Propagation of Vortex Hermite-Cosh-Gaussian Beams in a Gradient-Index Medium

M. Lazrek

Chouaïb Doukkali University

Z. Hricha

Chouaïb Doukkali University

A. Belafhal (✉ belafhal@gmail.com)

Chouaïb Doukkali University <https://orcid.org/0000-0003-2735-3108>

Research Article

Keywords: Vortex Hermite-cosh-Gaussian beams, Gradient-index-medium, Self-focusing, Propagation properties.

Posted Date: August 25th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-816932/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at Optical and Quantum Electronics on January 1st, 2022. See the published version at <https://doi.org/10.1007/s11082-021-03427-4>.

Propagation of vortex Hermite-cosh-Gaussian beams in a gradient-index medium

M. Lazrek, Z. Hricha*, A. Belafhal*

Laboratory LPNAMME, Laser Physics Group,

Department of Physics, Faculty of Sciences, Chouaib Doukkali University,

P. B 20, 24000 El Jadida, Morocco

** Corresponding authors. E-mail: belafhal@gmail.com, hrichazo66@gmail.com*

Abstract

Based on the Huygens–Fresnel integral, the propagation equation for a vortex Hermite-cosh-Gaussian beams (vHChGB) in gradient-index medium (GIM) is derived. From the obtained expression, the evolution of the intensity and the phase distributions of a vHChGB through a GIM are numerically demonstrated as a function of the gradient-index parameter β under the change of incident beam parameters. The results show that the characteristics of the output beam evolve periodically versus the propagation distance, and the period of evolution slows down when β is increased. Furthermore, it is demonstrated that the self-repeating properties of the intensity pattern and the phase distribution for the propagated vHChGB are altered by the incident beam parameters. The results obtained may be beneficial for applications in fiber communications and beam shaping.

Keywords: *Vortex Hermite-cosh-Gaussian beams; Gradient-index-medium; Self-focusing; Propagation properties.*

1. Introduction

The gradient index medium (GIM) is an optical material with continuous distribution and quadratic dependence of the refraction index. Such medium allows self-focusing of propagating light beams. Historically, the first model for a nonhomogeneous graded index optical medium is known as Maxwell fisheye lens, which was proposed by Maxwell in 1854 [1]. A lot of time after that date, the gradient index glass rods was introduced as imaging devices in 1970 [2]. Since then, the GIM has been the subject of extensive research in optics, and several works have been devoted to its practical applications, e.g., in imaging, optical communication, optical sensing and optical fibre manufacturing [3-5]. In the past few years, the propagation properties of various model beams through GIM have been investigated including those of cosine-Gaussian beams [6], Lorentz-Gaussian beams [7], Airy-Gaussian vortex beams [8], hollow sinh-Gaussian beams [9], chirped Airy beams [10], Gaussain vortex

beams [11] and higher-order cosh-Gaussian beams [12].

On the other hand, a beam model which is called as vortex Hermite-cosine hyperbolic Gaussian beam (vHChGB) has been recently introduced, and its propagation properties for free space have been reported by our research group [13]. The said beam can be regarded as the extended form of the vortex cosh-Gaussian beam (vChGB) [14]. In the waist plane, the amplitude of the vHChGB is characterized by three key parameters, namely 1) the beam order index n which is associated with the order of the Hermite polynomial, 2) the decentred parameter b which is associated with the cosh part and 3) the vortex charge number M associated with the vortex cofactor. The vHChGB has a squared-shaped cross section, and its profile depends crucially on the value of the parameter b . For a small value of b (b is typically equal 0.1), the vHChGB is hollow dark multi-petal-like beam; with a dark central region around by $4n$ lobes, while for a large value of b (typically, b is taken as 4), the beam profile is four petal-like. The intensity distribution of the vHChGB can be controlled broadly by adjusting the beam parameters. Under the special values of these parameters, the vHChGB reduces to the known forms of many laser beams, such as the hollow vortex-Gaussian beam [15], vortex Hermite-Gaussian beam [16] and vortex-cosh-Gaussian beam [14, 17-20]. The high degree of freedom in parametric control provides a prospective for beam shaping without the use of special optical media. Furthermore, the vortex charge embedded can be exploited in as micro-particle trapping. The vortex beam may be able to trap micro-particles as well as to set them into rotation by use of the orbital angular momentum of light [21]. Up to now, the propagation properties of the vHChGB in various optical media have been investigated [22-23], however, we note that those for the beam propagating through GIM have not been reported yet, to the best of our knowledge. Therefore, the present work is aimed at investigating the evolution properties of such beams when they are passing through a GIM. The rest of the manuscript is divided into two sections: the analytical expression for a vHChGB propagating in GIM is derived in Section 2, by using Huygens–Fresnel diffraction integral. In Section 3, the evolutions of the intensity and the phase distributions of the beam in GIM are analysed as functions of GIM factor β as well as the decentred parameter b , the beam order and the topological vortex charge with numerical examples. Finally, the main results are outlined in the conclusion part.

