 10 test particles should also help us confirm that remote structures with high redshifts began expanding before the expansion got initiated for local structures with low redshifts.

Velocity of particles (v) in m s^{-1} has been converted to redshift (z) by using the relation,

$$z = \frac{v}{c} \quad (3)$$

where c is the velocity of light in m s^{-1} . Similarly, the light-travel-time (t_c) in seconds corresponding to the distance (D) to particles in meters has been calculated by using the relation,

$$t_c \approx \frac{D}{c} \quad (4)$$

Just like a high-redshift remote supernova that we observe to be further away than expected as a result of cosmic acceleration (Figure 5 and Figure 6), we are observing the high-redshift

remote particle A which is also further away than expected (however, not because of acceleration, but because it began expanding before as per consecutive expansion (Figure 9)), at a distance (D) of $2.227813333 \times 10^{-5}$ light seconds (6683.44 m) with a redshift (z) of $1.172533333 \times 10^{-5}$ (3517.60 m s^{-1}).

The percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for particle A is $1.172533333 \times 10^{-3}\%$ (this also corresponds to the percentage of expansion that has occurred while the light from particle A has been in transit before reaching us), in other words, the Universe is $1.172533333 \times 10^{-3}\%$ larger now than it was when the light was emitted.

To get a factor (let us call it as expansion factor) which would help us calculate the time when the Universe was 100% smaller than now, we need to divide 100% by the percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for a particular particle (this percentage of shift in the spectral lines also corresponds to the percentage of expansion that has occurred while the light from that particular particle has been in transit before reaching us), therefore, the expansion factor for the remote particle A is,

$$\frac{100}{1.172533333 \times 10^{-3}} = 85285.4219 \quad (5)$$

(The expansion factor can also be obtained directly by taking an inverse of the redshift (z)). This factor suggests that we will have to reverse the expansion 85285.4219 times back into the past when the scale factor was zero and everything was at the same place – the Big Bang. (Since $t = D/v$, and $v = cz$, therefore, $t = D/cz$, on isolating we get, D/c which is the light-travel-time (t_c), and $1/z$ which is the expansion factor). Therefore, multiplying the expansion factor obtained for particle A (85285.4219) with its light-travel-time in seconds ($2.227813333 \times 10^{-5}$) gives back the original expansion time (t) for particle A (1.9 second), that is, the time (t) in the past when particle A began expanding (the time when expansion began for every particle has been obtained accurately; this also includes remote particles that deviate from linearity in Figure 9).

Figure 12. Plot of expansion factor versus time when expansion began (measured from past to present) for 10 test particles (remote and local particles from Figure 9) mimics an accelerating expansion (expansion factor increasing exponentially with time – red curve) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts. The green curve (overlapping the red curve) traces out an expansion that initially decelerates before accelerating (also see Figure 18).

Here in Figure 12, the time when expansion began for test particles has been obtained by multiplying their expansion factor with their light-travel-time in seconds, and, this is consistent with time when expansion began for these test particles obtained in Figure 11 by using Equation (2).

High-redshift remote particles that began expanding before are yielding a smaller expansion factor (expansion factor for remote particle A (85285.4219)) as compared to the expansion

factor that is increasing exponentially with time for low-redshift local particles that began expanding comparatively later (expansion factor for local particle J (565525.5618)).

We will now follow the same method for type Ia supernovae (remote and local supernovae from Figure 5) to see if they also exhibit a similar expansion factor versus time relationship. A similar relationship will further help us confirm that remote structures with high redshifts began expanding before the expansion got initiated for local structures with low redshifts.

We have remote supernova 1996H at a distance (D) of 14.5043 Gly with a redshift (z) of 0.62; this remote supernova is further away than expected as compared to local supernovae. The percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for this supernova is 62% (this also corresponds to the percentage of expansion that has occurred while the light from this remote supernova has been in transit before reaching us), in other words, the Universe is 62% larger now than it was when the light was emitted.

To get the expansion factor which would help us calculate the time when the Universe was 100% smaller than now, we need to divide 100% by the percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for a particular supernova (this percentage of shift in the spectral lines also corresponds to the percentage of expansion that has occurred while the light from that particular supernova has been in transit before reaching us), therefore, the expansion factor for the remote supernova 1996H is,

$$\frac{100}{62} = 1.612903226 \quad (6)$$

This factor suggests that we will have to reverse the expansion 1.612903226 times back into the past when the scale factor was zero and everything was at the same place – the Big Bang. Therefore, multiplying the expansion factor obtained for supernova 1996H (1.612903226) with its light-travel-time in years (14.510739×10^9 years) gives back the original expansion time (t) for supernova 1996H (23.4044×10^9 years), that is, the time (t) in the past when supernova 1996H began expanding.

Figure 13. Plot of expansion factor versus time when expansion began (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 5) shows an accelerating expansion (expansion factor increasing exponentially with time).

Here in Figure 13, the time when expansion began for supernovae has been obtained by multiplying their expansion factor with their light-travel-time in years, and, this is consistent with time when expansion began for these supernovae obtained in Figure 10 by using Equation (2).

High-redshift remote supernovae that are further away than expected as a result of cosmic acceleration are yielding a smaller expansion factor (expansion factor for remote supernova 1996H (1.612903226)) as compared to the expansion factor that is increasing exponentially with time for low-redshift local supernovae (expansion factor for local supernova 1994S (65.93696426)).

In Figure 12, high-redshift remote particles that are further away than expected are yielding a smaller expansion factor as compared to the expansion factor that is increasing exponentially with time for low-redshift local particles, and, such an exponential increase in expansion factor has occurred when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts, in other words, such an exponential increase in expansion factor has occurred without subjecting any test particle to acceleration or deceleration.

In Figure 13, high-redshift remote supernovae that are further away than expected are also yielding a smaller expansion factor as compared to the expansion factor that is increasing exponentially with time for low-redshift local supernovae – similar to what we observe in Figure 12 using test particles.

Such a similarity further confirms that remote structures with high redshifts began expanding before the expansion got initiated for local structures with low redshifts as per consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion.

6.4. Expansion factor versus light-travel-time relationship

In the previous section we obtained the expansion factor, the light-travel-time, and the time when expansion began. We plotted expansion factor versus time (time when expansion began) relationship and found expansion factor increasing exponentially with time (past to present – remote to local) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts (as per consecutive expansion), a similar relationship was also obtained for 5 type Ia supernovae. The time when expansion began was obtained by multiplying the expansion factor with the light-travel-time (also consistent with Equation (2)).

Here we will consider plotting expansion factor versus light-travel-time relationship for such a scenario when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts, we will then plot expansion factor versus light-travel-time relationship for 5 type Ia supernovae to see if they also exhibit a similar relationship. A similar relationship, if obtained, will further help us confirm that remote structures with high redshifts were the ones that began expanding before the expansion got initiated for local structures with low redshifts.

Figure 14. Plot of expansion factor versus light-travel-time (measured from past to present) for 11 test particles (remote and local particles from Figure 9) mimics an accelerating expansion (expansion factor increasing exponentially with time) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as expansion factor is increasing exponentially with time.

Plot of expansion factor versus light-travel-time relationship (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 5) shows a similar relationship in Figure 15 as obtained here in Figure 14 for 11 test particles that expanded consecutively – expansion factor increasing exponentially with time.

Figure 15. Plot of expansion factor versus light-travel-time (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 5) shows an accelerating expansion (expansion factor increasing exponentially with time).

6.5. Scale factor versus light-travel-time relationship

Evolution of Universe's scale factor with time plays an important role in determining the type of Universe we live in. Measuring the variation of Universe's scale factor with time (past to present – remote to local) would not only help us determine the type of Universe we live in, but it would also help us predict its ultimate fate.

The study conducted here based on the redshifts of 11 test particles should also help us confirm that remote structures with high redshifts began expanding before the expansion got initiated for local structures with low redshifts.

We have remote supernova 1996H at a distance (D) of 14.5043 Gly with a redshift (z) of 0.62. As a result of cosmic acceleration, this remote supernova is further away than expected as compared to the nearby, local supernovae. The scale factor ($a(t)$) which denotes the size of the Universe when the light was emitted is obtained by using the relation,

$$a(t) = \frac{1}{1+z} \quad (7)$$

where z is the redshift. The light-travel-time (t_c) in years corresponding to the distance (D) to the supernovae in meters has been calculated by using Equation (4).

Figure 16. Plot of scale factor versus light-travel-time (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 5) shows an accelerating expansion (scale factor increasing with time).

In Figure 16, high-redshift remote supernovae that are further away than expected are yielding a smaller scale factor (scale factor for remote supernova 1996H (0.61728395)) as compared to the scale factor that is increasing with time for low-redshift local supernovae (scale factor for local supernova 1994S (0.985060571)), in fact, scale factor evolution with time (past to present) exhibiting a curve as obtained in Figure 16 clearly depicts an accelerating expansion.

Now we need to plot the scale factor versus light-travel-time relationship when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts as per consecutive expansion.

Velocity (v) of particles in m s^{-1} has been converted to redshift (z) by using Equation (3). Similarly, the light-travel-time (t_c) in seconds corresponding to the distance (D) to the particles in meters has been calculated by using Equation (4).

Just like a high-redshift remote supernova that we observe to be further away than expected (as a result of cosmic acceleration) as shown in Figure 5 and Figure 6, we are observing the high-redshift remote particle A which is also further away than expected (however, not because of acceleration, but because it began expanding before as a result of consecutive expansion (Figure 9)) at a distance (D) of $2.227813333 \times 10^{-5}$ light seconds (6683.44 m), exhibiting a redshift (z) of $1.172533333 \times 10^{-5}$ (3517.60 m s^{-1}). The scale factor ($a(t)$) for these 11 test particles has been obtained by using Equation (7).

Figure 17. Plot of scale factor versus light-travel-time (measured from past to present) for 9 test particles (remote and local particles from Figure 9) mimics an expansion that accelerates (scale factor increasing with time) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as scale factor is increasing with time (in agreement with Figure 16).

As shown above in Figure 17, scale factor versus light-travel-time relationship for 9 test particles is consistent with an expansion that is accelerating with time (past to present – remote to local) even when the test particles have not been subjected to acceleration, whereas in Figure 18, scale factor versus light-travel-time relationship for 11 test particles is consistent with an expansion that first decelerates, and then accelerates even when the test particles have not been subjected to deceleration or acceleration. This apparent transition of particles' expansion from deceleration to acceleration as obtained in Figure 18 can also be seen in Figure 12 as traced by the green curve.

It is evidently clear from Figure 17 and Figure 18 that high-redshift remote particles that are further away than expected are yielding a smaller scale factor (scale factor for remote particle A (0.999988274)) as compared to the scale factor that is increasing with time for low-redshift local particles (scale factor for local particle K (0.99999914)). Why should test particles that were not at all subjected to deceleration or acceleration exhibit scale factor evolution consistent with a Universe whose expansion is accelerating over time?

Figure 18. Plot of scale factor versus light-travel-time (measured from past to present) for 11 test particles (remote and local particles from Figure 9) mimics an expansion that first decelerates, and then accelerates when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that first decelerates, and then accelerates (in agreement with Figure 19).

In Figure 17 and Figure 18, the observed increase in scale factor with time (past to present) for test particles (remote to local) has occurred when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts as per consecutive expansion, in other words, such an increase in scale factor has occurred without subjecting any test particle to deceleration or acceleration.

