Earth's Global Mean Energy Flow System as the Solution of Four Radiative Transfer Constraint Equations

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Abstract

Schwarzschild's equation in two-stream approximation provides a constraint on the net radiation at the lower boundary, as unequivocally connects it to fluxes at the upper boundary (top-of-atmosphere): surface net radiation, balanced by the non-radiative fluxes in radiative-convective equilibrium, is equal to half of the outgoing longwave radiation (OLR), independent of the optical depth. This paper controls this equation on satellite observations and proved to be valid within -2.19 W/m^2 for clear-sky, in the annual global mean. The all-sky version of the relationship can easily be constructed, separating atmospheric radiation transfer from the longwave cloud radiative effect (LWCRE); it is valid with a difference of 2.76 W/m^2 on the same data set. The equation provides a total (greenhouse) relationship as well, where the surface energy income is given as a function of OLR and the optical depth. This equation is controlled at a particular optical depth of two, and found to be verified with a difference of -2.72 W/m^2 for clear-sky in the global mean. An all-sky form is created by including LWCRE, justified within 2.42 W/m^2 on the same data set. The four equations together have a mean bias of 0.07 W/m^2 . This set of equations, using evident definitions between the all-sky and clear-sky fluxes has a solution for the involved components of the global mean energy flow system. Implications on the global mean clear-sky and cloudy energy flow systems are assessed, and the all-sky energy budget is constructed as a weighted sum of the clear and cloudy fluxes.

1 Introduction

Einstein read a paper at a meeting of the Prussian Academy of Sciences in Berlin, 1914 (chaired by Max Planck), in the absence of the author, Karl Schwarzschild, who served as a soldier in World War I. The paper introduced the equations of radiation transfer in an absorbing and emitting media. The two-stream approximation to the same problem was given by an earlier paper by Schwarzschild (1906, Eq. 11), reproduced by standard textbooks like E. A. Milne (1930, Eqs. 93–95); Goody (1964, Eq. 2.115); Houghton (1977, 2002, Eq. 2.13); Chamberlain (1978, Eq. 1.2.30); Goody and Yung (1989, Eq. 2.146 and Eq. 9.5); Andrews (2010, Eq. 3.50–3.51), etc. It was realized soon (Emden, 1913) that the solution in radiative equilibrium requires a discontinuity in the blackbody radiation at the surface; that is, a discontinuity in temperature, which is greatly diminished by the evaporation and heat convection (this way discovering radiative-convective equilibrium). Milne (1930) noted that the equations may be derived from first principles.

The original equation of Schwarzschild (1906, Eq. 11) consists of three terms:

$$(a) E = A_0(1 + \tau)/2; (b) A = A_0(2 + \tau)/2; (c) B = A_0 \tau/2,$$

where A is the upward beam at a layer, B is the downward beam at the layer, E is the blackbody emission of the layer, A_0 is the emerging flux at the upper boundary (TOA), and τ is the optical depth.

It is easy to see that the difference of terms (b) and (a) is

$$A - E = A_0/2, (I)$$

does not depend on the optical depth. This relationship is reproduced by several standard handbooks on atmospheric physics, for example Houghton (1977, 2002, Eq. 2.13) like

$$B_g - B_0 = \Delta B_g = \Phi/2\pi,$$

B_g being the blackbody radiation of the ground, B_0 the blackbody radiation of the lowest atmospheric layer near the ground, and Φ the incoming (and outgoing) flux at the upper boundary.

Since in the derivation of the relationship clouds are not considered, we regard this equation valid for the clear-sky, and will refer as the “net, clear-sky” equation. An all-sky version can be constructed by separating atmospheric radiation transfer from the longwave effect of clouds (LWCRE), represented by L (the “net, all-sky” equation):

$$A - E \text{ (all-sky)} = [A_0 \text{ (all-sky)} - L] 2. (II)$$

The term in (b) is called the greenhouse relationship (Houghton 1977, 2002, Eq. 2.15), providing the blackbody radiation of the ground as a function of the incoming solar (or outgoing longwave) flux at TOA and the optical depth. In the followings we will show that its accuracy in the global mean atmosphere of the Earth with the specific optical depth of $\tau = 2$ is in the same magnitude as the accuracy of the “net” equation. This particular feature requires explanation which will be given in the Discussion. The form of the equation is hence

$$A = 2A_0, (III)$$

which is regarded to be valid for the clear-sky in this form (“total, clear-sky” equation). Its all-sky version is

$$A(\text{all-sky}) = 2A_0 \text{ (all-sky)} + 1, (IV)$$

(“total, all-sky” equation).

For comparison with recent observations and global energy flow estimates, in CERES notation, using $OLR = TOA \text{ LW Up}$, they look like:

$$\text{Surface (SW net + LW net) (clear-sky)} = OLR(\text{clear-sky})/2 (1)$$

$$\text{Surface (SW net + LW net) (all-sky)} = [OLR(\text{all-sky}) - LWCRE]/2 (2)$$

$$\text{Surface (SW net + LW down) (clear-sky)} = 2OLR(\text{clear-sky}) (3)$$

$$\text{Surface (SW net + LW down) (all-sky)} = 2OLR(\text{all-sky}) + LWCRE. (4)$$

Surface SW net is solar radiation absorbed by the ground, which, unlike in the observations and energy budget models, in these equations is not resolved into its downward and upward (reflected) components.

Longwave net is regarded upward positive, that is, the difference of downward longwave radiation (DLR) from the atmosphere to the ground, and upward longwave radiation (ULW) from the surface, connected to surface temperature as blackbody, or Planck-radiation. Including non-radiative fluxes, balancing net radiation at the surface, as sensible heat (SH) and latent heat (LH), a more complete form of Eq. (1) would be in clear-sky:

$$\text{Surface [SW down} - \text{SW up} + \text{LW down} - \text{LW up}] = \text{SH} + \text{LH} = \text{OLR}/2.$$

The total energy absorbed (and, in equilibrium, emitted) by the surface, on the left-hand side of the third and fourth equations, is

$$\text{Surface [SW down} - \text{SW up} + \text{LW down}] = \text{ULW} + \text{SH} + \text{LH},$$

which is equated by 2OLR in clear-sky, and $2\text{OLR} + \text{LWCRE}$ in all-sky, provided the validity of the $\tau = 2$ assumption.