2. Propagation of vHChGB through GIM

In the present paper, we are interested in the radial gradient-index medium with the refraction-index distribution of the form [1]

$$n(r) = n_0 \left(1 - \frac{1}{2} \beta^2 r^2 \right), \quad (1)$$

where $r = \sqrt{x^2 + y^2}$ is the radial component and (x, y) are the transverse Cartesian coordinates, n_0 is the refractive index at the z axis, and β is the parameter associated with the parabolic dependence of the refractive index.

The optical matrix associated with GIM within the paraxial approximation is given as [24]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos(\beta z) & \frac{1}{\beta} \sin(\beta z) \\ -\beta \sin(\beta z) & \cos(\beta z) \end{pmatrix}. \quad (2)$$

In the Cartesian coordinates system, the electric field of a symmetrical vHChGB propagating along the z -axis can be expressed in the source plane $z=0$ as [13]

$$E(x_0, y_0, z=0) = E_p(x_0, z=0) E_p(y_0, z=0) (x_0 + iy_0)^M, \quad (3)$$

where

$$E_p(u, z=0) = H_p \left(\frac{\sqrt{2}u}{\omega_0} \right) \cosh \left(\frac{b}{\omega_0} u \right) \exp \left(-\frac{u^2}{\omega_0^2} \right), \quad (4)$$

with $u = x_0$ or y_0 , and (x_0, y_0) is the transverse coordinates at the source plane. The parameter p denotes the mode index associated with the Hermite polynomial $H_p(\cdot)$, $\cosh(\cdot)$ stands for the hyperbolic-cosine function, ω_0 is the waist size of the Gaussian part, b is the decentered parameter associated with the cosh part, and M denotes the topological vortex charge.

Within the frame work of paraxial approximation, the propagation of a vHChGB through a GIM is governed by the Huygens-Fresnel integral diffraction, which can be expressed as [25]

$$E(x, y, z) = \frac{ik}{2\pi B} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0(x_0, y_0, 0) \exp \left\{ -\frac{ik}{2B} [A(x_0^2 + y_0^2) - 2(xx_0 + yy_0) + D(x^2 + y^2)] \right\} dx_0 dy_0, \quad (5)$$

where $E_0(x_0, y_0, 0)$ and $E(x, y, z)$ are the beams at the source and receiver planes, respectively. z is the distance compted from the initial plane. A , B and D are the matrix elements associated with GIM and given in Eq.2, and $k = \frac{2\pi}{\lambda}$ is the wave number, with λ is

the wavelength of laser light in a vacuum.