In Figure 16 and Figure 19, far and near supernovae are also exhibiting a similar scale factor evolution as measured from past to present – high-redshift remote supernovae that are further away than expected are also yielding a smaller scale factor as compared to the scale factor that is increasing with time for low-redshift local supernovae – similar to what we observe in Figure 17 and Figure 18 using test particles.

Such a similarity further confirms that high-redshift remote structures were the ones that began expanding in preceding epochs of cosmic expansion before the expansion got initiated for low-redshift local structures in the current (or more recent) epoch of cosmic expansion – in agreement with consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion.

Figure 19. Plot of scale factor versus light-travel-time (measured from past to present) for type Ia supernovae helps depicting the expansion history of the Universe – the plot depicts Universe's expansion that first decelerates, and then accelerates. High-redshift remote supernovae that are further away than expected are yielding a smaller scale factor as compared to the scale factor that is increasing with time for the nearby, local supernovae with low redshifts, thereby indicating accelerating expansion of the Universe. Credit: Perlmutter S., Physics Today, Vol. 56, 53, page 57, year 2003, reproduced with the permission of the American Institute of Physics, Copyright (2003) American Institute of Physics. <https://doi.org/10.1063/1.1580050>

6.6. Redshift-distance and velocity-distance relationships

Figure 20. Redshift-distance relationship for 5 type Ia supernovae (local and remote supernovae from Figure 5). Remote supernovae are further away than expected as compared to the nearby, local supernovae.

Figure 21. Velocity-distance relationship for 5 type Ia supernovae (local and remote supernovae from Figure 5). Remote supernovae are further away than expected as compared to the nearby, local supernovae.

Here we have plotted redshift-distance relationship and velocity-distance relationship for 5 type Ia supernovae from Figure 5. Both these plots, just like Figure 5, demonstrate that the very distant, remote supernovae are further away than expected, thereby getting rendered 10% to 25% dimmer as compared to the nearby supernovae. This further-away-than-expected placement of remote supernovae is attributed to dark energy – an unknown energy component that has accelerated the expansion of the Universe – without this energy component there would have been no cosmic acceleration, and without cosmic acceleration, remote supernovae would have not been placed further away than expected.

In particular, the velocity-distance relationship for local and remote supernovae (Figure 21) is characteristically similar to the velocity-distance relationship obtained for local and remote test particles (Figure 9), that is, the expansion rate for remote supernovae (Figure 10) is lower even with high recession velocities as compared to the expansion rate for nearby, local supernovae even with low recession velocities – this attribute is exactly similar to what we have observed for test particles (Figure 11) without having subjected any of those test particles to deceleration or acceleration. Such a similar attribute provides

the most compelling and an independent confirmation in favour of consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion, rather than recent accelerated expansion and a previous decelerated expansion.

6.7. Distance-redshift relationship

Redshift-distance relationship already plotted for local and remote supernovae (Figure 5 and Figure 20) shows the deviation from linearity at high redshifts as remote supernovae are further away than expected. Here we will consider plotting distance-redshift relationship for 5 type Ia supernovae from Figure 5 (these are the same 5 type Ia supernovae that have been considered so far) we will then plot the same relationship for 11 test particles from Figure 9. A similar relationship will further help us confirm that remote structures with high redshifts were the ones that began expanding in preceding expansion epochs before the expansion got initiated for local structures with low redshifts in the current expansion epoch – in agreement with consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion.

Figure 22. Distance-redshift relationship for 5 type Ia supernovae (local and remote supernovae from Figure 5). High-redshift remote supernovae lie above the line; the deviation from linearity makes it clear enough that the distances to remote supernovae are larger than expected, thereby making them appear 10% to 25% dimmer as compared to the nearby, local supernovae. This further-away-than-expected placement of remote supernovae is attributed to a mysterious energy component that has accelerated the Universe's expansion.

Figure 23. Distance-redshift relationship for 11 test particles (local and remote particles from Figure 9). High-redshift remote particles lie above the line; the deviation from linearity also makes it clear enough that the distances to remote particles are larger than expected.

The deviation of distance-redshift relationship from linearity at large distances (at high redshifts) depends on how the expansion rate has changed over time. Since deviation from linearity implies distances to type Ia supernovae to be larger than expected and a change in the expansion rate, therefore, it is believed that such a feat can only be achieved if the Universe expanded slowly in the past than it does today.

As per Figure 23, there should also be an energy component that has accelerated the expansion of particles, thereby placing remote particles further away than expected – there has also been a change in the expansion rate for test particles (Figure 11). However, as per Section 6.1, we are well aware that not even a single test particle was subjected to deceleration or acceleration, therefore, the presence of an energy component responsible for placing remote particles further away than expected is completely out of question. It is only because of consecutive expansion of particles wherein expansion of one particle is followed by the next in a consecutive manner that we are getting a plot that is characteristically similar to the distance-redshift relationship for 5 type Ia supernovae as shown in Figure 22.

6.8. Distance modulus versus redshift relationship

Distance modulus (μ) is the difference between the apparent magnitude (m) and the absolute magnitude (M); it denotes distances on a logarithmic scale and is given as,

$$\mu = m - M = 5 \log(D) - 5 \quad (8)$$

where D is the luminosity distance in parsecs. If we know the distance (D), for instance, we already know the distances to 11 test particles, therefore, we can calculate and obtain the distance modulus (μ) for them by using the relation,

$$\mu = 5 \log(D) - 5 \quad (9)$$

or, by using the relation,

$$\mu = 5 \log\left(\frac{D}{10}\right) \quad (10)$$

If distance modulus (μ) is known, we can obtain the distance (D) in parsecs by using the relation,

$$D = 10^{\frac{\mu}{5} + 1} \quad (11)$$

Here in Figure 24, we have plotted distance modulus versus redshift relationship for 5 type Ia supernovae from Figure 5.

Figure 24. Distance modulus versus redshift relationship for 5 type Ia supernovae (local and remote supernovae from Figure 5). The red logarithmic curve shows that type Ia supernovae are further away than expected, this confirms the presence of an unknown energy component responsible for accelerating the Universe's expansion, thereby placing remote supernovae further away than expected.

Distance modulus versus redshift relationship for 11 test particles (local and remote particles from Figure 9) has also been plotted in Figure 25. The similar relationship obtained for test particles (red curve) further helps us confirm that remote structures with high redshifts began expanding before the expansion got initiated for local structures with low redshifts – consistent with consecutive expansion.

Figure 25. Distance modulus versus redshift relationship for 11 test particles (local and remote particles from Figure 9) is consistent with an accelerating Universe (in agreement with Figure 24) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts (red curve).

6.9. Replication of results

Replication of results plays an important role in scientific analysis. If same results are obtained after conducting an analysis on the original research, then the analysis is most likely to be correct.

Having considered 11 test particles to prove the credibility of the study conducted in this paper that happens to be most consistent with consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion, it would now be imperative to bring in another set of test particles to further confirm upon this undiscovered aspect that mimics cosmic acceleration so perfectly well.

We will consider another set of 113 test particles that expand consecutively – one particle after another – in agreement with consecutive expansion. All test particles have been assigned random recession velocities that decrease from remote particles to local particles (similar to what was incorporated in Section 6.1 for 11 test particles). Consecutive expansion of these 113 test particles is observed for 2 seconds with every test particle expanding consecutively after an interval of 0.01 second.

Figure 26. Velocity-distance relationship for 113 test particles (local and remote particles) expanding consecutively (one particle after another). Distances to remote particles are larger than expected with respect to local particles without acceleration. In other words, expansion initiated for remote particles before it did for local particles.

Figure 26 depicts the velocity-distance relationship for 113 test particles undergoing consecutive expansion. High-recession-velocity test particles that began expanding before as a consequence of consecutive expansion are further away than expected – these are the remote particles. The maximum recession velocity of one such remote particle (P1) is $95874.25 \text{ m s}^{-1}$ – this is the most distant particle in Figure 26.

Out of 113 test particles, last 13 test particles with low recession velocities (ranging between 199.478 m s^{-1} to 1 m s^{-1}) were the last particles to expand, they expanded exactly at the same time and were observed only for 1 second – these are the local particles and they exhibit a linear velocity-distance relationship as they expanded exactly at the same time – the value of slope and thus the expansion rate for these last 13 test particles is the same (P101 to P113).

As shown in Figure 26, the velocity-distance relationship for 113 test particles is sufficiently self-explanatory – a plot obtained only through consecutive expansion of particles, rather than acceleration or deceleration. Remote particles that began expanding before are not only further away than expected, but they are also yielding a slower rate of expansion (deceleration) even with high recession velocities as compared to local particles that are yielding a faster rate of expansion (acceleration) even with low recession velocities. For instance, the most distant, remote particle (P1) in Figure 26 with a whopping recession velocity of $95874.25 \text{ m s}^{-1}$ yields a slower expansion rate of just $0.5 \text{ m s}^{-1} \text{m}^{-1}$, whereas a local particle (P113) in Figure 26 with a minuscule recession velocity of 1 m s^{-1} ends up yielding a faster expansion rate of $1 \text{ m s}^{-1} \text{m}^{-1}$.

How can it scientifically be justified that high recession velocity of $95874.25 \text{ m s}^{-1}$ implies deceleration or a slower rate of expansion, whereas low recession velocity of 1 m s^{-1} implies a faster rate of expansion or acceleration? This paradoxical attribute is not something that can so easily be neglected and swept under the rug. With scientific responsibility one should be equipped with an explanation to such a paradoxical trend.

Figure 27. Plot of expansion rate versus time when expansion began (measured from past to present) for 113 test particles (remote and local particles from Figure 26) mimics an accelerating expansion (expansion rate appears to be increasing with time) when remote particles with high recession velocities began expanding before the expansion got initiated for local particles with low recession velocities. Expansion rate for remote particles that are further away than expected (see Figure 26) is lower even with high recession velocities as compared to the expansion rate for nearby, local particles even with low recession velocities.

Plot of expansion rate versus time for 113 test particles as shown above in Figure 27 gives an overall and much clearer picture how the expansion rate appears to have increased over time from past to present (remote to local) without having subjected any of those test particles to deceleration or acceleration. The plot is similar to what we have already witnessed for 11 test particles in Figure 11 and for 5 type Ia supernovae in Figure 10.

Figure 28. Plot of scale factor versus light-travel-time (measured from past to present) for 113 test particles (remote and local particles from Figure 26) mimics an expansion that accelerates (scale factor increasing with time) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as scale factor is increasing with time.

Scale factor evolution for 113 test particles shown above in Figure 28 is consistent with an expansion that is accelerating over time from past to present (remote to local). Surprisingly, not even a single test particle was subjected to deceleration or acceleration.

Figure 29. Distance modulus versus redshift relationship for 588 type Ia supernovae (local and remote supernovae from Figure 5). The deviation from linearity at high redshifts suggests that type Ia supernovae are further away than expected, this confirms the presence of an unknown energy component responsible for accelerating the Universe's expansion, thereby placing remote supernovae further away than expected (red curve).

As shown above in Figure 29, distance modulus versus redshift relationship has become the new Hubble diagram in cosmic literature. The plot is linear at low redshifts before deviating from linearity. The deviation from linearity at high redshifts clearly suggests that remote supernovae are further away than expected due to cosmic acceleration (red curve).