This study is ultimately about four plus one equations and their integer solution. Tables 1–3 provide the all-sky, clear-sky and cloudy global mean fluxes in integer values satisfying the set of the equations. Figures show global mean energy flow diagrams in integer ratios. Figure 1 and Fig. 2 present the theoretical arithmetic solutions for all-sky and clear-sky conditions, without any reference to the actual atmospheric gaseous composition. Figure 3 projects them on the most recently published present-day clear-sky and all-sky global mean energy budget diagrams. Figure 4 shows the theoretical cloudy structure as it follows from Eq. (5). Figures 5 and 6 offer a geometrical representation of the arithmetic solution, deducing the global mean energy flow system from the simplest two-stream single-slab greenhouse model and introduce the particular characteristics of Earth's atmosphere in the steps of the deduction. Figure 7 is the resulted all-sky system. Figures 8 and 9 present the basics of the clear-sky model; Figs. 10 and 11 display the fundamentals of the cloudy geometry. Figures 12 and 13 yield the all-sky energy flow system as the weighted sum of the clear and cloudy structures.

Table 1
Global mean energy flow components in Eq. (1)–(4), the integer ratio of their solution to the unit flux LWCRE = 1 = $26.68 \pm 0.01 \text{ Wm}^{-2}$ on the 21 years of CERES observations; the value of the integer position ($N \times \text{Unit}$) in Wm^{-2} , and their difference

Flux name	Value (Wm^{-2})	N	N x Unit (Wm^{-2})	Difference (Wm^{-2})
Clear-sky TOA LW	266.03	10	266.80	-0.77
Clear-sky Surface LW down	317.57	12	320.16	-2.59
Clear-sky Surface LW up	398.51	15	400.20	-1.69
Clear-sky Surface LW Net	-80.94	-3	-80.04	-0.90
Clear-sky Surface SW Net	211.76	8	213.44	-1.68
Clear-sky Surface SW + LW Net	130.83	5	133.40	-2.57
All-sky TOA LW	240.24	9	240.12	0.12
All-sky Surface LW down	345.07	13	346.84	-1.77
All-sky Surface LW up	398.70	15	400.20	-1.50
All-sky Surface LW Net	-53.64	-2	-53.36	-0.28
All-sky Surface SW Net	163.62	6	160.08	3.54
All-sky Surface SW + LW Net	109.98	4	106.72	3.26
TOA LW CRE	25.78	1	26.68	-0.90
G all-sky	158.46	6	160.08	-1.62
G clear-sky	132.48	5	133.40	-0.92

2 Data

2.1 CERES

NASA Clouds and the Earth's Radiant Energy System (CERES) (Wielicki et al. 1996) instruments flying on Terra and Aqua satellites contain scanning thermistor bolometer radiometric channels (Priestley et al. 2000) measuring filtered radiances in the shortwave (SW; wavelengths between 0.3 and 5 μm), total (TOT; 0.3 and 200 μm), and in the water vapor window, 8 and 12 μm) (Loeb et al. 2001). Longwave (LW) radiances are calculated as TOT minus SW. Radiance-to-flux conversion uses empirical angular distribution models (Su et al. 2015). Point-like measurements require time interpolation and spatial averaging to provide global coverage (Doelling et al. 2013).

Despite improvements, a large ($\sim 4.3 \text{ Wm}^{-2}$) imbalance persists in the average global net radiation at the TOA, in contrast to the expected ocean heating rate of $\sim 0.58 \pm 0.38 \text{ Wm}^{-2}$ (EBAF Ed2.8, July 2005-June 2010), or the more recent $0.71 \pm 0.10 \text{ Wm}^{-2}$ by in-situ observations (EBAF Ed4.1, Johnson et al. 2016). CERES Energy Balanced and Filled (EBAF) datasets are using an objective algorithm, applying a one-time adjustment on the entire record to adjust TOA SW and LW fluxes to the accepted range of imbalance (Loeb et al. 2009).

A new clear-sky flux parameter is introduced in EBAF Edition 4.1, providing clear-sky flux estimates not only for the cloud-free portion of the region but for the regions including cloudy portions (Loeb et al. 2020). Cloud radiative effects (CREs) are calculated using this new adjustment; all-sky fluxes were not changed from EBAF Ed4.0.

Surface fluxes cannot be directly observed by satellites, they are computed from TOA radiances using atmospheric states (temperature and humidity profiles and cloud and aerosol properties) from the MODIS and GEO imagers (Minnis et al. 2020) with the Langley Fu-Liou radiative transfer code.

We are going to control the global mean flux components in Eq. (1)–(4) on the CERES EBAF data: first on Edition 2.8 (the data product available at the time this study has started), March 2000 – February 2016, Rose et al. 2017) and on Edition 4.1, for 21 running years (252 monthly means), from March 2000 – February 2021.

2.2 NEWS

Estimates of the surface net radiation in different data sets reveal a discrepancy between the radiant energy and the corresponding turbulent fluxes (sensible heat and latent heat in the radiative-convective models) of about $13\text{--}24 \text{ Wm}^{-2}$ (L'Ecuyer et al. 2015). NASA Energy and Water-cycle Study (NEWS) presents a objectively-balanced reconstruction of the energy budget to reduce the uncertainty in the net surface energy imbalance to 0.45 Wm^{-2} . A second optimization on the same data set is introduced by Stephens and L'Ecuyer (2015) to constrain surface fluxes, including downward longwave radiation (DLR), to the values given by the CERES products.