Combining Eq. 3 and Eq. 2, and using the expanding series form of Hermite polynomial [26],

$$H_j(x) = \sum_{p=0}^{[j/2]} \frac{(-1)^p j!}{p!(j-2p)!} (2x)^{j-2p}, \quad (6a)$$

and the binomial formula

$$(x+iy)^M = \sum_{l=0}^M C_l^M x^l (iy)^{M-l}, \text{ with } C_l^M = \frac{M!}{l!(M-l)!}. \quad (6b)$$

Then recalling the following integral formula [27]

$$\int_{-\infty}^{+\infty} x^n e^{-px^2+2qx} dx = \sqrt{\frac{\pi}{p}} \exp\left(\frac{q^2}{p}\right) \left(\frac{1}{2i\sqrt{p}}\right)^n H_n\left(\frac{iq}{\sqrt{p}}\right), \quad (7)$$

and after doing lengthy and straightforward algebraic calculations, one obtains

$$\begin{aligned} E(x,y,z) = & \frac{ik}{2\pi \frac{1}{\beta} \sin(\beta z)} \exp\left[-\frac{ik \cos(\beta z)}{2B} (x^2 + y^2)\right] \left(\frac{\pi}{4\eta}\right) \sum_{l=0}^M C_l^M (i)^{M-l} \\ & \times \sum_{s=0}^{[p/2]} \frac{(-1)^s p!}{s!(p-2s)!} \left(\frac{2\sqrt{2}}{\omega_0}\right)^{p-2s} \left(\frac{1}{2i\sqrt{\eta}}\right)^{l+p-2s} \left[\begin{aligned} & \exp\left(\frac{f_+^2(x)}{\eta}\right) H_{l+p-2s}\left(\frac{if_+(x)}{\sqrt{\eta}}\right) \\ & + \\ & \exp\left(\frac{f_-^2(x)}{\eta}\right) H_{l+p-2s}\left(\frac{if_-(x)}{\sqrt{\eta}}\right) \end{aligned} \right] \\ & \times \sum_{t=0}^{[p/2]} \frac{(-1)^t p!}{t!(p-2t)!} \left(\frac{2\sqrt{2}}{\omega_0}\right)^{p-2t} \left(\frac{1}{2i\sqrt{\eta}}\right)^{M-l+p-2t} \left[\begin{aligned} & \exp\left(\frac{f_+^2(y)}{\eta}\right) H_{M-l+p-2t}\left(\frac{if_+(y)}{\sqrt{\eta}}\right) \\ & + \\ & \exp\left(\frac{f_-^2(y)}{\eta}\right) H_{M-l+p-2t}\left(\frac{if_-(y)}{\sqrt{\eta}}\right) \end{aligned} \right], \quad (8) \end{aligned}$$

where

$$\eta = \frac{1}{\omega_0^2} + \frac{ik\beta}{2 \tan(\beta z)}, \quad (9a)$$

and

$$f_{\pm}(u) = \frac{ik\beta u}{2 \sin(\beta z)} \pm \frac{b}{2\omega_0}. \quad (9b)$$

Eq. (8) is the propagation equation of a vHChGB in GIM. This formula indicates that the propagated beam is dependent on the incident beam parameters, the GIM factor β in addition to propagation distance z . This formula is convenient to investigate the evolution of intensity and the phase distributions of a vHChGB in GIM as functions of propagation distance z , and to highlight the effects of the parameter β and the incident beam parameters on the self-repeating properties of the beam.

One can see from Eq. (8) that the evolution field amplitude of vHChGB in GIM is periodic with propagation distance z , and the period is $L = \frac{2\pi}{\beta}$.

The intensity $I(x, y, z)$ of the output beam is defined as the squared modulus of the field

$$I(x, y, z) = |E(x, y, z)|^2. \quad (10)$$

When Eq. (8) is replaced into Eq. (10), one can directly obtain the analytical expression of intensity for a vHChGB propagating in GIM.

According to the symmetry of the expression of the output beam obtained in Eq. (8), and the periodicity of the terms therein one may expect that the period of evolution of the intensity distribution during propagation will be less than or equal to $\frac{L}{2}$.