In Figure 30, distance modulus versus redshift relationship has been plotted for 113 test particles undergoing consecutive expansion, the plot is similar to what has been plotted for 588 type Ia supernovae here in Figure 29 – the plot is linear at low redshifts before deviating from linearity. The deviation from linearity at high redshifts (red curve) clearly suggests that

remote particles that began expanding before as per consecutive expansion are further away than expected; not even a single test particle was subjected to acceleration or deceleration.

Figure 30. Distance modulus versus redshift relationship for 113 test particles (local and remote particles from Figure 26) is consistent with an accelerating Universe (in agreement with Figure 29) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts (red curve).

Perhaps we should introduce more randomness in recession velocities for a few randomly-targeted remote particles. The resultant velocity-distance relationship for 113 test particles in such a scenario turns out to be as shown below in Figure 31.

Figure 31. Velocity-distance relationship for 113 test particles (local and remote particles) expanding consecutively (one particle after another). Recession velocities have been made more random for a few randomly-targeted remote particles. Distances to remote particles are still larger than expected with respect to local particles without acceleration. In other words, expansion initiated for remote particles before it did for local particles.

Figure 31 is still consistent with consecutive expansion of particles – high-recession-velocity remote particles began expanding before the expansion got initiated for low-recession-velocity local particles. Overall recession velocities are still increasing with distances, however, there is more randomness now in recession velocities and hence the distances to a few random remote particles. Such randomness has made the plot much similar to the redshift-distance relationship for 588 type Ia supernovae plotted in Figure 5. Local test particles that

expanded exactly at the same time and were observed only for 1 second still continue exhibiting a linear velocity-distance relationship consistent with nearby, local supernovae.

Although random recession velocities were introduced for a few randomly-targeted remote particles, the plot of expansion rate versus time relationship for these 113 test particles still remains conserved as shown below in Figure 32.

Figure 32. Plot of expansion rate versus time when expansion began (measured from past to present) for 113 test particles (remote and local particles from Figure 31) still mimics an accelerating expansion (expansion rate appears to be increasing with time) when remote particles with high recession velocities began expanding before the expansion got initiated for local particles with low recession velocities. Expansion rate for remote particles that are further away than expected (see Figure 31) is lower even with high recession velocities as compared to the expansion rate for nearby, local particles even with low recession velocities.

Figure 33. Distance modulus versus redshift relationship for 113 test particles (local and remote particles from Figure 31) is still consistent with an accelerating Universe (in agreement with Figure 29) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts (red curve).

After introducing more random recession velocities for a few randomly-targeted remote particles, distance modulus versus redshift relationship for 113 test particles as shown above in Figure 33 has also become much similar to distance modulus versus redshift relationship for 588 type Ia supernovae plotted in Figure 29. Figure 33 is still consistent with consecutive

expansion of particles, rather than acceleration or deceleration of those test particles – the plot is consistently linear at low redshifts before deviating from linearity (red curve).

6.10. Scale factor versus distance relationship

Having already plotted scale factor versus time relationship for 5 type Ia supernovae (Figure 16), 11 test particles (Figure 17 and Figure 18), and 113 test particles (Figure 28), we shall now consider plotting scale factor versus distance relationship for 588 type Ia supernovae from Figure 5 and then the same relationship for 113 test particles from Figure 31. We will also plot the scale factor versus distance relationship for 31 (binned) type Ia supernovae from Betoule et al. (2014) followed by the plotting of the same relationship for 11 test particles from Figure 9. This will help us confirm the results based on another aspect altogether.

Since distances provide an estimate of the past and the present, therefore, the scale factor evolution with distance should also help us take a look into how the scale factor has evolved from past to present (remote to local).

Figure 34. Plot of scale factor versus distance (distance giving an estimate of scale factor evolution from past to present – remote to local) for 588 type Ia supernovae (remote and local supernovae from Figure 5) shows an accelerating expansion (scale factor is increasing over time).

Figure 35. Plot of scale factor versus distance (distance giving an estimate of scale factor evolution from past to present – remote to local) for 113 test particles (remote and local particles from Figure 31) also shows an accelerating expansion (scale factor is increasing over time) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts.

As depicted in Figure 34, Figure 35, Figure 36, and Figure 37, scale factor evolution from past to present shows a prominent increase in scale factor in the more recent epoch. For type Ia

supernovae this suggests a recent accelerated expansion. Similar relationship obtained for test particles should also suggest the same – recent accelerated expansion. However, the scale factor versus distance relationship for test particles (Figure 35 and Figure 37) has been obtained through consecutive expansion of those test particles, and not through deceleration or acceleration.

Figure 36 shown below depicts the scale factor evolution with distance for 31 (binned) type Ia supernovae from Betoule et al. (2014) with redshifts ranging from $z = 0.01$ to $z = 1.3$. Scale factor evolution from past to present (remote to local) shows an expansion that first decelerates, and then accelerates.

Figure 36. Plot of scale factor versus distance (distance giving an estimate of scale factor evolution from past to present – remote to local) for 31 (binned) type Ia supernovae from Betoule et al. (2014) with redshifts ranging from $z = 0.01$ to $z = 1.3$ shows an expansion that first decelerates, and then accelerates. Inner box shows the redshift-distance relationship for the same 31 (binned) type Ia supernovae.

Figure 37 shown below depicts the scale factor evolution with distance for 11 test particles from Figure 9. Scale factor evolution from past to present (remote to local) also shows an expansion that first decelerates, and then accelerates (in agreement with Figure 36). However, as per Figure 9, not even a single test particle was subjected to deceleration or acceleration. All these similarities make it clear enough that such a relationship can also be obtained through consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion, rather than cosmic deceleration that preceded the current epoch of cosmic acceleration.

Figure 37. Plot of scale factor versus distance (distance giving an estimate of scale factor evolution from past to present – remote to local) for 11 test particles (remote and local particles from Figure 9) also shows an expansion that first decelerates, and then accelerates when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts.

6.11. Deceleration parameter

Measuring the deceleration parameter (q_0) helps probing the expansion history of the Universe. We can obtain the deceleration parameter by using the following approximation,

$$q_0 \approx - \frac{2DH - 2cz - cz^2}{cz^2} \quad (12)$$

where H is the expansion rate ($\text{m s}^{-1} \text{m}^{-1}$), D is the distance (m), c is the velocity of light (m s^{-1}), and z is the redshift. The above equation for deceleration parameter (q_0) is valid for $z < 0.5$ and has been obtained using the following approximation from Schmidt (2012).

$$D_L \approx \frac{c}{H_0} \left[z + z^2 \frac{(1 - q_0)}{2} \right] \quad (13)$$

The surprising transition of the Universe's expansion from deceleration to acceleration has already shown that the deceleration parameter, as per the “current acceleration of the expansion”, is negative, that is, $q_0 < 0$ (Riess et al. 1998).

We have already obtained the expansion rate versus time relationship for 5 type Ia supernovae in Section 6.2 (Figure 10) – we get an overall picture how the expansion rate has increased over time (transition from deceleration to acceleration – remote to local – past to present).

In this section we will see how the deceleration parameter values vary for the remote type Ia supernova (SN 1995K) while using evolving values of the expansion rate (H) obtained from 4 type Ia supernovae (remote to local – past to present), we will then see if the most distant, remote test particle from Figure 26 (particle P1) also exhibits a similar attribute for the deceleration parameter, that is, $q_0 < 0$ while using more recent or evolving values of the expansion rate (H).

Table 1

Deceleration parameter (q_0) values for the remote supernova (SN 1995K) obtained by using evolving values of expansion rate (H) (past to present – remote to local).

SN	H ($\text{km s}^{-1} \text{Mpc}^{-1}$)	q_0 values for SN 1995K
1995K	48.19194335	+ 1.000000000
1999DU	53.07308350	+ 0.577096460
1992BS	59.99558432	– 0.022671218
1994S	69.05436111	– 0.807526529

Table 1 represents the expansion rate for 4 type Ia supernovae (remote to local) and the deceleration parameter values for the remote type Ia supernova SN 1995K ($z = 0.479$, $D = 9.7211$ Gly). The deceleration parameter is getting “more and more negative” while using evolving values of H obtained from 4 type Ia supernovae (evolving values of H signify how the expansion rate has increased over time from remote to local – past to present, while the departure of deceleration parameter from positive ($q_0 > 0$) to increasingly negative values ($q_0 < 0$) suggests the transition of the Universe's expansion from deceleration to acceleration) (SN 1995K yields $q_0 = -0.6$ while using H as $67 \text{ km s}^{-1} \text{Mpc}^{-1}$; this is the same q_0 value obtained by the High-Z Team for SN 1995K (Schmidt 2012)).

In a Universe that is dominated by matter, one would rightfully expect the expansion rate to slow down with time due to the gravitational attraction of matter, in such a case, the deceleration parameter would simply be given as,

$$q_0 = \frac{\Omega_M}{2} \quad (14)$$

where Ω_M denotes the matter density. However, the increasingly negative values for deceleration parameter obtained for the remote supernova (SN 1995K) while using more recent or evolving values of the expansion rate (H) – from past to present (remote to local) is disturbing as it necessitates the introduction of “negative mass” exhibiting anti-gravitational effect that acts against the gravitational attraction of matter and accelerates the expansion of the Universe. According to Riess (2012), “The only way to match the change in the expansion rate was to allow the Universe to have a “negative” mass”.

Now, since negative mass is practically impossible, therefore, the only solution to quench out this impossible paradigm of negative mass was to reintroduce the abandoned cosmological

constant “ Λ ”. “From our working definition of q_0 , negative values for the current deceleration (i.e., accelerations) are generated only by a positive cosmological constant and not from unphysical, negative mass density” (Riess et al. 1998). Consequently, the significance of the cosmological constant was calculated, “99.7% to 99.8% confidence no matter what the mass density” (Riess 2012). Therefore, Equation (14) now gets written as,

$$q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda \quad (15)$$

where Ω_Λ denotes the energy density of empty space – the history of this term helps us recall Section 1. In 1917, Einstein had introduced the cosmological constant (Λ) into his gravitational field equations to represent the energy associated with empty space. The term was necessary to account for a “static Universe” – a Universe that neither contracts nor expands. Cosmological constant was introduced to overcome the gravitational attraction of matter that would have otherwise caused the Universe to contract and collapse under gravity.

However, the revolutionizing discovery by Hubble in 1929 from his observations of distant galaxies through the 100-inch Hooker telescope atop Mount Wilson in California showed that the Universe was expanding; it was not at all “static” as was being assumed. This observation of expanding Universe led to the abandoning of the cosmological constant.

Cosmological constant denotes the energy density of empty space or vacuum (vacuum energy density). However, when the value of the cosmological constant is obtained according to the quantum field theory, a huge discrepancy is introduced – a discrepancy involving a mismatch by 120 orders of magnitude (10^{120}). Such a colossal value of the cosmological constant would have led to a vacuum catastrophe.

Having obtained the increasingly negative values for the deceleration parameter for the remote supernova (SN 1995K) by using evolving values of expansion rate (H) obtained from 4 type Ia supernovae (remote to local – past to present), we will now bring the test particles into the picture and study how the deceleration parameter (q_0) values vary for the remote particle (P1) while using evolving values of expansion rate (H) obtained from 4 test particles (remote to local – past to present) when test particles expanded consecutively – one particle after another as per consecutive expansion.