3 Results

3.1 The components and the accuracy of the equations

3.1.1 CERES EBAF Edition 2.8, global means, March 2000 – February 2016 (Rose et al. 2017)

$$\text{Eq (1) SFC (SW net + LW down - LW up) (clear) = TOA LW (clear) / 2} \quad \text{Wm}^{-2}$$

$$214.32 + 316.27 - 398.40 = 265.59 / 2 - 0.60$$

$$\text{Eq (2) SFC (SW net + LW down - LW up) (all) = [TOA LW (all) - LWCRE] / 2}$$

$$162.34 + 345.15 - 398.27 = (239.60 - 25.99) / 2 + 2.41$$

$$\text{Eq (3) SFC (SW net + LW down) (clear) = 2 TOA LW (clear)}$$

$$214.32 + 316.27 = 2 \times 265.59 - 0.59$$

$$\text{Eq (4) SFC (SW net + LW down) (all)} = 2 \text{ TOA LW (all)} + \text{LWCRE}$$

$$162.34 + 345.15 = 2 \times 239.60 + 25.99 + 2.70$$

$$\text{Mean bias of the four equations} = 0.98$$

As it seems, the clear-sky equations, Eq. (1) and (3) are valid with a difference equal to the accepted value of that time for the Earth's energy imbalance. There is no reference to any adjustments to these equations (not even to these equations themselves), so we must accept the excellent accuracy of the data product. The all-sky equations are more biased, leading to a mean bias for the four constraint equations of close to 1 Wm⁻².

3.1.2 CERES EBAF Edition 4.1, global means, March 2000 – February 2021

$$\text{Eq (1) SFC (SW net + LW down – LW up) (clear)} = \text{TOA LW (clear)} / 2 \quad \text{Wm}^{-2}$$

$$211.76 + 317.57 - 398.51 = 266.03 / 2 - 2.19$$

$$\text{Eq (2) SFC (SW net + LW down – LW up) (all)} = [\text{TOA LW (all)} - \text{LWCRE}] / 2$$

$$163.62 + 345.07 - 398.70 = (240.24 - 25.78) / 2 + 2.76$$

$$\text{Eq (3) SFC (SW net + LW down) (clear)} = 2 \text{ TOA LW (clear)}$$

$$211.76 + 317.57 = 2 \times 266.03 - 2.71$$

$$\text{Eq (4) SFC (SW net + LW down) (all)} = 2 \text{ TOA LW (all)} + \text{LWCRE}$$

$$163.62 + 345.07 = 2 \times 240.24 + 25.78 + 2.42$$

$$\text{Mean bias of the four equations} = 0.07$$

Here the clear-sky and all-sky biases are more balanced, resulting in a net mean bias of only 0.07 Wm⁻².

Since these equations are two-stream (plane-parallel) approximations of Schwarzschild's general integral equation, the question naturally arises how much difference might come from this approximation. Loeb et al. (2002) show that a satellite-based TOA 20 km flux reference level corresponds to the effective radiative "top of atmosphere" so there is no need to correct the plane-parallel estimations.

The accuracy of these equations makes it possible to calculate the exact value of the surface temperature discontinuity in the radiative equilibrium model, and its realistic value in the radiative-convective framework. Emden (1913) calculated the temperature of the lowest-level atmospheric layer near the ground as 15.9 °C, and the total energy income of the surface as 36 °C, finding a theoretical 20°C temperature jump, being diminished by evaporation and heat conduction. Shepherd (2003) mentions the ground-air temperature discontinuity as 10 or 20 °K or more, depending on local conditions. Graphical representation of the discontinuity in radiative equilibrium is given by Houghton (1977), Chamberlain (1987), Marshall and Plumb (2008), Andrews (2000). Pierrehumbert (2008) mentions the behavior of the discontinuity in the two (optically thin and thick) limits, Eqs. 4.44 – 4.45). Now we are able to compute its theoretical global mean value from Eq. (2) as if the total energy income of the surface were balanced by Planck radiation, it would be $163.62 + 345.07 = 508.69 \text{ Wm}^{-2}$, equivalent to 307.76 °K, while the observed surface upward longwave radiation is 398.7 Wm⁻², that is 289.58 °K, with a difference of 110 Wm⁻² for the surface net radiation, allowing a 18.2 °K temperature difference to be equilibrated by the non-radiative

fluxes (sensible heat and latent heat) in the annual global mean. Now this value by Eq. (2) is constrained to $(OLR - LWCRE)/2$, having 107.23 Wm⁻² in the CERES EBAF Ed4.1 product (2.76 Wm⁻² bias). The equation is able to constrain the all-sky hydrological activity within 0.5 °K. Notice that the clear-sky net equation has a negative 2.19 Wm⁻² bias, the two net equations, Eq. (1), clear-sky and Eq. (2), all-sky, together have a mean bias of 0.28 Wm⁻².

3.2 Integer ratio solution

Regarding the flux components of the global mean energy flow system as independent variables in the equations, we write

$$\text{SFC SW net (clear-sky)} = X_1$$

$$\text{SFC LW down (clear-sky)} = X_2$$

$$\text{SFC LW up (clear-sky)} = X_3$$

$$\text{TOA LW (clear-sky)} = X_4$$

$$\text{SFC SW net (all-sky)} = X_5.$$

Definitions:

$$\text{LWCRE at SFC} = \text{LWCRE at TOA}$$

$$\text{SFC LW down (all-sky)} = \text{SFC LW down (clear-sky)} + \text{LWCRE}$$

$$\text{SFC LW up (clear-sky)} = \text{SFC LW up (all-sky)}$$

$$\text{TOA LW up (all-sky)} = \text{TOA LW up (clear-sky)} - \text{LWCRE}.$$

From now, the equations will be scaled to LWCRE = 1.

With these, the four equations look like this:

$$X_1 + X_2 - X_3 = X_4/2 \quad (1)$$

$$X_5 + (X_2 + 1) - X_3 = ((X_4 - 1) - 1)/2 = X_4/2 - 1 \quad (2)$$

$$X_1 + X_2 = 2X_4 \quad (3)$$

$$X_5 + (X_2 + 1) = 2(X_4 - 1) + 1 = 2X_4 - 1 \quad (4)$$

We have four equations for five independent variables. The constraint will be that we accept only integer ratio solutions. From now, LWCRE will be shown in bold typeface as LWCRE = 1 unit. With this, the set of

solution is given in Table 1. The best fit of the unit of the integer ratio system on 21 years of CERES data is 1 unit = 26.68 Wm⁻². The fourth column of Table 1 gives the theoretical positions using this unit, and the fifth presents the differences from the observed CERES value.