3. Numerical calculations and analysis

In this section, the propagation properties of a vHChGB in GIM are illustrated based on the calculation of Eq. (8), and the intuitive numerical analyses are provided. In the numerical calculations, the parameters of the vHChGB are set as $\omega_0 = 1 \text{ mm}$, $\lambda = 632.8 \text{ nm}$ and $\beta = 0.3 \text{ mm}^{-1}$, except it is indicated. In order to analyse the influences of the beam parameters on the evolution behavior of the vHChGB through GIM, we illustrate numerically the evolution of the transverse intensity and phase distributions with the propagation distance z under different beam parameters conditions

Since the initial vHChGB profile is strongly dependent on the value of b , and can in general have two distinct types of profiles; i.e., small b configuration and large b configuration, in the following we will present the numerical examples for the beam in each profile configuration.

Figs. 1 and 2 illustrate the behavior of the intensity and the phase distributions of a vHChGB propagating at different propagation distance z , with $M=1$, and $p=1$ or 2 for the small b and large b beam configurations.

From Fig. 1 one can easily see that although the QIM cannot support the vHChGB as stationary solitons, it allows the evolution of their intensity distributions to be periodic with the propagation distance z , with the period equal $L/2$ (with $L = \frac{2\pi}{\beta}$). Within the range

$0 < z \leq L/4$, the energy of the beam converges, and the beam lobes are gradually focused towards the center. The beam lobes bonds progressively and a new dark hollow structure is formed and the beam attains its minimum spot size at the distance $z = L/4$. Within the second

half of the period, i.e., $L/4 < z \leq L/2$, the beam is defocused and the beam behavior is an reverse process of that observed for the former half of period. At $z=L/2$, the beam retrieves its initial shape, and repeats the same evolution behavior periodically. One can note that the profile of the beam obtained at half of the period depends crucially on the beam parameters (b , p , and M).

Figure 1: The intensity distribution of a vHChGBs in GIM at different propagation distances z , for $M=1$, with $\omega_0 = 1\text{mm}$, $\lambda = 632.8\text{nm}$ and $\beta = 0.3\text{mm}^{-1}$. Rows (a-b) for $p=1$ and (c-d) for $p=2$.

Fig. 2 shows that the equiphase contours take initially on diverging rays and morph gradually into distorted concentric rings (for $p=1$) or spiral pattern ($p=2$).

Figure 2: The phase distributions of vHChGB in the GIM. The conditions are the same as those in Fig.1.

With the increase of the propagation distance, the phase distributions undergo rotation and combination. One can notice that the self-repeating period of the phase pattern evolution is the double of the intensity distributions one. One can see that the vHChGBs has the same pattern at the positions of $z=0$, $z=L/2$ and for $z=L/8$, $z=5L/8$. Upon propagation, the rotation of the phase spiral changes from a clockwise to counterclockwise sense.

The effects of the topological charge M on the evolution of the intensity and phase distributions of vHChGBs in GIM are illustrated in Figs. 3 and 4, respectively.

Figure 3: The normalized intensity distribution of vHChGBs in the GIM at different propagation distances z , with $\omega_0 = 1\text{mm}$, $\lambda = 632.8\text{ nm}$, $p=1$ and $\beta = 0.3\text{mm}^{-1}$. Lines (a-b) for $M=2$, and lines (c-d) for $M=3$.

Figure 4: Phase distributions of vHChGBs in GIM at different propagation distances z .

The conditions are the same as those in Fig.3.

It is seen from Fig. 3 that the evolution of the intensity distributions is quite similar for the three values of the vortex charge M ($M=1, 2$ and 3), except at $z=L/4$ and $z=3L/4$ where the beam profile is strongly related to the value of M . Furthermore, one can note that when ($z=L/4$ and $z=3/4L$), the output beam presents a bright center for $M=2$, in contrast with the case $M=3$, where the output beam has a dark central region.

Fig. 4 shows that the period of phase distributions is dependent on the value of M . When $M=3$ (odd value), the period is the distance $L/2$, whereas for $M=2$ (even value), the period equals L .

Fig. 5 displays the intensity distributions of vHChGBs for three values of β , $\beta=0.1, 0.2$, and 0.3 mm^{-1} . One observes that the evolution period of the vHChGBs in GIM decreases as β is increased.