We have already obtained the expansion rate versus time relationship for 11 test particles in Section 6.2 (Figure 11) and then for 113 test particles in Section 6.9 (Figure 27) – we get an overall picture how the expansion rate appears to have increased over time (transition from deceleration to acceleration – remote to local – past to present) without having subjected any of those test particles to deceleration or acceleration.

Since we have not subjected any test particle to deceleration or acceleration, therefore, it would be very interesting to see if such a similar feat would also be traced out by the most distant, remote test particle (P1) (Figure 26), that is, $q_0 < 0$ while using more recent or evolving values of the expansion rate (H). A similar behaviour would help us confirm the study conducted in this paper to an unprecedented level that supports, not acceleration, but consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion.

Table 2

Deceleration parameter (q_0) values for the remote particle (P1) obtained by using evolving values of expansion rate (H) (past to present – remote to local).

Particle	H (m s ⁻¹ m ⁻¹)	q_0 values for P1
P1	0.5	+ 1.000000000
P41	0.625	– 1563.549397
P76	0.8	– 3753.918553
P113	1.0	– 6257.197589

Table 2 represents the expansion rate for 4 test particles (remote to local) and the deceleration parameter values for the remote particle P1 ($z = 3.195808333 \times 10^{-4}$, $D = 191748.5$ m). It must again be noted that remote particle P1 is the first particle that expanded, whereas local particle P113 is the last or more recent particle that underwent expansion as per consecutive expansion.

It is very clear from Table 2 that the deceleration parameter is getting “more and more negative” while using evolving values of H obtained from 4 test particles (evolving values of H signify how the expansion rate has increased over time from remote to local – past to present, while the departure of deceleration parameter from positive ($q_0 > 0$) to increasingly negative values ($q_0 < 0$) suggests the transition of particles’ expansion from deceleration to acceleration). This is exactly what we have obtained from 4 type Ia supernovae in Table 1.

The increasingly negative values for deceleration parameter ($q_0 < 0$) obtained for the remote test particle (P1) while using more recent or evolving values of the expansion rate (H) – from past to present (remote to local) also necessitates the introduction of negative mass responsible for accelerating the expansion of test particles over time. Now, since there is no such thing as negative mass as already discussed, therefore, the only way to explain those increasingly negative values of deceleration parameter would be to introduce an energy component responsible for accelerating the expansion of those test particles with time from past to present (remote to local). However, as we are well aware that not even a single test particle was subjected to deceleration or acceleration, therefore, the presence of an energy component responsible for the increasingly negative values of deceleration parameter, thereby indicating current acceleration of the expansion is again completely out of question. It is only because of consecutive expansion of particles wherein expansion of one particle is followed by the next in a consecutive manner that we are getting increasingly negative values for deceleration parameter for the remote test particle (P1) while using more recent or evolving values of the expansion rate (H) – from past to present (remote to local).

6.12. Reproducibility of results

Not just replication, but reproducibility of results also plays an important role in scientific analysis. If “same results are obtained again” after conducting an analysis on the original research, then reproducibility along with replication of those results adds further credence to the analysis and helps warrant its validity.

Having considered 11 test particles and then 113 test particles to prove the validity of the study conducted in this paper that happens to be most consistent with “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”, rather than “cosmic deceleration that preceded the current epoch of cosmic acceleration”, we will now bring in another set of test particles to further confirm upon this undiscovered aspect that mimics cosmic acceleration so perfectly well.

We will consider a set of 400 test particles that expand consecutively – one particle after another – in agreement with consecutive expansion. This time we will assign the redshifts of 400 type Ia supernovae from Figure 5 to these 400 test particles (redshifts ranging from 0.015 to 0.498 will help keep Equation (12) and Equation (13) valid, this will also make the analysis more relatable in terms of actual redshifts observed). As observed for type Ia supernovae, the redshifts decrease from remote particles to local particles (similar to what has been incorporated so far). Consecutive expansion of these 400 test particles is observed for 4.99 seconds with every test particle expanding consecutively after an interval of 0.01 second.

Initially, particle SN1 ($z = 0.498$) begins expanding, 0.01 second later, particle SN2 ($z = 0.497$) begins expanding, the expansion of particle SN2 is followed by the expansion of particle SN3 ($z = 0.497$) after another 0.01 second. Consecutive expansion of particles continues in the same way for remaining particles – SN4 ($z = 0.496$), SN5 ($z = 0.495$), SN6 ($z = 0.495$), SN7 ($z = 0.493$), and so on. Particle SN400 ($z = 0.015$) is the last particle to expand, and it is observed only for 1 second. By the time this last particle (SN400) expands and is observed for 1 second, particle SN1 has already been expanding for 4.99 seconds, particle SN2 for 4.98 seconds, particle SN3 for 4.97 seconds, and so on, this becomes their respective observation time or the expansion time.

Figure 38 depicts the velocity-distance relationship for 400 test particles undergoing consecutive expansion. High-redshift test particles that began expanding before as a consequence of consecutive expansion are further away than expected – these are the remote particles. The maximum redshift of one such

remote particle (SN1) is 0.498 – this is the most distant particle in Figure 38. Out of 400 test particles, test particle SN400 with low redshift ($z = 0.015$) was the last particle to expand and was observed only for 1 second – this is the local particle.

Figure 38. Velocity-distance relationship for 400 test particles (local and remote particles) expanding consecutively (one particle after another). Distances to remote particles are larger than expected with respect to local particles without acceleration. In other words, expansion initiated for remote particles before it did for local particles.

Velocity-distance relationship for 400 test particles is again sufficiently self-explanatory – a plot obtained only through consecutive expansion of particles, rather than acceleration or deceleration. Remote particles that began expanding before are not only further away than expected, but they are also yielding a slower rate of expansion (deceleration) even with high recession velocities as compared to local particles that are yielding a faster rate of expansion (acceleration) even with low recession velocities as shown below in Figure 39.

Figure 39. Plot of expansion rate versus time when expansion began (measured from past to present) for 400 test particles (remote and local particles from Figure 38) mimics an accelerating expansion (expansion rate appears to be increasing with time) when remote particles with high recession velocities began expanding before the expansion got initiated for local particles with low recession velocities. Expansion rate for remote particles that are further away than expected (see Figure 38) is lower even with high recession velocities as compared to the expansion rate for nearby, local particles even with low recession velocities.

Plot of expansion rate versus time for 400 test particles as shown in Figure 39 gives an overall and much clearer picture how the expansion rate appears to have increased over time from past to present (remote to local) without having subjected

any of those test particles to deceleration or acceleration. The plot is similar to what we have already witnessed for 11 test particles in Figure 11, for 5 type Ia supernovae in Figure 10, and for 113 test particles in Figure 27, and Figure 32.

We need to ask ourselves an important question here to which we already know the answer – has the expansion rate really increased for test particles over time from past to present (remote to local) as a consequence of accelerated expansion, or is it that the expansion rate “appears” to have increased for test particles over time as a consequence of consecutive expansion?

Figure 40. Plot of scale factor versus light-travel-time (measured from past to present) for 400 test particles (remote and local particles from Figure 38) mimics an expansion that accelerates (scale factor increasing with time) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as scale factor is increasing with time.

Scale factor evolution for 400 test particles as shown above in Figure 40 is again consistent with an expansion that is accelerating over time from past to present (remote to local) (in agreement with Figure 16). Surprisingly, not even a single test particle was subjected to deceleration or acceleration.

Figure 41. Distance modulus versus redshift relationship for 400 test particles (local and remote particles from Figure 38) is consistent with an accelerating Universe (again in agreement with Figure 29) when remote particles with high redshifts began expanding before the expansion got initiated for local particles with low redshifts (red curve).

As shown in Figure 41, distance modulus versus redshift relationship has been plotted for 400 test particles undergoing consecutive expansion, the plot is again similar to distance

modulus versus redshift relationship for 588 type Ia supernovae plotted in Figure 29 – the plot is linear at low redshifts before deviating from linearity (red curve). The deviation from linearity at high redshifts clearly suggests that remote particles that began expanding before as per consecutive expansion are further away than expected; again, not even a single test particle was subjected to acceleration or deceleration.

In Section 6.11 we witnessed the increasingly negative values for the deceleration parameter for the remote supernova (SN 1995K) while using evolving values of expansion rate (H) obtained from 4 type Ia supernovae (remote to local – past to present), we also witnessed the increasingly negative values for the deceleration parameter for the remote test particle (P1) while using evolving values of expansion rate (H) obtained from 4 test particles (remote to local – past to present).

We will now bring these 400 test particles into the picture and study how the deceleration parameter (q_0) values vary for the remote particle (SN5) while using evolving values of expansion rate (H) obtained from 8 test particles (remote to local – past to present) when test particles expanded consecutively – one particle after another as per consecutive expansion.

We have already obtained the expansion rate versus time relationship for 400 test particles in this section (Figure 39) – we get an overall picture how the expansion rate appears to have increased over time (transition from deceleration to acceleration – remote to local – past to present) without having subjected any of those test particles to deceleration or acceleration.

Since we have not subjected any of those 400 test particles to deceleration or acceleration, therefore, it would be very interesting to see again if such a similar feat would be traced out by the fifth most distant, remote test particle (SN5) (Figure 38), that is, $q_0 < 0$ while using more recent or evolving values of the expansion rate (H). A similar behaviour if obtained again will further help us confirm the study conducted in this paper to an unprecedented level that supports “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”, rather than “cosmic deceleration that preceded the current epoch of cosmic acceleration”.

Table 3

Deceleration parameter (q_0) values for the remote particle (SN5) obtained by using evolving values of expansion rate (H) (past to present – remote to local).

Particle	H (m s ⁻¹ Mpc ⁻¹)	q_0 values for SN5
SN5	0.202020202	+ 1.00000000
SN100	0.25	+ 0.04040404
SN180	0.3125	– 1.20959596
SN250	0.4	– 2.95959596
SN300	0.5	– 4.95959596
SN340	0.625	– 7.45959596
SN375	0.8	– 10.95959596
SN400	1.0	– 14.95959596

Table 3 represents the expansion rate for 8 test particles (remote to local) and the deceleration parameter values for the remote particle SN5 ($z = 0.495$, $D = 735075000$ m). It must again be noted that remote particle SN5 is the fifth particle that expanded, whereas local particle P400 is the last or more recent particle that underwent expansion as per consecutive expansion.

It is again clear from Table 3 that the deceleration parameter is getting “more and more negative” while using evolving values of H obtained from 8 test particles (evolving values of H signify how the expansion rate has increased over time from remote to local – past to present, while the departure of deceleration parameter from positive ($q_0 > 0$) to increasingly negative values ($q_0 < 0$) suggests the transition of particles’ expansion from deceleration to acceleration). This is exactly what we have obtained from 4 type Ia supernovae in Table 1.

According to Equation (14), if q_0 is negative, then Ω_M also gets rendered negative while solving for it. Therefore, the increasingly negative values for deceleration parameter ($q_0 < 0$) obtained for the remote test particle (SN5) while using more recent or evolving values of the expansion rate (H) – from past to present (remote to local) again necessitates the introduction

of negative mass responsible for accelerating the expansion of test particles over time. Negative mass being practically impossible, therefore, the only way to account for those increasingly negative values of the deceleration parameter (q_0) would be to consider the presence of an overwhelming amount of cosmological constant or the energy associated with empty space (Equation (15)) that has accelerated the expansion of particles over time.