Since the mean bias of the four equations is 0.07 Wm⁻², and the integer ratios are the solution of the four equations not depending on anything else, we expect that the mean bias of the integer solution is the same value. The largest deviation of the CERES value from its theoretical (integer) position is All-sky surface SW net, 3.54 Wm⁻², still in the magnitude of the instrument calibration uncertainty of 3.4 Wm⁻². It seems that the clear-sky TOA outgoing LW radiation, OLR (clear) is reduced by the accepted TOA imbalance of 0.77 Wm⁻² related to its theoretical position, while the all-sky OLR has the smallest deviation (0.12 Wm⁻²) from the integer ratio. An important composite quantity, longwave cooling of the atmosphere has excellent accuracy, because of cancellations from the same shift of surface LW up and down: ATM cooling = SFC LW up – SFC LW down – TOA LW up = -186.61 Wm⁻², compared to its theoretical position at -7 units as -186.76 Wm⁻² (0.15 Wm⁻² difference). All-sky surface LW net has very small bias, less than 0.5 Wm⁻² (0.28 Wm⁻²) as well. The normalized greenhouse effect in the all-sky is $g(\text{all}) = G(\text{all})/\text{SFC LW up (all)} = 158.46/398.7 = 0.3974$ and in the clear-sky $g(\text{clear}) = 132.48/398.51 = 0.3324$, while their theoretical integer ratio positions are $g(\text{all, theory}) = 6/15 = 0.4$ and $g(\text{clear, theory}) = 5/15 = 1/3$.

The four equations now can be presented in their integer form:

$$\text{Eq (1) SFC (SW net + LW down – LW up) (clear) = TOA LW (clear) / 2}$$

$$8 + 12 - 15 = 5 = 10 / 2$$

$$\text{Eq (2) SFC (SW net + LW down – LW up) (all) = [TOA LW (all) – LWCRE] / 2}$$

$$6 + 13 - 15 = 4 = (9 - 1) / 2$$

$$\text{Eq (3) SFC (SW net + LW down) (clear) = 2 TOA LW (clear)}$$

$$8 + 12 = 20 = 2 \times 10$$

$$\text{Eq (4) SFC (SW net + LW down) (all) = 2 TOA LW (all) + LWCRE}$$

$$6 + 13 = 19 = 2 \times 9 + 1$$

3.3 Extension of the integer ratio system

The integer ratios come directly from the arithmetic solution of the four equations without any further theoretical considerations. The question arises: if these fundamental fluxes have given round-number ratios to each other, are there any further elements in the system with the same inherited characteristics?

3.3.1 SFC (SW + LW) net (NEWS)

A fundamental characteristic of the system in radiative-convective equilibrium is the non-radiative flux in the annual global mean, being balanced by turbulent fluxes, defining the hydrological activity as the sum of “thermals” and “evapotranspiration”, but they are not separated either in the CERES data product or in the four radiative transfer equations. The NASA Energy and Water cycle study, assesses the global components using evaporation and precipitation (run-off) data, in order to reconcile existing large

imbalance between the radiative computations and the water cycle observations (Rodell et al. 2015, L'Ecuyer et al. 2015). The observed (unconstrained) global energy budget yields $23 \pm 5 \text{ Wm}^{-2}$ for the sensible heat and $75 \pm 7 \text{ Wm}^{-2}$ for evaporation, with a net absorbed energy of 16 Wm^{-2} at the surface. After applying relevant energy and water cycle constraints, the corresponding quantities become $\text{SH} = 25 \pm 4 \text{ Wm}^{-2}$ and $\text{LH} = 81 \pm 4 \text{ Wm}^{-2}$, with a net surface imbalance of $0.45 \pm 0.4 \text{ Wm}^{-2}$. Introducing a second optimization to more tightly constrain some flux components like downward longwave radiation (DLR) at the surface to CERES observations (Stephens and L'Ecuyer 2015), their best estimate become $\text{SH} = 26 \pm 5 \text{ Wm}^{-2}$ and $\text{LH} = 82 \pm 7 \text{ Wm}^{-2}$. Now let us recognize that the integer position for sensible heat is 1 unit = 26.68 Wm^{-2} and for latent heat is 3 units = 80.04 Wm^{-2} , differing only by 1.68 Wm^{-2} and 0.96 Wm^{-2} from the estimate of L'Ecuyer et al. 2015), and by 0.68 Wm^{-2} and 1.96 Wm^{-2} , resp., from the second-optimized proposal.

Since clear-sky data were not taken into account in these studies, to control the validity of the all-sky equations, Eq. (2) net and Eq. (4) total in these data sets, we have to utilize the contemporary estimate of LWCRE from Stephens et al. (2012) as $26.7 \pm 4 \text{ Wm}^{-2}$. With this, Eq. (2) in the data set of L'Ecuyer et al. (2015) is

$$186 - 22 + 341 - 399 = (238 - 26.7)/2 \text{ with a difference of } 0.35 \text{ Wm}^{-2},$$

and Eq. (4) looks like

$$186 - 22 + 341 = 2 \times 238 + 26.7, \text{ with a difference of } 2.30 \text{ Wm}^{-2}.$$

The two equations on the second-optimized version of Stephens and L'Ecuyer (2015) are:

$$185 - 22 + 344 - 399 = (240 - 26.7)/2, \text{ the difference is } 1.35 \text{ Wm}^{-2},$$

$$185 - 22 + 344 = 2 \times 240 + 26.7, \text{ difference} = 0.30 \text{ Wm}^{-2}.$$

This accuracy shows the consistency of these estimates, enhancing further our trust in the values of their sensible and latent heat components, which will now being introduced into the integer ratio system as $\text{SH} = 1$ and $\text{LH} = 3$.

3.3.2 SFC SW Down (CERES)

Although our four equations do not separate surface net (absorbed) solar radiation into its downward and upward components, it can be seen in the above-mentioned studies that SW radiation downward to the surface is estimated to be 186 Wm^{-2} and 185 Wm^{-2} , resp. The corresponding CERES EBAF Ed4.1 value for 21 years of observations is SFC SW down = 186.80 Wm^{-2} in the all-sky, notice that the corresponding integer ratio position is 7 units = 186.76 Wm^{-2} , differing only by 0.04 Wm^{-2} . Therefore let us insert SFC SW down = 7 into our all-sky integer ratio tables.