Figure 5: The normalized intensity distribution of vHChGBs in GIM at different propagation distances z , with $M=1$, $\omega_0 = 1 \text{ mm}$, $\lambda = 632.8 \text{ nm}$, $p=1$. (a) for $\beta = 0.1 \text{ mm}^{-1}$, (b) for $\beta = 0.2 \text{ mm}^{-1}$ and (c) for $\beta = 0.3 \text{ mm}^{-1}$.

In Fig. 6, we investigate the effect of the beam waist ω_0 widths on the intensity distributions in the x-direction of the vHChGB in the GIM at the self-focus plane ($z=0.3m$). The curves show that the dark hollow region of beam structure at self-focus plane increases as ω_0 is increased. From the above numerical results, it follows that the self-repeating and the intensity pattern and phase distributions evolutions of a vHChGB in GIM are altered by the combined effects of the decentred parameter b , beam order p , topological charge M , waist width ω_0 , in addition to the gradient-index parameter β . The richness of the beam profile, which is assured by controlling the above beam parameters can be used for beam shaping applications.

Figure 6: The normalized intensity in x-direction of a vHChGB in the GIM as follow:

$z=0.3$ m, $M=1$, $p=1$, $\beta=0.3\text{mm}^{-1}$ and $\lambda=632.8$ nm, (a): $b=0.1$, (b): $b=4$, and for three values of the waist.

4. Conclusion

In this paper, an analytical expression for a vHChGB propagating through GIM has been derived in detail. From the obtained formula, the evolution of the intensity and the phase distributions of the beam in GIM are illustrated numerically as functions of propagation distance under the change of the initial beam parameters and the gradient-index parameter β . The changes of the intensity and phase distributions are shown to highlight the self-repeating characteristics of vHChGBs in the inhomogeneous media. It is demonstrated that the medium parameter β determines the period of self-repeating of the intensity distributions and phase distributions. The period evolution becomes shorter as β increases. The period of the phase distributions can also be influenced by the topological charge M . When M is odd, the period is $L/2$, and the period changes to L when M is even. The findings of this article might have applications in fiber communication and laser optics.

References

- [1] D. T. Moore, “Gradient-index optics: a review”. *Appl. Opt.* 19, 1035-1038 (1980).
- [2] T. Uchida, M. Furukawa, I. Kitano, K. Koizumi, H. Matsumura, “Optical characteristics of a light-focusing fiber guide and its applications”. *IEEE J. Quant. Elect.* 6, 606–612 (1970)
- [3] S.H. Song, S. Park, C.H. Oh, P.S. Kim, M.H. Cho, Y.S. Kim, “ Gradient-index planar optics for optical interconnections”. *Opt. Lett.* 23, 1025–1027 (1998).
- [4] J.R. Leger, W.M. Kunkel, “Gradient-index design for mode conversion of diffracting beams”. *Opt. Express* 24, 13480–13488 (2016).
- [5] C. Gomez-Reino, M.V. Perez, C. Bao, “Gradient-index optics”. *Fundamentals and Applications*. Springer-Verlag Berlin Heidelberg (2002).
- [6] H. Y. Song, T. R. Song, S. H. Chen, Y. C. Huang, Y. T. Li, W. L. Zhang, “Propagation properties of cosine-Gaussian beams in gradient-index medium”. *High Power Laser Part. Beams*, 23, 890–894 (2011).
- [7] G. Zhou, “Propagation of a Lorentz–Gauss beam in gradient-index medium”. *High Power Laser Part. Beams*, 25 (1), 42-46 (2013).
- [8] R. Zhao, F. Deng, W. Yu, J. Huang, D. Deng, “Propagation properties of Airy-Gaussian vortex beams through the gradient-index medium”. *J. Opt. Soc. Am. A* 33, 1025–1031 (2016).
- [9] D. Zou, X. Li, X. Pang, H. Zheng, Y. Ge, “Propagation properties of hollow sinh-Gaussian beams in quadratic-index medium”. *Opt. Commun.* 401, 54–58 (2017).
- [10] L. Feng, J. Zhang, Z. Pang, L. Wang, T. Zhong, X. Yang, D. Deng, “Propagation properties of the chirped Airy beams through the gradient-index medium”. *Opt. Commun.* 402, 60–65 (2017).
- [11] S. Yang, J. Wang, M. Guo, Z. Qin, J. Li, “Propagation properties of Gaussian vortex beams through the gradient-index medium”. *Opt. Commun.* 465, 125559-125563 (2020).
- [12] F. Saad, A. Belafhal, “Propagation properties of Hollow higher-order cosh-Gaussian beams in quadratic index medium and Fractional Fourier transform,” *Opt. Quant. Elect.* 53(1), 28-43 (2021).
- [13] Z. Hricha, M. Yaalou, A. Belafhal, “Introduction of the vortex Hermite-Cosh-Gaussian beam and the analysis of its intensity pattern upon propagation”. *Opt. Quant. Elect.* 53, 80-87 (2021).
- [14] Z. Hricha, M. Yaalou, A. Belafhal, “Introduction of a new vortex cosine-hyperbolic-Gaussian beam and the study of its propagation properties in Fractional Fourier Transform optical system”. *Opt. Quant. Elect.* 52, 296-302 (2020).
- [15] G.Q. Zhou, Y. Cai, C.Q. Dai, “Hollow vortex Gaussian beams”. *Sci. in Chin.* 56(5), 896-