However, as we are well aware that not even a single test particle was subjected to deceleration or acceleration, therefore, the presence of an overwhelming amount of cosmological constant responsible for the increasingly negative values of deceleration parameter, thereby indicating current acceleration of the expansion of those test particles is again completely out of question. It is only because of consecutive expansion of particles wherein expansion of one particle is followed by the next in a consecutive manner that we are getting increasingly negative values for deceleration parameter for the remote test particle (SN5) while using more recent or evolving values of the expansion rate (H) – from past to present (remote to local).

As was already discussed in Section 4, if there can be an epoch of “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), then there can also be epochs of “consecutive expansion that preceded the current epoch of cosmic expansion”, in fact, all similarities encountered so far in this paper for test particles and for type Ia supernovae (replication and reproduction of results) help confirming that remote structures began expanding before the expansion got initiated for local structures – the study conducted in this paper using test particles is therefore most consistent with consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion, rather than cosmic deceleration that preceded the current epoch of cosmic acceleration.

6.13. Hubble residuals

Figure 42. Distance modulus versus redshift relationship for 31 (binned) type Ia supernovae from Betoule et al. (2014) (from $z = 0.01$ to $z = 1.3$) showing the transition of the Universe’s expansion from deceleration to acceleration (red curve) (see Figure 36 (inner box)).

A very nearby, local supernova plotted above in Figure 42 at a redshift (z) of 0.01 is yielding an expansion rate of ≈ 77 km s⁻¹ Mpc⁻¹. The expansion rate obtained from this local supernova has been used to plot a linear expansion in the above figure (blue curve); the deviation from linearity at high redshifts (red curve) makes it obvious that distances to remote supernovae are larger than expected as a result of cosmic acceleration.

Figure 43 A. Hubble residual plot for 31 (binned) type Ia supernovae from Figure 42 (from $z = 0.01$ to $z = 1.3$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch (linear expansion ≈ 77 km s⁻¹ Mpc⁻¹).

Hubble residual plots ($\Delta\mu = \mu_{\text{Cosmic Acceleration}} - \mu_{\text{Linear Expansion}}$) obtained for SNe Ia and SNe Ia + GRBs (Figure 43 A, Figure

43 B, Figure 43 C, Figure 43 H, Figure 43 I) and the Hubble residual plots ($\Delta\mu = \mu_{\text{Consecutive Expansion}} - \mu_{\text{Linear Expansion}}$) obtained for test particles (Figure 43 D, Figure 43 E, Figure 43 F, Figure 43 G, Figure 43 J, Figure 43 K) are consistent with an expansion that has sped up over time.

Figure 43 B. Hubble residual plot for 31 (binned) type Ia supernovae from Figure 42 (from $z = 0.01$ to $z = 1.3$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch (linear expansion $\approx 69 \text{ km s}^{-1} \text{ Mpc}^{-1}$).

Figure 43 C. Hubble residual plot for 585 of 588 type Ia supernovae from Figure 1 (from $z = 0.015$ to $z = 1.39$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 D. Hubble residual plot for 11 test particles from Figure 25 confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 E. Hubble residual plot for 113 test particles from Figure 30 confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 F. Hubble residual plot for 113 test particles from Figure 33 confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 G. Hubble residual plot for 16 of 400 test particles from Figure 41 (from $z = 0.015$ to $z = 0.498$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 H. Hubble residual plot for 588 type Ia supernovae (from $z = 0.015$ to $z = 1.7$) and for 216 GRBs (from $z = 0.414$ to $z = 9.3$) from Figure 3 confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 I. Hubble residual plot for 588 type Ia supernovae from Figure 1 (from $z = 0.015$ to $z = 1.7$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 J. Hubble residual plot for 10 of 400 test particles from Figure 41 (from $z = 0.015$ to $z = 0.25$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Figure 43 K. Hubble residual plot for 7 of 400 test particles from Figure 41 (from $z = 0.015$ to $z = 0.108$) confirming slower expansion in the past and faster expansion in the current (or more recent) epoch.

Hubble residual plots add further credence and help confirm the study conducted in this paper that happens to be most consistent with “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”, rather than “cosmic deceleration that preceded the current epoch of cosmic acceleration”.

7. DISCUSSION AND CONCLUSIONS

The study conducted in this paper provides the first conclusive and strong evidence for “consecutive expansion epochs of the Universe that preceded the current (or more recent) epoch of cosmic expansion”. The key aspects of the study conducted in this paper have been summarized as follows:

(1) Direct and the best observational evidence for an accelerating Universe comes from the comparison of redshift-distance relationship for high and low-redshift type Ia supernovae. As compared to nearby, local supernovae, remote supernovae appear 10% to 25% dimmer as they are further away than expected. There are some astrophysical effects that can imitate this direct evidence for an accelerating Universe from type Ia supernovae, for instance, a pervasive screen of extragalactic grey dust could dim the supernovae (Aguirre 1999a, 1999b), thereby tricking us into believing that remote supernovae are further away than expected, luminosity evolution of type Ia supernovae can also impact the measurements if high-redshift type Ia supernovae are intrinsically fainter as compared to the low-redshift ones (Drell et al. 2000). However, according to Riess et al. (2004), “To date, no evidence for an astrophysical origin of the apparent faintness of SNe Ia has been found (Riess 2000, Coil et al. 2000, Leibundgut 2001, Sullivan et al. 2003)”. More recent studies conducted by Betoule et al. 2014, Scolnic et al. 2018, and Kenworthy et al. 2019 have also confirmed cosmic acceleration based on type Ia supernovae observations. The study conducted by Scolnic et al. 2018 provided evidence ($> 6\sigma$) for an accelerating Universe using type Ia supernovae alone.

(2) Just like type Ia supernovae, quasars (Risaliti and Lusso 2015) and gamma-ray bursts (GRBs) (Demianski et al. 2017 and Dirirsa et al. 2019) have also been used as standard candles to study how the expansion of the Universe has changed over time. Studies conducted using quasars and GRBs, just like type Ia supernovae observations, are also consistent with cosmic acceleration, that is, recent acceleration at low redshifts and past deceleration at high redshifts.

(3) Since direct and the best observational evidence for an accelerating Universe came from type Ia supernovae observations (Riess et al. 1998 and Perlmutter et al. 1999); not to mention that the evidence for an accelerating Universe from type Ia supernovae-only sample is $> 6\sigma$ (Scolnic et al. 2018), therefore, the analysis conducted in this paper deals with the comparison of low-redshift type Ia supernovae ($z = 0.01$, $z = 0.015$) with a high-redshift type Ia supernova ($z = 1.7$), quasar ($z = 6$), and GRB ($z = 9.3$).

(4) By finding type Ia supernovae at different distances and measuring their redshifts, we can plot the expansion rate of the Universe against time. In Figure 5, the redshift of a remote supernova ($z = 1.7$) is 112 times higher than the redshift of a local supernova ($z = 0.015166$). In Figure 6, the redshift of the most distant supernova ($z = 0.479$) is 14 times higher than the redshift of a local supernova ($z = 0.0333$). Similarly, in Figure 3, the redshift of the most distant GRB ($z = 9.3$) is 930 times higher than the redshift of a local supernova incorporated by Betoule et al. 2014, Scolnic et al. 2018, and Kenworthy et al. 2019 ($z = 0.01$). Studies that are in perfect agreement with an accelerating Universe have made it very clear to us that the expansion rate obtained for local supernovae is higher with low redshifts as compared to the expansion rate obtained for remote supernovae, quasars, and GRBs with high redshifts. Since observed redshifts in an expanding Universe provide an estimate of recession velocities as shown in Figure 6, Figure 7, and Figure 8, therefore, it is very disturbing to find that low recession velocities indicate a faster rate of expansion (acceleration), whereas high recession velocities indicate a slower rate of expansion (deceleration). Such an aspect cannot be ignored and swept under the rug, because according to Davis and Lineweaver (2004), “Recession velocities exceed the speed of light in all viable cosmological models for objects with redshifts greater than 1.5”.

(5) The evidence for an accelerating Universe came from measuring how the expansion rate ($\text{km s}^{-1} \text{Mpc}^{-1}$) of the Universe has changed over time. Since expansion rate for local Universe is found to be higher than the expansion rate for remote Universe, therefore, we say that the Universe is expanding faster now and had a slower expansion in the past.

This apparent transition of the Universe’s expansion from deceleration to acceleration (remote to local – past to present) is explained by invoking dark energy – a mysterious and hypothetical energy of unknown origin having no explanation in fundamental physics. As pointed out by Durrer (2011), “our single indication for the existence of dark energy comes from distance measurements and their relation to redshift. Supernovae, cosmic microwave background anisotropies and observations of baryon acoustic oscillations simply tell us that the observed distance to a given redshift z is larger than the one expected from a Friedmann–Lemaître universe with matter only and the locally measured Hubble parameter”. Also, according to Mohayaee et al. (2021), “it was inferred (based on type Ia supernovae observations) that the Hubble expansion rate is accelerating as if driven by a positive Cosmological Constant Λ in Einstein’s theory of gravity. This is still the only *direct* evidence for the ‘dark energy’ that is the dominant component of today’s standard Λ CDM cosmological model. Other data such as baryon acoustic oscillations (BAO) in the large-scale distribution of galaxies, temperature fluctuations in the cosmic microwave background (CMB), measurement of stellar ages, the rate of growth of structure, *etc* are all ‘concordant’ with this model but do not provide independent evidence for accelerated expansion”.

(6) Theoretical calculation for the value of dark energy believed to be the intrinsic energy associated with empty space or the vacuum energy according to the quantum field theory results in a huge 120-orders-of-magnitude (10^{120}) discrepancy. Such a colossal value of the cosmological constant would have resulted in a vacuum catastrophe. The fact that there is no fundamental explanation available in physics for mysterious and hypothetical dark energy, suggests that it has only been introduced to account for the apparent transition of the Universe’s expansion from deceleration to acceleration.

(7) One should also consider that “expansion of gas molecules in a vacuum chamber by the virtue of vacuum energy or dark energy” has never been heard before. In fact, it is worth noting that an experiment conducted by Sabulsky et al. (2019) by using atom interferometry to detect dark energy acting on a single atom inside an ultra-high vacuum chamber showed no trace of any mysterious energy. Dark energy believed to be stronger in high-vacuum environments should have easily been detected acting on a minuscule mass – a single atom.

(8) Distance modulus versus redshift relationship for near and far objects (Figure 1, Figure 2, and Figure 3) is the new Hubble diagram. Supernovae observations ($z > 1$) confirmed the result that the Universe was decelerating in the past before it began accelerating (Riess 2012). Therefore, by just looking where the objects lie on the Hubble diagram (including Figure 5), one gets to know if the Universe is accelerating or decelerating over time. High-redshift remote objects ($z = 1.7$, $z = 6$, $z = 9.3$) that are further away than expected are expanding at a slower rate (decelerating) as compared to the low-redshift local objects ($z = 0.015$), and this is where a paradoxical trend gets uncovered.