3.3.3 CERES TOA SW up

The reflected solar radiation at the top of the atmosphere is the sum of reflections from the surface, from the clear atmosphere and from the clouds. They do not have any theoretical equations in the radiation transfer system. Yet, observations tell us that the 21-yr CERES TOA global mean SW up equals 99 Wm⁻² in the all-sky, and 53.74 Wm⁻² in the clear-sky. The former is in integer ratio position of 15/4 with a difference of 1.04 Wm⁻², the latter is 2 units, with a difference of 0.38 Wm⁻² from its arithmetic position of 53.36 Wm⁻². Both of these recognitions points to the direction that incoming solar radiation, 340.02 Wm⁻², is an element in the system, with a value of 51/4 units. Total solar irradiance (TSI) is 1360.68 ± 0.5 Wm⁻², according to the SORCE observation's full period mean. The all-sky system at the intercepting cross-section disk to solar radiation is: TSI = 51, Absorbed Solar Radiation (ASR) = 36, Emitted Thermal radiation = 36, Reflected Solar Radiation (RSR) = 15, the clear-sky system is: TSI = 51, ASR = 43, OLR = 40, RSR = 8, leaving a TOA net imbalance between absorbed SW and emitted LW in the clear-sky of 3. Values at TOA can be given after division by 4 for spherical weighting: ASR (clear-sky) = 43/4 = 286.81 Wm⁻², resulting a 3/4 = 20.01 Wm⁻² solar absorption surplus over the emitted thermal radiation of 10 = 266.80 Wm⁻² in the clear-sky.

3.3.4 WIN clear

Although CERES instruments do have a window (WIN) channel to measure radiances in the water vapor mid-infrared 8-12 μm spectral range, it is not a well representation of the infrared transparency of our atmosphere since there are transmissions outside this region and there are absorptions within this range as well. To determine Earth's real window radiation, Costa and Shine (2012) performed detailed radiative transfer computations of the outgoing longwave radiation due to directly transmitted surface emission, without making assumptions on the wavelengths at which radiation from the surface can reach the TOA. Their surface transmitted irradiance (STI) was computed using a line-by-line radiative transfer code on 3D climatologies of humidity, temperature and cloudiness profiles. They found that the global mean STI is indeed largely confined to the mid-infrared window, but the 18-25 μm spectral range also has contributions in drier atmospheric regions. Their best estimate in the clear-sky is 65 Wm⁻² for their model-OLR of 259 Wm⁻², leaving 194 Wm⁻² for the atmospheric net upward emission. Assuming proportionality, with our theoretical clear-sky OLR = 266.80 Wm⁻² this means STI(clear) = 66.96 Wm⁻². It is easy to recognize that there is a close integer ratio position within 0.26 Wm⁻² only, at 66.70 Wm⁻², which equals to OLR(clear)/4. Accepting this value, the known integer ratio system in the clear-sky looks like this (Table 2):

Table 2. Clear-sky global mean fluxes, expressed in Wm⁻² (second row), in all-sky units (third and fourth rows) and in clear-sky units (sixth row). WIN(clear) is from Costa and Shine (2012), scaled to our OLR(clear).

WIN(clear)	SFC SW+LW net	ATM up (clear)	OLR(clear)	SFC LW up	SFC total(clear)
66.70	133.40	200.10	266.80	400.20	533.60
2.5	5	7.5	10	15	20
10/4	20/4	30/4	40/4	60/4	80/4
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>6</u>	<u>8</u>

The second row gives the theoretical values in Wm⁻², the third row presents them using the unit (1 = LWCRE = 26.68 Wm⁻²), the fourth row indicates the values after spherical weighting from the intercepting disk, while the bottom row introduces a new measure, shown in underlined bold typeface, as WIN(clear-sky) = 1 = 66.70 Wm⁻², what we will call the clear-sky unit, while 1 = LWCRE = 26.68 Wm⁻² will be called from now the all-sky unit.

Using this new unit, our clear-sky equations, Eq. (1) net and Eq. (3) total, can be expressed as

$$\text{Eq (1) SFC (SW + LW) net (clear) = TOA LW (clear) / 2}$$

$$\underline{2} = \underline{4} / 2$$

$$\text{Eq (3) SFC (SW + LW) total (clear) = 2 TOA LW (clear)}$$

$$\underline{8} = 2 \times \underline{4}$$

The greenhouse effect in the clear-sky is G(clear) = 2, and the normalized greenhouse effect is g(clear) = 2/6 = 1/3.

An important feature of the new unit that it can be used to express total solar irradiance in mixed units:

$$\text{TSI} = 51 \text{ all-sky units} = 20 \text{ clear-sky units} + 1 \text{ all-sky-unit, TSI} = 50 + 1 = \underline{20} + 1.$$

Now we are in the position to summarize our results in all-sky and clear-sky global mean energy flow diagrams, in the former using all-sky units, in the latter, mixed all-sky and clear-sky units, the fundamental clear-sky system as defined by the equations can be shown in own clear-sky units only. We indicated also the TOA net imbalance in the clear-sky, and expressed the fluxes in Wm⁻² as well. In the clear-sky diagram, we advanced a new equation, Eq. (5), to be introduced in the next sub-section.

These time-independent and greenhouse gas-independent clear-sky and all-sky theoretical arithmetic systems, as solutions of the four equations without reference to the actual atmospheric gaseous composition can be compared to clear-sky and all-sky global energy balance assessments based on direct observations and climate models for the present day climate at the beginning of the 21st century. In our Fig. 3 we project our values on the diagrams in Fig. 14 of Wild et al. (2019). Though these equations do not describe climatic processes, they provide relevant constraints on the possible evolution of the climate dynamics.