903 (2013).

[16] V.V. Kotlyar, A.A. Kovalev, A.P. Porfirev, “Vortex Hermite–Gaussian laser beams”. *Opt. Lett.* 40 (5), 701-704 (2015).

[17] Z. Hricha, M. Yaalou, A. Belafhal “Propagation properties of vortex cosine-hyperbolic-Gaussian beams in strongly nonlocal nonlinear media”. *J. Quant. Spectrosc. Radiat. Transf.* 5, 107554-107561 (2021).

[18] Z. Hricha, E.M. El Halba, M. Lazrek, A. Belafhal, “Focusing properties and focal shift of a vortex cosine-hyperbolic Gaussian beam”, *Opt. Quant. Elect.* 53(8), 449-465 (2021).

[19] Z. Hricha, M. Lazrek, E. El Halba, A. Belafhal, “ Parametric characterization of vortex cosine-hyperbolic-Gaussian beams”, *Results in Optics* 5, 100120-100127 (2021).

[20] Z. Hricha, M. Lazrek, M. Yaalou, A. Belafhal, “Propagation of vortex cosine-hyperbolic-Gaussian beams in atmospheric turbulence”, *Opt. Quant. Elect* 53(7), 383-398 (2021).

[21] N.B. Simpson, K. Dholakia, L. Allen, M.J. Padgett, “Mechanical equivalence of spin and orbital angular momentum of light: an optical spanner”. *Opt. Lett.* 22, 52-54 (1997).

[22] Z. Hricha, M. Lazrek, M. Yaalou, A. Belafhal “Propagation of vortex Hermite-cosine-hyperbolic-Gaussian beams in turbulent atmosphere”. Submitted to *Opt. Quant. Elect.* (April, 2021).

[23] Z. Hricha, E. M. El Halba, A. Belafhal “Focusing properties and focal shift of vortex Hermite-cosh- Gaussian beams”. Submitted to *Opt. Quant. Elect.* (April, 2021).

[24] J. N. McMullin, “ The abcd matrix in arbitrarily tapered quadratic-index waveguides”. *Appl. Opt.* 25 (13), 2184-2187 (1986).

[25] S.A. Collins, “Lens-system diffraction integral written in terms of matrix optics”. *J. Opt. Soc. Am* 60, 1168-1177 (1970).

[26] I.S. Gradshteyn, I.M. Ryzhik, “Tables of Integrals Series, and Products”. 5th Edition, Academic Press, New York, (1994).

[27] A. Belafhal, Z. Hricha, L. Dalil-Essakali, T. Usman, “A note on some integrals involving Hermite polynomials and their applications”. *Adv. Math. Mod. and Appl.* 5 (3), 313-319 (2020).