(9) It is a well-known fact, and also according to Davis and Lineweaver (2004), “higher redshift galaxies are more distant from us and receding faster than lower redshift galaxies”. Moreover, only superluminal expansion can cause a distant object to exhibit an observational redshift (z) of 1.7. According to Davis and Lineweaver (2004), “Recession velocities exceed the speed of light in all viable cosmological models for objects with redshifts greater than $z \sim 1.5$ ”. In fact, in the Λ CDM concordance model itself, all galaxies beyond a redshift of $z = 1.46$ are receding faster than the speed of light (Davis and Lineweaver 2004). Furthermore, it has also been acknowledged by Nugent (Research News 2001) while referring to this highly-redshifted supernova with a redshift (z) of 1.7, “highly redshifted objects are moving away from us so fast that time dilation is large”. However, something very strange that does not go hand in hand simultaneously with the above statement is then advocated while referring to this same highly-redshifted supernova with a redshift of 1.7, “the supernova allows us to glimpse an era when matter in the universe was still relatively dense and expansion was still slowing under the influence of gravity. More recently the dark energy has begun to predominate and expansion has started to speed up” (Nugent, Research News 2001). These statements clearly imply that the “highly-redshifted type Ia supernova with a redshift (z) of 1.7 is moving away from us so fast (of course, due to expansion) that

it is also providing us an indication of past deceleration or slowing down under the gravitational influence of matter” – this sounds paradoxical; how can “moving away from us so fast” simultaneously be an indication of “slowing down” under the gravitational influence of matter? Intuitively, “moving away from us so fast” and “slowing down” do not go hand in hand simultaneously. It is very disturbing to accept that superluminal remote expansion ($z = 1.7$) also indicates a slower rate of expansion (deceleration) as compared to subluminal local expansion ($z = 0.01$, $z = 0.015$) – completely counterintuitive (a paradox). Moreover, based on the velocity versus luminosity-distance relationship for type Ia supernovae (Figure 6 and Figure 8) as well as the plot by the High-Z Supernova Search Team (Figure 7), it is very disturbing to find that further-away-than-expected remote supernovae even with high recession velocities (60% of speed of light) are yielding a slower rate of expansion (deceleration) as compared to the nearby, local supernovae that are yielding a faster rate of expansion (acceleration) even with low recession velocities (just 1% of speed of light). Confidently enough, remote expansion at 60% of speed of light is way faster than local expansion at just 1% of speed of light. Then why should such a paradoxical trend even be observed wherein remote expansion at 60% of speed of light indicates a slower rate of expansion (deceleration), whereas local expansion at just 1% of speed of light indicates a faster rate of expansion (acceleration)?

(10) Such a paradoxical trend as discussed above in (9) is not limited only to type Ia supernovae observations. To study how the expansion of the Universe has changed over time, Risaliti and Lusso (2015) used quasars as standard candles (up to $z \approx 6$) as quasars are available in large numbers especially at much higher redshifts as compared to type Ia supernovae. The Hubble diagram (distance modulus vs. redshift relationship) obtained by them for those quasars (extending up to $z > 6$) is in perfect agreement with the Hubble diagram for type Ia supernovae in the redshift (z) range of 0.01 to 1.4 as shown in Figure 2. Therefore, the study conducted by them using quasars is also consistent with cosmic deceleration that preceded the current epoch of cosmic acceleration.

(11) Studies incorporating gamma-ray bursts (GRBs) have gone even beyond by focusing at much higher redshifts (up to $z \approx 8$ and $z \approx 9$ (Dirirsa et al. 2019 and Demianski et al. 2017 respectively)). GRBs, just like type Ia supernovae, have also been used as standard candles to probe the expansion history of the Universe. The results obtained using GRBs, just like quasars and type Ia supernovae observations, are also consistent with cosmic acceleration, that is, past deceleration at high redshifts and recent acceleration at low redshifts as shown in Figure 3.

(12) Now, according to Riess et al. (2001), “If the cosmological acceleration inferred from SNe Ia is real, it commenced rather recently, at $0.5 < z < 1$. Beyond these redshifts, the universe was more compact and the attraction of matter dominated the repulsion of dark energy. At $z > 1$, the expansion of the universe should have been decelerating”. Quasars, as depicted by the Hubble diagram (Figure 2) which extends up to $z > 6$, are way beyond, GRBs, as depicted by the Hubble diagram (Figure 3) which extends up to $z > 9$, are even way beyond, at these redshifts, the Universe was more compact, and the expansion of the Universe should have been decelerating according to Riess et al. (2001). However, we know that only superluminal expansion can cause distant objects to exhibit such high observational redshifts ($z \approx 6$ for a quasar, and $z \approx 9$ for a GRB), therefore, it is impossible to digest that superluminal expansion is simultaneously an indication of slowing down under the gravitational attraction of matter, furthermore, we measure how fast the Universe is expanding now, and how fast the Universe was expanding in the past based on the distance and the redshift of remote and local objects, for instance, we compare the distance and the redshift of a remote object with the distance and the redshift of a local object before coming to this conclusion that the Universe is expanding faster now than it was in the past, in other words, we are literally comparing superluminal remote expansion with subluminal local expansion, therefore, advocating that very distant, remote expansion (which happens to be superluminal) is an indication of cosmic deceleration or slowing down under the gravitational attraction of matter (since $z > 1$) sounds very disturbing as it is completely counterintuitive (a paradox). Redshifts exhibited by a supernova ($z = 1.7$), a quasar ($z = 6$),

and a GRB ($z = 9.3$) are much higher than the redshifts (z) specified by Davis and Lineweaver (2004) for which recession velocities exceed the speed of light ($z > 1.46$ for the Λ CDM concordance model, and $z > 1.5$ for all viable cosmological models). It is therefore very disturbing (surprising even) that discoveries at high redshifts ($z > 1$) get advocated as evidence for past deceleration or “slowing down” under the gravitational attraction of matter (Riess et al. 2001, Riess et al. 2004, Riess 2012) as compared to subluminal local expansion (a minuscule redshift (z) of 0.01). Even more disturbing (shocking even) is the official acceptance of such an impossible notion without raising even a single question when it is already known that “higher redshift galaxies are more distant from us and receding faster than lower redshift galaxies” (Davis and Lineweaver 2004). One should answer, how can superluminal remote expansion even be scientifically justified as deceleration (“slowing down” under gravity), or a slower rate of expansion as compared to subluminal local expansion?

(13) Being unable to answer this question, one would readily put forward a lame excuse to get this paradoxical trend swept under the rug by suggesting that recession velocities are meaningless and so is superluminal expansion. However, as pointed out by Davis and Lineweaver (2004), “Expansion has no meaning without well-defined concepts of velocity and distance. If recession velocity were meaningless we could not refer to an ‘expanding universe’ and would have to restrict ourselves to some operational description such as ‘fainter objects have larger redshifts’”. It is a well-known observational fact, and also according to Davis and Lineweaver (2004), “The host of low redshift distance measures and the multitude of available evidence for the Big Bang model all suggest that higher redshift galaxies are more distant from us and receding faster than lower redshift galaxies”. Clearly, “*expanding Universe*”, “*accelerating Universe*”, “*Universe’s expansion is not slowing down with time, as expected, but is speeding up*”, “*remote supernovae, quasars, and GRBs are not receding as quickly as expected from observations of the local ones*” (as per the discovery of accelerating Universe), and then, the unit of expansion rate “ $\text{km s}^{-1} \text{Mpc}^{-1}$ ”, all these necessarily require a velocity description to make sense in an expanding Universe.

(14) As a matter of fact, the original discovery of accelerating Universe itself was based on measuring the recession velocities of supernovae from their observed redshifts (see Figure 6 and Figure 7); here are some notable excerpts that vouch for this – “Team members (HZT) measured the *speed at which these distant supernovae are moving away*. The rate was then compared with the motion of supernovae much closer to Earth” – “How far away a supernova is, and how *fast it’s moving away from us*, tells us how fast the universe is expanding, Riess says” (CNN 1998). Another excerpt from Science News (1998) states this more specifically, “The team (HZT) also measured the supernovas’ *recession velocity*”. Here is another excerpt describing the discovery of the most distant supernova ever seen in 1992 ($z = 0.458$), “The scientists (SCP) are planning to use the light from this supernova – and from others like it – to make a precise measurement of its distance from Earth and the *velocity at which it is receding from us in the expanding universe*” (Currents 1992). Riess et al. (1998) clearly mention the incorporation of seven nearby SNe Ia within 7000 km s^{-1} (108 Mpc for $H_0 = 65 \text{ km s}^{-1} \text{Mpc}^{-1}$ – an expansion rate that is 6% greater than that measured for the more distant objects as noted by Zehavi et al. (1998)). According to Kirshner (2004), “Astronomers measure the velocity of a galaxy from its spectrum”. “Radial velocities show up as shifts in the wavelengths of the lines from the galaxy compared with the spectra of the same atoms at rest in the observatory: blueshifts for objects approaching us and redshifts for objects receding” (Kirshner 2004). Here is another excerpt from the 2007 Gruber Cosmology Prize Press Release, “they (HZT and SCP) not only needed to be able to *measure the speed with which distant objects are travelling away from us*, but also how far away they are”. Let us also recall and take a look at the excerpt that describes this astounding discovery of cosmic acceleration from “Written in the stars” by The Nobel Committee for Physics – The Royal Swedish Academy of Sciences, “By establishing the distance to the supernovae and the *speed at which they are moving away from us*, scientists hoped to reveal our cosmic fate. This is exactly where the notion of deceleration or “slowing down under gravity” at high redshifts ($z > 1$) fits in

as per the startling discovery of cosmic acceleration, that is, remote supernovae were found to be receding slowly (not as fast) as compared to local supernovae. Therefore, a problem should not arise if redshifts are interpreted in terms of recession velocities, after all, the more shifted a spectral line is towards the red end of the spectrum (redshift), the faster that object is receding from us. However, within the framework of accelerating Universe itself, a problem soon arises once redshifts get interpreted in terms of recession velocities and the notion of deceleration or “slowing down under gravity” at high redshifts becomes absurd, paradoxical, and counterintuitive. Therefore, with respect to the very surprising discovery of accelerating Universe (the apparent transition of the Universe’s expansion from deceleration to acceleration), how can every high-redshift (superluminal) remote expansion ($z = 1.7$, $z = 6$, and $z = 9.3$) sensibly be termed as, or scientifically be justified as deceleration or “slowing down under gravity” as compared to low-redshift (subluminal) local expansion ($z = 0.01$)?

(15) Since redshifts as low as 0.01 (Betoule et al. 2014, Scolnic et al. 2018, Kenworthy et al. 2019) can also be used to determine Universe’s expansion rate to confirm recent acceleration at low redshifts and past deceleration at high redshifts, therefore, in such a case, the above question becomes even more serious and rightfully demands an answer, rather than being swept under the rug.

(16) Studies that are in perfect agreement with an accelerating Universe (using SNe Ia, QSOs, and GRBs) have made it very clear to us that as compared to a minuscule redshift of 0.01 (SN Ia) that indicates acceleration, a high redshift of 1.7 (SN Ia) indicates deceleration, an extreme redshift of 6 (QSO) also indicates deceleration, and surprisingly, a whopping redshift of 9.3 (GRB) also indicates deceleration – this sounds weird; it makes no sense that every superluminal remote expansion can simultaneously be an indication of deceleration (slowing down under the gravitational influence of matter) as compared to subluminal local expansion (a mere redshift of 0.01). A crucial aspect appears to have gone unnoticed, thereby making us conclude that the Universe is accelerating now at low redshifts under the influence of hypothetical dark energy having no explanation in fundamental physics and was decelerating (“slowing down”) in the past at high redshifts under the gravitational influence of matter!