3.4 Equation (5)

Equation (3), connecting the total energy at the surface to TOA fluxes, was originally derived for general conditions, like radiative transfer in the sun's atmosphere, where evidently no "clouds" were taken into consideration, therefore we regarded it valid in clear-sky conditions. But on the right-hand side the flux can be either the incoming or the emerging solar radiation, or it can be the outgoing LW flux as well. But in the Earth's atmosphere, the condition that the incoming net solar flux equals the outgoing LW radiation is valid under all-sky conditions, in the clear-sky, a large imbalance of 20 Wm⁻² exists. Therefore, in our atmosphere, the total energy absorption (and the equivalent emission) at the surface, πB_g cannot be equal 2ASR since WIN is lost and only LWCRE is gained back. Here the valid condition is this:

$$\pi B_g = 2ASR - WIN + LWCRE = 2OLR$$

Let us have this as our fifth equation, for three different conditions: all-sky, clear-sky and cloudy-sky.

$$2ASR - WIN + LWCRE = 2OLR \quad (5)$$

$$2ASR \text{ (all)} - WIN \text{ (all)} + LWCRE = 2OLR \text{ (all)} \quad (5a)$$

$$2ASR \text{ (clear)} - WIN \text{ (clear)} + LWCRE = 2OLR \text{ (clear)} \quad (5b)$$

$$2ASR \text{ (cloudy)} - WIN \text{ (cloudy)} + LWCRE = 2OLR \text{ (cloudy)} \quad (5c)$$

The all-sky version tells us that under equilibrium conditions, when absorbed solar radiation is balanced by emitted thermal radiation, $ASR = OLR$, the all-sky window equals the longwave cloud effect. From a surface perspective: the surface emitted energy being lost in the all-sky window is gained back by the longwave effect of clouds, creating this way an "effectively IR-opaque" atmosphere. Is this condition possible? The numerical equivalence require $WIN(\text{all-sky}) = LWCRE = 26.68 \text{ Wm}^{-2}$. The referred study of window computation (Costa and Shine 2012) found that the all-sky STI is about 22 Wm⁻², the same as resulted from the assumption that the observed annual global mean cloud area fraction of 67% is completely IR-opaque: $STI(\text{all}) = (1 - 0.67) \times 65$. But we know that this assumption is not valid, an effective (IR-opaque) cloud area fraction of about 60% would result in a 26.7 Wm⁻² all-sky window radiation from the computed clear-sky value. Although we do not have full confidence in this result, both theoretical considerations and approximate calculations allow us to assume a clear-sky global mean window radiation of 66.70 Wm⁻², an effective cloud area fraction of 0.6 and all-sky window radiation around 26.7 Wm⁻².

Now if these values are not impossible, let us calculate Eq. (5b), the clear-sky version. It easily gives us $ASR(\text{clear-sky}) = 43/4$ with $OLR(\text{clear-sky}) = 40/4$, $WIN(\text{clear-sky}) = 10/4$ and $LWCRE = 4/4$. This way Eq. (5b) arithmetically reproduces the found value of $ASR(\text{clear}) = 286.81 \text{ Wm}^{-2}$ and the TOA net clear-sky imbalance of $3/4 = 20.01 \text{ Wm}^{-2}$, with $WIN(\text{clear}) = 66.70 \text{ Wm}^{-2}$ and $OLR(\text{clear}) = 266.80 \text{ Wm}^{-2}$.

The third version, for cloudy-skies, is to be used under a single IR-opaque effective cloud layer, where the window is zero by definition: $WIN(\text{cloudy}) = 0$. Hence Eq. (5c) tells us that

$$\text{ASR}(\text{cloudy}) - \text{OLR}(\text{cloudy}) = -\text{LWCRE}/2.$$

Knowing the positive imbalance in the clear-sky, $\text{ASR}(\text{clear}) - \text{OLR}(\text{clear}) = 3\text{LWCRE}/4$ is transported to the cloudy part of the atmosphere and arrives as $\text{LWCRE}/2$, this defines the clear/cloudy area ratio as $2/3$, that is, 40 / 60 percent, we have the effective cloud area fraction of $\beta_{\text{eff}} = 0.6$. This way, the (effective) global mean cloudy fluxes can be also constructed. One way is area-weighting: we know the all-sky values, the clear-sky values and β_{eff} , so the evident definition equation is

$$F(\text{all}) = (1 - \beta_{\text{eff}}) \times F(\text{clear}) + \beta_{\text{eff}} \times F(\text{cloudy}). \quad (6)$$

The total longwave cloud radiative effect also can be defined: as $\text{LWCRE} = \text{OLR}(\text{clear}) - \text{OLR}(\text{cloudy})$, $\text{LWCRE}(\text{total})$ will be $\text{LWCRE}_{\text{Tot}} = \text{OLR}(\text{clear}) - \text{OLR}(\text{cloudy}) = \text{LWCRE} / \beta_{\text{eff}} = \text{LWCRE}/0.6 = 44.466 \text{ Wm}^{-2}$. Therefore $\text{OLR}(\text{cloudy}) = 222.33 \text{ Wm}^{-2}$. Similarly, $\text{DLR}(\text{cloudy}) = \text{DLR}(\text{clear}) + \text{LWCRE}_{\text{Tot}} = 364.63 \text{ Wm}^{-2}$.

Now $\text{LWCRE}_{\text{Tot}}$ will be our unit flux in the cloudy global mean energy flow system, shown in bold, underlined, italic typeface: $\text{LWCRE}_{\text{Tot}} = \underline{\underline{1}}$.

The cloudy equations look like the all-sky ones, with $\text{LWCRE}_{\text{Tot}}$ at the place of LWCRE :

$$\text{SFC (SW+LW) net, cloudy} = [\text{OLR}(\text{cloudy}) - \text{LWCRE}_{\text{Tot}}]/2 \quad (\text{Eq 2, cld})$$

$$\text{SFC (SW net + LW down), cloudy} = 2 \text{ OLR}(\text{cloudy}) + \text{LWCRE}_{\text{Tot}} \quad (\text{Eq 4, cld})$$

Using Eq. (6) and taking into account the area-weighted energy transfer from the clear-sky part of the atmosphere, the energy flow system can easily be constructed, resulting in the integer solution for the cloudy energy flows as shown in Table 3.