(17) According to Durrer (2011), “Dark energy is very disturbing. On the one hand, the fact that such an unexpected result has been found by observations shows that present cosmology is truly data driven and not dominated by ideas that can be made to fit sparse observations. On the other hand, a small cosmological constant is so unexpected and difficult to bring into agreement with our ideas about fundamental physics that people have started to look into other possibilities”.

(18) One such idea that helps to take a look into another possibility that happens to mimic cosmic acceleration has therefore been presented here. The surprising discovery of accelerating Universe is the result of an undiscovered aspect that has been unravelled in this paper. Based on the study conducted and the results obtained in this paper, this undiscovered aspect perfectly mimics cosmic acceleration. The most compelling evidence that provides an independent confirmation in favour of this undiscovered aspect also comes from the observation of actual observable, that is, the redshift; a well-known observational fact, and also according to Davis and Lineweaver (2004), “higher redshift galaxies are more distant from us and receding faster than lower redshift galaxies”, in fact, as discussed earlier in (14), the original discovery of cosmic acceleration itself was based on measuring the recession velocities of supernovae from their observed redshifts (notable excerpts that vouch for this have already been quoted in (14)). According to ESO (1998), “The observed redshift of a distant galaxy or quasar gives a direct estimate of the universal expansion (i.e. the “recession velocity”). For instance, a redshift of $z = 0.1$ corresponds to a velocity of $30,000 \text{ km s}^{-1}$. A direct and official acknowledgement that “highly-redshifted objects are receding faster” has been provided by Nugent (Research News 2001) while referring to a highly-redshifted supernova with a redshift (z) of 1.7; Nugent remarks, “highly redshifted objects are moving away from us so fast” (of course, due to expansion), moreover, “Recession velocities exceed the speed of light in all viable cosmological models for objects with redshifts greater than $z \sim 1.5$ ” (Davis

and Lineweaver 2004). Therefore, it is very disturbing to find that “moving away from us so fast” or every high-redshift (superluminal) remote expansion (expansion $\gg c$ ($z = 1.7$, $z = 6$, $z = 9.3$)) indicates a slower rate of expansion (deceleration or “slowing down” under gravity) as compared to low-redshift (subluminal) local expansion (expansion $\ll c$ ($z = 0.01$, $z = 0.015$)) that indicates a faster rate of expansion (acceleration or speeding up) – completely counterintuitive (a paradox).

(19) If there can be an epoch of “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), then there can also be epochs of “consecutive expansion that preceded the current epoch of cosmic expansion”. It remains undiscovered that an object that begins expanding before in the preceding epoch of cosmic expansion (as a consequence of consecutive expansion) will not only be further away than expected, but it will also be yielding a lower value of slope (or a slower rate of expansion) even with high recession velocity as compared to an object with low recession velocity that begins expanding comparatively later in the current (or more recent) epoch of cosmic expansion.

(20) Logically, an object that begins expanding before (as a consequence of consecutive expansion) in the preceding epoch of cosmic expansion has an utmost probability of being further away than expected; the observational fact, that such an object, which happens to be further away than expected, yields a lower value of slope (a slower rate of expansion (deceleration or slowing down)) even with high recession velocity as compared to an object that yields a higher value of slope (a faster rate of expansion (acceleration or speeding up)) even with low recession velocity is the most compelling (independent) evidence in favour of this undiscovered aspect.

(21) There is absolutely no other reason for an object with high recession velocity to yield a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration or slowing down) and then be further away than expected as compared to an object with low recession velocity, unless it began expanding before in preceding expansion epoch.

(22) Plotting together the high-recession-velocity remote structures that began expanding before in preceding expansion epochs, and the low-recession-velocity local structures that began expanding comparatively later in the current (or more recent) expansion epoch, causes the Hubble diagram to deviate from linearity.

(23) Comparing the slope and thus the expansion rate of high-recession-velocity remote structures that began expanding before in preceding expansion epochs, with the slope and thus the expansion rate of low-recession-velocity local structures that began expanding comparatively later in the current (or more recent) expansion epoch, causes the high-recession-velocity remote structures to appear as if they are receding slowly (not as fast) as compared to the low-recession-velocity local structures. For this reason, the recession velocity (and hence the redshift) of a remote structure appears to be lower than the recession velocity (and hence the redshift) predicted by the Hubble’s law for a local structure, that is, Universe appears to be expanding slowly (decelerating) in the past and faster (accelerating) now. Just like recession velocity, the redshift of a remote structure for which expansion initiated before in preceding expansion epoch will also appear to be less when compared to the redshift of a local structure for which expansion initiated comparatively later in the current expansion epoch. One would therefore be forced into believing that the Universe is expanding faster now than it was in the past. This satisfactorily explains why remote supernovae, quasars, and GRBs even with very high redshifts (superluminal expansion) appear as if they are not receding as quickly as expected from observations of the local ones.

(24) An object with high recession velocity (no matter how high) that began expanding before in the preceding expansion epoch will be further away than expected and will appear to be decelerating, whereas an object with low recession velocity (no matter how low) that began expanding comparatively later in the current expansion epoch will appear to be accelerating.

(25) Intuitively, objects that began expanding exactly at the same time (in the same expansion epoch) will only yield exactly the same value of slope (will fall exactly on the same line), and will therefore have exactly the same expansion rate (see Figure 9 and Figure 11 – particle J and particle K are the only particles that have exactly the same value of slope (they fall exactly on

the same line), and have exactly the same expansion rate as they began expanding exactly at the same time). Same applies for 13 local test particles (P101 to P113) in Figure 26 and Figure 31.

(26) Velocity-distance relationship for objects (particles and supernovae) (Figure 9, Figure 26, Figure 31, and Figure 38 for particles, Figure 6, Figure 8, and Figure 21 for supernovae) shows distances to remote objects to be larger than expected, that is, “the observed distance to a remote object is larger than the one expected from the locally-measured expansion rate” (consistent with the description by Durrer (2011)). Remote objects with high recession velocities are not only further away than expected, but they are also yielding a slower rate of expansion (deceleration) as compared to local objects that are yielding a faster rate of expansion (acceleration) even with low recession velocities (apparent transition of the expansion from deceleration to acceleration (past to present – remote to local)).

(27) Expansion rate versus time (time when expansion began) relationship for objects (supernovae and particles) (Figure 10 for supernovae, Figure 11, Figure 27, Figure 32, and Figure 39 for particles) shows expansion rate increasing with time, that is, further-away-than-expected remote objects even with high recession velocities are yielding a slower rate of expansion (deceleration) as compared to local objects that are yielding a faster rate of expansion (acceleration) even with low recession velocities (apparent transition of the expansion from deceleration to acceleration (past to present – remote to local)).

(28) Observed trends discussed above in (26) and (27) are based on recession velocity interpretation and are possible only when objects with high recession velocities began expanding in preceding expansion epochs before the expansion got initiated for objects with low recession velocities in the current (or more recent) expansion epoch, that is, recession velocity interpretation alone provides the most compelling and an independent evidence in favour of “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”.

(29) Expansion factor versus time (time when expansion began) relationship for objects (particles and supernovae, Figure 12 and Figure 13 respectively) shows expansion factor increasing exponentially with time (past to present – remote to local).

(30) Expansion factor versus light-travel-time relationship for objects (particles and supernovae, Figure 14 and Figure 15 respectively) shows expansion factor increasing exponentially with time (past to present – remote to local).

(31) Scale factor versus light-travel-time relationship for objects (supernovae and particles) (Figure 16 for supernovae, Figure 17, Figure 28, and Figure 40 for particles) shows scale factor evolution consistent with a Universe that accelerates with time (past to present – remote to local).

(32) Scale factor versus light-travel-time relationship for objects (particles and supernovae, Figure 18 and Figure 19 respectively) shows scale factor evolution consistent with a Universe that first decelerates, and then accelerates (past to present – remote to local).

(33) Scale factor versus distance relationship for objects (supernovae and particles, Figure 34 and Figure 35 respectively) shows scale factor evolution consistent with a Universe that accelerates with time (past to present – remote to local).

(34) Scale factor versus distance relationship for objects (supernovae and particles, Figure 36 and Figure 37 respectively) shows scale factor evolution consistent with a Universe that first decelerates, and then accelerates (past to present – remote to local).

(35) Distance-redshift relationship for objects (supernovae and particles, Figure 22 and Figure 23 respectively) makes it obvious that high-redshift objects lie above the line; the deviation from linearity makes it clear enough that the distances to remote objects are larger than expected.

(36) Distance modulus versus redshift relationship for objects (supernovae, quasars, supernovae + GRBs, and particles) (Figure 1, Figure 24, Figure 29, and Figure 42 for supernovae, Figure 2 for quasars, Figure 3 for supernovae + GRBs, Figure 25, Figure 30, Figure 33, and Figure 41 for particles) shows linearity at low redshifts and a deviation from linearity at high redshifts.

(37) Hubble residuals for objects (supernovae, supernovae + GRBs, and particles) (Figure 43 A, Figure 43 B, Figure 43 C,

and Figure 43 I for supernovae, Figure 43 H for supernovae + GRBs, Figure 43 D, Figure 43 E, Figure 43 F, Figure 43 G, Figure 43 J, and Figure 43 K for particles) help confirm a slower expansion in the past and faster expansion in the current (or more recent) epoch.

(38) Observed trends discussed in (29), (30), (31), (32), (33), (34), (35), (36), and (37) are based on redshift interpretation and are possible only when remote objects with high redshifts began expanding in preceding expansion epochs before the expansion got initiated for local objects with low redshifts in the current (or more recent) expansion epoch, that is, redshift interpretation also provides the most compelling evidence in favour of “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”.

(39) Moreover, the departure of deceleration parameter (q_0) from positive ($q_0 > 0$) to increasingly negative values ($q_0 < 0$) for the remote type Ia supernova SN 1995K (Table 1) while using more recent or evolving values of expansion rate (H) obtained from 4 type Ia supernovae (remote to local – past to present) suggests previous deceleration and recent acceleration. Exact attribute, that is, the departure of deceleration parameter from positive ($q_0 > 0$) to increasingly negative values ($q_0 < 0$) for the remote particles – particle P1 (Table 2) and particle SN5 (Table 3) while using more recent or evolving values of expansion rate (H) obtained from 4 test particles (Table 2) and 8 test particles (Table 3) (remote to local – past to present) should also suggest the same – previous deceleration and recent acceleration.

(40) It might be worth pointing out again that test particles were not at all subjected to acceleration or deceleration, however, we still obtained plots and the deceleration parameter values for test particles that are consistent with an expansion that is not at all slowing down with time, but is speeding up (apparent transition of the expansion from deceleration to acceleration (past to present – remote to local)); remote particles even with high recession velocities (high redshifts) appear as if they are not receding as quickly as expected from observations of the local ones; this is exactly what is observed for supernovae, quasars, and gamma-ray bursts. Therefore, such multiple similarities obtained for test particles strongly confirm the study conducted in this paper that happens to be most consistent with consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion, rather than cosmic deceleration that preceded the current epoch of cosmic acceleration. In agreement with consecutive expansion, remote structures began expanding in preceding expansion epochs before the expansion got initiated for local structures in the current (or more recent) expansion epoch. Remote supernovae, quasars, and gamma-ray bursts are therefore not only further away than expected, but they also happen to yield a slower rate of expansion, thereby suggesting their deceleration or “slowing down” even with “superluminal expansion”.