Table 3. Cloudy fluxes in cloudy units, and their value in Wm^{-2}

TOA LW (OLR(cloudy))	<u>5</u>	222.33
SFC LW Up	<u>9</u>	400.20
SFC (SW+LW) net	<u>2</u>	88.93
SFC (SW net + LW down)	<u>11</u>	489.13
Eq. (2, cloudy)	<u>2</u> = (<u>5</u> - <u>1</u>)/2	88.93
Eq. (4, cloudy)	<u>2</u> + <u>9</u> = 2 × <u>5</u> + <u>1</u>	489.13

Total solar irradiance can be expressed in mixed cloudy and all-sky units as $\text{TSI} = 50 + 1 = \underline{\underline{30}} + 1$. The proposed effective cloudy global mean energy flow system is shown in Fig. 4. Important feature of the system that having the clear-sky fluxes in clear-sky units and the cloudy fluxes in cloudy units, adding them the all-sky system is resulted in all-sky units.

4 Discussion

To reach a better understanding of the principles beyond the equations, let's try to explain the arithmetic relationships through their geometric representation. The simplest textbook greenhouse geometry is the single-slab model, see for example Hartmann (1994, Fig. 2.3); Marshall and Plumb (2008, Fig. 2.7); or Liou (1980, Fig. 8.20 with $A = 0$ and $\epsilon = 1$), consisting of a black-radiating surface and an atmospheric layer that is transparent for solar radiation but opaque to terrestrial radiation; and there is no other energy exchange between the surface and the atmosphere than radiative. This is the geometry where the longwave energy radiated up from the surface is completely absorbed by the atmospheric layer and radiated upward and downward equally, therefore the LW upward radiation from the lower boundary is twice the outgoing longwave radiation at the upper boundary; see our Fig. 4, reproduced from Marshall and Plumb (2008, Fig. 2.7).

This is the simplest theoretical model representing Eq. (3). Earth's atmosphere is not like that, but we use it as a starting point, and will deduce the integer flow system as determined by the five equations by introducing some solar absorption, infrared transparency, and convective fluxes during the steps of the deduction, see Fig. 6, (a) – (f) panels. The final result is given in Fig. 7.

Finally, close the balance with convective fluxes. Since we added altogether 2 units of LWCRE (one up, one down), to keep the 1/2 ratio of the model for atmospheric/surface radiations, we have to introduce $CONV = 4$ units, Fig. 7. This is the global mean all-sky energy flow structure we have seen in Fig. 1.

Now let's have a look at the clear-sky system within this framework. In Fig. 8 we present the fundamental clear-sky fluxes in their own unit projected on the intercepting cross section disk (before spherical weighting). All-sky units represent the clear-sky TOA surplus to be transported to the cloudy part of the atmosphere.

Clouds, that is the single infrared-opaque effective cloud layer represents the genuine single-slab model, so the energy flow system can be easily determined. The only task is to handle the energy transport from the clear-sky part, which has to be re-distributed; see Fig. 10.

Finally, the area-weighted sum of the clear and cloudy structures in their own units results the all-sky system in all-sky units. Fundamental fluxes and the basic equations are displayed. Some further features of this model:

- The all-sky greenhouse effect (6 all-sky units) equals the solar radiation absorbed by the surface (a characteristic being reserved from the initial single-slab model during the course of deduction, *cf.* Figure 4);
- The clear-sky greenhouse effect equals the net radiation at the surface (5 all-sky units = 2 clear-sky units);
- The greenhouse effect in the cloudy part is 4 cloudy units, and in the clear-sky part is 2 clear-sky units; hence the contribution of the cloudy part to the all-sky greenhouse effect (4 all-sky units) is

twice the contribution of the clear-sky part (2 all-sky units);

- The emitted LW energy by the cloud-covered surface (15 all-sky units $\times$ 3/5 = 9 all-sky units) equals the all-sky outgoing LW radiation;
- The surface net radiation of the cloudy part (2 cloudy units) contributes to the all-sky mean (4 all-sky units) by the same amount as the contribution of the clear-sky part (2 clear-sky units), resulting $88.93 \times 0.6 + 133.40 \times 0.4 = 106.72 \text{ Wm}^{-2}$ (= 2 + 2 all-sky units);
- If the longwave cloud effect at TOA equals the longwave cloud effect at the surface (CERES: 25.78 Wm^{-2} and 27.30 Wm^{-2} ; Wild et al. (2019): 28 Wm^{-2} and 26 Wm^{-2} , resp.), then the longwave cooling of the atmosphere is the same in both regions, $\text{ULW} - \text{OLR} - \text{DLR} = -7$ all-sky units. Since the contribution to the LW cooling by the surface is -2 all-sky units from both regions, this way the total LW emission is everywhere secured as $\text{OLR}(\text{all}) = 9$ all-sky units;
- Ultimately, the five equations together define us an atmospheric geometry consisting of three regions: a cloudy-opaque (with an area-fraction of 9/15), a GHG-opaque (5/15) and a transparent (1/15), with area-weighted contributions of 5, 3 and 1 all-sky units adding up the 9 all-sky units of OLR.

5 Conclusions

Schwarzschild's original equations in the plane-parallel approximation, under radiative equilibrium conditions, define a discontinuity in the blackbody radiation at the surface, equal to half of the TOA outgoing longwave radiation, independent of the optical depth of the atmosphere. In radiative-convective equilibrium this surface net radiation is balanced by non-radiative energy flows like convection and evaporation. This feature of radiation transfer is reproduced by standard textbooks on atmospheric physics, but according to our best knowledge the accuracy of this equation was not controlled on observations. In this study we use the best CERES EBAF Ed4.1 datasets on the available full 21 running years, and found the equation valid in the clear-sky with a global mean difference of -2.19 Wm^{-2} . We created the all-sky version of the equation by separating atmospheric radiation transfer from the longwave effect of clouds, and this equation proved to be valid with a difference of 2.76 Wm^{-2} . This way, the clear-sky and all-sky net equations together are justified within 0.28 Wm^{-2} . Schwarzschild's equation for the total flux at the surface depends on the optical depth; as a first approximation we controlled this "greenhouse" equation with an optical depth of 2, and found it satisfied with a difference of -2.72 Wm^{-2} in the clear-sky global mean. The evident all-sky form of this equation is also introduced, with a validation of 2.42 Wm^{-2} . The accuracy of the total equations with optical depth of 2 is therefore -0.14 Wm^{-2} , showing that this assumption on the global mean optical depth is appropriate. The four equations together have a 0.07 Wm^{-2} mean bias. Regarding the global mean energy flow components is the equations as independent variables, with evident definitions the set of equations was solved, resulting in integer ratio expressions for the fluxes, related to the longwave cloud radiative effect (LWCRE) as unit flux. Since the integer ratio system is an arithmetic consequence of the set of equations without dependence on anything else, the mean accuracy of the flux integer values must be the same (0.07 Wm^{-2}). We determined the best fit of the unit flux on the full data set as $\text{LWCRE} = 26.68 \pm 0.01 \text{ Wm}^{-2}$.

Examining other energy flow components in the global energy budgets, we recognized that some fluxes that are directly not involved in the equations (e.g., downward solar radiation, components of the turbulent energy flow, TOA solar reflection, or the window radiation) still have definite positions in the integer system. Utilizing the independent computed value of the clear-sky atmospheric window and the two clear-sky equations only, the involved clear-sky flux components are arranged in a clear-sky global mean energy flow system, expressed in own clear-sky units, which turned out to be the clear-sky window radiation, WIN(clear). Based on the greenhouse geometry beyond the system with optical depth of two, we proposed a fifth equation as well that helped to establish an effective (infrared-opaque) cloud area fraction as 60%, and this way to create a complete cloudy-sky energy flow structure, expressed in own cloudy-sky units. Now the all-sky energy budget is evidently emerges as the weighted sum of the clear and cloudy structures.

Declarations

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Figures

Figure 1

The all-sky global mean energy flow system as the solution of the four time-independent radiative transfer constraint equations, without referring to the actual atmospheric greenhouse gas composition. The all-sky unit is shown in red.

Integer solution of the four radiative transfer constraint equations (red bold typeface) and their value in Wm^{-2} (green) is projected on the diagrams in Fig. 14 of Wild et al. (2019). Left: clear-sky, right: all-sky. The greenhouse effects $G = \text{ULW} - \text{OLR}$ is shown (ULW is surface Upward LW emission). Eq. (5) is also indicated. The theoretical normalized greenhouse effect is $g(\text{theory}) = G/\text{ULW}$. The theoretical greenhouse effect in the clear-sky is $g(\text{theory, clear}) = 5/15 = 1/3$, in the all-sky $g(\text{theory, all-sky}) = 6/15 = 0.4$. The estimate of the diagram for the present-day climate at the beginning of the 21st century gives $g(\text{clear-sky}) = (398 - 267)/398 = 0.329$ and $g(\text{all-sky}) = (398 - 239)/398 = 0.3995$.

Figure 4

The effective cloudy global mean energy flow system, expressed in mixed cloudy and all-sky units. Energy transfer from the clear-sky part transports the absorbed surplus energy to the cloudy part. The cloudy unit is shown in brown, the all-sky unit in red. Flux values in Wm^{-2} are to be compared to the global means in CERES FluxByCldTyp data product when it becomes publicly available

Figure 5

The simplest greenhouse model, after Marshall and Plumb (2008, Fig. 2.7). Solar energy absorbed by the system is completely absorbed by the surface, energy radiated up from ground is completely absorbed by the atmospheric layer and radiated up (A) and down (A) equally, therefore $S = 2A$. This is the geometry where Eq. (3) is valid: $\pi Bg = S = 2OLR$. Notice also that the greenhouse effect equals OLR and also equals the absorbed solar energy by the surface: $G = S - A = A = S_0(1 - \alpha)/4$.

Figure 6

Panel (a), upper left: The starting point: a shortwave-transparent, longwave-opaque, non-turbulent atmospheric model. The unit is not yet specified here. Panel (b), upper right: introduce a change in the unit, from 1 to 3. Nothing else was modified. Panel (c), middle row left: Introduce 1 unit for solar atmospheric absorption, $SAA = 1$. Therefore solar absorption surface, $SAS = 2$, $ULW = 5$, DLR remains 3. Panel (d), middle row right: perform again a unit change, now from 3 to 9. Everything else remained the same. Panel (e), lower left: Allow 1 unit for atmospheric longwave transparency (window). Longwave atmospheric absorption $LAA = 14$, atmospheric upward emission $ATM = 8$. Panel (f), lower right:

introduce clouds by 1 unit for longwave cloud radiative effect (LWCRE) both at TOA and at the surface, now they are included in ATM and DLR.

Figure 7

The resulted stationary greenhouse-gas independent global mean all-sky energy flow system, as determined by the five radiative transfer constraint equations. Greenhouse effect = $ULW - OLR = 6$ units, for flux values in Wm^{-2} , compare to Fig. 1.

Figure 8

The clear-sky system on the disk in their own clear-sky units. Left panel: TOA clear-sky energy imbalance to be transported to the cloudy part is indicated in all-sky units. By evident arithmetic identity coming from their definitions, Right panel: To have the necessary surplus in solar absorption, reflected solar radiation must be reduced. The energy surplus in solar absorption is transported to the cloudy part of the atmosphere. The validity of the clear-sky equations Eq. (1) and (3) is shown, Eq. (5b) is also satisfied. TOA clear-sky imbalance = $-$ TOA Net Cloud Radiative Effect

Figure 9

The complete clear-sky system on the disk. Shortwave energy is re-distributed to satisfy the constraint equation (6), equations (1), (3) and (5) are satisfied.

Figure 10

Left panel: Cloudy fluxes on the disk in their own units, with effective cloud area fraction of 3/5. Right panel: The energy income from the clear-sky part of the atmosphere is re-distributed, computed using the definition equation (6). The shortwave fluxes are only approximative since they belong to the observed cloud area fraction of 67% rather than to the effective IR-opaque clud area fraction of 60%.

Figure 11

The cloudy system on the disk in cloudy and all-sky units, computed using the definition Equation (6). The corresponding cloudy transfer equations (2), (4), and (5c) also satisfied.

Figure 12

Cloudy and clear global mean integer energy flow systems on the disk, satisfying the corresponding net and total equations in their own units. After spherical weighting, their sum results the all-sky integer system, see Fig. (13)

Figure 13

Compendium: all-sky values in all-sky units can be given by adding the corresponding clear and cloudy fluxes expressed in their own units.