(41) Since consecutive expansion epochs of the Universe “preceded” the current epoch of cosmic expansion, as proved in this paper, therefore, it is prudent to consider that the current epoch of cosmic expansion is definitely not a special epoch; consecutive expansion epochs “succeeding” the current epoch of cosmic expansion are also expected. The expansion rate obtained from such consecutive expansion epochs that will “succeed” the current epoch of cosmic expansion will also appear to be higher than the expansion rate of the current expansion epoch, again, not because of acceleration, but because the expansion initiated comparatively later. To be more precise, expansion epochs “succeeding” the current epoch of cosmic expansion will make the Universe appear to us as if it is not only expanding faster than before, but is also younger than before.

(42) The undiscovered aspect unravelled in this paper also helps alleviate the “Hubble tension” that has necessitated an immediate need for some new “physics beyond Λ CDM” (Riess et al. 2019) to explain the apparent discrepancy in the expansion rates of early and late Universe. Based on the undiscovered aspect unravelled in this paper, only those objects that began expanding exactly at the same time (in the same epoch of cosmic expansion) will be yielding exactly the same expansion rate, in other words, the expansion rate obtained from an object whose expansion initiated before in the preceding expansion epoch will be lower than the expansion rate obtained from an object whose expansion initiated comparatively later in the

current (or more recent) expansion epoch, for instance, an object that began expanding 14.3 billion years ago (in the preceding epoch of cosmic expansion) will be yielding a lower expansion rate of $\approx 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$, an object that began expanding comparatively later 13.8 billion years ago (in the current or more recent epoch of cosmic expansion) will be yielding a higher expansion rate of $\approx 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and, an object that began expanding 12.2 billion years ago (in the expansion epoch “succeeding” the current epoch of cosmic expansion) will be yielding even a higher expansion rate of $\approx 80 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The time obtained from such “expansion rates” should therefore be taken not as the age of the Universe, but (more precisely) as the time when expansion began for those particular objects – this would help alleviate not only the Hubble tension, but also an additional perplexing problem that makes the Universe appear to us as if it is not only expanding faster than before, but is also younger than before. Particularly, the study conducted in this paper also helps alleviate the tension pointed out through the study conducted by Riess et al. (2016) that showed that the Universe is not only expanding 9% faster than before, but is also younger than before. However, as just discussed here, as well as in (41), expansion epochs “succeeding” the current epoch of cosmic expansion will also have the same effect of making the Universe appear to us as if it is not only expanding faster than before, but is also younger than before.

(43) To prove what has been discussed above in (41) and (42), let us go back and take a look at Section 5. The distance covered by particle B in 1 second with a recession velocity of 0.4 m s^{-1} was 0.4 m. According to Equation (1), particle B ended up yielding a slope or an expansion rate of $1 \text{ m s}^{-1} \text{ m}^{-1}$; according to Equation (2), the inverse of this slope or expansion rate gives back the time when expansion began for particle B (1 second ago). Since particle B expanded only for 1 second, therefore, the expansion of particle B can be considered as an expansion that occurred very, very recently – just 1 second ago in the most recent epoch of cosmic expansion, that is, expansion of particle B “succeeds” all expansion epochs of the Universe so far – it “succeeds” an expansion that occurred 14.3 billion years ago ($\approx 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$), it “succeeds” an expansion that occurred 13.8 billion years ago ($\approx 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$), and, it also “succeeds” an expansion that occurred 12.2 billion years ago ($\approx 80 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Consequently, the corresponding value of slope or the expansion rate for particle B in terms of $\text{km s}^{-1} \text{ Mpc}^{-1}$ turns out to be $3.0857 \times 10^{19} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (for comparison, a slope or an expansion rate of $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ corresponds to just $2.268529021 \times 10^{-18} \text{ m s}^{-1} \text{ m}^{-1}$). Why would particle B with a minuscule recession velocity of 0.4 m s^{-1} end up yielding such a colossal value of expansion rate? Moreover, the deceleration parameter (q_0) value for the remote supernova SN 1995K (Table 1) using this colossal value of the expansion rate turns out to be $-2.673460323 \times 10^{18}$. A negative value obtained for the deceleration parameter so large makes it imperative to introduce the most overwhelming amount of cosmological constant to account for this “apparent acceleration” of the Universe’s expansion even though this particular particle was not subjected to acceleration at all. This makes it evidently clear that expansion epochs “succeeding” the current (or more recent) epoch of cosmic expansion will also have the same effect of making the Universe appear to us as if it is not only expanding faster than before, but is also younger than before, that is, based on the slope or the expansion rate obtained from particle B that underwent expansion in the most recent epoch of cosmic expansion (just 1 second ago – “succeeding” all expansion epochs of the Universe so far), one would believe (from the inverse of the slope or the expansion rate obtained from particle B) that the Universe is just 1 second old (younger than ever) and is expanding faster than ever at a ground-breaking rate ($3.0857 \times 10^{19} \text{ km s}^{-1} \text{ Mpc}^{-1}$). This further and again confirms the study conducted in this paper that happens to be most consistent with “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion” – an object that began expanding before in the preceding epoch of cosmic expansion (as a consequence of consecutive expansion) will not only be further away than expected, but it will also be yielding a lower value of slope (or a slower rate of expansion, thereby suggesting its deceleration or “slowing down”) even with high recession velocity as compared to an object with low

recession velocity that began expanding comparatively later in the current (or more recent) epoch of cosmic expansion.

(44) The study conducted in this paper does not advocate in any way that we are located in a special or privileged position within the Universe. The study conducted in this paper does not advocate in any way that we are located in a region where the expansion is “literally faster” than the remote expansion. Instead, the study conducted in this paper clearly explains why the local expansion “appears to be faster” than the remote expansion. To be more precise, the study conducted in this paper is able to justify scientifically why subluminal local expansion “appears to be faster” than superluminal remote expansion. The study conducted in this paper advocates “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion” which is perfectly analogous to “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004). According to Starkman et al. (1999), “direct measurements of the cosmic expansion using supernova at redshifts from $z = 0 - 1.2$, suggest that the expansion of our portion of the universe is *accelerating*”. The study conducted in this paper perfectly fits with this given description by showing how “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion” can make it “appear” to us “that the expansion of our portion of the universe is *accelerating*”. As long as there are epochs of consecutive expansion (one expansion epoch following the next in a consecutive manner) an observer in any expansion epoch will observe expansion attributes that appear similar to cosmic acceleration, that is, “cosmic deceleration that preceded the current epoch of cosmic acceleration” – to an observer in any expansion epoch, preceding expansion epochs will appear to be decelerating or “slowing down”, whereas the observer’s current epoch of cosmic expansion that happens to “succeed” those preceding expansion epochs will appear to be accelerating. In reality, however, such an observation would be attributed not to deceleration or acceleration, but to consecutive expansion epochs wherein one expansion epoch is followed by the next in a consecutive manner. The expansion rate of preceding expansion epoch appears to be lower (decelerating) as compared to the expansion rate of the expansion epoch that succeeds it – a natural feature of consecutive expansion.

(45) Some of the primary advantages and salient features of the study conducted in this paper are summarized here, these include: (i) the study does not question the observations carried out by the Nobel Prize-winning teams (the observations that showed remote supernovae are further away than expected (Riess et al. 1998 and Perlmutter et al. 1999)); in fact, the study conducted in this paper takes into consideration the observations carried out by the Nobel Prize-winning teams and helps provide a robust and a novel explanation/interpretation why remote supernovae, quasars, and GRBs would end up being further away than expected without acceleration, (ii) the study does not make any modifications to gravity, (iii) the study is free from complications associated with an accelerating Universe and hypothetical dark energy of unknown origin having no explanation in fundamental physics, (iv) the study, “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion” is perfectly analogous to “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), therefore, the study is most likely to provide the “subtle reason” (Filippenko 2001) that the researchers have been looking for ever since the unexpected discovery of cosmic acceleration, (v) the study is able to justify why subluminal local expansion “appears to be faster” than superluminal remote expansion, or why high-recession-velocity (high-redshift) remote expansion “appears to be decelerating or slowing down” as compared to low-recession-velocity (low-redshift) local expansion, (vi) an additional aspect unravelled in this paper based on “expansion epochs of the Universe “succeeding” the current (or more recent) epoch of cosmic expansion” helps alleviate the Hubble tension that has necessitated an immediate need for some new “physics beyond Λ CDM” (Riess et al. 2019), (vii) this additional aspect discussed above in (vi) also helps alleviate an additional perplexing problem that makes the Universe appear to us as if it is “not only expanding faster than before, but is also younger than before” (Riess et al. 2016).

(46) Since the apparent transition of the Universe’s expansion from deceleration to acceleration still cannot be explained –

“Unfortunately, no obvious breakthrough in our understanding has yet occurred – cosmic acceleration remains the same mystery that it was in 1998” (Schmidt 2012) – “probably one of the greatest challenges in modern physics” (Ezquiaga & Zumalacárregui 2017), mysterious and hypothetical dark energy of unknown origin having no explanation in fundamental physics, the colossal 120-orders-of-magnitude discrepancy involved while calculating the theoretical value of the energy associated with empty space, and then, the “mounting evidence that the standard Λ CDM model is in tension at more than 4σ with local direct measurements of the Hubble constant” (Lusso et al. 2019) suggesting new “physics beyond Λ CDM” (Riess et al. 2019); a tension that is confirmed to be “at $> 4\sigma$ with the whole SNe Ia + quasars + GRB data set” (Lusso et al. 2019), we therefore need to consider the study conducted in this paper that perfectly mimics cosmic acceleration (“cosmic deceleration that preceded the current epoch of cosmic acceleration”) based on multiple similarities (replication and reproduction of results) obtained in a single paper while plotting velocity-distance relationship, expansion rate vs. time relationship, expansion factor vs. time relationship, scale factor vs. time relationship, scale factor vs. distance relationship, distance-redshift relationship, distance modulus vs. redshift relationship, and Hubble residuals, moreover, and most importantly, deceleration parameter (q_0) was also found to be departing from positive ($q_0 > 0$) to increasingly negative values ($q_0 < 0$) while using more recent or evolving values of the expansion rate (H) as measured from past to present (remote to local).

(47) We should keep this in mind and not forget that the discovery of accelerating Universe came as a complete surprise to the research teams (HZT and SCP) – it was an unexpected discovery. According to Filippenko (2001), “This was not the answer we had expected, and many members of the HZT were worried that a subtle error had been made; indeed, Bob Kirshner reflected that “deep in our hearts, we know this can’t be right”. “The conclusion reached by the HZT and SCP is so dramatic, so “crazy” in some respects, that it behooves us to find an alternative explanation” (Filippenko 2001).

(48) Had the study conducted in this paper (that happens to be most consistent with “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion”) not been in agreement with “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), then results obtained in this paper would have vastly been different; replication and reproduction of results would have not worked. Velocity/redshift-distance relationships obtained through “consecutive expansion epochs of the Universe that preceded the current epoch of cosmic expansion” are so identical to velocity/redshift-distance relationships obtained through actual observations pertaining to “cosmic deceleration that preceded the current epoch of cosmic acceleration” (Riess et al. 2004), that it is impossible to differentiate one plot from the other.

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CONTRIBUTION

The author (Karan R. Takkhi) participated in the analysis and in writing this paper.

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